

# Infrared Quantum Gravity

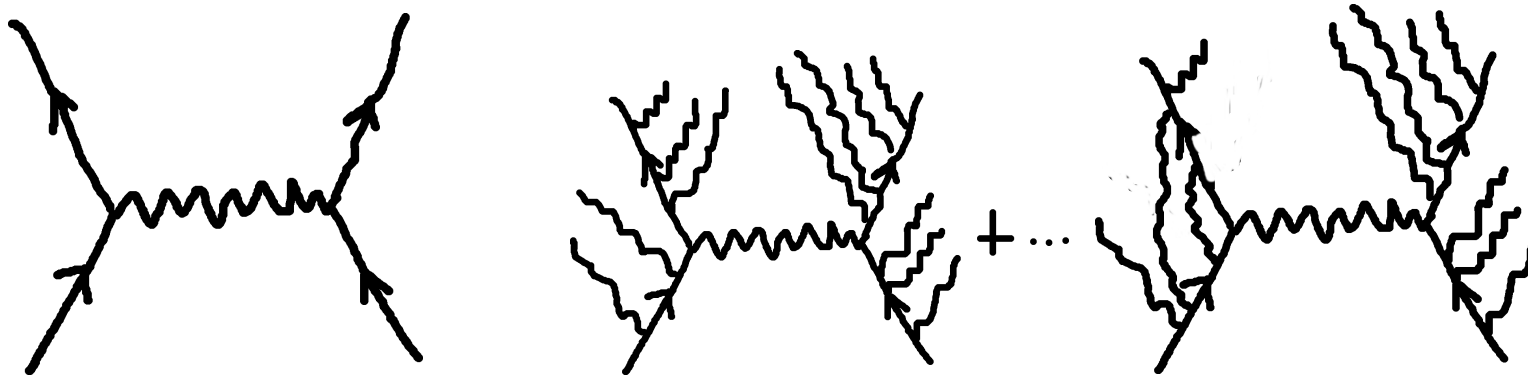
*Gordon W. Semenoff*

*University of British Columbia*

Gravity: Past, Present and Future, Vancouver, September 1-4, 2017



Central to the solution of the infrared problem in quantum electrodynamics is the fact that a scattering experiment with charged particles is accompanied by the production of an infinite number of arbitrarily soft photons ( $k^\mu = (|\vec{k}|, \vec{k})$ ,  $k \sim 0$ )



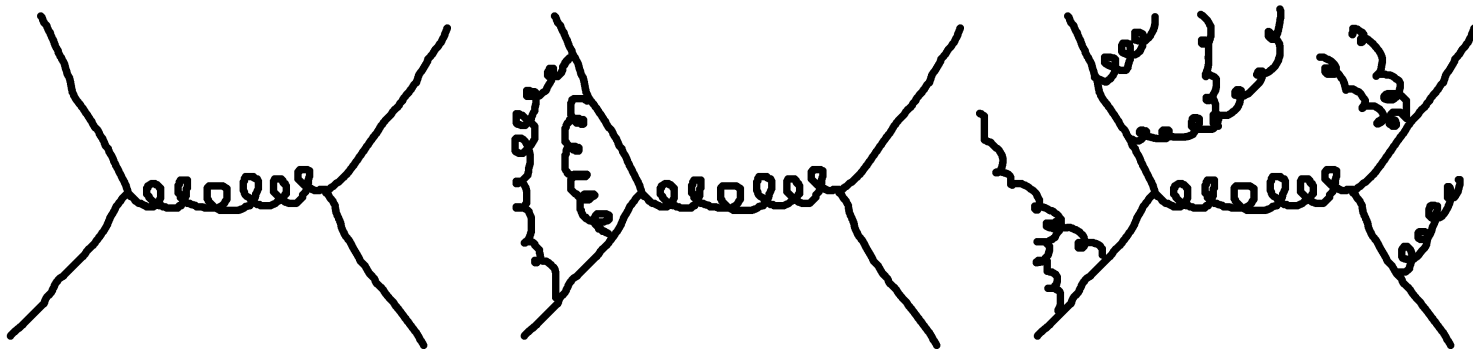
**F. Bloch and A. Nordsieck, Phys. Rev. 52, 54 (1937)**

**D. R. Yennie, S. C. Frautschi, H. Suura, Ann. Phys. 13, 379 (1961)**

**soft photon theorems**

The same phenomenon occurs in perturbative quantum gravity where *soft gravitons* are produced.

