More is the Same
Less is the Same, too;
Mean Field Theories and Renormalization

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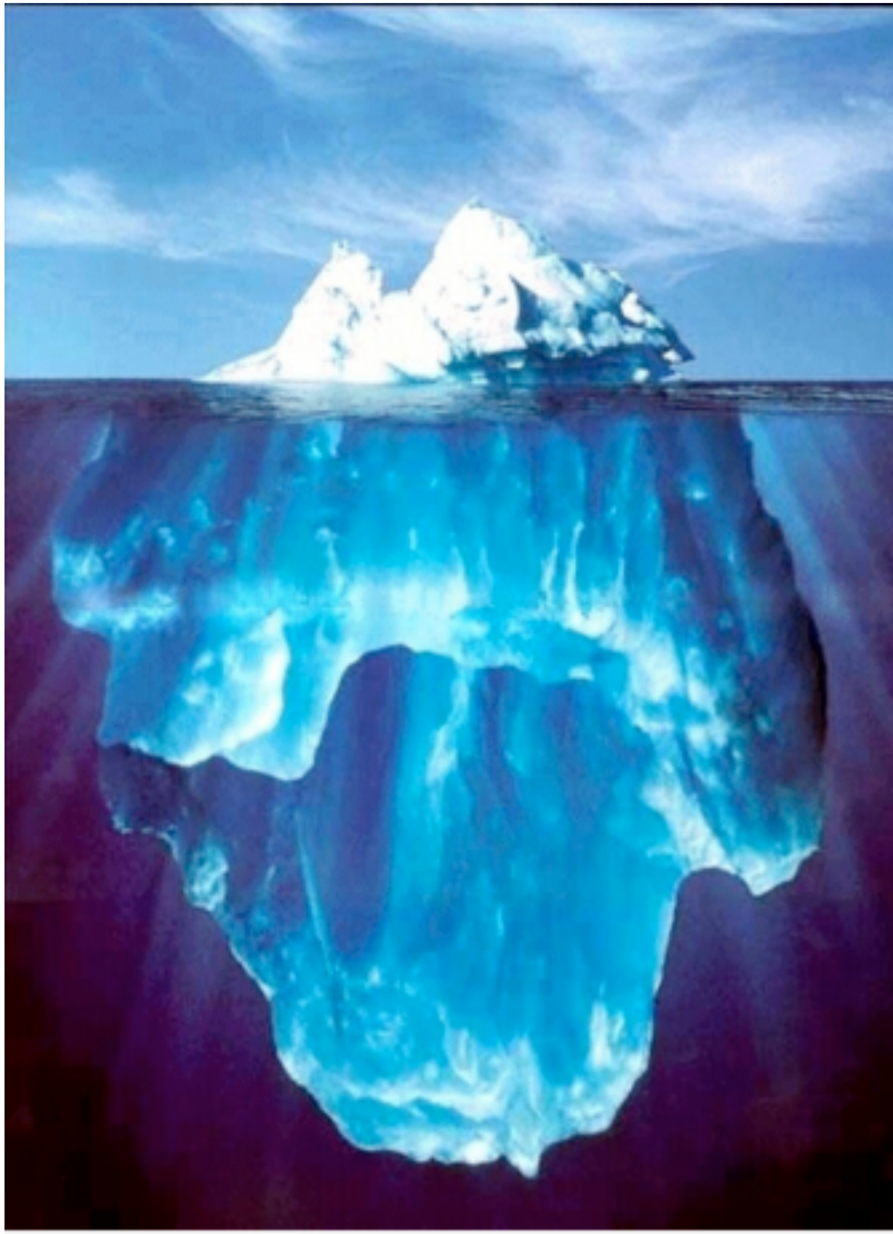
Abstract

This talk summarizes concepts derived from condensed matter physics. They key ideas go under the names of "mean field theory", "phase transitions", "universality", "variational method", and "renormalization".

Issues

Matter exists in different phases, different states of matter with qualitatively different properties: These phases are interesting in modern physics and provocative to modern philosophy. For example, no phase transition can ever occur in a finite system. Thus, in some sense phase transitions are not products of the finite world but of the human imagination.



Gibbs: A phase transition is a singularity in thermodynamic behavior. This occurs only in an infinite system

