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Almost Light-Cones and Applications

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Details on line, gr-qc.
Several published papers,
CQG, GERG & Living Reviews.

abstract

In studies of gravitational radiation, Bondi-type of null surfaces and their associated Bondi coordinates have been almost exclusively used for calculations. It turns out that some surprising relations arise if instead of the Bondi coordinates, one uses ALCs and their associated coordinate systems in the analysis of the Einstein-Maxwell equations near null infinity. The asymptotic Bianchi Identities turn directly into many of the standard relations and equations of classical mechanics coupled with Maxwell's equations. These results greatly extend and generalize the beautiful results of Bondi and Sachs.

They do leave a serious enigma.

1. Real Einstein & Einstein-Maxwell Eqs.

a. **NO NEW PHYSICS** is introduced -
Just **standard** GR Equations.
Recently found Structures ARE described.

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b. Complex Ideas are used - but in end all REAL

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c. Misuse term "*congruence*" - a 2-parameter family of curves rather than 3-parameter family.
e.g., generators of a single light-cone

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d. An awful lot is left out

2. Shear-Free & Asymptotic Shear Free Null Surfaces - BASIC

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A. **In Minkowski space**:

Light-Cones

- each cone labeled by 4 real coordinates x^a at apex.

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Almost Complex Light-Cones - **REAL** congruences - Constructed from Complex Light Cones in Complex Minkowski space

- labeled by 4 complex coordinates z^a at apex.

Twisting Congruence => later, related to **spin**

Example: **Kerr congruence**

4. **B. Asymptotically-Flat Einstein-Maxwell SpaceTimes**

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a. In the past **Bondi Null surfaces** have been used almost exclusively for the study of Asymptotic solutions of Einstein-Maxwell Space-Times.

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b. **New Structures**:

Asymptotically Shear-Free Null Surfaces

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(**Theorem**) **EACH** surface is **Labeled** (again) by 4 complex numbers,

points in 4-complex dimensional space.

5. **Asy. Shear-free Surfaces** referred to as **Almost Light-Cones, ALC**

Asy. Shear-free, diverging & labeled by 4 complex coordinates. (**Defining H-space**.)

Flat-space-limit: *Ordinary Light-Cones*

[**Aside**: *H-space, complex self-dual Vacuum metric*]

To study the fields near **future null infinity**, we **Replace** the Bondi coordinates by the **Almost Light-Cones** coordinates.

Interesting results follow.

6. **Review of NULL Asymptopia**

a. Coordinates on Future Null Infinity, $[S^2 x R]$, $(u,\zeta,\overline{\zeta})$,

$$\zeta = \cot(\theta/2)e^{i\phi}$$

b. 5 Complex Weyl Tensor components, $\Psi_n(r,u,\zeta,\zeta)$

Peeling Theorem

$$\Psi_0 = 0(r^{-5}),$$

$$\underline{\Psi_1} = 0(r^{-4}) = \Psi_1{}^0(u,\zeta,\overline{\zeta})\, r^{-4} + \ldots,$$

$$\underline{\Psi_2} = 0(r^{-3}) = \Psi_2{}^0(u,\zeta,\overline{\zeta})\, r^{-3} + \ldots$$

$$\Psi_3 = 0(r^{-2}), \qquad \Psi_4 = 0(r^{-1})$$

(*real*), $\Psi_2{}^0(u,\zeta,\overline{\zeta}) = \overline{\Psi_2{}^0(u,\zeta,\overline{\zeta})}$.

c. **Asymptotic Bianchi Identities-Evolution Eqs**

★ $\partial_u \Psi_1^0(u,\zeta,\zeta) = -\eth\Psi_2^0 + 2\sigma^0\Psi_3^0 + 2k\varphi_1^0\overline{\varphi}_2^0,$

★ $\partial_u \Psi_2^0(u,\zeta,\zeta) = -\eth\Psi_3^0 + \sigma^0\Psi_4^0 + k\varphi_2^0\overline{\varphi}_2^0,$

$k = 2Gc^{-4},$

φ's = Maxwell fields, (mod angular terms)

$\sigma^0(u,\zeta,\zeta)$ = free radiation data

$D_{E\&M}$ = complex Dipole Moments = $D_E + iD_M$

$\varphi_1^0 = q + \dot{D}_{E\&M}$ ...., $\varphi_2^0 = \ddot{D}_{E\&M} + \ldots\ldots$

d. **Definitions** - Spherical Harmonic Coefficients

[known **constant coefficients omitted** here]

(1) $$\Psi_2^{0} = M + P_i\, Y^{0}_{1i}(\zeta,\bar{\zeta}) + \ldots$$

(M, P_i) = **Bondi-Sach Mass & Linear Mom.**

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(2) $$\Psi_1^{0}(u,\zeta,\bar{\zeta}) = \Psi_1^{0}{}_{1i}\, Y^{1}{}_{1i}(\zeta,\bar{\zeta}) + \ldots$$

with

Complex Dipole Moment: $\Psi_1^{0}{}_{1i} = D_i + i\, J_i$

D_i = Mass Dipole, J_i = Total Angular Momentum

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ϕ's => **Maxwell Field**, (standard definition)

with q = charge

(3) $D_{E\&M}$ = complex Dipole Moments = $D_E + iD_M$

Special => Center of mass coincides w center of charge

9. <u>Modus Operandi</u>

i. **Start in a Bondi system**.

ii. **Transform** to <u>one-parameter</u> family of **Almost Null-Cones**,

with '<u>world-line</u>', $z^a = \xi^a(\tau) = \xi^a{}_R(\tau) + i\xi^a{}_I(\tau)$

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iii. World-Line $\xi^a(\tau)$ **<u>determined</u>** by **<u>condition</u>** on 'line'

$$\Psi^*{}_1{}^0{}_{1i} = D_i + i\,J_i = 0.$$

<u>Defines</u>: <u>Complex CofMass World-Line</u> $\xi^a(\tau) = \xi^a{}_{CofM}(\tau)$.