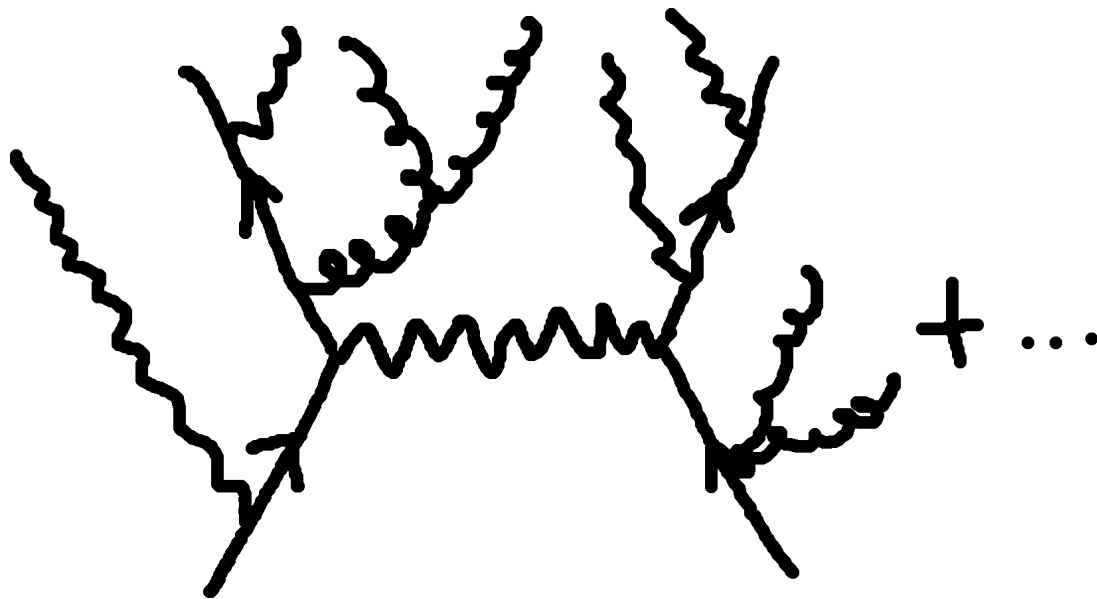
$$k^\mu = (|\vec{k}|, \vec{k}) , k \sim 0$$



**soft graviton theorem**

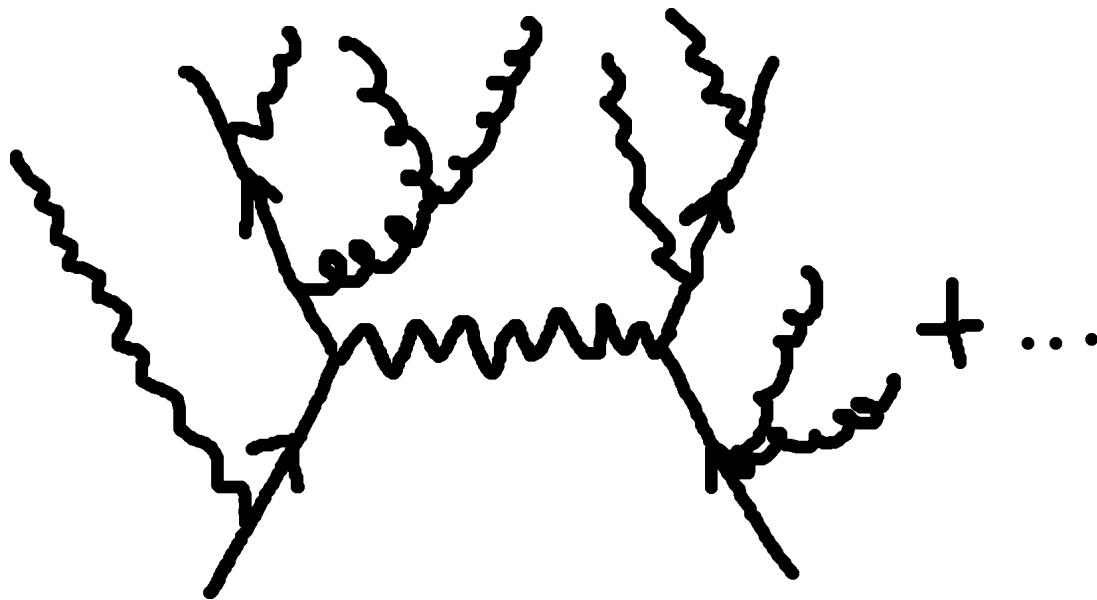
**S. Weinberg, Phys. Rev. 140, B516 (1965)**