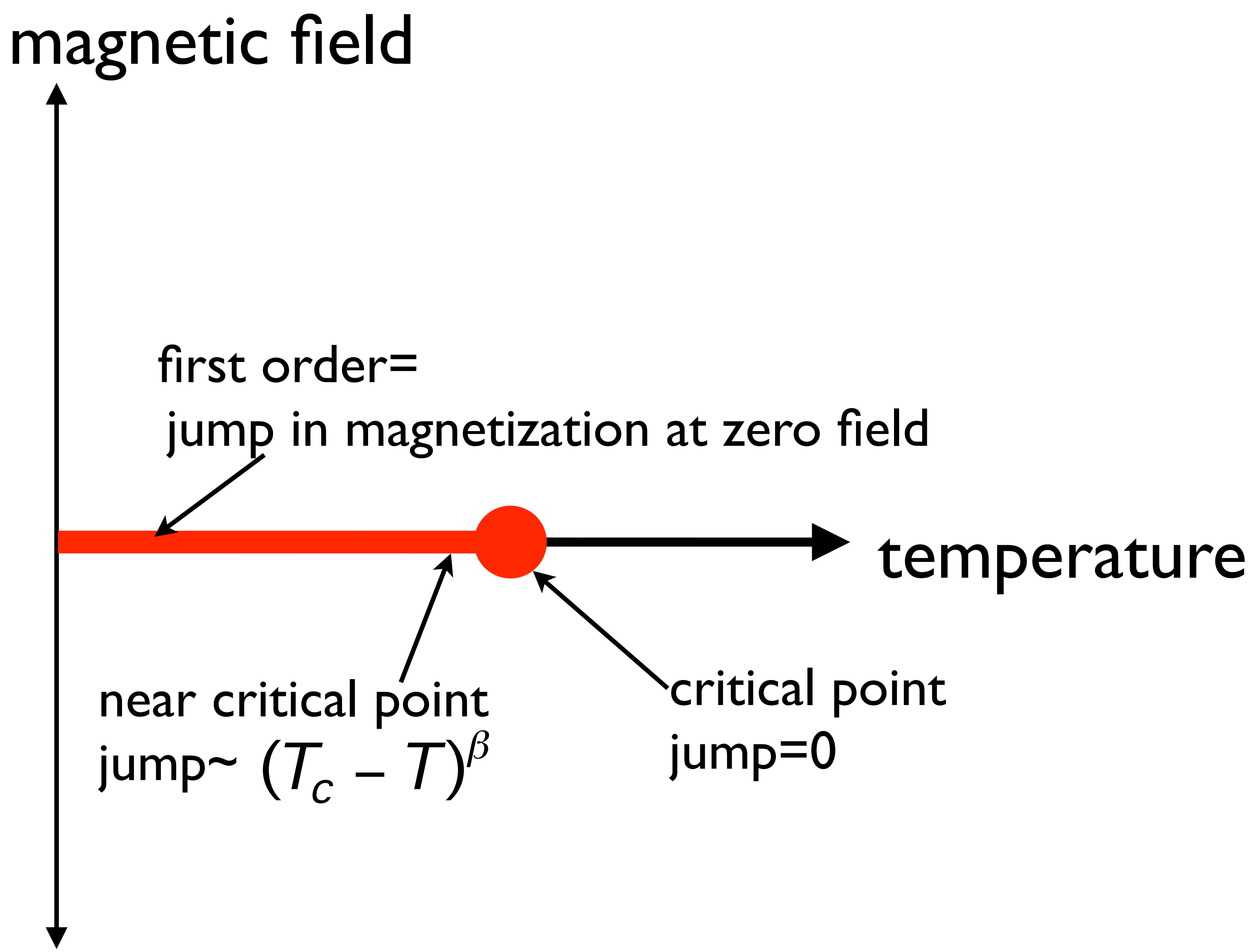
P. Ehrenfest

Ehrenfest:

- First order= discontinuous jump in thermodynamic quantities.

- Second order has continuous thermodynamic quantities, but infinity in derivative of thermodynamic quantities.

Magnetic Phase Diagram



theory starts from: phase transitions are singularities in free energy

J. Willard Gibbs:

gets to: phase transitions are a property of infinite systems

proof: .... consider Ising model for example

problem defined by

$$-H/(kT) = K\sum_{nn}\sigma_r\sigma_s + h\sum_r \sigma_r$$

free energy defined by

$$-F/(kT) = \ln \sum_{\{\sigma_r=\pm 1\}} \exp -H\{\sigma_r\}/(kT)$$

H is a smooth function of K and h.
Since a finite sum of exponentials of smooth functions is a positive smooth function, it follows that the free energy is smooth too.

Mean Field Theory: more is the same

one spin

Ising model, spin, simplified atom

$$\sigma = \pm 1$$

one spin in a magnetic field

$$H = -\sigma \mu B = -kT\sigma h$$

statistical average:

