The Motion of Small Bodies in Space-Time

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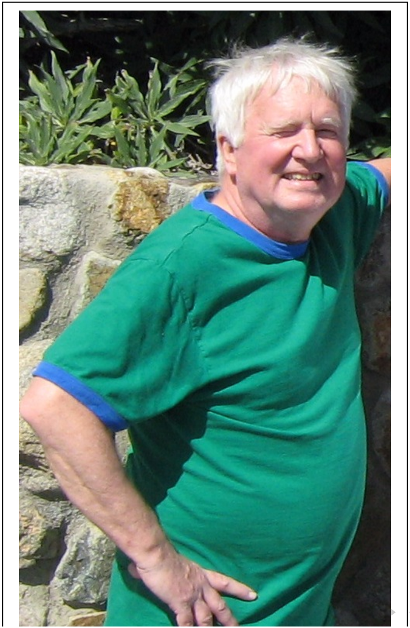

The Problem of Motion

Question

How do small bodies move in general relativity?

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Geodesic Principle

Free massive point particles traverse timelike geodesics. Light rays traverse null geodesics.

Talk Overview

1. Two Approaches, and Their Discontents

2. The Miracle of Tracking

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Distributions

As first observed by Mathisson, and developed by Souriau, Sternberg, Guillemin, and others, there is a very short argument for geodesic motion using **distributions**.

Distributions

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It follows that $\mathbf{T}^{ab} = m\delta_\gamma u^a u^b$, where m is a number, δ_γ is the delta distribution supported on γ, and u^a is the unit tangent to γ; **and γ is a geodesic.**

Advantages of Distributions

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For instance, a body with timelike worldline γ, subject to an arbitrary force $\mathbf{f}^a = \nabla_b \mathbf{T}^{ab}$ compatible with $\mathbf{T}^{ab}$ order zero, can be described by an energy-momentum $\mathbf{T}^{ab} = \mu u^a u^b$ satisfying

$$\boldsymbol{\mu} u^n \nabla_n u^a = q^a{}_b \mathbf{f}^b$$
$$\nabla_b(\boldsymbol{\mu} u^b) = -\mathbf{f}^b u_b$$

where $\boldsymbol{\mu}$ is an order zero distribution supported on γ.

Advantages of Distributions

Likewise, fix a background electromagnetic field F_{ab}. Represent a charged body by an energy-momentum distribution $\mathbf{T}^{ab}$ supported on timelike γ and an (order zero) charge current density $\mathbf{J}^a$ supported on γ. Assume $\nabla_a J^a = 0$ and $\mathbf{f}^a = F^a{}_b \mathbf{J}^b$.

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Then $\mathbf{J}^a = e\delta_\gamma u^a$, $\mathbf{T}^{ab} = m\delta_\gamma u^a u^b$, and γ is a e/m Lorentz force curve.

One can solve for the general case, where $\mathbf{J}^a$ is order one (the highest order compatible with $\mathbf{T}^{ab}$ order zero); one finds contributions to the motion arising from electric and magnetic dipoles.

Disadvantages of Distributions

But the situation concerning distributions is not entirely satisfactory.

Why order zero?

Concern 1: The assumption that $\mathbf{T}^{ab}$ is order zero is physically obscure.

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In fact, it can be justified by the following argument.

Why order zero?

We will say that a smooth test field $\texttt{T}_{ab}$ satisfies the **dual energy condition** at a point p if $\texttt{T}_{ab}$ can be written as a sum of symmetrized outer products of pairs of co-oriented causal covectors. A symmetric distribution $\mathbf{T}^{ab}$ satisfies the **dominant energy condition** if, for every test field $\texttt{T}_{ab}$ satisfying the dual energy condition, $\mathbf{T}\{\texttt{T}\} \geq 0$.

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Proposition

Let $\mathbf{T}^{ab}$ *be a symmetric distribution satisfying the dominant energy condition. Then* $\mathbf{T}^{ab}$ *is order zero.*

Relation to “realistic” matter?

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Thus one cannot generally have a distributional energy-momentum associated (even) with distributional fields supported on a curve. **This is because multiplication of distributions is ambiguous.**

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Geroch & Traschen showed that there are no metrics satisfying certain weak conditions compatible with distributional sources supported on a curve.

