Gravitational Theory and Cosmology: Past, Present, and Future

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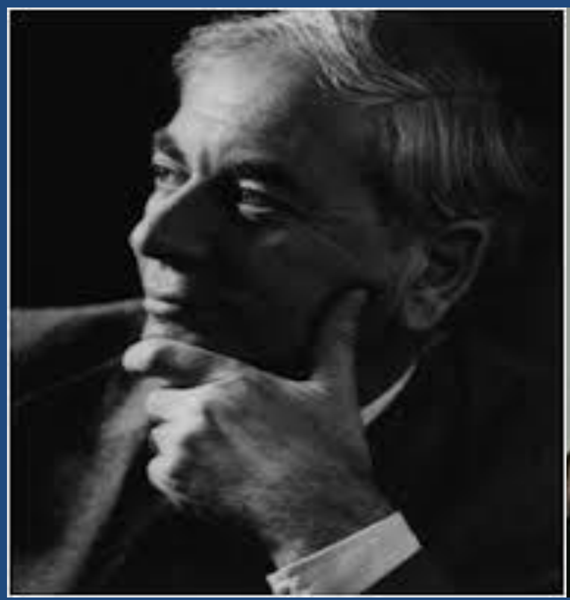
Gravity: past, present, and future:

University of British Columbia, 2017

Because gravity underlies cosmological dynamics,
the two are intimately related.

1. Bottom up effects: Local gravity everywhere implies global dynamics (Einstein) but not topology. A hidden scale underlies such models.
2. Gravity implies motion: cosmology is dynamic (Friedmann, Lemaitre, Eddington) and testable (Hubble, McVittie, Sandage)
3. Local physics determines the evolution (Tolman, Guth) unifying physics and cosmology at different epochs to some degree
4. Top down dynamics: Global variables control local evolution (Lifshitz) giving testable relations (Sachs and Wolfe, Peebles)
5. Classical GR plus energy conditions imply a start to the universe (Lemaitre, Hawking) which is major crisis (Wheeler). QG predictions??
6. Inflation and dark energy imply energy conditions not satisfied; vacuum energy puzzle and gravitational theories (e.g. unimodular)
7. Is universe FLRW? Why was its start so smooth? (Penrose) Was gravity then the same as now? (Greene)

Not with us …

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Major discoveries 1

- Local gravity relations applied everywhere determines the effect of gravity on the universe as a whole

  EFE + matter model + symmetries → large scale structure

- Consistent models of gravitational effect on cosmos can be made on the basis of general relativity (Einstein)

- But does not determine the topology of the 3-spaces in cosmology. We still don’t have a principle that could determine it.

- Maybe an integral rather than differential formulation can do this ? The boundary terms will differ according to topology ..

The hidden averaging scale

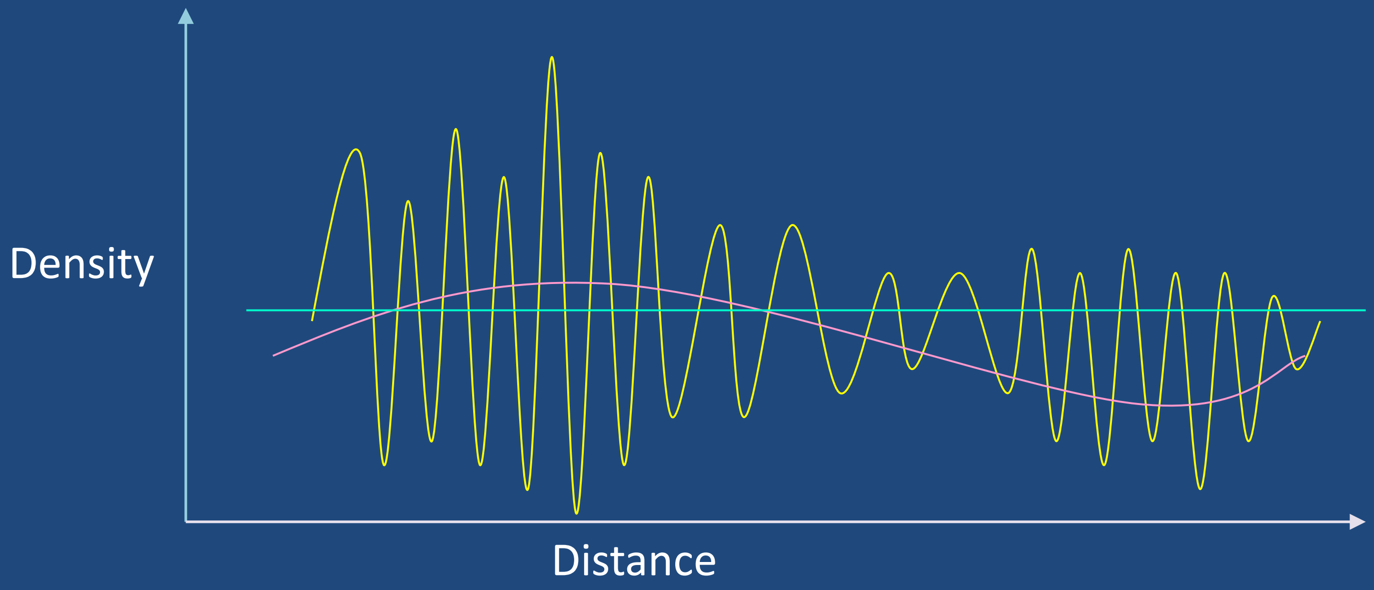
Any mathematical description of a physical system depends on an averaging scale characterizing the nature of the envisaged model. This averaging scale is usually hidden from view: it is taken to be understood.

Thus, when a fluid is described as a continuum, this assumes one is using an averaging scale large enough that the size of individual molecules is negligible. If the averaging scale is close to molecular scale, small changes in the position or size of the averaging volume lead to large changes in the measured density and velocity of the matter, as individual molecules are included or excluded from the reference volume. Then the fluid approximation is no longer applicable.

Local inhomogeneity: description

Multiple scales of representation of ***same*** system

Different averaging scales



Stars, clusters, galaxies, universe

Global and local variables

Global variables are defined by coarse-graining local variables:

$$\rho_{cosm}(t) = \int_{V(cosm)} \rho(x,t) d^3V$$

where $V(cosm)$ is a cosmolgical averaging scale, and $\rho(x,t)$ is an astrophysical scale density measure, defined by averaging over a smaller scale volume V_{astro}. It is $\rho_{cosm}(t)$ that enters the Friedman equation and so determines the volume behaviour $a(t)$ of the universe - itself a coarse-grained variable. It is $\rho(x,t)$ defined for smaller averaging scales that enters perturbations studies.

Averaging effects

Metric tensor: $g_{ab} \longrightarrow \hat{g}_{ab} = \langle g_{ab} \rangle$

Inverse Metric tensor: $g^{ab} \longrightarrow \hat{g}^{ab} = \langle g^{ab} \rangle$

but not necessarily inverse …

need correction terms to make it the inverse

Connection: $\Gamma^a{}_{bc} \longrightarrow \langle \Gamma^a{}_{bc} \rangle + C^a{}_{bc}$

new is average plus correction terms

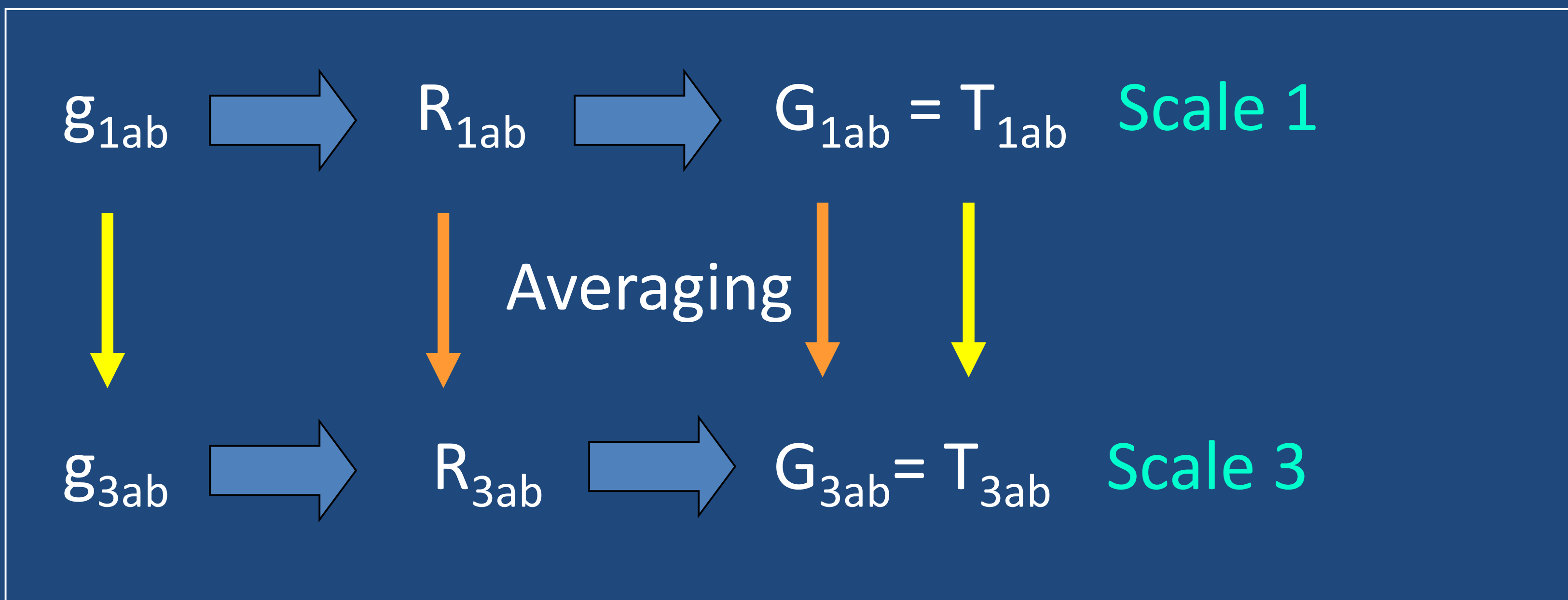
Curvature tensor $\longrightarrow$ plus correction terms