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iv. **NOW -** Using known $\xi^a{}_{CofM}(\tau)$, **<u>transform</u> <u>to</u>**
<u>straight</u> 'world-line', *i.e.*, ***Lorentzian-like coordinates***

via a <u>one-parameter family</u> of Almost Null-Cones

$$z^a = \xi^a(\tau^*) = \tau^*\delta^a{}_0.$$

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FINALLY with Lorentzian-like coordinates

v. Express $\Psi_2{}^0$, $\Psi_1{}^0$ & B.I.
in terms of Physical Definitions & $\xi^a{}_{CofM}$.

vi. LONG complicated calculations to get here -
Now, No MORE Calculations
=> immediate RESULTS

v. Just Collect Terms => and LOOK

11. **Results: I**

From $\Psi_{10i}^{*}=0$, **center of Mass Condition**

with $\xi_{CofM}{}^{i} = \xi_R{}^{i} + i\xi_I{}^{i}$

$$\Psi_{10i} = -\,Gc^{-2}\, M_B \xi_{CofM}{}^{i} + i\, G P_B{}^{k}\, \xi_{CofM}{}^{j}\, \epsilon_{kji}$$

$$D_i + i\, J_i = M_B \xi_R{}^{i} + i\, M_B \xi_I{}^{i} + i\, P_B{}^{k}(\xi_R{}^{j} + i\xi_I{}^{j})\, \epsilon_{kji}$$

OR

Dipole and Angular momentum

$$D^i = M\xi^i{}_R - c^{-1}P^k\xi^j{}_I\, \varepsilon_{jki} + \ldots,$$

$$J^i = cM\xi^i{}_I + P^k\xi^j{}_R\,\varepsilon_{jki} + \ldots.$$

or

$\mathbf{D} = M\mathbf{r} + c^{-2}M^{-1}\mathbf{P}\times\mathbf{S}$, $\mathbf{S} = cM\xi_I$, $\mathbf{J} = \mathbf{S} + \mathbf{r}\times\mathbf{P}$.

Beautiful !!!!!?

12. **Results: II**

B.I. : $\partial_u \Psi_1{}^0(u,\zeta,\zeta) = -\eth\Psi_2{}^0 + 2\sigma^0\Psi_3{}^0 + 2k\varphi_1{}^0\varphi_2{}^0$

$$M\dot{\xi}^i_R = P^i + \text{H.O.} + q\ddot{\xi}^i_R$$

or

$$P^i = M\dot{\xi}^i_R - 2q^2/(3c^3)\ddot{\xi}^i_R$$

Real Part: **Kinematic Momentum & Rad.Reaction** Term

Imaginary Part: **Conservation of Ang. Momentum**

$\dot{J}^i$ = **Landau-Lifschitz** terms + **spin-loss**(new)

PERFECT - EXACT

Numerical Factors Work out Correctly

VErY VErY Beautiful !!!!!

13. **2nd B.I.**

$$\partial_u \Psi_2^0(u,\zeta,\zeta) = -\eth\Psi_3^0 + \sigma^0\Psi_4^0 + k\varphi_2^0\varphi_2^0,$$

$\ell = 0$ terms

$\dot{M}$ = **standard Quadrupole+spinloss**(new)

+ E&M dipole and quadrupole loss

Exact, Fabulous !!!!.

$\ell = 1$ terms

$$M\ddot{\xi}^i_R = \dot{M}\dot{\xi}^i_R + 2q/3c^3\dddot{\xi}^i_R + F_{recoil}{}^i$$

Rocket Force and Radiation Reaction

A very pleasant surprise

14Comments & Extras !!!!

a. **Dirac value; gyromagnetic ratio**, i.e., **g=2** comes from centers of mass and charge equality.

b. **Relativist angular moment tensor** there.

c. **Note**: There is **No** Space-Time in the analysis,
- Just the space of shear-free congruences,

A Major Enigma ????

d. **Imaginary Part of C**of**Mass coordinate** z^a is **physical spin**. !!!!!!????

15 e. **All relations** found **either agree** with standard theory - or - {if any of any of this is physically sensible} - **are actually new predictions**.

f. This is sitting in GR- what does it say- if anything - about Quantum Gravity????

g. Where is the BMS group? It is there. The Space of the Almost Light-Cones transforms under the BMS group - with an unusual representation of the Lorentz subgroup.

16.

h. The Radiation Reaction Force is There -
Including, presumably,
the well-known instabilities - runaway behavior.

Question: Is it possible that the additional terms, neglected here, Could Stabilze the Situation?

To BE Studied!!!!!!

Is all this just a strange coincidence

or is there something deeper?

I do not know. Any Ideas?

Thank you!