$$<\sigma>= \tanh(h)$$

many spins

spin in a magnetic field, dimension d

$$-H / kT = K \sum_{nn} \sigma_r \sigma_s + h \sum_r \sigma_r$$

focus on one spin

in average field of others

$$-H_{eff} / (kT) = \sigma_r [h_r + K \sum_s <\sigma_s>]$$

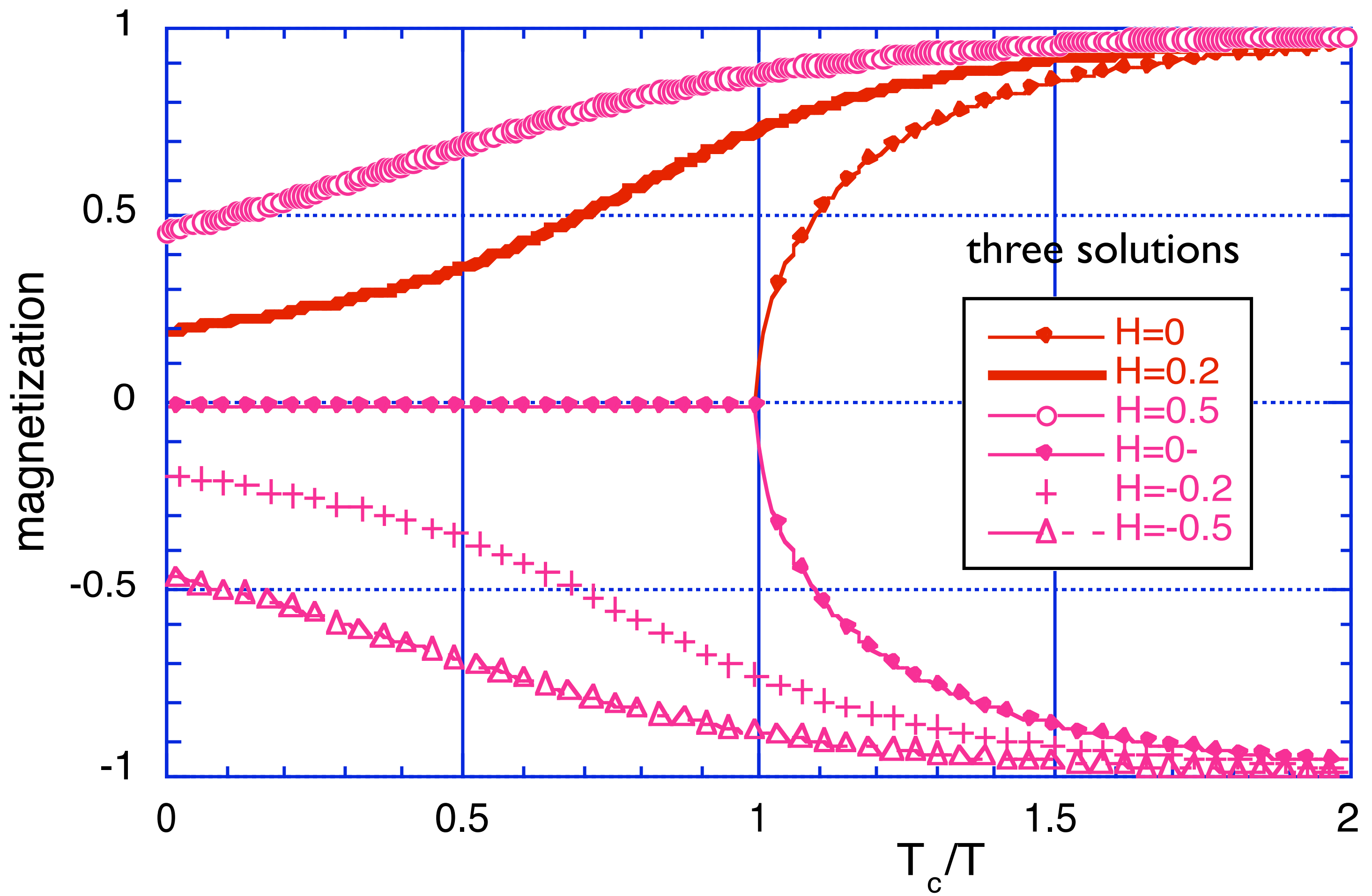
$$h_{eff} = [h + Kz <\sigma>]$$

statistical average:

$$<\sigma>= \tanh(h_{eff})$$

I took averages and assumed all spatial position were the same

order parameter in mean field transition



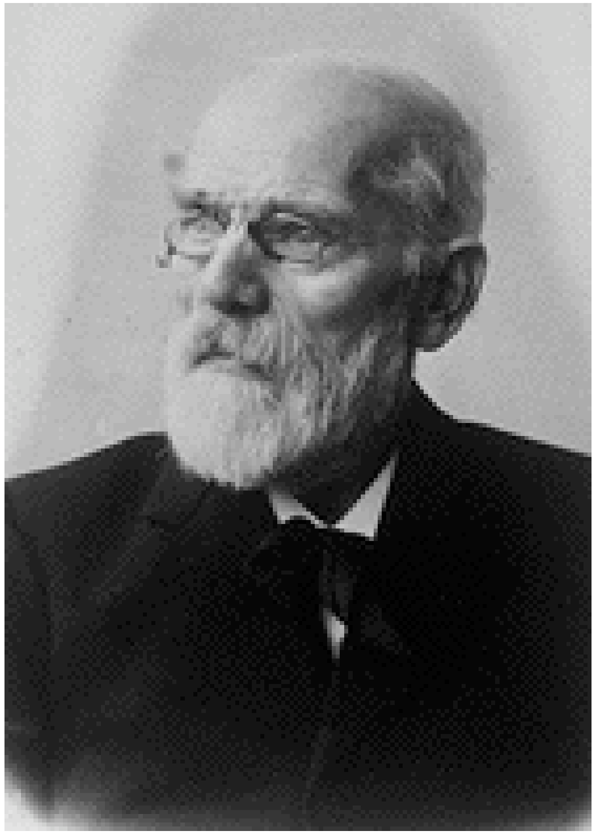
Many Different Phase Transitions:

- liquid -gas
- paramagnetic to ferromagnetic
- ...

van der Waals: (1873) Different simple liquids-gas transitions have very similar thermodynamic properties. Derives mean field theory of liquid.

