Hawking Raidiation and Information Loss – insights from the semi-classical stress-energy tensor

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Outline

1. Constructing the Unruh state semi-classical stress-energy tensor (SCSET) for large Schwarzschild black holes
   - A. basic assumptions
   - B. interpretation of numerical results, old and new
   - C. myths about the origin of Hawking radiation debunked
   - D. first-order backreaction on geometry

2. Implications for the black hole information paradox
   - A. black hole horizons are locally just Rindler horizons
   - B. no storage of quantum information in a "stretched horizon"
   - C. Hawking radiation is entangled with fields in the deep interior of the black hole
   - D. no firewalls

3. How might the semi-classical picture break down by the Page time?

4. Is a quantum black hole, assuming an initial non-compact Cauchy hypersurface, necessarily singular in its interior?

The context

The focus of this talk is on large Schwarzschild black holes with mass $M \gg m_\mathrm{p}$, m_p the Planck mass, formed by spherically symmetric gravitational collapse in an asymptotically flat spacetime, such that in the distant past the spacetime is approximately Minkowskian. The collapse leads to the formation of trapped surfaces and an *apparent* horizon at $r \cong 2M$ that persists for many dynamical times. There may or may not be a true *event* horizon, since the classical energy conditions that establish its existence are not valid in quantum field theory.

Quantum fields are considered small perturbations on a classical background spacetime, whose metric is a solution of the classical Einstein equations. There is every reason to think that this semi-classical approximation is extraordinarily well justified on and outside the horizon. For the black holes known to exist, with $M > 1 M_\odot$, the quantum corrections are of order $m_\mathrm{p}^2 / M^2 < 10^{-76}$.

My units are $G = c = 1$, $\hbar = m_\mathrm{p}^2$.

Hawking radiation

Hawking's landmark result from 1974-1975 showed that in the semi-classical approximation black holes radiate with a quasi-thermal spectrum at the Hawking temperature

$$T_{\mathrm{H}} = \kappa m_{\mathrm{p}}^2 / 2\pi;$$

the surface gravity $\kappa = 1/4M$ for Schwarzschild. The luminosity for a spin s massless field has the form

$$L_{\mathrm{H}} = 4\pi M^2 \sigma T_{\mathrm{H}}^4 k_s = 6\pi M^2 P_0 k_s .$$

The coefficients k_s, calculated by Page for $s = 1,2$ and Elster for $s = 0$, are $k_0 = 14.36$, $k_1 = 6.4928$, $k_2 = 0.7404$.

The prospect that black holes might evaporate down to the Planck scale and disappear on a time scale of order M^3 / m_{p}^2 has raised profound questions about quantum gravity and quantum field theory that are still unresolved after more than 40 years. Would this conflict with unitarity? What would it take to preserve unitarity? Is there some breakdown of local quantum field theory at the horizon?

How is Hawking radiation created? By pair creation or tunneling very close to the horizon, or tidal disruption of vacuum fluctuations in the general vicinity of the horizon?

The semi-classical stress-energy tensor

The renormalized expectation value of the quantum stress-energy tensor. Spherical symmetry implies 4 independent components. In a static frame: energy density $E = -T_t^t$, energy flux $F = -(1-2M/r)^{-1} T_t^r$, radial stress $P_r = T_r^r$, transverse stress $P_t = T_\theta^\theta = T_\varphi^\varphi$.

Conservation equations $\nabla_\nu T_\mu^\nu = 0$, with no t-dependence and $x \equiv 2M/r$:

energy $F = \left[\left(\pi M^2\right)^{-1} L_{\mathrm{H}} = (3/8) k_s P_0 \right] x^2 / (1-x)$,

momentum $P_{\mathrm{r}} = \dfrac{x^2}{1-x} \displaystyle\int_1^x \left[\frac{(3x'-2)}{x'^3} P_{\mathrm{t}}(x') - \frac{1}{2x'^2} T(x') \right] dx' + CF.$

No singularities in a free-fall frame if radiation is purely *ingoing* at the horizon in the static frame: $E \simeq P_{\mathrm{r}} \to -F$ as $x \to 1$ $(C = -1)$. Then $E^{\mathrm{reg}} \equiv E + F$ and $P_{\mathrm{r}}^{\mathrm{reg}} \equiv P_{\mathrm{r}} + F$ are finite on the horizon.