Thus it is difficult to evaluate, for instance, backreaction arising from a distributional $\mathbf{T}^{ab}$.

TL;DR

Question

In what sense do distributional $\mathbf{T}^{ab}$ *represent realistic matter?*

Curve-First

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Theorem (Geroch-Jang)

Let γ be a smooth, timelike curve in a spacetime (M, g_{ab}). Suppose that, in any neighborhood O of γ, there exists a smooth, symmetric, divergence-free, and non-vanishing tensor field T^{ab} satisfying the dominant energy condition whose support lies in O. Then γ is a geodesic.

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Moreover, smooth T^{ab} fields may be sources in Einstein's equation, and so this method may be adapted to consider backreaction.

Advantages of Curve-First

Theorem (Ehlers-Geroch)

Let γ be a smooth, timelike curve in a spacetime (M, g_{ab}). Suppose that, for any (closed) neighborhood O of γ, and any $C^1[O]$ neighborhood $\hat{O}$ of g_{ab}, there exists a Lorentzian metric $\hat{g} \in \hat{O}$ whose Einstein tensor is non-vanishing, which satisfies the (g) dominant energy condition, and whose support lies in O. Then γ is a geodesic.

Disadvantages of Curve-First

But again, the situation is not totally satisfactory.

How to Generalize?

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What further constraints are needed?

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But for hyperbolic systems, **this is not generally possible.**

Embarrassment: The geodesic principle theorems do not establish that Maxwell fields follow null geodesics, even in the optical limit!

Talk Overview

Goal

Question

Can we combine the distributional and curve-first approaches in a way that allows us to extend both?

Tracking

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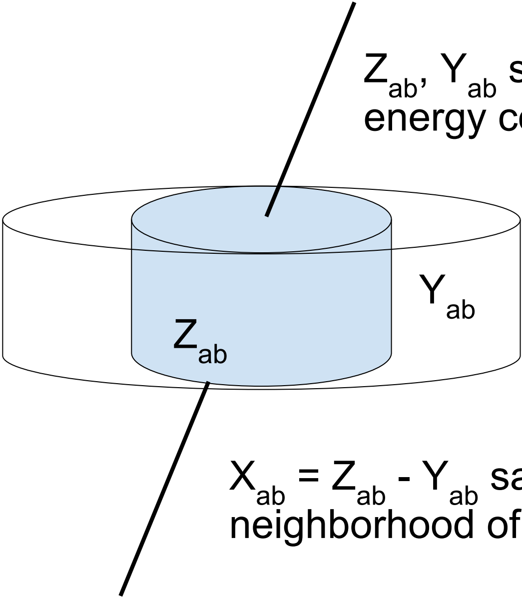
Let us suppose that we are given, in a spacetime (M, g_{ab}), a collection $\mathcal{C}$ of smooth, symmetric fields T^{ab}, each satisfying the dominant energy condition.

Each of these fields defines a distribution, whose action on test fields X_{ab} is $T\{\mathrm{X}\} = \int_M T^{ab}\mathrm{X}_{ab}$.

We will say that this collection **tracks** a timelike curve γ if, for every smooth test field X_{ab} satisfying the dual energy condition in a neighborhood of γ and "generic" at some point of γ, there is a field T^{ab} in $\mathcal{C}$ such that $T\{\mathrm{X}\} > 0$.

Z_{ab}, Y_{ab} satisfy the dual energy condition.



$X_{ab} = Z_{ab} - Y_{ab}$ satisfies it in a neighborhood of the curve.

X_{ab} measures the degree to which matter is concentrated near the curve.

Tracking

Theorem

Let (M, g_{ab}) be a spacetime, γ a timelike curve therein, and $\mathcal{C}$ a collection of symmetric fields T^{ab}, each satisfying the dominant energy condition, that tracks γ. Suppose each of these fields is conserved. Then there exists a sequence of fields $T_1^{ab}, T_2^{ab}, \ldots$, each a positive multiple of some element of $\mathcal{C}$, that converges, in the sense of distributions, to $\delta_\gamma u^a u^b$.