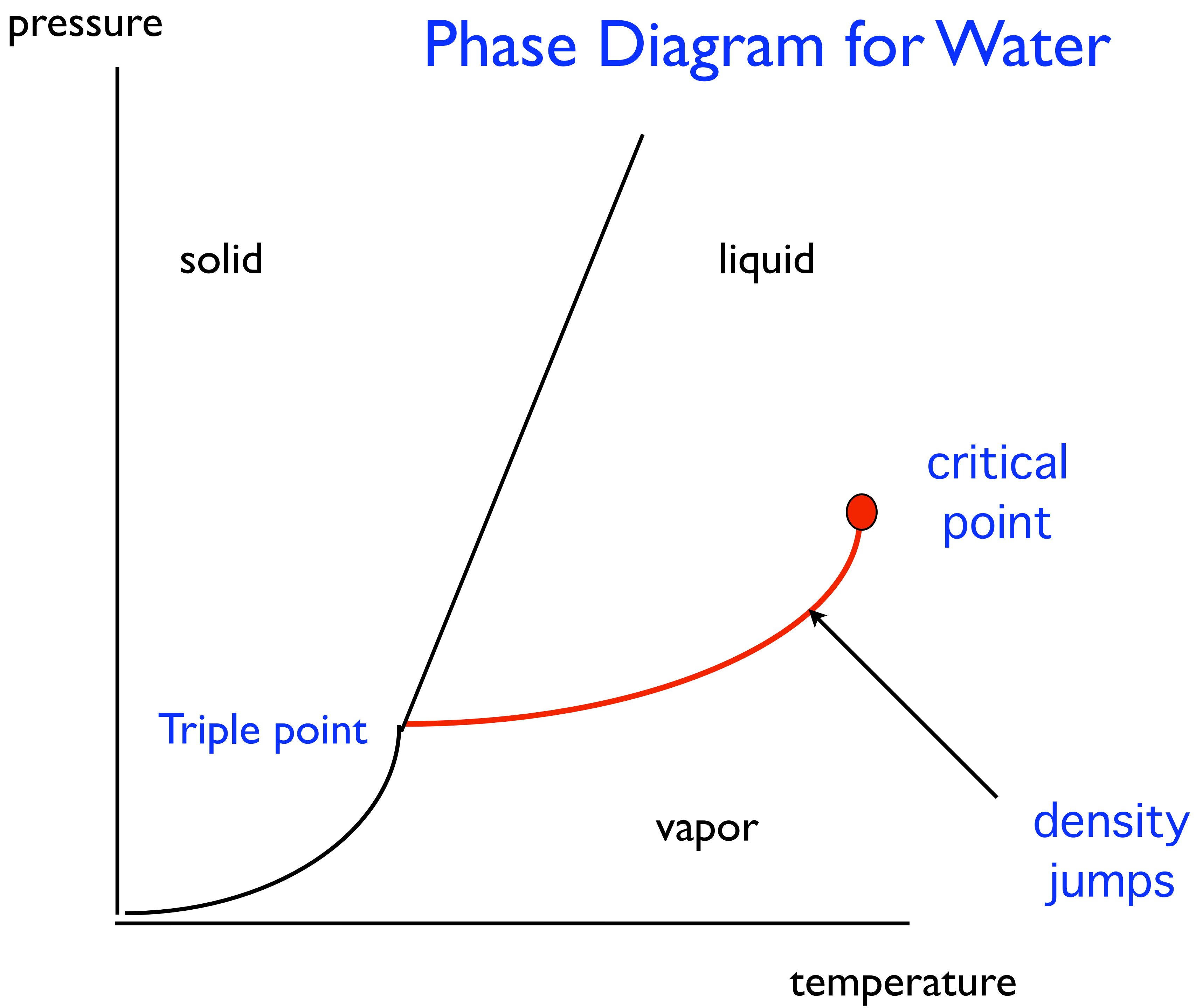
Curie-Weiss (1907) mean field theory of magnets.

But, each different phase transition calls for its own theory.



Johannes Diderik van der Waals

Phase Diagram for Water



Order Parameter, generalized

- Landau (~1937)suggested that phase transitions were manifestations of a broken symmetry, and used the order parameter to measure the extent of breaking of the symmetry.

- in ferromagnet, parameter = magnetization
- in fluid, parameter = density

L.D. Landau

Generalized Mean Field Schemes I

Many different mean-field schemes developed:. Each one has an order parameter, an average of a microscopic quantity. Landau generalized this by assuming an expansion of the free energy in an order parameter, M(r)=magnetization

$$F = \int d\mathbf{r}\ [a + hM + bM^2 + cM^4]$$

expansion assumes a small order parameter (works near critical point) and small fluctuations (works far away?!)

h is magnetic field

b is proportional to $(T\text{-}T_c)$

minimize F in M: result General Solution $M(h, (T\text{-}T_c))$

singularity as b,h go through zero!