The trace of the SCSET $T = -E + P_{\mathrm{r}} + 2P_{\mathrm{t}} = -E^{\mathrm{reg}} + P_r^{\mathrm{reg}} + 2P_{\mathrm{t}}$ is given by the trace anomaly $T^{\mathrm{anom}} = 96 q_s P_0 x^6$ for conformally coupled fields, with $q_0 = 1$, $q_1 = -13$, $q_2 = 212$.

Numerical results

Based on point-splitting renormalization (Christensen, Christensen and Fulling 1970s)

Quantum state: Hartle-Hawking (HH) or Unruh, with only the Unruh state directly relevant for an evaporating black hole.

Conformally coupled fields: only need to calculate P_{t} from scratch, results published for $0.4 \le x \le 1$ $\left(1 \le r/2M \le 2.5\right)$ as graphs.

- Spin 0 HH, Howard (1984) and Anderson, et al (1993).
- Spin 1 HH, Jensen and Ottewill (1989).
- Spin 0 and spin 1 Unruh, Jensen, et al (1991).
- Table of HH and Unruh spin 0 data preserved in Visser (1997).
- Unpublished table of $1 \le r/2M \le 3$ Unruh spin 1 data recently obtained from Visser.

Can try to extrapolate to $x = 0$, subject to the constraint (for the Unruh state) that $\left(x^{-2} P_{\mathrm{r}}^{\mathrm{reg}}\right)_{x=0} \equiv r_2 = 2\left(x^{-2} F\right)_{x=0}$, using a polynomial fit to P_{t} data. It was usually assumed that the lowest power in the Unruh P_{t} is x^4 (as in Visser 1997). But for spin 0 a significantly better fit is obtained including a x^3 term.

Polynomial fits for conformally coupled spin 0 Unruh:

$$P_{\mathrm{t}} = P_0\left(0.2484x^3 + 25.5877x^4 - 57.7703x^5 + 37.6939x^6\right),$$

$$P_{\mathrm{r}}^{\mathrm{reg}} = P_0\begin{pmatrix} 10.769x^2 + 10.2722x^3 - 14.9429x^4 \\ + 49.1583x^5 - 13.0164x^6 \end{pmatrix},$$

$$Z \equiv 0.5\left(E^{\mathrm{reg}} + P_{\mathrm{r}}^{\mathrm{reg}}\right)/\left(1-x\right)$$