Ricci tensor $\longrightarrow$ plus correction terms

Field equations $G_{ab} = T_{ab} + P_{ab}$

Local inhomogeneity: averaging dynamic effects

Averaging and calculating the field equations do not commute
In general



averaging process

averaging gives different answer

*Averaging scal*e *relations*

Relations between scales: lower level relations underlie higher ones, ***but*** There is in principle a non-commutativity of averaging with dynamics and observations

It is not necessarily the case that the same relations apply at larger averaging scales as at smaller scales

Averaging leads to extra terms in effective higher level equations

GR is tested on the solar system scale but we are applying it at the cosmological scale

Need to check what the coarse grained resulting equations are

It is probably not important in this case! But testing is important

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Major discoveries 2

- Local gravity everywhere determines the evolution of the universe as a whole
- Dynamic models are possible (Friedmann, Lemaître)
- Static solutions are unstable so expansion (or collapse) is inevitable (Eddington)
- Unifies falling apples, motion of Moon round Earth and Earth round Sun, and the expansion of the cosmos
- These models are observationally testable
- An initial singularity is indicated, but might be due to symmetry

Large scale equations: FLRW

- *Energy conservation equation*

  $d\rho/dt + 3H(\rho+p)=0$ (1)

- *Friedmann equation*:

  $3H^2 = \kappa\rho + \Lambda - 3k/a^2$ (2)

- Raychaudhuri equation:

  $3(d^2S/dt^2)/S = -(\kappa/2)(\rho+3p) + \Lambda$ (3)

Any two implies the third. Global variables!

NB: $\rho+3p$ is active gravitational mase

Static model: $\kappa\rho + \Lambda = 3k/a^2 > 0$, $(\kappa/2)(\rho+3p) = \Lambda > 0$

Cosmological observations

Magnitude-redshift curves

Number counts

Source isotropy

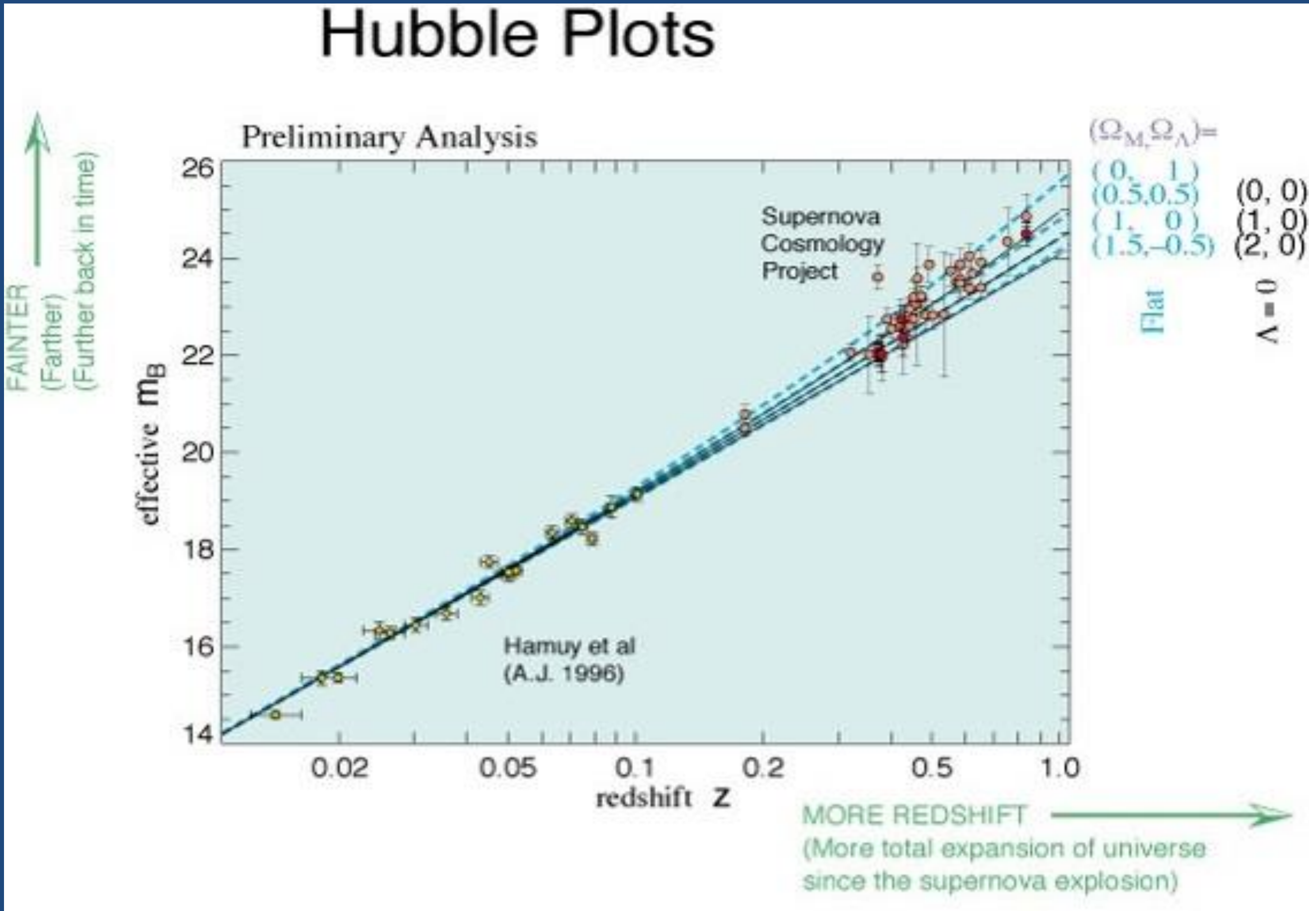
➔ "Just 2 numbers": q_0, H_0

But standard candles problem:

?? Source evolution (Tinsley)

Particularly radio sources

Type 1a SN to the rescue



Universe is accelerating

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Conservation equations

Energy conservation equation for perfect fluid in general

i

$$d\rho/dt + (\rho+p)\,(3/S)dS/dt = 0$$

- Expansion changes energy density,
- Depending on equation of state:

$p = 0 \Rightarrow \rho = \rho_0/S^3$, $p = \rho/3 \Rightarrow \rho = \rho_0/S$

Enters Friedmann equation and changes outcome

Momentum conservation equation for perfect fluid in general

$$(\rho+p)a^d + {}^{(3)}\nabla^d p = 0$$

Pressure gradient causes acceleration

- $(\rho+p)$ is inertial mass density and also determines $d\rho/dt$
- Major problems if it becomes negative

Local Physics at each epoch determines background evolution

Radiation dominated era (Tolman):

$P = \rho a/3 \Rightarrow a(t) = t^{1/2}$

Hot Big Bang: nucleosynthesis, element abundances

Decoupling, CMB existence and spectrum

Matter dominated Era (Einstein-de Sitter):

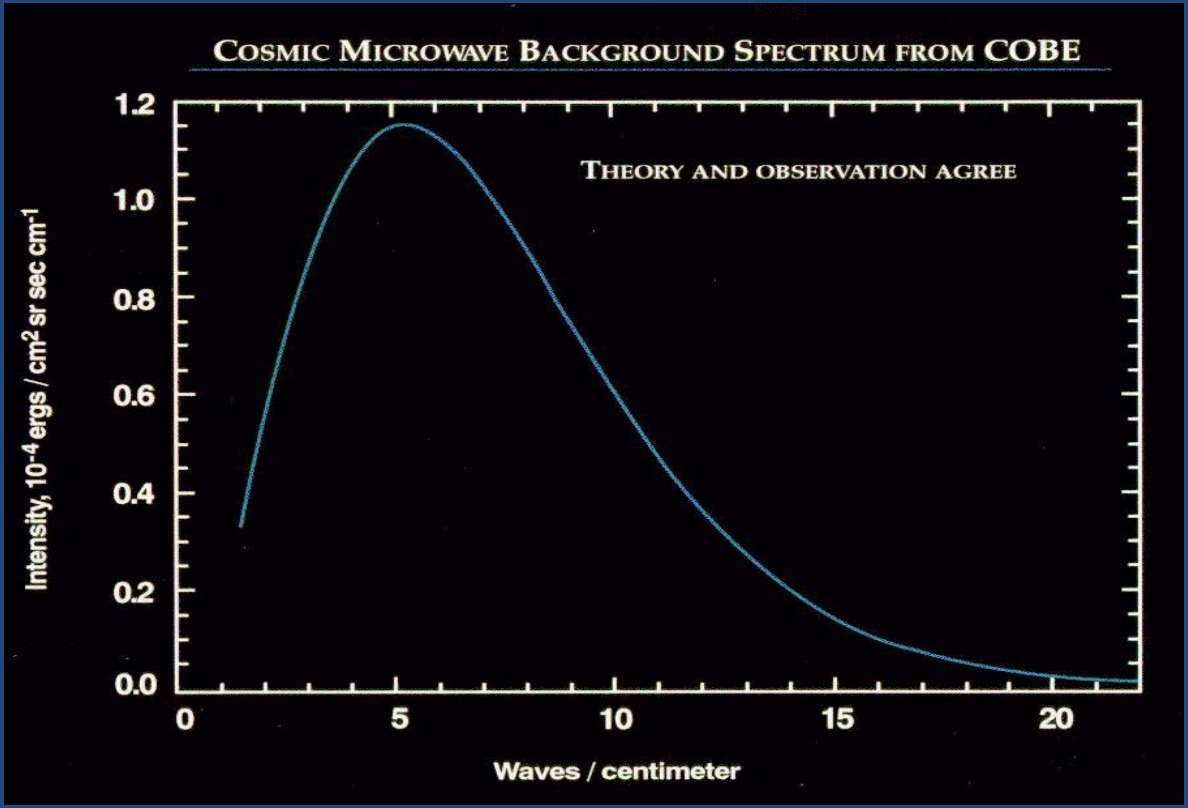
$P = 0 \Rightarrow a(t) = t^{2/3}$

Cosmological constant/dark energy dominated era (de Sitter):