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Corollary

The curve γ is a geodesic.

Tracking

With a small modification, this holds for null curves as well.

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Conversely, if a collection of conserved T^{ab} fields satisfying the dominant energy condition tracks a curve γ, then γ is timelike or null.

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In other words, **every** sequence of smooth, symmetric, divergence-free fields, satisfying the dominant energy condition, whose support approaches a timelike curve γ, converges, up to rescaling, to a multiple of the δ distribution on γ.

This captures the sense in which the distribution $\delta_\gamma u^a u^b$ represents the energy-momentum of realistic (extended) matter: it is the essentially unique accumulation point for energy-momentum tensors of small bodies.

Tracking

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Proposition

If $\mathcal{C}$ contains, for every neighborhood O of a curve γ, a smooth, symmetric, non-vanishing, divergence-free field T^{ab} that satisfies the dominant energy condition and vanishes outside of O, then $\mathcal{C}$ tracks γ.

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One can recover the Ehlers-Geroch theorem in a similar manner.

Tracking

This new approach allows us to extend the curve-first approach in two important ways.

Electromagnetism

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In particular, we need to exert enough control on the collection $\mathcal{C}$ to specify a limit up to overall scaling.

For instance, in the case of a charged body, we must control the electric and magnetic dipole moments.

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A number $\kappa > 0$ bounds the charge-to-mass ratio of the elements of $\mathcal{C}$ if, for any unit timelike vector t^a at a point, and any pair $(T^{ab}, J^a) \in \mathcal{C}$,

$$|J^a t_a| \leq \kappa T^{ab} t_a t_b.$$

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Theorem

Let (M, g_{ab}) be a space-time, F_{ab} an antisymmetric tensor field on M, and γ a timelike curve. Let $\mathcal{C}$ be a collection of pairs, (T^{ab}, J^a), of tensor fields on M, where each T^{ab} satisfies the dominant energy condition, each J^a satisfies $\nabla_a J^a = 0$, and each pair satisfies $\nabla_b T^{ab} = F^a{}_b J^b$. Suppose the collection has charge-mass ratio bounded by $\kappa \geq 0$ and that it tracks γ. Then there exists a sequence of pairs, $(\overset{n}{T}{}^{ab}, \overset{n}{J}{}^a)$, each a multiple of some element of $\mathcal{C}$, that converges to $(u^a u^b \boldsymbol{\delta}_\gamma, \kappa' u^a \boldsymbol{\delta}_\gamma)$, for some number κ' satisfying $|\kappa'| \leq \kappa$.

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Corollary

γ is a Lorentz force curve with charge-to-mass ratio κ'.

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As noted, the theorems described substantially **strengthen the consequent** of curve-first results.

They also **weaken the premises**.

In particular, they permit matter to be non-vanishing far from γ, as long the quantity of such matter can be made arbitrarily small.

Hence, these results apply to solutions of hyperbolic systems, such as Maxwell and Klein-Gordon.

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It follows that $\mathcal{C}$ can track **only** timelike and null geodesics.

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Claim: $\mathcal{C}$ tracks all null geodesics; it tracks no timelike geodesics.

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This captures the sense in which light rays follow null geodesics.

Note that we do not require the electromagnetic fields themselves to converge to any distribution.

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Similar results hold for charged Klein-Gordon fields, for all $m \geq 0$ and fixed charge-to-mass ratio κ.

Inertia and Geometry

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It identifies a geometrically privileged class of curves with a privileged class of motions—hence giving physical significance to the notion of "geodesy".

Conversely, all metric geometry is encoded in the class of inertial trajectories: if two Lorentzian metrics agree on all null and timelike geodesics, up to reparameterization, then they are constant multiples of one another.

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New Geodesic Principle

Energy-momentum tensors associated with solutions to source-free matter field equations track (only) timelike or null geodesics.

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It holds for a system of field equations whenever the energy-momentum tensors associated with source-free solutions:

1. are divergence-free w.r.t. the **spacetime derviative operator**; and
2. satisfy the dominant energy condition.

The end

Thank you![1]

arXiv:1707.04222 [gr-qc]

[1] Based on work supported by the National Science Foundation.