$$= P_0\left(10.769x^2 + 21.2896x^3 + 31.9344x^4 + 23.3224x^5\right).$$

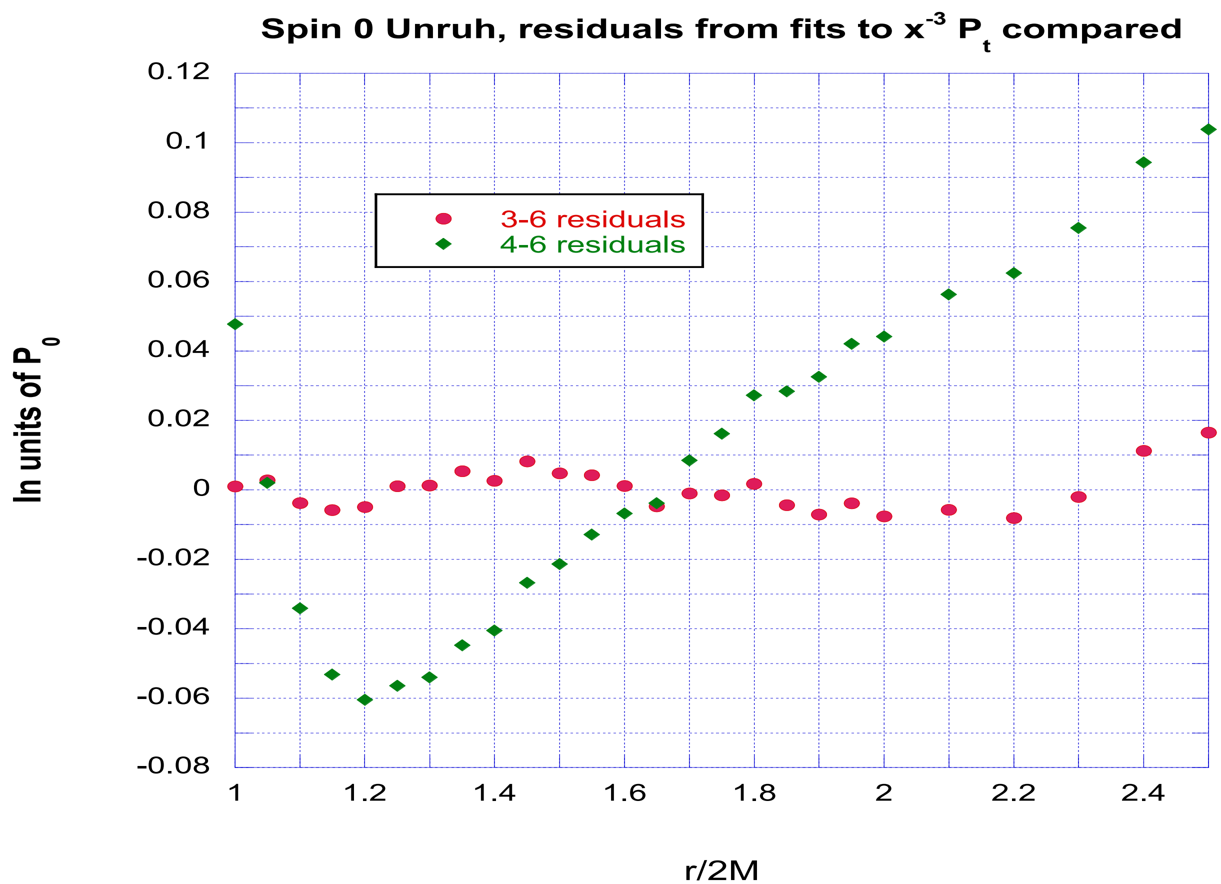
From Levi and Ori (2016) $0.04 \le x \le 1$ data, polynomial fits for minimally coupled spin 0 Unruh:

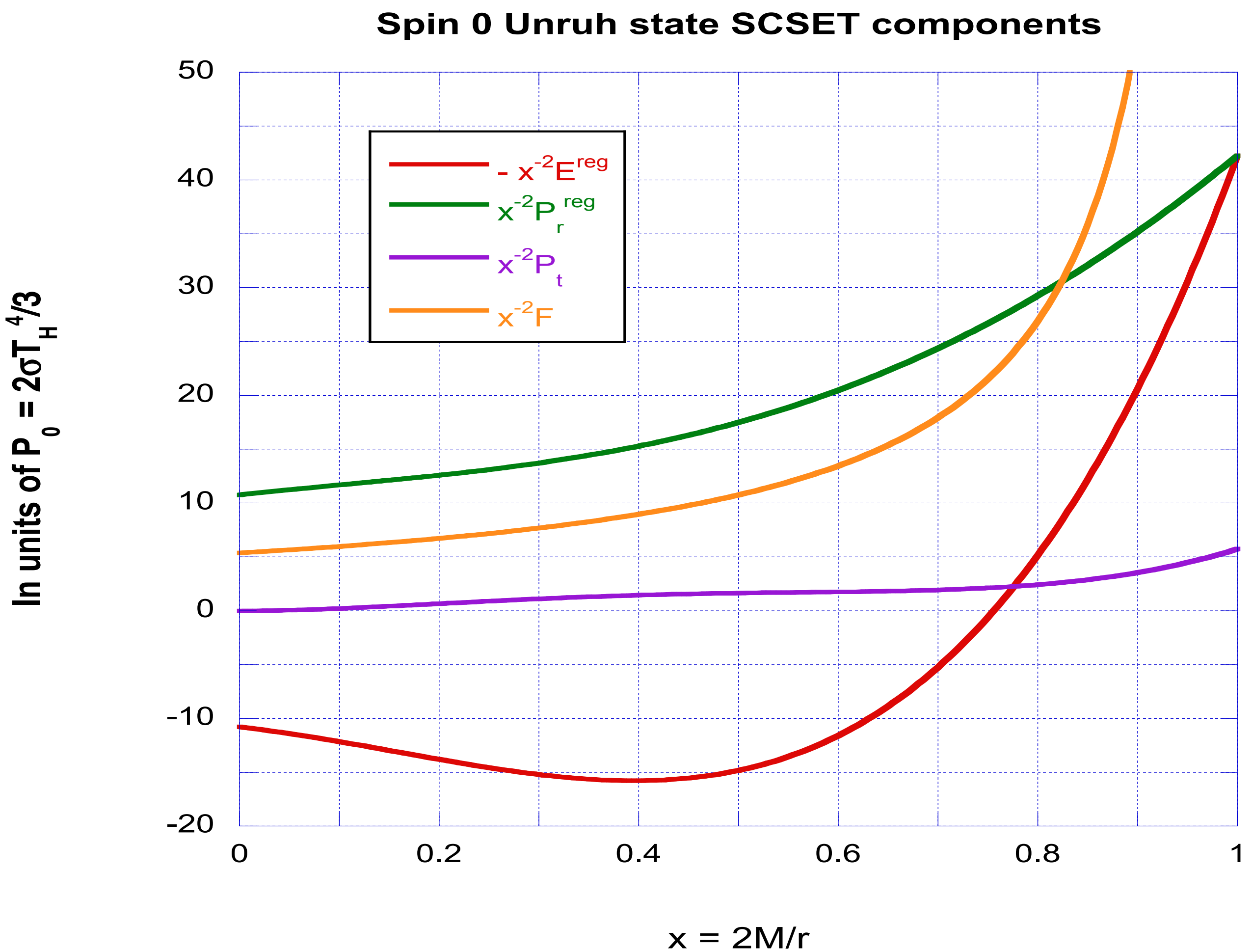
$$P_{\mathrm{t}} = P_0\left(1.5242x^3 - 145.91x^4 + 108.19x^5 + 153.8558x^6\right),$$