The *soft photons* and *soft gravitons* which escape detection have polarizations and directions of propagation.



The *soft photons* and *soft gravitons* which escape detection have polarizations, directions of propagation.

**How much information do they carry away?**



**D.Carney, L.Chaurette, D.Neuenfeld, GWS,  
arXiv:1706.03782**

**G.Grignani and GWS, Phys. Lett. B 772 (2017) 699**

## Information loss due to entanglement:

Composite system of two qubits:  $|\psi\rangle_1 \otimes |\psi\rangle_2$

If subsystem  $|\psi\rangle_2$  becomes inaccessible to us, how much information about  $|\psi\rangle_1$  do we lose?

*Unentangled state*  $|\psi\rangle = |\uparrow\rangle_1 \otimes |\downarrow\rangle_2$

*Entangled state*  $|\psi\rangle = \frac{1}{\sqrt{2}} |\uparrow\rangle_1 \otimes |\downarrow\rangle_2 + \frac{1}{\sqrt{2}} |\downarrow\rangle_1 \otimes |\uparrow\rangle_2$

Reduced density matrix  $\rho = \text{Tr}_2 |\psi\rangle\langle\psi|$

*Unentangled state*  $\rightarrow \rho = |\uparrow\rangle_1\langle\uparrow| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

*Entangled state*  $\rightarrow \rho = \frac{1}{2} \{ |\uparrow\rangle_1\langle\uparrow| + |\downarrow\rangle_1\langle\downarrow| \} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$

Entanglement entropy  $S = -\text{Tr} \rho \ln \rho$

*Unentangled state*  $S = 0$       *Entangled state*  $S = 2 \ln 2$

We want to estimate the entanglement of the “hard” particles and the “soft” radiation composed of very long wavelength photons and gravitons in a generic scattering experiment.