$P = - \rho \Rightarrow a(t) = e^{Ht}$

No curvature dominated era? (if no dark energy decay)

Data: CMB spectral data



Together with primordial element abundances

Key feature: scalar fields possible

Scalar field $\phi(t)$ obeying Klein Gordon Equation

$$d^2\phi/dt^2 + 3\,H\,\phi + dV/d\phi = 0$$

Energy density $\rho = (1/2)\,(d\phi/dt)^2 + V(\phi)$

Pressure $p = (1/2)\,(d\phi/dt)^2 - V(\phi)$

$$\rho + p = (d\phi/dt)^2 > 0$$

$$\rho + 3p = (d\phi/dt)^2 - V(\phi)$$

Hence active gravitational mass can be negative

- So scalar fields can cause an exponential acceleration
- Which smooths out and flattens universe (Guth)

Relation to physics

Unifications			
Gravitation	Universe Expansion Dynamics	Hubble diagram	Apple, moon, universe
Atomic physics	Equilibrium and Decoupling	CBR spectrum	Planck black body, CBR
Nuclear physics	Nucleosynthesis	Element abundances	Nuclear reactors, nucleosynthesis
Particle physics	Inflation	CBR anisotropies	LHC and inflation iff Higgs
Quantum gravity	??	??	??

Higgs Inflaton

The Standard Model Higgs boson as the inflaton
Fedor Bezrukov, Mikhail Shaposhnikov arxiv.0710.3755

Jerome Martin, Christophe Ringeval, Vincent Vennin
Encyclopaedia Inflationaris
arxiv:1303.3787.

If you want to add a Bayesian prior: this is it!
G Ellis and J-P Uzan: Inflation and the Higgs Particle
Astronomy & Geophyiscs • February 2014 • Vol. 55

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Perturbation solutions

Perturb FLRW models linearly

Study growth of structure (Lifshitz 1946)

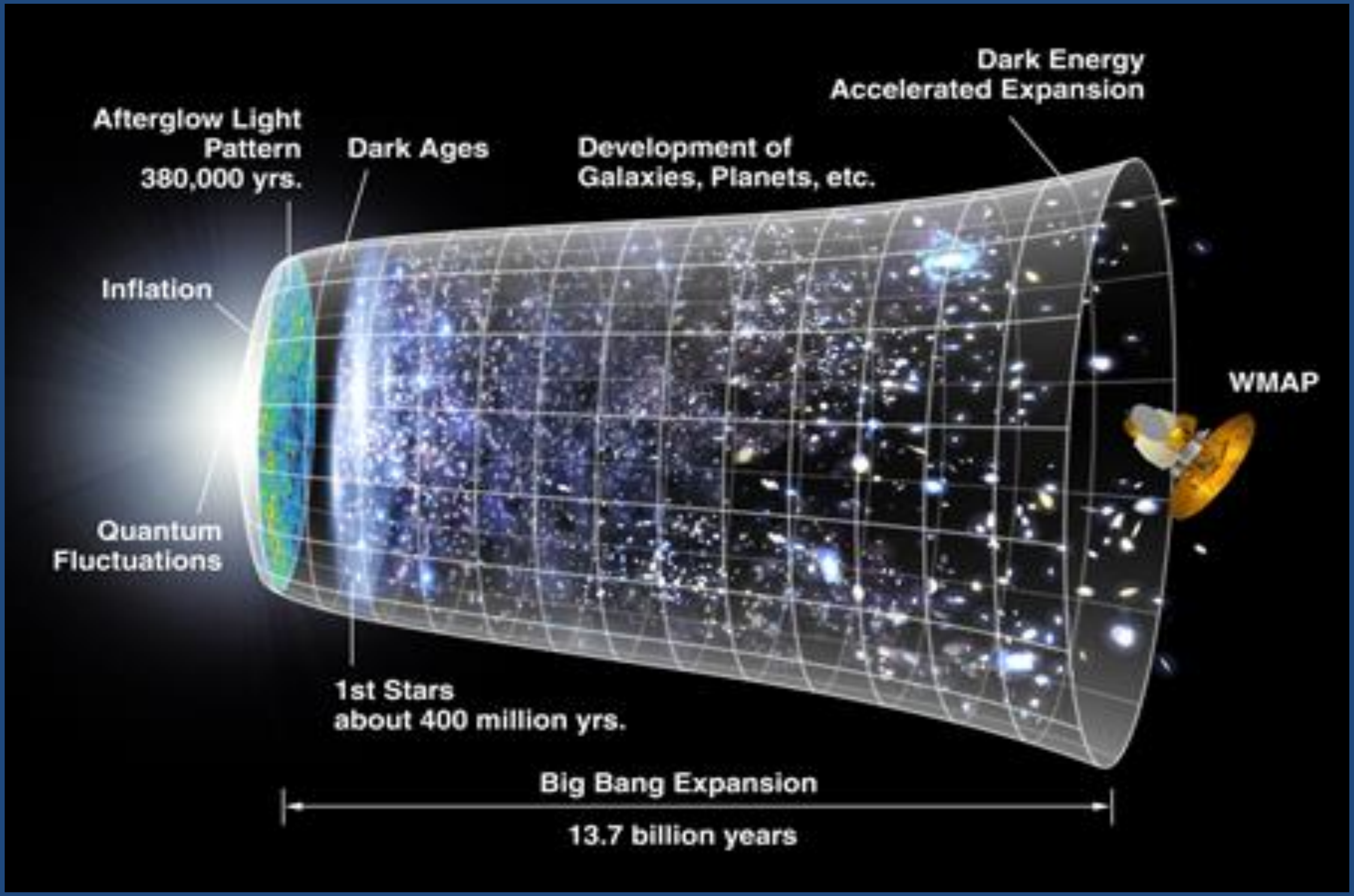
- Scalar, vector, tensor modes
- Fourier analyse

Result depends on nature of matter and radiation interactions

- Inflation: generation of almost scale-free seeds
- Baryon acoustic oscillations (pressure)
- Gravitational instability (baryons/dark matter)

→ Theory of structure formation

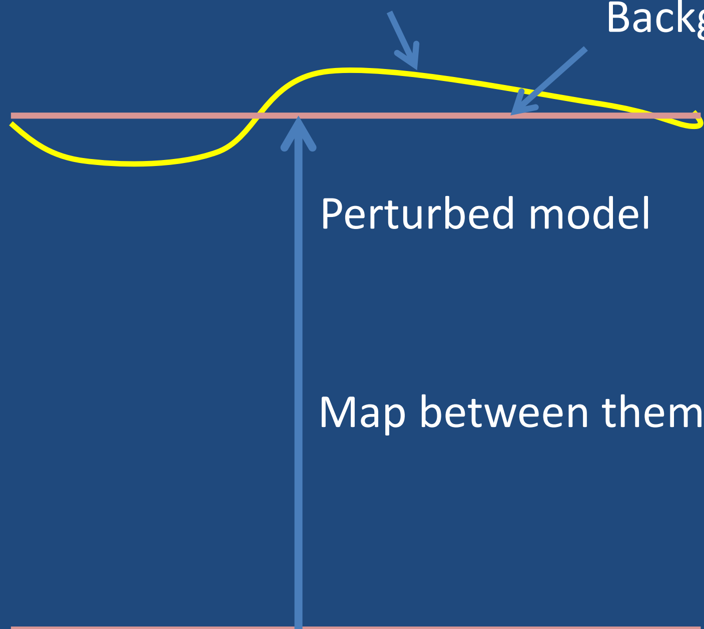
Expansion history



time

Gauge problem for cosmology:

Background spacetime not unique



Perturbation

$$\Delta\rho = \rho - \rho_0$$

Arbitrariness :

e.g. $\rho_0 = \rho$ implies

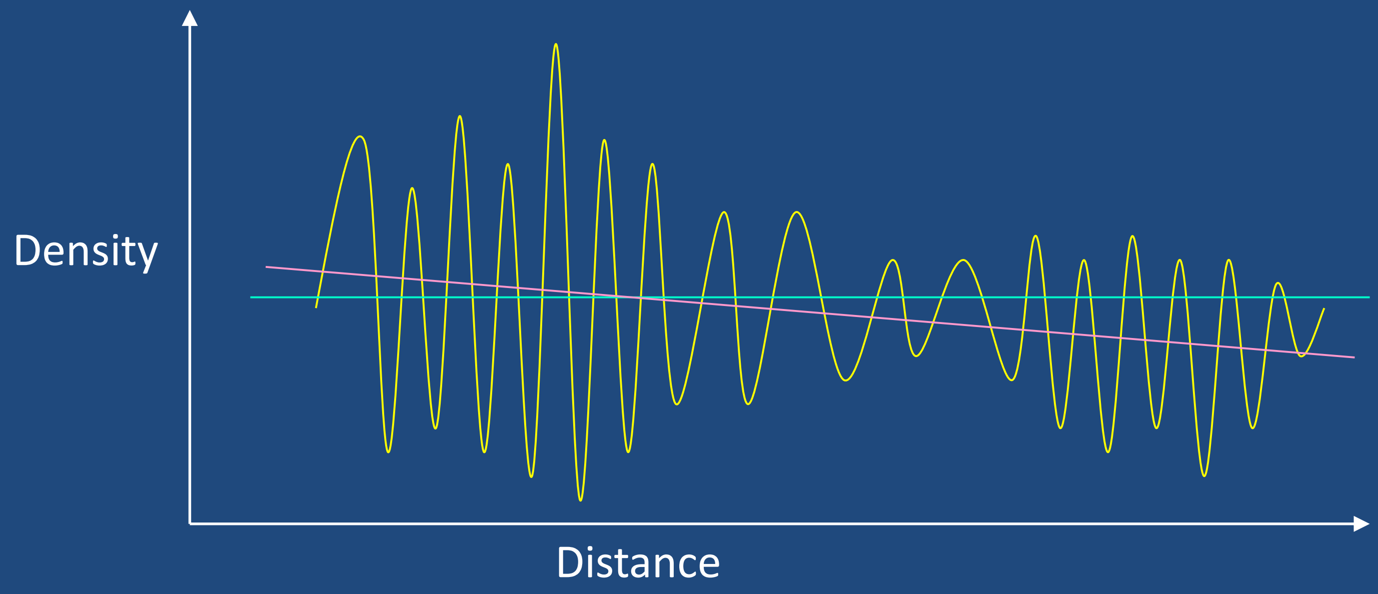
$$\Delta\rho = \rho - \rho_0 = 0$$

Bardeen variables

Local inhomogeneity: the fitting problem

Which background model to use?