$$T = P_0\left(2.2196x^3 - 273.02x^4 + 365.55x^5 + 336.63x^6\right),$$

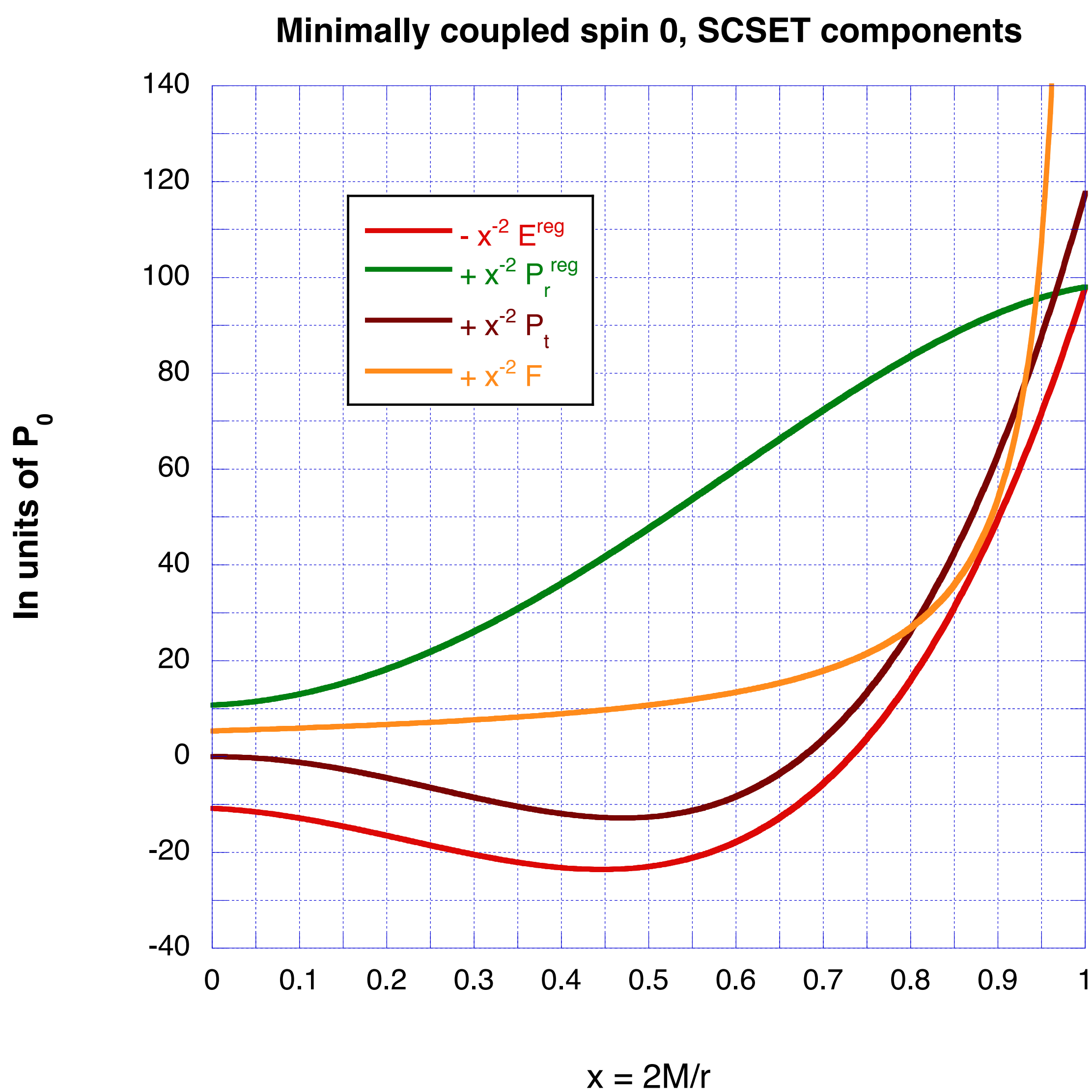
$$P_{\mathrm{r}}^{\mathrm{reg}} = P_0\begin{pmatrix} 10.769x^2 + 7.7206x^3 + 155.362x^4 \\ - 17.1713x^5 - 58.6505x^6 \end{pmatrix}.$$