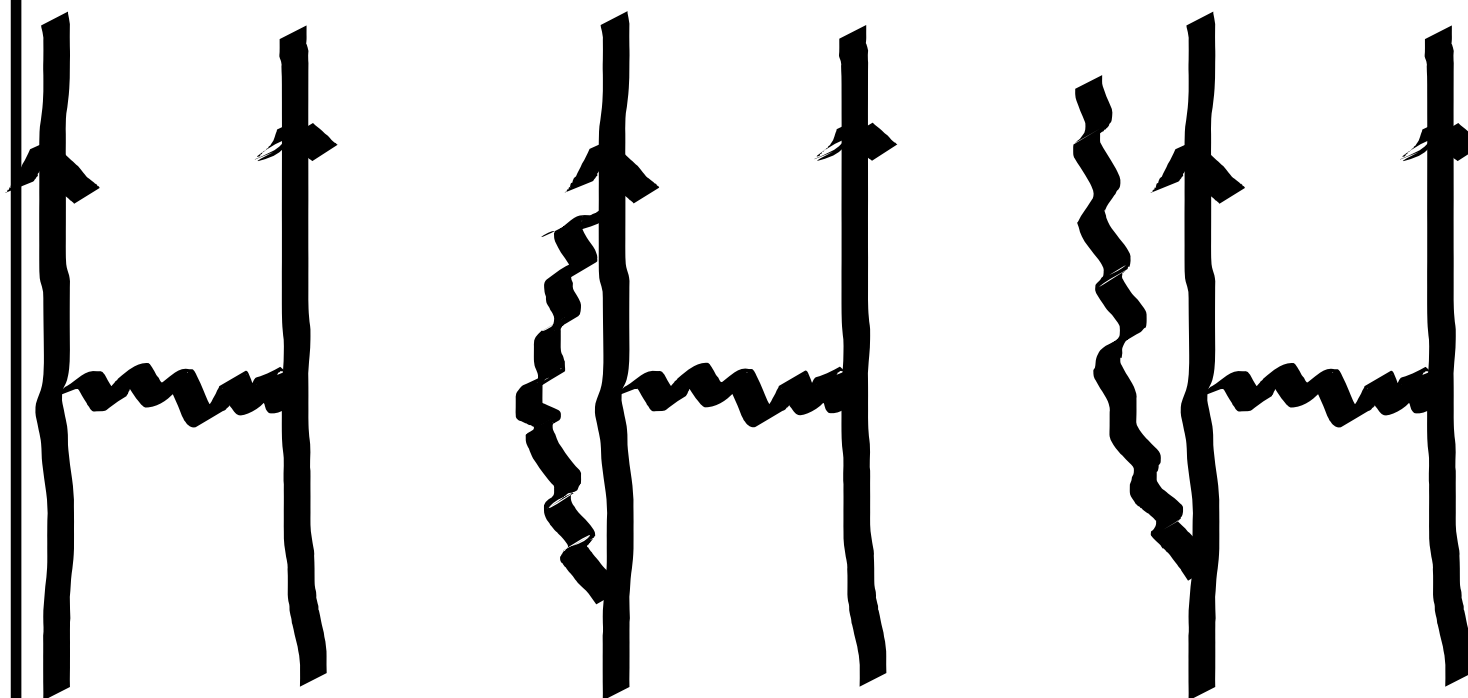
We shall find that they are highly entangled.

The result is decoherence of the final state.

**D.Carney, L.Chaurette, D.Neuenfeld, GWS,  
arXiv:1706.03782**



In **quantum electrodynamics**, the computation of the  $S$ -matrix for the scattering of charged particles encounters infrared divergences



Inclusive probability

$$\left| \text{tree} + \text{1-loop} + \dots \right|^2 +$$

$$\sum_{\text{soft}} \left| \text{tree} + \text{1-loop} + \dots \right|^2$$

= finite

Cancellation of infrared divergences is guaranteed by unitarity of infrared cutoff S-matrix.

## Outgoing density matrix

Scattering event

$$|\alpha\rangle \rightarrow S^\dagger |\alpha\rangle = \sum_{\beta\gamma} |\beta\gamma\rangle \langle \beta\gamma | S^\dagger | \alpha \rangle$$

The states  $|\beta\gamma\rangle_{\text{in}}$  contain both hard ( $\beta$ ) and soft ( $\gamma$ ) particles.

Form the out-going density matrix

$$\rho = \text{Tr}_{\text{soft}} S^\dagger |\alpha\rangle \langle \alpha | S$$

$$\rho_{\beta\beta'} =$$

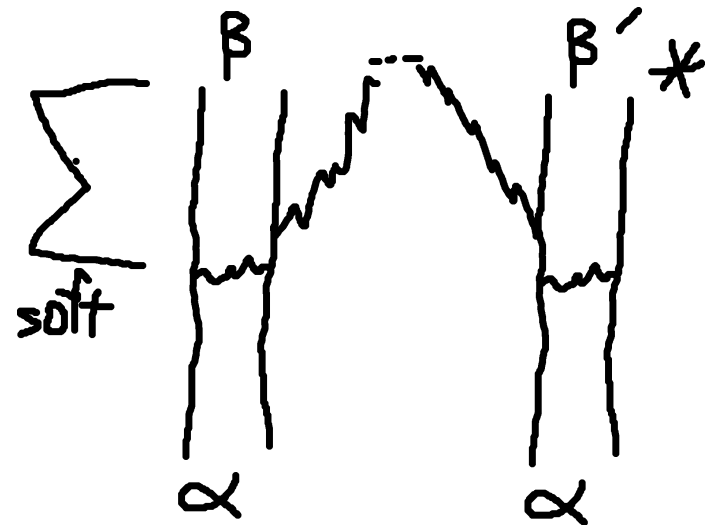
$$= \sum_{\hat{\gamma}} \langle \hat{\gamma} | \left[ \sum_{\beta\gamma} |\beta\gamma\rangle \langle \beta\gamma | S^\dagger | \alpha \rangle \sum_{\beta'\gamma'} \langle \alpha | S | \beta'\gamma' \rangle \langle \beta'\gamma' | \right] | \hat{\gamma} \rangle$$

Diagonal elements are probabilities of outcomes of scattering experiments.