G.F.R. Ellis, W.R. Stoeger: ``The fitting problem in cosmology", *Class. Quant. Grav*. 4 (1987) 1697.



Covariant and Gauge Invariant variables

Vanish in FLRW models: $F_0 = 0$: then $\Delta F = F - F_0 = F$.

Given a preferred 4-velocity vector (Ricci eigenvector):

Anisotropy:

- Vorticity ω_{ab}, shear σ_{ab}, acceleration a_b
- Weyl tensor E_{ab}, H_{ab}

Inhomogeneity: Spatial gradient of energy density, pressure, and expansion

$$X^a = {}^{(3)}\nabla^d \rho, \quad Y^a = {}^{(3)}\nabla^d p, \quad Z^a = {}^{(3)}\nabla^d \Theta$$

Propagation equations: take spatial gradient of energy conservation, Raychaudhuri: commute derivatives

Top down effect on density growth

Covariant and gauge invariant comoving fractional density variation orthogonal to a physically preferred 4-velocity u^a $(u^a u_a = -1)$ is

$$\mathcal{D}_a = a(t)\left(h_a^b \frac{\nabla_b \rho}{\rho}\right) \text{ where } h_b^a = g_a^b + u^a u_b$$

then in the pressure-free case

$$\ddot{\mathcal{D}}_a + \frac{2}{3}\theta\dot{\mathcal{D}}_a - \frac{1}{2}\kappa\rho\mathcal{D}_a = 0$$

where $\theta = 3\dot{a}/a$ is the cosmological expansion. Hence the cosmological variable $a(t)$, whose evolution is determined by the cosmological density parameter $\rho(t)$, determines the local growth of inhomogeneity and so structure formation.

In the static case,

$$\ddot{\mathcal{D}}_a - \frac{1}{2}\kappa\rho\mathcal{D}_a = 0 \;\Rightarrow\; \mathcal{D}_a$$

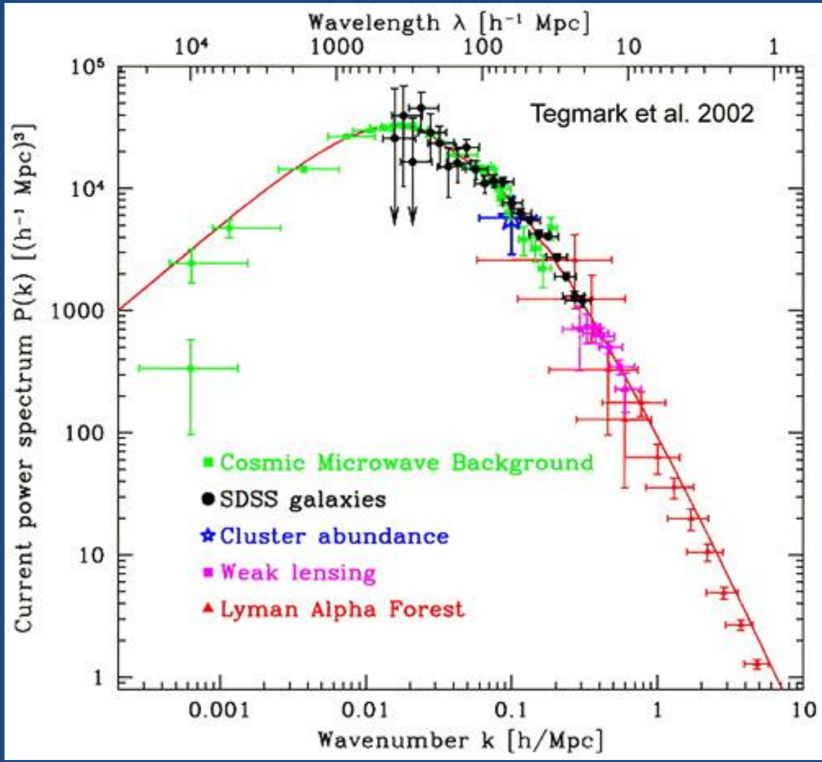
so $\mathcal{D}_a$ grows exponentially. But if $a(t) = t^{2/3}$ (the Einstein-de Sitter case) then

$$\mathcal{D}_a = c_1 t^{2/3} + c_2 t^{-1}$$

has a power law growth.

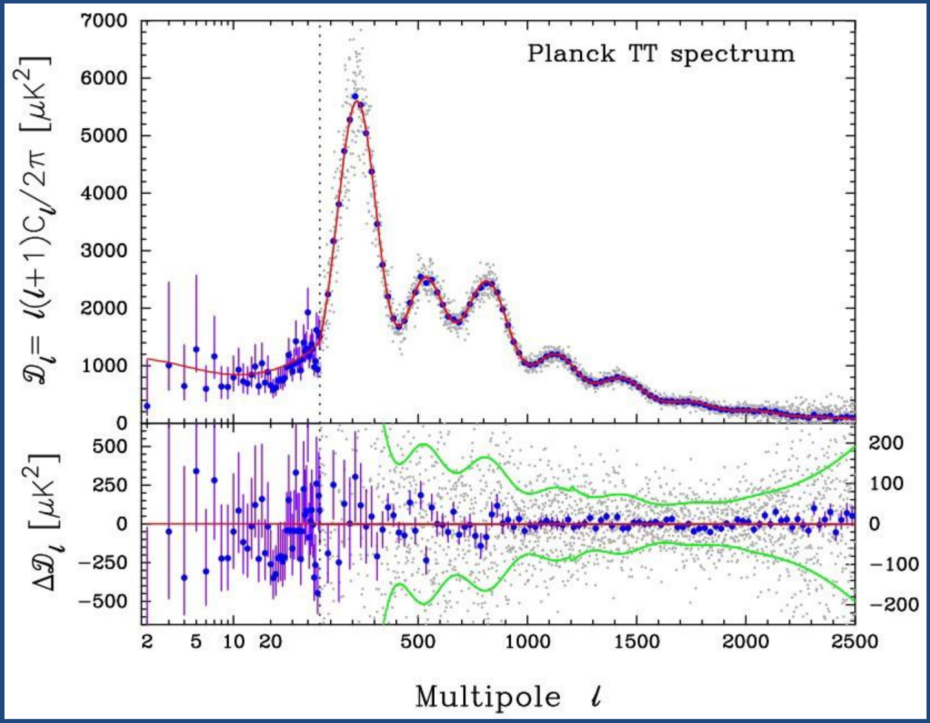
Global parameters determine outcome of local evolution

Perturbation data

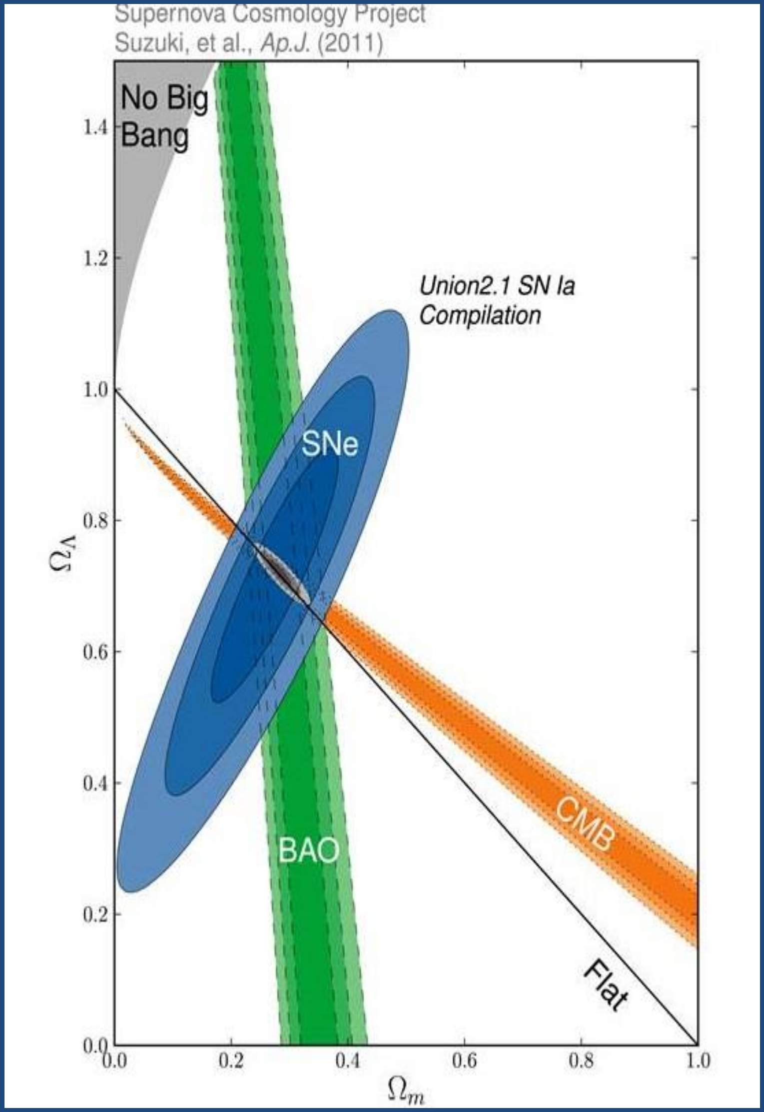


Matter correlation functions (Peebles)
and power spectra, BAO

CMB anisotropy patterns because of matter inhomogeneity
(Sachs and Wolfe)



Cosmic parameters



From direct measurement (SN1a) and from structure formation (BAO) and its effects on the CMB

It is the latter, due to top down effects on structure formation, that give the tightest limits on cosmic parameters (which set the context for structure formation)

Other Top down effects (Sciama)

- Nucleosynthesis → primordial element abundances (also constrain cosmology models)
- Olber's paradox: the dark night sky, necessary for life on Earth (resolved, night sky temperature: 2.75K)
- Mach's principle: the origin of inertia (resolved??)
- The arrow of time: special initial conditions at the start of the universe give a preferred time direction out of time symmetric physics (resolved??)