There is no satisfactory polynomial fit to the spin 1 Unruh data.


Spin 0 Unruh, residuals from fits to x^{-3} P_t compared
3-6 residuals
4-6 residuals
In units of P_0
r/2M
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
1
1.2
1.4
1.6
1.8
2
2.2
2.4



Spin 0 Unruh state SCSET components
- x^-2 E^reg
x^-2 P_r^reg
x^-2 P_t
x^-2 F
In units of P_0 = 2σT_H^4/3
x = 2M/r
50
40
30
20
10
0
-10
-20
0
0.2
0.4
0.6
0.8
1


Minimally coupled spin 0, SCSET components
In units of P_0
x = 2M/r
$- x^{-2} E^{reg}$
$+ x^{-2} P_r^{reg}$
$+ x^{-2} P_t$
$+ x^{-2} F$
140
120
100
80
60
40
20
0
-20
-40
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Interpretation

The myth:

Hawking radiation is originates very close to the horizon by pair creation or tunneling. A Hawking partner with negative Killing energy is left inside the horizon, decreasing the mass of the black hole.

If so, close to the horizon the singular energy flux there should represent an outflow of positive energy, with $F \cong +E \cong +P_{\mathrm{r}}$. This would imply $r_2 = \lim_{x\to 0}\left(x^{-2}P_{\mathrm{r}}^{\mathrm{reg}}\right) = 0$, which is inconsistent with both the spin 0 P_{t} and the direct calculation of P_{r} by Levi and Ori. Even more seriously, this would be an extreme violation of conservation of energy and momentum in a local inertial frame, where both the Hawking particle and its partner would have an enormous energy and outward momentum, coming from nothing.

The alternative, as argued originally by Unruh (1977) and Fulling (1977), is that the Hawking radiation originates from vacuum fluctuations propagating along and straddling the horizon. As these are radially stretched by geodesic deviation, they contribute positive energy to the SCSET at larger radii and are interpreted as particles excited from the vacuum at $r \gg 3M$. There is negative energy inflow near the horizon that does not have any clear particle interpretation.