singularity as h goes through zero for $T < T_c$

Generalized Mean Field Schemes II

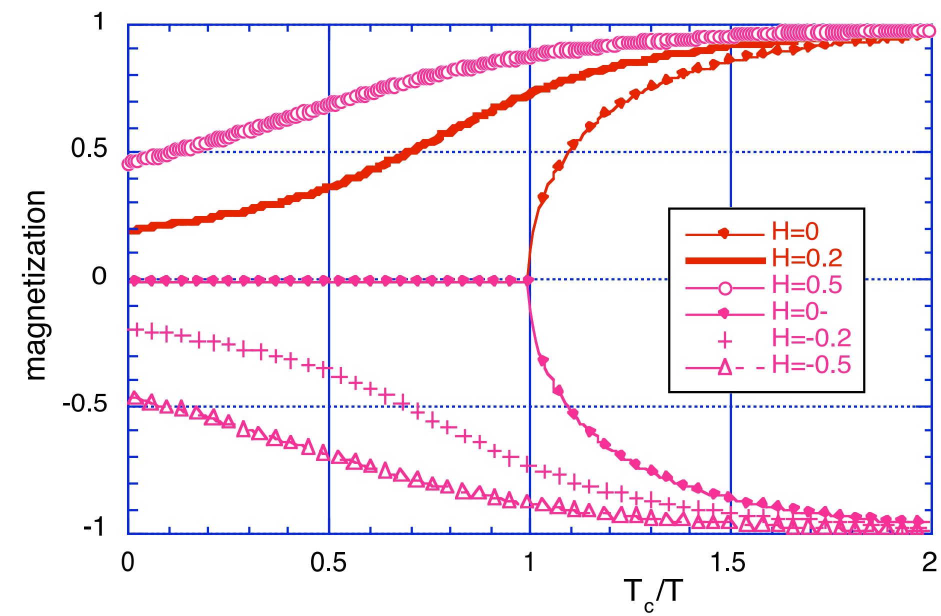
For example for $T < T_c$, jump in order parameter goes as

$$M \sim (T_c - T)^{\beta}$$

This square root (β=1/2) appears to be a Universal result.

Mean Field Theory predicts all near-critical behavior

order parameter and free energy were crucial concepts



order parameter in mean field transition

……..order parameter and free energy were crucial concepts

free energy could be expressed in terms on any descriptors of systems behavior. It is a minimized by the corrrect value of any one of them, We have thus come loose from the particular thermodynamic variables handed to us by our forefathers,

order parameter could be anything which might jump in the transition.

other variables could be anything at all.

In the meantime Schwinger was working on electromagnetic fields for World War II radar. He use variational methods and effective fields (“lumped variables”) to build electromagnetic circuits.

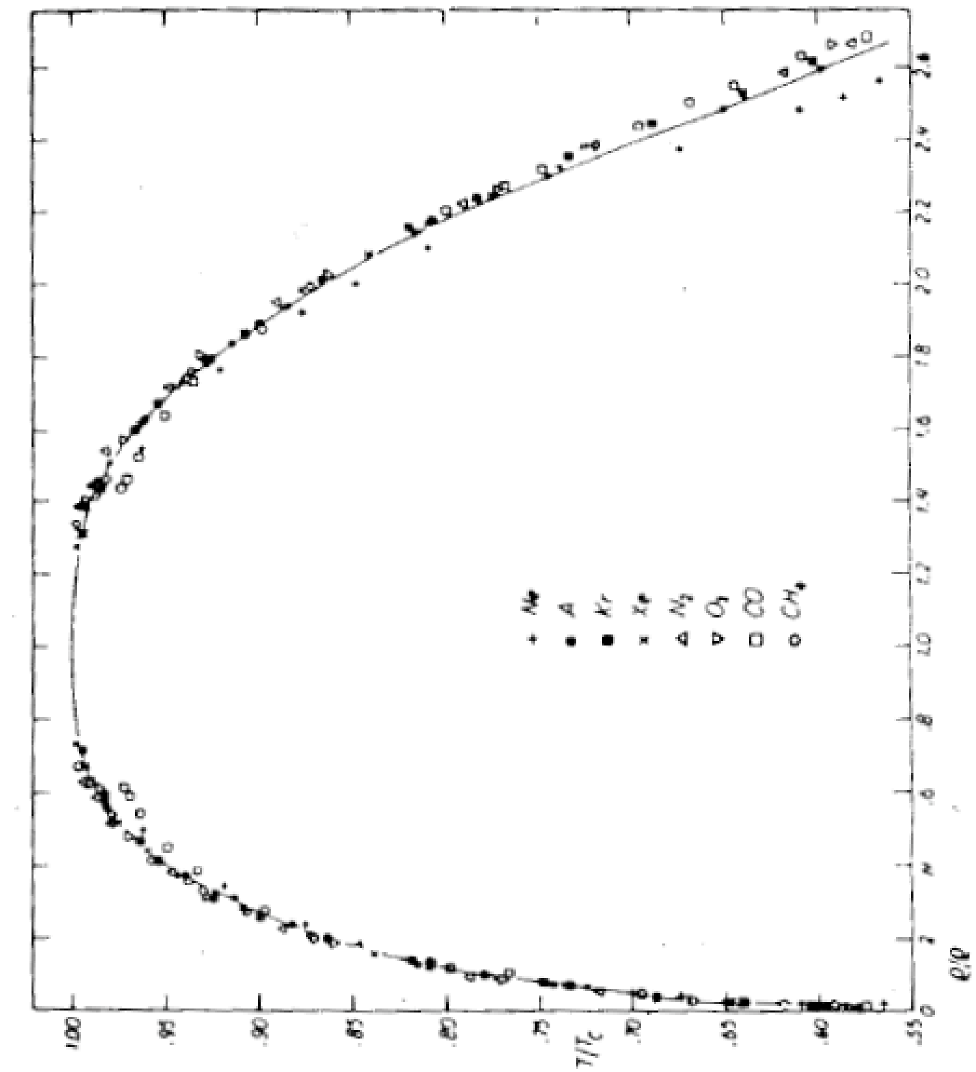
A worry?