Weyl tensor as gravity variable

Action at a distance ➔ fields!! (Maxwell case): Gravity??

Pirani, Trümper, Newman and Penrose: The possibility of using the Weyl tensor (free Gravitational field) as the key gravitational variable rather than the metric

Curvature tensor R_{abcd} comprises Ricci tensor R_{ab} and Weyl tensor C_{abcd} (= trace free part of curvature tensor)

Ricci tensor determined pointwise by Matter tensor

$$R_{ab} - \tfrac{1}{2} R\, g_{ab} + \Lambda\, g_{ab} = \kappa\, T_{ab} \qquad (1)$$

The Weyl tensor determined non-locally by matter elsewhere

- There is no similar equation for C_{abcd}

Weyl tensor: electric and magnetic trace-free parts

$$E_{ac} = C_{abcd}\, u^b u^d = E_{<ac>}, \quad E^a{}_{a=0,}\ E_{ac} u^c = 0$$
$$H_{ac} = {}^*C_{abcd}\, u^b u^d = H_{<ac>}, \quad H^a{}_{a=0,}\ H_{ac} u^c = 0$$

Bianchi identities as consistency equations:

$$\nabla^d C_{abcd} = \nabla_{[a} \left(-R_{b]c} + (1/6)\, R g_{b]c}\right)$$

EFE algebraically substitute for R_{bc} in terms of T_{bc}

(M Trumper; S Hawking, J Kristian and R Sachs)

Described by 1+3 Equations:

G F R Ellis Varenna Lectures, 1971 [*Gen. Rel. Grav.* **41**: 581 (2009)]

R Maartens and B A. Bassett: arXiv:gr_qc/9704059

Maxwell equation-like Constraints:

$$D^b E_{ab} = -\,3\omega^b H_{ab} + [\sigma,H]_a + (1/3) D_a \rho$$
$$D^b H_{ab} = 3\omega^b E_{ab} - [\sigma,E]_a + (\rho + p)\omega_a$$

Maxwell equation-like Evolution Equations:

$$dE_{<ab>}/dt - \text{curl } H_{ab} = -\,\Theta E_{ab} + 3\sigma_{c<a} E_{b>}{}^c$$
$$-\,\omega^c \varepsilon_{cd(a} E_{b)}{}^d + 2a^c \varepsilon_{cd(a} H_{b)}{}^d - (1/2)\,(\rho + p)\sigma_{ab}$$

$$dH_{<ab>}/dt + \text{curl } E_{ab} = -\,\Theta H_{ab} + 3\sigma_{c<a} H_{b>}{}^c$$
$$-\,\omega^c \varepsilon_{cd(a} H_{b)}{}^d - 2a^c \varepsilon_{cd(a} E_{b)}{}^d$$

Gives wave equations for E_{ab}, H_{ab} ➔ gravitational waves;

Gives Gauss law (constraint) ➔ generates E_{ab}

➔ tidal forces acting at distance

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Was there a start to the universe?

FLRW model Raychaudhuri equation:

$$3(d^2S/dt^2)/S = -(1/2)(\rho+3p) + \Lambda$$

- Directly gives deceleration due to matter
- Active mass density is $(\rho+3p)$:
- positive for all ordinary matter so $q_0 > 0$
- Cosmological constant causes acceleration

- Singularity at time $t_0 < 1/ H_0$ ago if $(\rho+3p)>0$, $\Lambda=0$
- Major crisis for physics (Wheeler): it predicts its own end

Was there a start to the universe?

But the FLRW model has very high symmetry:

Is that the reason for the singularity?

Raychaudhuri/Ehlers: rotation/acceleration might stop it:

$$(1/S)d^2S/dt^2 = 2(\omega^2 - \sigma^2) - (1/2)(\rho+3p) + \Lambda + a^b{}_{;b}$$

No! Penrose, Hawking:

- Closed trapped surfaces (time reversed) imply singularity according to classical GR,
- If suitable energy and causality conditions hold

But that can just be taken as a statement that a quantum gravity epoch must have occurred

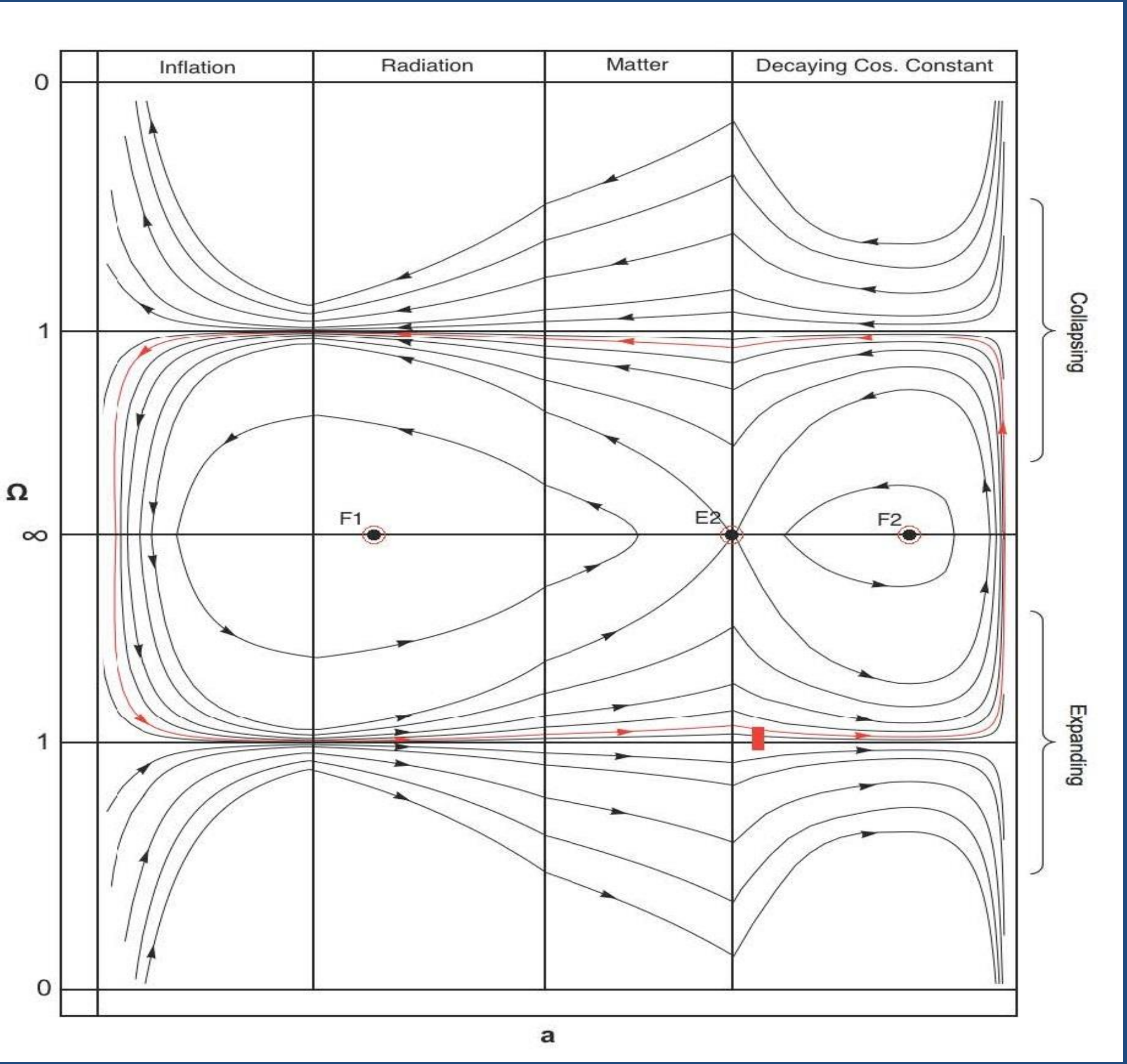
Was there a start to the universe?

- Quantum gravity may allow non-singular start. Loop Quantum Gravity, No Boundary suggests this.

- But remember issue of non-commutativity across scales. QM validity at small scales does not necessarily imply it is valid at larger scales e.g. wave function of the universe.