Backreaction

The general spherically symmetric metric in advanced Eddington-Finkelstein coordinates is

$$ds^2 = -e^{2\psi}\left(1-2m/r\right)dv^2 + 2e^{\psi}dvdr + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right).$$

The inverse metric has

$$g^{vv} = 0, \quad g^{vr} = e^{-\psi}, \quad g^{rr} = 1-2m/r.$$

The stress-energy tensor in terms of the static frame components is

$$T_v^{\,v} = -E^{\mathrm{reg}}, \quad T_r^{\,r} = P_r^{\mathrm{reg}}, \quad T_\theta^{\,\theta} = T_\varphi^{\,\varphi} = P_{\mathrm{t}},$$

$$T_v^{\,r} = -\left(1-x\right)F, \quad T_r^{\,v} = \left(1-x\right)^{-1}\left(E^{\mathrm{reg}} + P_r^{\mathrm{reg}}\right) = 2Z_s.$$

The Einstein equations include an evolution equation

$$\left(\partial m/\partial v\right)_r = -\left(3/2\right)\pi k_s\left(2M\right)^2 P_0.$$

and two initial value equations,

$$\left(\partial m/\partial r\right)_v = -4\pi r^2 T_v^{\,v} \text{ and } \left(\partial e^{-\psi}/\partial r\right)_v = -4\pi r T_r^{\,v}..$$

Strictly speaking the semi-classical approximation is only valid as long as the background geometry is essentially unchanged. However, note that $\left(\partial m/\partial v\right)_r$ is independent of r. As long as $\left|\partial m/\partial v\right| \ll 1$ and $\left|m\partial\psi/\partial r\right| \ll 1$ for $r \approx 2m$, the geometry remains Schwarzschild with $M = m$ to an excellent approximation in the vicinity of the horizon, even as M decreases substantially from its initial value.

A black hole information paradox?

Allowing quantum information to propagate acausally in a macroscopic spacetime seems an extremely serious violation of basic principles.

Entanglement of fields outside the black hole with fields in the deep interior is a generic property of non-degenerate black holes. There is no way the entanglement can be restricted to a small region near the horizon, since "outward" radial null geodesics diverge away from the horizon on both sides. The degrees of freedom entangled with the Hawking radiation do *not* propagate along the horizon, as assumed in complementarity (Susskind, et al 1993).

Locally a large black hole's event horizon (assuming one exists) is almost indistinguishable from a Rindler horizon in Minkowski spacetime. No one would suggest that quantum information cannot cross a Rindler horizon, or that it can be "copied" into modes propagating along the Rindler horizon. An accelerating detector hanging just above a black hole horizon or a Rindler horizon sees thermal radiation, the Unruh heat bath, but this is just a property of the detector interacting with the local vacuum. The Hawking radiation that distinguishes a black hole horizon from a Rindler horizon is a *nonlocal* effect.

Firewalls?

What if the quantum information necessary to preserve unitarity for an external observer starts leaking out at some point, through subtle correlations between the Hawking particles? This hypothesis requires acausal propagation of quantum information, as long as the black hole is still essentially Schwarzschild.

Almheiri, et al (2012) argued that entanglement of late Hawking radiation with early Hawking radiation and the emission of the Hawking radiation from near the horizon, consistent with low energy effective field theory, implies the existence of a "firewall", because from the no-cloning theorem the late Hawking particles cannot also be entangled with their partners inside the horizon. Absence of the latter entanglement implies very high energy excitations at the horizon.

Forming a firewall would require enormous amounts of energy suddenly appearing from nothing in a free-fall frame. Putting the firewall right at the apparent horizon, so gravitational potential energy cancels the local energy, is of no help, as argued earlier. Therefore, I consider a firewall to be impossible unless one supposes that spacetime suddenly ends and general relativity breaks down at the horizon, as the result of some cataclysmic phase transition (e.g., a Mathur fuzzball).

Entropies

What is the entropy of a black hole? The Bekenstein-Hawking entropy $S_{\mathrm{BH}} = A/(4\hbar) = 4\pi\left(M^2/m_{\mathrm{p}}^2\right)$, where A is the area of the event horizon (assuming one exists), has the role of thermodynamic entropy. But what about the microscopic (von Neumann) entropy S_{vN}? Normally, in local quantum field theory this is infinite, unless it is renormalized so as not to include the short-range correlations of the vacuum across the boundary of the region being considered. Such renormalization carried out at the horizon of a black hole yields a net S_{vN} equal to the number of entangled degrees of freedom involved in the initial collapse and any subsequent accretion, plus the trapped vacuum fluctuations or Hawking "partners" entangled with qbits of the emitted Hawking radiation.