Mean field theory gives $M \sim (T_c - T)^{\beta}$ and $\beta = 1/2$

This power is, however, wrong. Experiments are closer to

$$M \sim (T_c - T)^{1/3} \quad \text{in 3-D}$$

1880-1960: No one worries much about discrepancies



order parameter: density versus Temperature in liquid gas phase transition. After E.A. Guggenheim J. Chem. Phys. **13** 253 (1945)

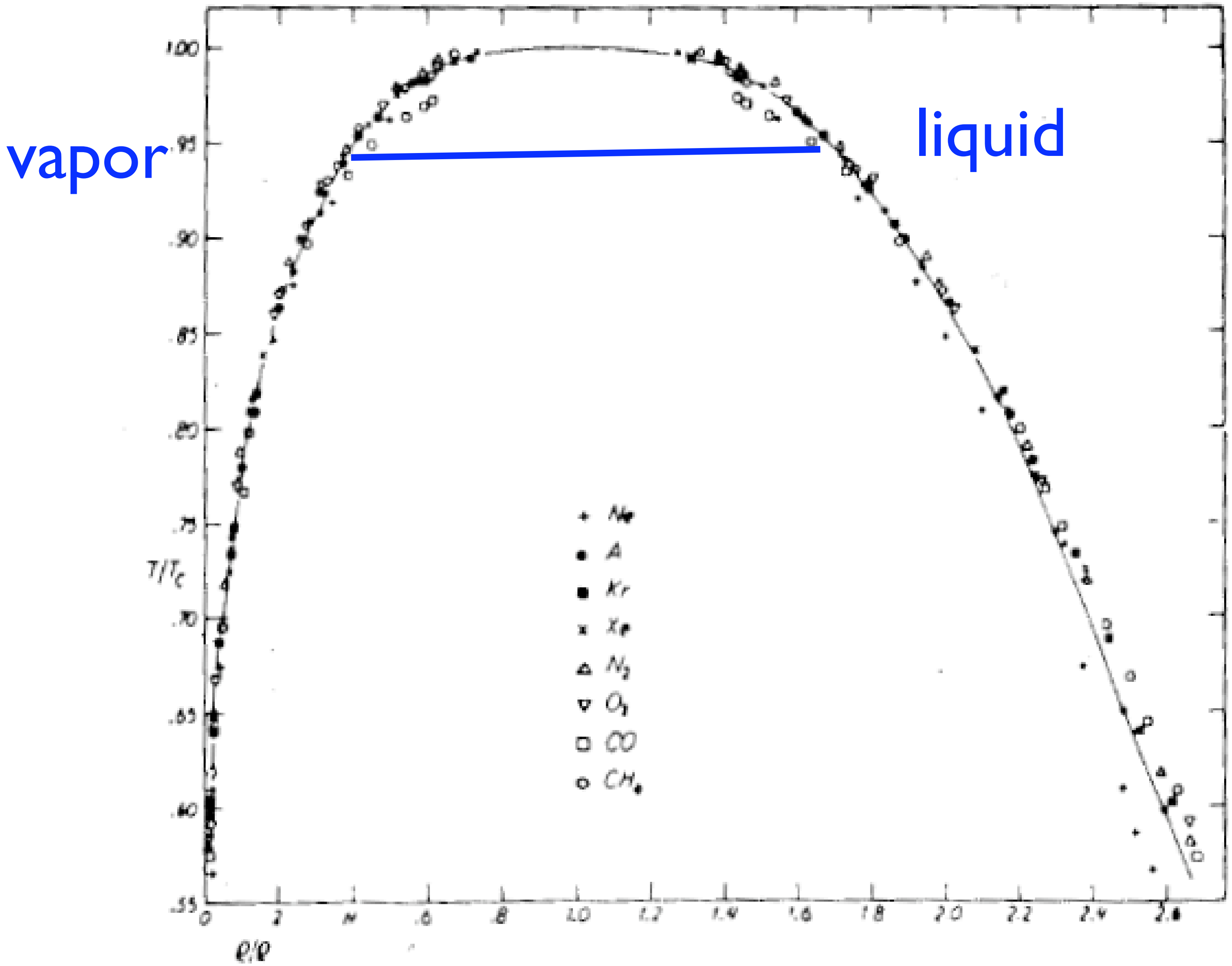


Figure 1.6 Reduced densities of coexisting liquid and gas phases for a number of simple molecular fluids (Guggenheim 1945). The experimental points support a law of corresponding states, but the universal curve is cubic rather than quadratic as required by van der Waals' theory.