- But in any case the inflationary field also allows this, if we have a bounce following decay of dark energy

- We can in principle have non-singular bouncing models that never enter a quantum gravity regime



Cyclic models
Are possible
That have no
Quantum
Gravity epoch

The field that
Causes inflation
Can avoid a
Singularity
(because it
Can violate
The timelike
 energy
Conditions)

arXiv:
1511.03076

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The issue of vacuum energy

- Problem of vacuum energy: QFT vacuum energy suggests Λ huge, discrepant with GR if vacuum gravitates

* MAJOR PROBLEM * $\rho_{vac} > 10^{70}$ (many contributions)

- Issue: ***Does the vacuum gravitate?*** If so we have a problem:

The vacuum energy disaster: *The Raychaudhuri equation shows that* (ρ + 3p) *is the active gravitational mass density, while the Friedman Equation shows the energy equation for expansion has* ρ *as source term. If we include the vacuum energy densities as source terms in the Einstein equations, both equations and are a disaster*.

The trace free EFE
as an alternative to the EFE

- ***Problem of vacuum energy: QFT vacuum energy suggests Λ huge, discrepant with GR if vacuum gravitates***

NB - two of the best tested physics theories

- Any full quantum gravity theory should have this as a limit

- Vacuum does not gravitate if we use TFE plus separate conservation equations ("unimodular gravity")
- Then vacuum does not gravitate

Solves profound contradiction arising between WFT and EFE if we join them in the obvious way

$R_{ab} - \frac{1}{2} R\, g_{ab} + \Lambda\, g_{ab} = \kappa\, T_{ab}$ (1)

(10 equations) implies

$T^{ab}{}_{;b} = 0$ (2)

Instead, take trace free part:

$R_{ab} - 1/4\, R\, g_{ab} = \kappa\, (T_{ab} - 1/4\, R\, g_{ab})$ (3)

(9 equations) and assume (2) separately

The vacuum energy has been disempowered:

→ Has no gravitational effect.

- Solves problem. Related to Unimodular gravity [Finkelstein, Unruh]
- Variation principle? [Alvarez arXiv:1204.6162]

Trace Free Einstein Equations:

$$R_{ab} - 1/4\, R\, g_{ab} = \kappa\, (T_{ab} - 1/4\, R\, g_{ab}) \qquad (3)$$

$$T^{ab}{}_{;b} = 0 \qquad (2)$$

What about standard EFE?

$$R_{ab} - ½\, R\, g_{ab} + \Lambda\, g_{ab} = \kappa\, T_{ab} \qquad (1)$$

Take divergence of (3), use (2) and $G^{ab}{}_{;b} = 0$:

Recovers (1): but now Λ is a constant of integration and has nothing to do with vacuum energy: which does not gravitate [Weinberg 1989]

Recover standard GR: Schwarzschild, gravitational waves, etc.

Cosmology

- *Energy conservation equation*

  $$d\rho/dt + 3H(\rho+p)=0 \qquad (1)$$

- *Friedmann equation*:

  $$3H^2 = \kappa\rho + \Lambda - 3k/a^2, \qquad (2)$$

- Raychaudhuri equation:

$$3(d^2S/dt^2)/S = -(\kappa/2)(\rho+3p) + \Lambda \qquad (3)$$

Subtract

$$3(d^2S/dt^2)/S = -(3/2)\,\kappa(\rho+p) + 3H^2 + 3k/a^2 \qquad (4)$$

TFE: Equations (1) and (4) are the cosmological equations

What does QFT version of gravity say?

[Feynman, Deser, Weinberg, Zee]

- Should also give trace free version!
- Because graviton is symmetric trace free

Needs to be revisited

Assume energy momentum conservation separate from gravity equations

Should get only trace free equations as the graviton can’t get a handle on trace equation

E.g. $L = T^{ab} h_{ab} = T^{ab} h_{<ab>} = T^{<ab>} h_{<ab>}$

- **Should necessarily give Trace Free version of EFE**

- these have a good claim to be the correct equations

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2. Gravity implies motion: cosmology is dynamic (Friedmann, Lemaitre, Eddington) and testable (Hubble, McVittie, Sandage)
3. Local physics determines the evolution (Tolman, Guth) unifying physics and cosmology at different epochs to some degree

Is universe FLRW?

4. Top down dynamics: Global variables control local evolution (Lifshitz) giving testable relations (Sachs and Wolfe, Peebles)
5. Classical GR plus energy conditions imply a start to the universe (Lemaitre, Hawking) which is major crisis (Wheeler). QG predictions??
6. Inflation and dark energy imply energy conditions not satisfied; vacuum energy puzzle and gravitational theories (e.g. unimodular)
7. Inhomogeneity issues: 1. Is universe FLRW? 2. Why was its start so smooth? (Penrose) Was gravity then the same as now? (Greene)

Issue 1: Large scale inhomogeneity instead of dark energy?

Perhaps there is a large scale inhomogeneity of the observable universe

such as that described by the Lemaitre-Tolman-Bondi pressure-free spherically symmetric models.

We are near the centre of a void (comment later)

Challenging dark energy with exact inhomogeneous models (Celerier)

One can reproduce the (M,z) and (N,z) relations, with or without Lambda, by an inhomogeneous model

LTB (Lemaitre-Tolman Bondi models

Metric: In comoving coordinates,

$$ds^2 = -dt^2 + B^2(r,t) + A^2(r,t)(d\Theta^2+\sin^2\Theta\, d\Phi^2)$$

where

$$B^2(r,t) = A'(r,t)^2\ (1-k(r))^{-1}$$

and the evolution equation is

$$(\mathring{A}/A)^2 = F(r)/A^3 + 8\pi G\rho_\Lambda/3 - k(r)/A^2$$

with $F'\ (A'A^2)^{-1} = 8\pi G\rho_{M.}$

Two arbitrary functions: k(r) (curvature) and F(r) (matter).

Can we use the freedom in these models to fit the supernova observations? Yes we can!

Theorem:

We can always choose a LTB model that will simultaneously satisfy any (magnitude, redshift) relation and any source number count relations that may be observed, for any chosen value of Λ (including zero) and for any source evolution.

N Mustapha, C Hellaby and G F R Ellis: Mon Not Roy Ast Soc 292: 817-830 (1997)

This follows from existence of the free functions in the LTB models.

Measuring Curvature in FLRW

- in FLRW we can combine Hubble rate and distance data to find curvature at present time from null cone data

$$\Omega_k = \frac{[H(z)D'(z)]^2 - 1}{[H_0 D(z)]^2}$$

$$\left[d_L = (1+z)D = (1+z)^2 d_A\right]$$

- independent of *all* other cosmological parameters, including dark energy model, and theory of gravity
- can be used at single redshift
- what else can we learn from this?

- FLRW: must be same for all z!

Clarkson Bassett Lu arXiv:0712.3457

Generic Consistency Test of FLRW

- since Ω_k independent of *z* we can differentiate to get consistency relation
- $$\mathscr{C}(z) = 1 + H^2\left(DD'' - D'^2\right) + HH'DD' = 0$$
- depends *only* on FLRW geometry:
  - **independent of curvature, dark energy, theory of gravity**
- should expect $\mathscr{C}(z) \approx 10^{-5}$ in FLRW
- In non-FLRW case this will not be true. $\mathscr{C}(z) \neq 0$

Copernican Test!

- Errors may be estimated from a series expansion

$$\mathscr{C}(z) = \left[q_0^{(D)} - q_0^{(H)}\right] z + \mathcal{O}(z^2)$$

deceleration parameter measured from *distance* measurements

deceleration parameter measured from *Hubble* measurements

[Percival et al]

- simplest to measure *H(z)* from BAO: SDSS

- time drift of redshifts over many years gives it

[Uzan Clarkson & Ellis]

[in FLRW] arXiv:0801.0068

$$\dot{z}(z) = H_0(1+z) - H(z)$$

Best tests of Copernican Principle: kinematic SZ effect

SZ effect (scattering of CMB off hot gas at high redshift) tests isotropy of CMB at those redshifts

→ Limits on anisotropy then

→ Hence almost limits deviation from FLRW models

(in effect application of almost-EGS theorem)

Confirms Copernican Principle Observationally

(Caldwell, Stebbins)

Changes philosophical assumption into observationally tested theory

But then why is this the case?

Issue 2: Inflation and Entropy: Homogeneity of the Universe

- Penrose: why is gravitational entropy so low at the start of the universe? Inflation does not solve this

- Thermal entropy is at a maximum, gravitational entropy very low

- Implies that the inflationary scenario is incredibly unlikely

Resolution 1: Conformal Cyclic Cosmology

Resolution 2: Gravity turns on after being turned off at the start of the expansion of the universe (Greene et al)

Ongoing issues

1. Dark Matter and Dark Energy
- Structure formation details, CMB observations and source surveys

2. Inflaton?? – can it be Higgs?

3. Start to the universe???

- Extrapolating known physics to where physics did not exist; else no start to the universe