As the black hole evaporates, the net S_{vN} increases and S_{BH} decreases. They become equal at the Page time, when the black hole has lost about 1/2 of its original mass. What happens at that point? If the black hole continues emitting Hawking radiation that is entangled with the interior of the black hole, eventually $S_{\mathrm{vN}} \gg S_{\mathrm{BH}}$. On the other hand, if this late Hawking radiation is assumed to be entangled with the earlier Hawking radiation, to keep $S_{\mathrm{vN}} \le S_{\mathrm{BH}}$, one is faced with the need to propagate quantum information acausally and the issue of "firewalls" (Almheiri, et al 2012). This is the information paradox.

Possible resolutions?

1) A phase transition to a "fuzzball" (Mathur) or to a "bose condensate of gravitons" (Dvali), or some other breakdown of the notion of a spacetime described to good approximation by GR, at and inside the horizon of a large black hole. Drastic revisions of conventional quantum field theory. I find all of these extremely distasteful and implausible for any large black hole.

2) Evaporation continues, with quantum information remaining trapped, until the black hole approaches the Planck scale, when it dissolves into vacuum fluctuations containing all the trapped information. Unruh and Wald have argued that there are no compelling reasons to reject this scenario, though they seem to be in the minority.

3) The Bekenstein-Hawking entropy is a measure of the total number of quantum degrees of freedom associated with the interior and horizon of the black hole, and the *unrenormalized* von Neumann entropy on the horizon is $S_{\mathrm{vN}}^{\mathrm{bare}} = S_{\mathrm{BH}}$. At the Page time the Hawking radiation stops, because there are no more quantum degrees of freedom to be "pulled out of the vacuum". The black hole survives as a massive non-radiating remnant. (But this seems inconsistent with inflation.)

4) There is a connection between entanglement and geometry that prevents the macroscopic entanglement entropy on a light sheet normal to a compact two-surface from exceeding 1/4 its area in Planck units, by modifying the geometry as this limit is approached. See Bardeen (2014, 2017) for a toy model implementing this idea.

A quantum singularity theorem?

Bousso, et al (2017) argue that, under rather general assumptions, there is a theorem valid in quantum theory similar to the Penrose singularity theorem of classical GR. The area of a null geodesic congruence is replaced by a generalized entropy

$$S_{\text{gen}} = S_{\text{out}} + \frac{1}{4}\frac{A(\sigma)}{m_{\text{p}}^2},$$

where $A(\sigma)$ is the area of a compact 2-surface σ dividing a Cauchy hypersurface into two regions, with S_{out} the von Neumann entropy of the "external" non-compact region. The "theorem" is based on a "quantum focusing conjecture" (QFC), that deforming σ along a congruence of normal null geodesics, $d^2 S_{\text{gen}} / d\lambda^2 \leq 0$.

Is the QFC valid in the context of an evaporating Schwarzschild black hole? As the initial 2-surface, take a sphere with radius r_0 such that $\varepsilon = (2M - r_0)/4M$ is in the range $(m_{\text{p}}/M)^2 \ll \varepsilon \ll m_{\text{p}}/M$, inside the apparent horizon but outside the event horizon. The radius of the light sheet as a function of advanced time v is

$$r \cong r_0 - \varepsilon v - 4\pi M T_v^r v^2 \cong r_0 - \varepsilon v + L_{\text{H}} \frac{v^2}{4M}.$$

The emission of Hawking radiation, when averaged over $\Delta v \gg 2M$, $dS_{\text{out}}/dv \sim (700M)^{-1}$. Then

$$dS_{\text{gen}}/dv \approx (4\pi M / m_{\text{p}}^2)\left[O(m_{\text{p}}/M)^2 - \varepsilon + O(m_{\text{p}}/M)^2 v/2M\right],$$

which is initially less than zero and becomes positive while the approximations are still valid at $v/2M = O(M/m_{\text{p}})^2 \varepsilon \gg 1$.