Theoretical work

Onsager solution (1943) for 2D Ising model gives infinity in specific heat and order parameter index $\beta=1/8$(Yang)--contradicts Landau theory which has $\beta=1/2$

L. Onsager, Phys. Rev. **65** 117 (1944)

C.N, Yang, Phys. Rev. **85** 808 (1952)

Kings' College school (Domb, Fisher, ... (1949-)) calculates indices using series expansion method. Gets values close to $\beta=1/8$ in two dimensions and $\beta=1/3$ in three and not the Landau\van der Waals value, $\beta=1/2$.

Still Landau's no-fluctuation theory of phase transitions stands

Turbulence Work

Kolmogorov theory (1941) uses a mean field argument to predict velocity in cascade of energy toward small scales.

Result: velocity difference at scale r behaves as

$$\delta v(r) \sim (r)^{1/3}$$

(N.B. First Scaling theory) Landau criticizes K's work for leaving out fluctuations. Kolmogorov modifies theory (1953) by assuming rather strong fluctuations in velocity.

Still Landau's no-fluctuation theory of phase transitions stands

Pre-revolutionary era

Set our qualitative nature of the phase diagram.

Thought up many of the important theoretical concepts.
Did not produce a satisfactory theoretical synthesis.

Experimenters and series expanders measured many of the critical indices.
Theory got wrong values.

Explained phase transition as a symmetry breaking.
Was not much concerned about how information on "which phase" was transfered.

A Revolutionary Period:

1960s and early 70s

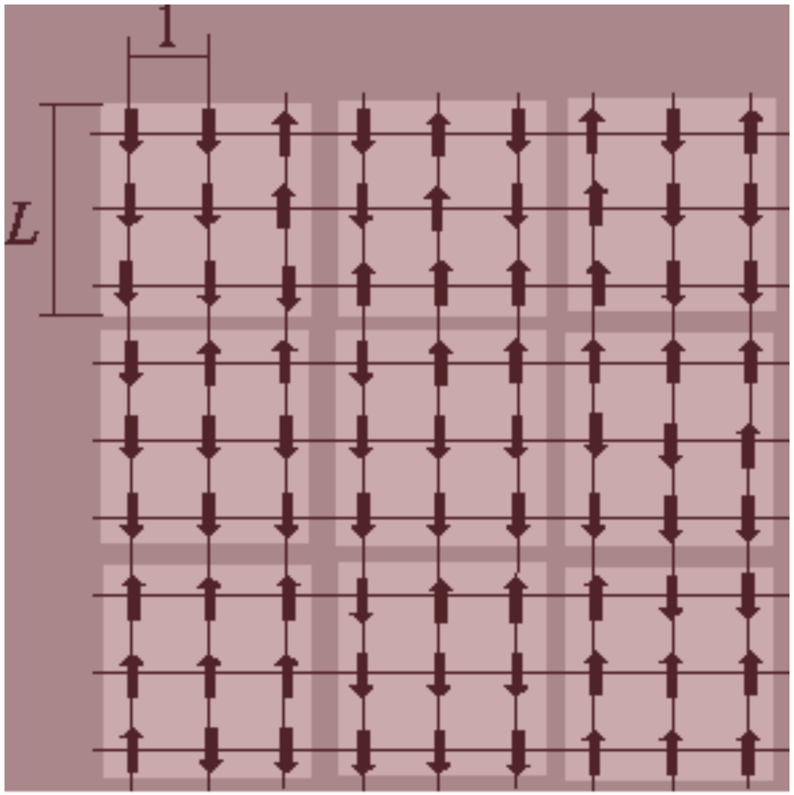
Step I: The phenominology

Pokrovskii & Patashinskii and Ben Widom noticed that near-critical behavior had an important invariance property, but did not detail the nature of the invariance.

Step II, Half-way there

Less is the same too

Kadanoff considers invariance properties of critical point and asks how description might change if one replaced a block of spins by a single spin, changing the length scale. Answer: the system could equally well be decribed in the old way by effective values of (T-Tc) and magnetic field. Fewer degrees of freedom imply no change at all. This approach justifies the phenominology of step I.



fewer degrees of freedom produces “block renormalization”

More rumbles before the revolution

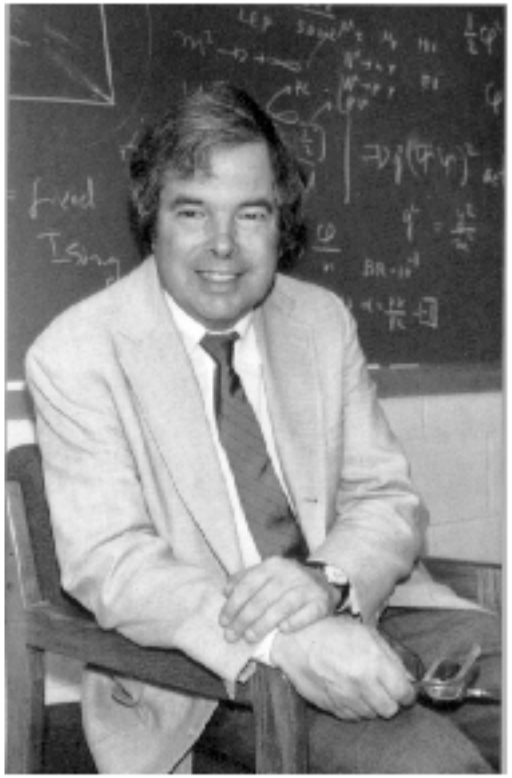
Recognize that ‘critical phenomena’ is a subject US NBS conference 1965.