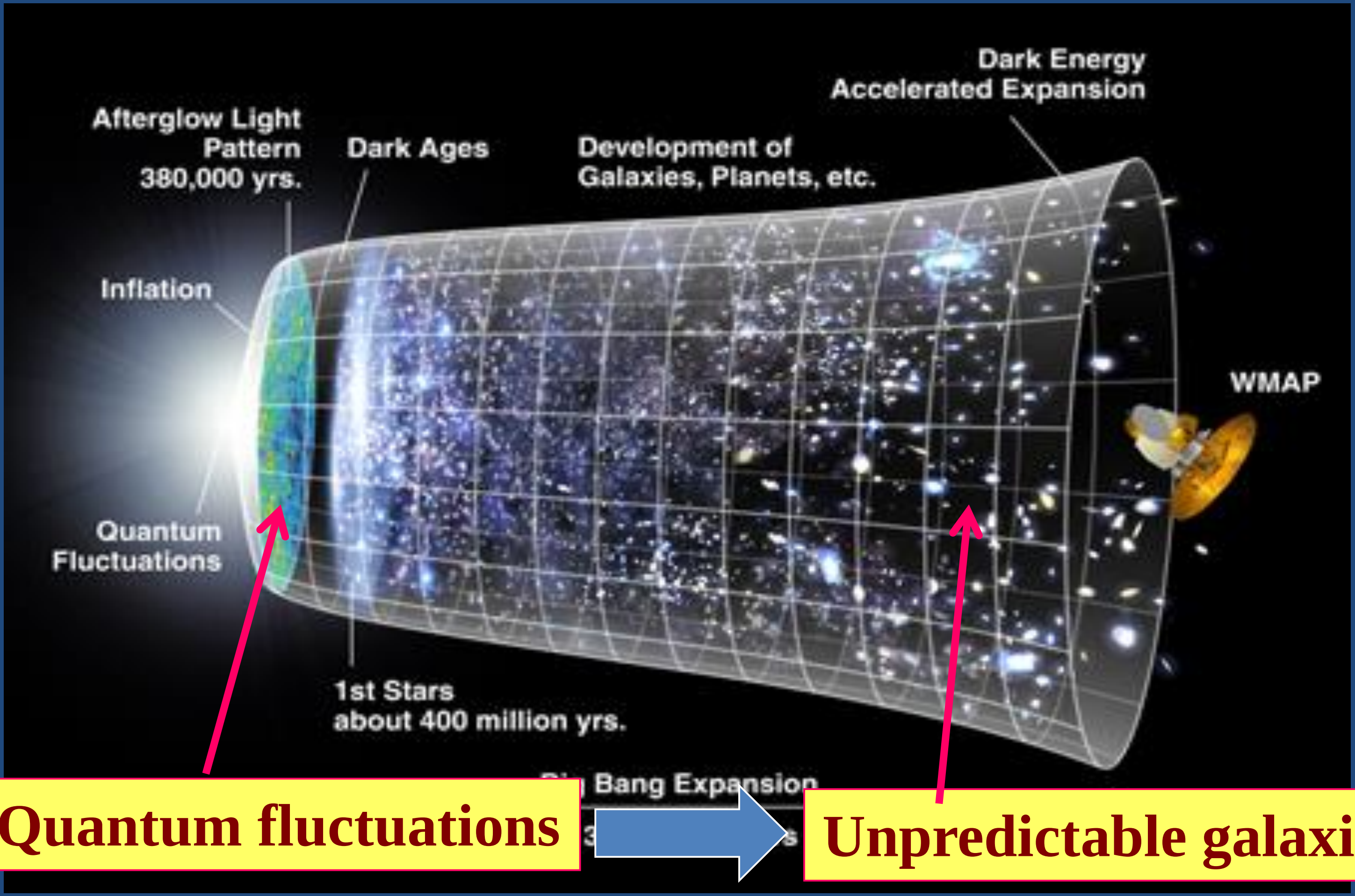
4. Multiverse and testability

? Needed for Λ? (Rees, Weinberg)

- Unruh explanation of vacuum energy density

5. End of quantum fluctuations????

Inflation perturbations



Quantum fluctuations

Unpredictable galaxies

time

Issue: No infinities

- **Infinity is an unattainable state rather than a number**

- **Hilbert's Golden Rule:**

  David Hilbert: "the infinite is nowhere to be found in reality, no matter what experiences, observations, and knowledge are appealed to."

Unacceptable infinity:

Uncountable infinity of physically existing points between my fingers

Issue: The often claimed existence of *physically existing infinities*

will infinities occur in the theory?

If so will they play an essential or inessential role?

Thesis: finiteness:

infinities are essentially unattainable by their very definition, should not occur as essential elements in any physical theory, including cosmology.

Infinity is an unattainable state rather than a number

(David Hilbert: "the infinite is nowhere to be found in reality, no matter what experiences, observations, and knowledge are appealed to.")

not a scientific statement – if science involves testability by either observation or experiment.

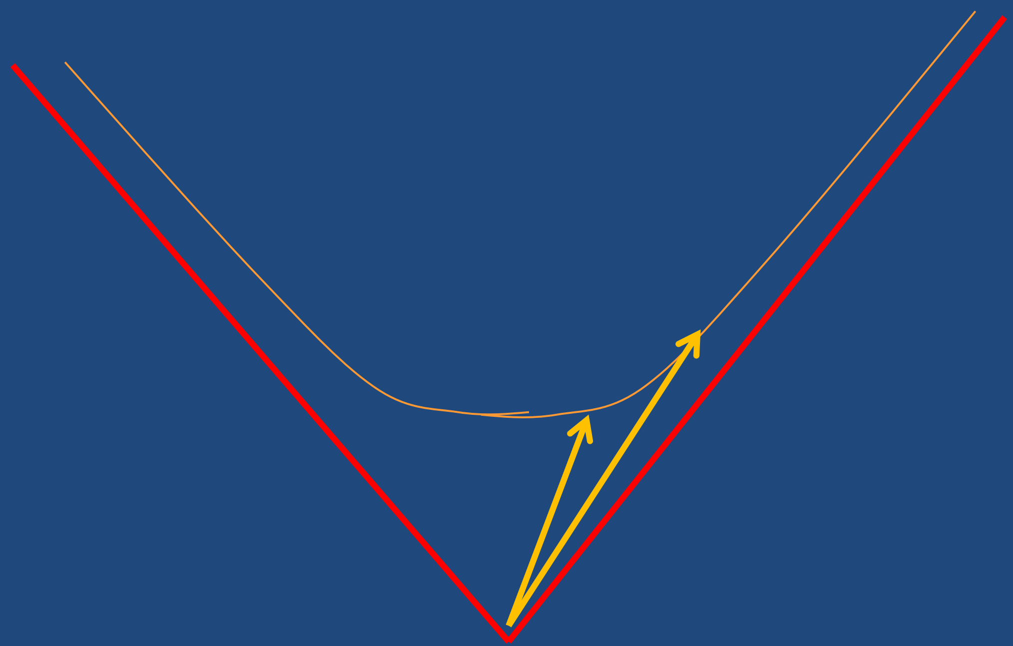
It is a huge act of hubris to extrapolate from one small domain to infinity – NEVER encountering a limit

(remember the conformal diagram; problems with measure).

Infinity from tunnelling?

Susskind et al: arXiv:hep-th/0505232

Ellis and Stoeger: arXiv:1001.4590



These are the same proper time:
So an infinite space section appears at once

Infinity from tunnelling?

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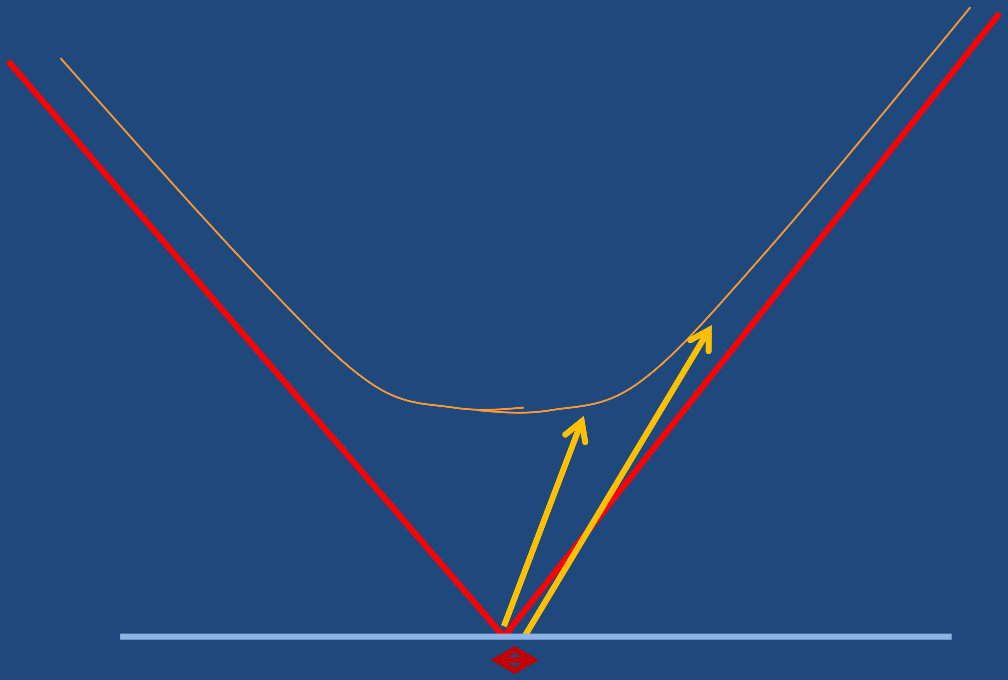


Not true if we remember that the origin
can't be exactly a point: takes an eternity
to complete, no matter how small the nucleus is

TFE 4 possibilities:

$G_{ab} = \kappa\, T_{ab}$ (a)

$G_{<ab>} = \kappa\, T_{ab}$ (b)

$G_{ab} = \kappa\, T_{<ab>}$ (c)

$G_{<ab>} = \kappa\, T_{<ab>}$ (d)

Only first and last OK

Last solves GR $\Longleftrightarrow$ QFT incompatibility!

Cosmology ok: even though only inertial mass density in EFE; Ok as regards inflation [arXiv:1306.3021]

- Related to Unimodular gravity [Finkelstein, Unruh]
- Variation principle? [Alvarez arXiv:1204.6162]

TFE: What about Inflation?

Scalar field $\phi(t)$ obeying Klein Gordon Equation

$$d^2\phi/dt^2 + 3\ H\ \phi + dV/d\phi = 0 \qquad (1)$$

Energy density $\rho = (1/2)\ (d\phi/dt)^2 + V(\phi)$

Pressure $p = (1/2)\ (d\phi/dt)^2 - V(\phi)$

$$\rho + p = (d\phi/dt)^2 > 0$$

$$3(d^2S/dt^2)/S = -(3/2)\ \kappa\ (\rho+p) + 3H^2 + 3k/a^2 \qquad (2)$$

$$d\rho/dt + 3H(\rho+p)=0 \qquad (3)$$

Paradox 1: (2), (3) do not involve $V(\phi)$! Can't give inflation!