Off-diagonal elements are correlations between outcomes.

**Entanglement entropy**  $S = -\text{Tr } \rho \ln \rho \equiv -\sum_i \rho_i \ln \rho_i$

$$\rho = \left( \sum_{\beta} |\beta\rangle S_{\beta\alpha}^\dagger \right) \left( \sum_{\beta'} S_{\alpha\beta'} \langle\beta'| \right) +$$



Eigenvalues  $\left| \sum_{\beta} |\beta\rangle S_{\beta\alpha}^\dagger \right|^2$  and 0 perturbed by  $\sim \left[ \frac{e^2}{4\pi} \right]^3 \ln \frac{\Lambda}{m_{\text{ph}}}$

$\rho_i$  logarithmically infrared divergent at order  $e^6$

## The entanglement entropy is infrared divergent

$$S \sim - \left[ \frac{e^2}{4\pi} \right]^3 \ln \frac{\Lambda}{m_{\text{ph}}} * \ln \left[ \left[ \frac{e^2}{4\pi} \right]^3 \ln \frac{\Lambda}{m_{\text{ph}}} \right] \\ - \left[ \frac{e^2}{4\pi} \right]^2 \left[ \frac{\tilde{\Lambda}^2}{M_{\text{Pl}}^2} \right] \ln \frac{\tilde{\Lambda}}{m_{\text{grav}}} * \ln \left[ \left[ \frac{e^2}{4\pi} \right]^2 \left[ \frac{\tilde{\Lambda}^2}{M_{\text{Pl}}^2} \right] \ln \frac{\tilde{\Lambda}}{m_{\text{grav}}} \right]$$

**Big logarithm = “amplification” of small quantum corrections!!**

**Observable quantum gravity???**

But, such infrared effects are non-perturbative.

We must do better.

Soft photon theorems.

Soft graviton theorems.

**The density matrix:** (D.Carney, L.Chaurette, D.Neuenfeld, GWS, arXiv:1706.03782) We can use soft photon and soft graviton theorems to show

$$\rho_{\beta\beta'} = S_{\beta\alpha}^{\Lambda_1\tilde{\Lambda}_1} S_{\beta'\alpha}^{\Lambda_1\tilde{\Lambda}_1*} \mathcal{F} \left[ \Lambda_i, \tilde{\Lambda}_i, E_T, m_{\text{ph}}, m_{\text{grav}} \right]$$

where

$$\begin{aligned} \mathcal{F} = & \left( \frac{\Lambda_2}{m_{\text{ph}}} \right)^{\tilde{A}_\alpha^{\beta\beta'}} \left( \frac{m_{\text{ph}}}{\Lambda_1} \right)^{\frac{A_\alpha^\beta + A_\alpha^{\beta'}}{2}} \left( \frac{\tilde{\Lambda}_2}{m_{\text{grav}}} \right)^{\tilde{B}_\alpha^{\beta\beta'}} \left( \frac{m_{\text{grav}}}{\tilde{\Lambda}_1} \right)^{\frac{B_\alpha^\beta + B_\alpha^{\beta'}}{2}} \times \\ & \times f\left(\frac{\Lambda_2}{E_T}, \tilde{A}_\alpha^{\beta\beta'}\right) f\left(\frac{\tilde{\Lambda}_2}{E_T}, \tilde{B}_\alpha^{\beta\beta'}\right) \\ & \sim m_{\text{ph}}^{\Delta A} m_{\text{grav}}^{\Delta B}, \quad \Delta A, \Delta B \geq 0 \end{aligned}$$