Don’t look at the entire phase diagram, focus on the region near the critical point.

Get a whole host of new experiments,
embrace a new phenomenology,

broad use of effective this and effective that:
effective medium theory in complex materials
effective masses throughout solid state
effective energies in Landau’s theory of $Helium^3$
particle physics: some renormalization Gell-Mann & Low, Stuckelberg & Peterman
recall Schwinger

The Renormalization Revolution:



Kenneth G. Wilson
synthesizes new theory

Wilson converts a phenominology into a calculational method.

- Adds concept of fixed point
- Instead of using a few fields (T-Tc, h), he conceptualizes the use of a whole host of fields

This then provides a method which can be used by many people for many real calculations. The most notable is the epsilon expansion of Wilson and Fisher.

note that new theory matches experiments
everything in critical phenomena seems to be explained

The Crucial Ideas-for the revolution

Ideas:

- Criticality: recognize a subject in itself
- Scaling: Behavior has invariance as length scale is changed
- Universality: Expect that critical phenomena problems can be divided into different "universality classes"
- Running Couplings: Depend on scale. Cf. standard model based on effective couplings of Landau & others.
- effective fields of all sorts: Running couplings are but one example of this.
- Fixed Point: Singularities when couplings stop running. K. Wilson
- Renormalization Group: K. Wilson (1971), calculational method based on ideas above.

Each Item, except RG & fixed point, is a "consensus" product of many minds

The Outcome of Revolution

Excellent quantitative and qualitative understanding of phase transitions in all dimensions. Information about

- **Universality Classes**

All problems divided into “Universality Classes” based upon dimension, symmetry of order parameter, ...

Different Universality Classes have different critical behavior

e.g. Ising model, ferromagnet, liquid-gas are in same class

XYZ model, with a 3-component spin, is in different class

To get properties of a particular universality class you need only solve one, perhaps very simplified, problem in that class.

moral: theorists should study simplified models. They are close to the problems we wish to understand

Conceptual Advances

First order phase transition represent a choice among several available states or phases. This choice is made by the entire thermodynamic system.

Critical phenomena are the vacillations in decision making as the system chooses its phase.

Information is transferred from place to place via local values of the order parameter.

There are natural thermodynamic variables to describe the process. The system is best described using these variable.

Each variable obeys a simple scaling.

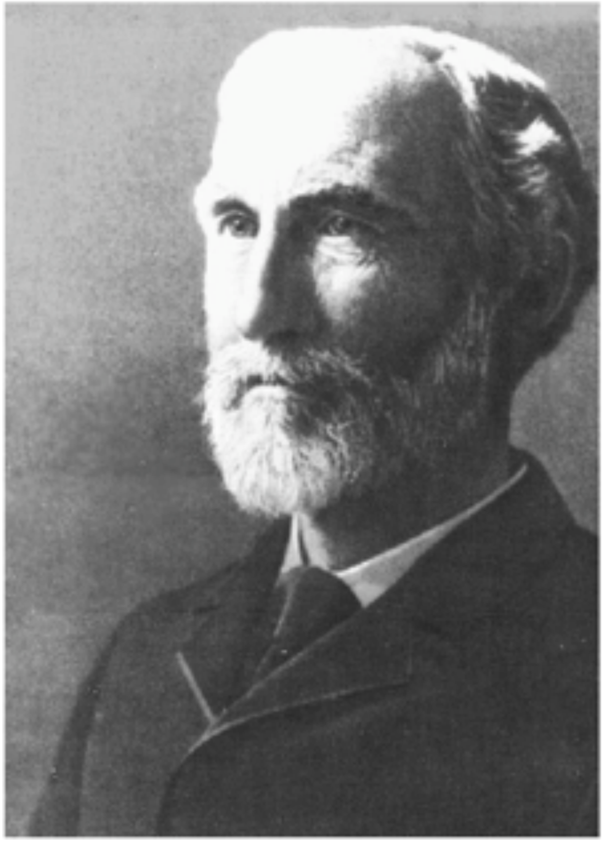
Next: After the Revolution:

Summary

Critical behavior occurs at but one point of the phase diagram of a typical system. It is anomalous in that it is usually dominated by fluctuations rather than average values. These two facts provide a partial explanation of why it took until the 1960s before it became a major scientific concern. Nonetheless most of the ideas used in the eventual theoretical synthesis were generated in this early period.

Around 1970, these concepts were combined with experimental amd numerical results to produce a complete and beautiful theory of critical point behavior.

In the subsequent period the "revolutionary synthesis" radiated outward to (further) inform particle physics, mathematical statistics, various dynamical theories....



JW Gibbs

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