What about Inflation?

Consistency check gives back standard equations by use of conservation equations:

$d\rho/dt + 3H(\rho+p)=0$

$\rho = (1/2)\,(d\phi/dt)^2 + V(\phi)$

So

$D[(d\phi/dt)^2 + 2V(\phi)\,]/dt + 6H(\rho+p)=0$

Integrate to get ϕ^2 get back equations with $V(\phi)$

[conservation equation equivalent to Klein Gordon]

Detail: Multiply (2) by an integrating factor ($6\dot{S}/S$), use the Klein Gordon equation , and integrate to get the Friedman equation

$$3(\dot{S}/S)^2 + 3k/S^2 - \underline{\Lambda} = 8\pi G\,(1/2\,(\dot{\phi})^2 + V(\phi)) \quad (27)$$

where $\underline{\Lambda}$ is a constant of integration. Substituting back will give the effective Raychaudhuri equation

$$3(\ddot{S}/S) - \underline{\Lambda} = -4\pi G\,(2\dot{\phi}^2 - 2V(\phi)) \qquad (28).$$

Now $V(\phi)$ does indeed appear in both (27) and (28) and does affect spacetime curvature.

But now the second paradox arises:

- **Second paradox**: *If this works for* V(ϕ), *why does the same not happen for the vacuum energy* Λ_{vac}*? Why does this huge energy not re-appear, Phoenix like, in the effective EFE, as happens for* V(ϕ)*?*

The point here is that the causal chain whereby V (ϕ) enters the equation only works when dϕ/dt ≠0. When dϕ/dt = 0, the term V (ϕ) is constant and drops out of D[(dϕ/dt)^2 + 2V(ϕ)]/dt + 6H(ρ+p)=0.

- **Non-reappearance of vacuum energy** vac *in the effective field equations is because it is a constant, and so has no effect.*

But then what determines the present day cosmological constant?

There are ***two*** constants of integration in the theory:

- one from integrating the gravitational equations,
- one from integrating the Klein Gordon equation

[If there are multiple fields: one for each field]

It is the ***summed effect*** of these constants that determine the present day value of CC

It is the ***difference*** of $V(\phi)$ between today and the time of inflation that determines what the value was then

- Radiation: 1+3 covariant CBR (Cargese (236))

Photon momentum

$$p^a = E\,(u^a + e^a)\ ,\ e^a e_a = 1\ ,\ e^a u_a = 0$$

Distribution function multipole expansion

$$f(x, p) = f(x,E,e^a)$$
$$= F + F_a e^a + F_{ab} e^a e^b + F_{abc} e^a e^b e^c + \ldots$$

where coefficients are PSTF:

$$F_{abc}=F_{(abc)},\ \ F_{abc}u^b=0,\ \ F_{abc}h^{ab} = 0$$

Then ρ, p are integrals only of F (with coefficients of E); q_a only of F_a; π_{ab} only of F_{ab}.

Higher F_{abcd} .. do not enter field equations

- Radiation: CBR (Cargese (236))

For example

$$\mu = 4\pi \int E^3 F \, dE,$$

$$q^a = (4\pi/3) \int E^3 F^a \, dE \, ,$$

$$\pi^{ab} = (8\pi/15) \int E^3 F^{ab} \, dE \, ,$$

Boltzmann equation becomes a chain of equations linking the quantities F_{abcd}...

Their energy integrals link the moments such as μ, q^a, π^{ab}

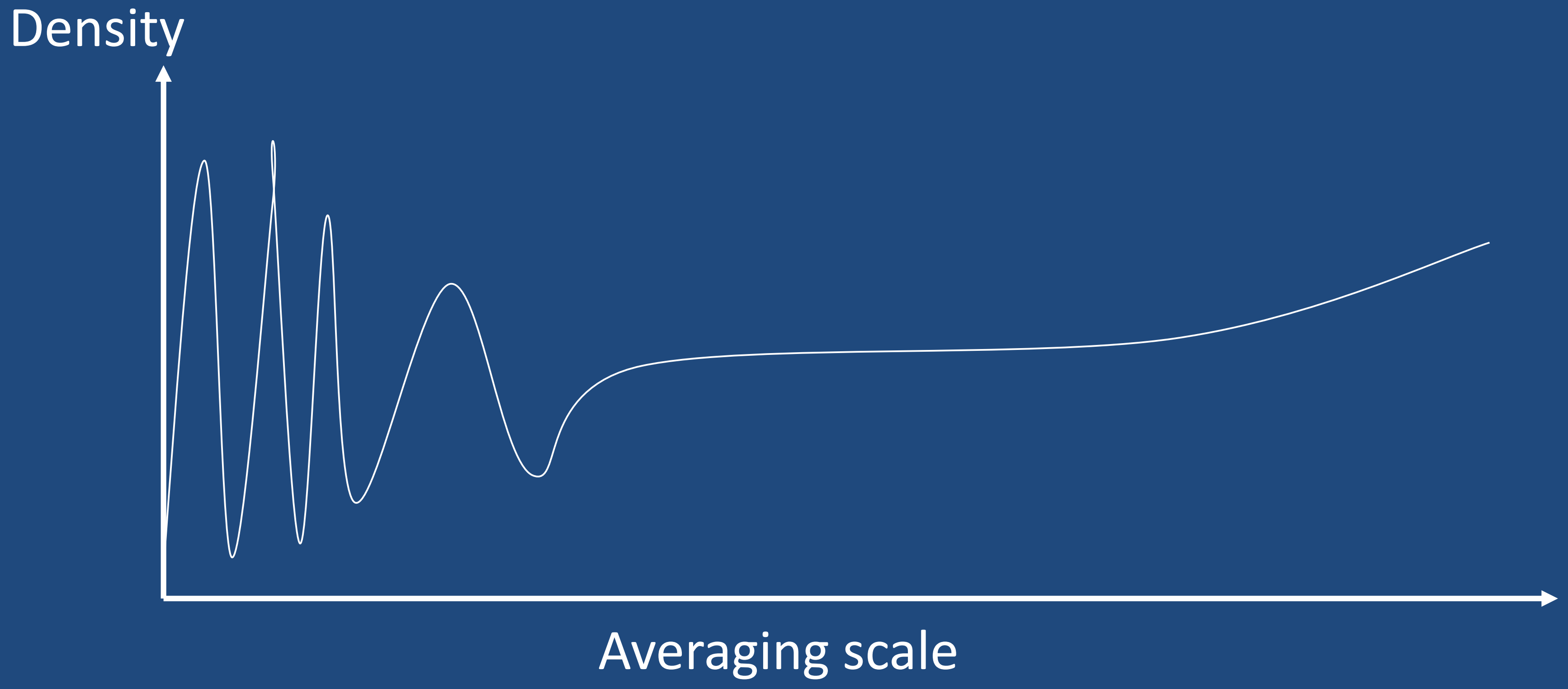
The first two are energy and momentum conservation

Can write down the fully non-linear equations and linearise to get linear equations

*The hidden averaging scal*e

Each variable definition hides an averaging scale:

e.g. density of gas (George Batchelor)



Raychaudhuri equation

Fundamental equation for gravitational attraction in general

$$(1/S)d^2S/dt^2 = 2(\omega^2 - \sigma^2) - (1/2)(\rho+3p) + \Lambda + a^b{}_{;b}$$

- $(\rho+3p)$ is active gravitational mass
- ω^2 and Λ cause divergence (acceleration)
- σ^2 and $(\rho+3p) > 0$ cause convergence
- Singularity in past for irrotational dust unless Λ is large enough to prevent it

Because gravity underlies cosmological dynamics, the two are intimately related. That relation has changed in major ways over time, partly due to the assumptions made about cosmology.

- Firstly the view that the universe was necessarily static has given way to the view that it is necessarily dynamic, because of stability issues plus observations.
- Secondly the view that the universe is necessarily homogeneous and isotropic has given way to the view that it is generic. This has been supported by perturbation calculations, which play a crucial role in the third transition: from studying only the dynamics of spatially homogeneous models to studying the growth of structure in the expanding universe, which has been a major success.
- But issues that need clarification abound: the nature of the inflaton, dark matter, and dark energy in particular, details of structure formation, the Hubble constant value, and the nature of the earliest universe: why the gravitational degrees of freedom were not highly excited, like the matter degrees were.
- Maybe cosmology will show us we need a generalised classical theory of gravity to resolve some of these issues, for example a form of unimodular gravity. We will have to wait for a good quantum theory of gravity to get some kind of convincing view of the origin of the universe - if that is possible.
- Underlying all of this is the issue of averaging: we are applying a theory developed and tested at one scale to a completely different scale. In principle extra terms arise in the equations when one takes averaging between scales into account.

Amongst the contributions to vacuum energy are that from the electroweak field

- $\rho_{vac}^{(EW)} \simeq (200\text{GeV})^4 \simeq 3 \times 10^{47}$erg/cm3,

that from quantum chromodynamics

- $\rho_{vac}^{(QCD)} \simeq (0.3\text{GeV})^4 \simeq 1.6 \times 10^{36}$erg/cm3,

and that from fluctuations at the Planck scale

- $\rho_{vac}^{(PL)} = M^4_{pl} \simeq (10^{19}\ \text{GeV})^4 \simeq 2 \times 10^{110}$erg/cm3.

These will all contribute to the total effective cosmological constant term:

- $\Lambda_{eff}\, g_{ab} = (\Lambda + \Lambda_{vac})\, g_{ab} = (g_{ab} + 8\pi G\, \Sigma_{(i)}\rho_{vac\ (i)})\, g_{ab}$

where the sum is over all such vacuum energy contributions.

The total term is still Lorentz invariant.

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