**RESULT:**  $\rho_{\beta\beta'} \neq 0$  as  $m_{\text{ph}} \rightarrow 0$  and  $m_{\text{grav}} \rightarrow 0$  only if  $\Delta A = 0$  and  $\Delta B = 0$

density matrix element  $\sim m_{\text{ph}}^{\Delta A} m_{\text{grav}}^{\Delta B}$

inequalities saturated and density matrix element nonzero only when

i)  $\Delta A = 0$  if the set of out-going single particle currents

$$\left\{ e_1 \frac{p_{1\mu}}{2\omega_{p_1}}, e_2 \frac{p_{2\mu}}{2\omega_{p_2}}, \dots \right\} \text{ equals } \left\{ e'_1 \frac{p'_{1\mu}}{2\omega_{p'_1}}, e'_2 \frac{p'_{2\mu'}}{2\omega_{p'_2}}, \dots \right\}$$

ii)  $\Delta B = 0$  if the outgoing single particle stress-energies

$$\left\{ \frac{p_{1\mu} p_{1\nu}}{2\omega_{p_1}}, \frac{p_{2\mu} p_{2\nu}}{2\omega_{p_2}}, \dots \right\} \text{ equals } \left\{ \frac{p'_{1\mu} p'_{1\nu}}{2\omega_{p'_1}}, \frac{p'_{2\mu} p'_{2\nu}}{2\omega_{p'_2}}, \dots \right\}$$

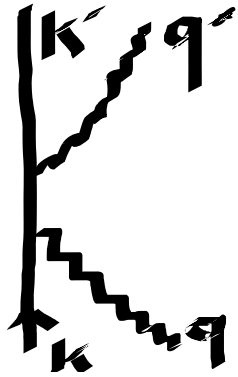
---

**A matrix element of the density matrix  $\rho_{\beta\beta'}$  is nonzero if the distributions of charge currents and stress-energy currents in the states  $|\beta\rangle$  and  $|\beta'\rangle$  are identical.**

---

**Decoherence!**

## Example: Compton scattering in QED



$$\rho_{k'_1, q'_1; k'_2, q'_2} \sim (m_{\text{ph}})^{\frac{e^2}{4\pi^2} \left[ \frac{1}{2\beta} \ln \frac{1+\beta}{1-\beta} - 1 \right]}, \quad \beta = \text{relative electron velocity}$$

Exponent  $\geq 0$ . Exponent = 0 *only when*  $\beta = 0$

As  $m_{\text{ph.}} \rightarrow 0$ ,  $\rho_{k', q'; k', q'} \neq 0$  but  $\rho_{k'_1, q'_1; k'_2 \neq k'_1, q'_2} \rightarrow 0$ .

### Implication:

Probability of  $|k, q\rangle \rightarrow |k' q'\rangle$  as in QFT textbooks

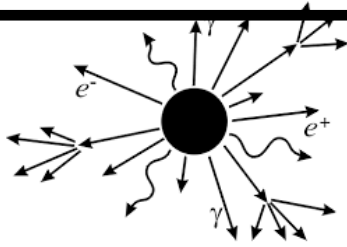
But  $|kq\rangle \rightarrow \frac{1}{\sqrt{2}} \{ |k'_1 q'_1\rangle + |k'_2 q'_2\rangle \}$  is  $\frac{1}{2} (P_{kq \rightarrow k'_1 q'_1} + P_{kq \rightarrow k'_2 q'_2})$

### Decoherence of out-state!



## Conclusions

- The solution of the infrared problem in quantum electrodynamics and in perturbative quantum gravity leads to a fundamental decoherence of final states in scattering experiments.
- Could such a decoherence coming from quantum gravity be observable?
- Applications to Hawking evaporation of black holes with “soft hair”.
- What if the photon/graviton has a small mass?



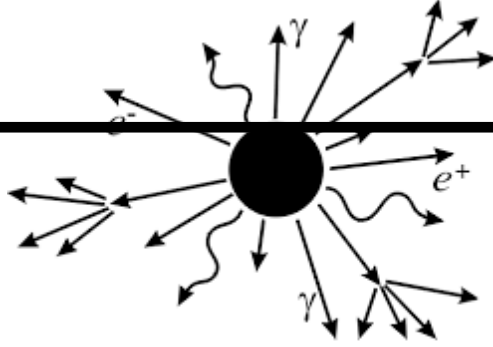
## Information paradox

In a theory of quantum gravity, the collision of two high-energy particles (i.e. gravitons) could produce a black hole which would then evaporate by emitting Hawking radiation.

Pure quantum state of two incoming particles evolves to Hawking radiation.

$$|\alpha\rangle \rightarrow \sum_E e^{-\beta_H E} |E\rangle \otimes |\tilde{E}\rangle, \quad \rho = \sum_E e^{-\beta_H E} |E\rangle\langle E|$$

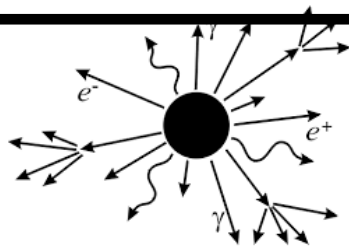
In a unitary quantum mechanical theory, a pure state should not evolve to a mixed state.



**Strominger's idea** ([A.Strominger, arXiv:1706.07143](#)):  
soft gravitons purify the Hawking radiation

$$|\Psi\rangle = \sum_E |E, \text{soft}\rangle$$

$$\rho = \text{Tr}_{\text{soft}} |\Psi\rangle\langle\Psi| = \sum_E e^{-\beta_H E} |E\rangle\langle E|$$



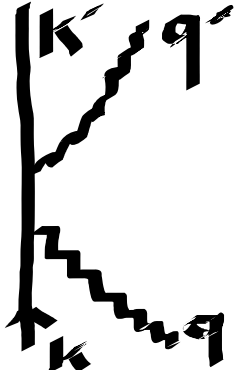
## Information paradox

Pure quantum state of two incoming particles evolves to Hawking radiation which subsequently emits soft bremsstrahlung.

$$|\alpha\rangle \rightarrow \sum_E e^{-\beta_H E} |E\rangle \otimes |\tilde{E}\rangle \rightarrow \sum_{\beta\gamma E} e^{-\beta E} S_{\beta\gamma E}^\dagger |\beta\rangle \otimes |\gamma\rangle \otimes |\tilde{E}\rangle$$

**Monogamy of entanglement:** since  $|\beta\rangle$  and  $|\gamma\rangle$  are highly entangled,  $|\beta\rangle$  and  $|\tilde{E}\rangle$  can no longer be entangled. Information is still lost, but the loss is due to the soft gravitons and photons which escape detection.

## What if the photon has a mass?



$$\rho_{k'_1, q'_1; k'_2, q'_2} \sim (m_{\text{ph.}}) \frac{e^2}{4\pi^2} \left[ \frac{1}{2\beta} \ln \frac{1+\beta}{1-\beta} - 1 \right]$$

$$\sim (m_{\text{ph.}}) \frac{e^2}{8\pi^2} \beta^2 \quad (\beta \ll 1) \quad , \quad \sim (m_{\text{ph.}}) \frac{e^2}{8\pi^2} \ln \frac{2}{1-\beta} \quad (\beta \rightarrow 1)$$

Experimental bound on photon mass  $m_{\text{ph}} < 10^{-32} m_{\text{el.}}$

$$\sim e^{-0.04\beta^2} \quad (\beta \ll 1) \quad , \quad \sim \left( \frac{1-\beta}{2} \right)^{0.04} \quad (\beta \rightarrow 1)$$

Gravity is even more weakly coupled.

## Soft photon theorem (oversimplified)



$$1 - \left( \frac{e^2}{4\pi} \gamma(p_i) \ln \frac{\Lambda}{m_{\text{ph.}}} \right) + \frac{1}{2!} \left( \frac{e^2}{4\pi} \gamma(p_i) \ln \frac{\Lambda}{m_{\text{ph.}}} \right)^2 + \dots = \left( \frac{m_{\text{ph.}}}{\Lambda} \right)^{\frac{e^2}{4\pi} \gamma(p_i)}$$

$$m_{\text{ph.}} (\rightarrow 0) \ll \Lambda \ll p_i, m_{\text{el}}$$

\*Amplitude for process which does not emit photons vanishes.

\*Amplitude for process which emits a finite number of photons also vanishes.

\*Same for gravitons