# Quantum control of levitated massive mechanical systems

- a new approach for gravitational

quantum physics

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## Summary

• The predictions of **quantum theory** and of **general relativity**, our current theory of gravity, are **extremely well confirmed by experiment**.

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- There are few table-top experiments to date that probe the interface between quantum physics and gravity. In all these experiments, Earth's gravity acts as a constant classical background field. Loosely speaking, the quantum system is used as a "test particle" in an external gravitational field.
- On the other hand, the last decade has seen large progress in controlling the quantum regime of massive micro-mechanical oscillators. These systems may lead the way to a new class of gravitational quantum physics experiments, in which the quantum system itself, e.g. the center of mass degree of freedom of a massive sphere, serves as a gravitational source mass.

## Quantum theory works, as does GR...

#### Example from quantum theory: validity of the quantum superposition principle for

- orbital angular momentum states of photons up to a few hundred quantum numbers (1)
- $\mu$ A-level current states carrying up to 10<sup>6</sup> electrons (2,3)
- collective spin degrees of freedom of 10<sup>12</sup> Rubidium atoms (4).
- macromolecules (up to 10<sup>4</sup> amu) (5,6)
- vibrational degrees of freedoms of mechanical resonators (up to 10<sup>16</sup> amu) (7,8)

#### Examples from GR:

- dynamics of binary pulsars (9)
- satellite tests of the Lense-Thirring effect (11,12).
- tests of the weak equivalence principle to an accuracy of \_ better than10<sup>-13</sup> (13)
- measurements of Newton's constant G to 10<sup>-4</sup> (14).
- atomic clocks for gravitational redshift to 10<sup>-6</sup> (15).

→ strong relativistic fields and gravitational radiation

→ solar-system scale experiments in the weak relativistic regime

→ earth-based high-precision tests of gravity





## OUTLINE

- Quantum systems as "test masses" a brief (very incomplete) survey on table-top quantum experiments that probe gravity
- Quantum systems as "source masses"? ,what prevents this from becoming a practical experiment?'
- Quantum control of levitated massive systems towards a "quantum Cavendish" experiment







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grandationel polential ( on Earth : \$ = g b)

VOLUME 34, NUMBER 23

PHYSICAL REVIEW LETTERS

9 JUNE 1975



**Observation of Gravitationally Induced Quantum Interference\*** 

R. Colella and A. W. Overhauser Department of Physics, Purdue University, West Lafayette, Indiana 47907

and

S. A. Werner Scientific Research Staff, Ford Motor Company, Dearborn, Michigan 48121 (Received 14 April 1975)

We have used a neutron interferometer to observe the quantum-mechanical phase shift of neutrons caused by their interaction with Earth's gravitational field.



FIG. 1. Schematic diagram of the neutron interferometer and <sup>3</sup>He detectors used in this experiment.







Laser-cooling of atoms and atom-trapping are finding increasing application in many areas of science<sup>1</sup>. One important use of lasercooled atoms is in atom interferometers<sup>2</sup>. In these devices, an atom is placed into a superposition of two or more spatially separated atomic states; these states are each described by a quantum-mechanical phase term, which will interfere with one another if they are brought back together at a later time. Atom



0

Phase (rad)

2π

π

 $-2\pi$ 

2 atomic fountains at different locations
 → differential acceleration measurement
 → Measure G through additional test mass

#### Science 2007

## Atom Interferometer Measurement of the Newtonian Constant of Gravity

J. B. Fixler,<sup>1</sup> G. T. Foster,<sup>2</sup> J. M. McGuirk,<sup>3</sup> M. A. Kasevich<sup>1</sup>\*

We measured the Newtonian constant of gravity, *G*, using a gravity gradiometer based on atom interferometry. The gradiometer measures the differential acceleration of two samples of laser-cooled Cs atoms. The change in gravitational field along one dimension is measured when a well-characterized Pb mass is displaced. Here, we report a value of  $G = 6.693 \times 10^{-11}$  cubic meters per kilogram second squared, with a standard error of the mean of  $\pm 0.027 \times 10^{-11}$  and a systematic error of  $\pm 0.021 \times 10^{-11}$  cubic meters per kilogram second squared. The possibility that unknown systematic errors still exist in traditional measurements makes it important to measure *G* with independent methods.

#### Nature 2014

## Precision measurement of the Newtonian gravitational constant using cold atoms

G. Rosi<sup>1</sup>, F. Sorrentino<sup>1</sup>, L. Cacciapuoti<sup>2</sup>, M. Prevedelli<sup>3</sup> & G. M. Tino<sup>1</sup>



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PRL 98, 111102 (2007)

PHYSICAL REVIEW L

Paul Hamilton<sup>1</sup>, Matt Jaffe<sup>1</sup>, Philipp Haslinger<sup>1</sup>, Quinn Simmons<sup>1</sup>, Holger Müller<sup>1,2\*</sup>,

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and Justin Khoury<sup>3</sup>

Testing General Relativity with Ato...

Savas Dimopoulos, Peter W. Graham, Jason M. Hogan, and Mark A. Kasevich Department of Physics, Stanford University, Stanford, California 94305, USA (Passived 10 October 2006: published 15 March 2007)



PRL 110, 093602 (2013)



Interferometry with Bose-Einstein Condensates in Microgravity

H. Müntinga,<sup>1</sup> H. Ahlers,<sup>2</sup> M. Krutzik,<sup>3</sup> A. Wenzlawski,<sup>4</sup> S. Arnold,<sup>5</sup> D. Becker,<sup>2</sup> K. Bongs,<sup>6</sup> H. Dittus,<sup>7</sup> H. Duncker,<sup>4</sup> N. Gaaloul,<sup>2</sup> C. Gherasim,<sup>8</sup> E. Giese,<sup>5</sup> C. Grzeschik,<sup>3</sup> T. W. Hänsch,<sup>9</sup> O. Hellmig,<sup>4</sup> W. Herr,<sup>2</sup> S. Herrmann,<sup>1</sup> E. Kajari,<sup>5,10</sup> S. Kleinert,<sup>5</sup> C. Lämmerzahl,<sup>1</sup> W. Lewoczko-Adamczyk,<sup>3</sup> J. Malcolm,<sup>6</sup> N. Meyer,<sup>6</sup> R. Nolte,<sup>8</sup> A. Peters,<sup>3,11</sup> M. Popp,<sup>2</sup> J. Reichel,<sup>12</sup> A. Roura,<sup>5</sup> J. Rudolph,<sup>2</sup> M. Schiemangk,<sup>3,11</sup> M. Schneider,<sup>8</sup> S. T. Seidel,<sup>2</sup> K. Sengstock,<sup>4</sup> V. Tamma,<sup>5</sup> T. Valenzuela,<sup>6</sup> A. Vogel,<sup>4</sup> R. Walser,<sup>8</sup> T. Wendrich,<sup>2</sup> P. Windpassinger,<sup>4</sup> W. Zeller,<sup>5</sup> T. van Zoest,<sup>7</sup> W. Ertmer,<sup>2</sup> W. P. Schleich,<sup>5</sup> and E. M. Rasel<sup>2,\*</sup>



**Fig. 1.** Cuts through the ZARM drop tower facility in Bremen (**A**) and the capsule (**B**) containing the heart of the BEC experiment (**C**). The capsule is released from the top of the tower (**D**) and is recaptured after a free fall of 4.7 s through an evacuated stainless steel tube at the bottom of the tower by a 8-m-deep pool of polystyrene balls (**E**). In the process of recapturing the capsule, the experiment has to survive decelerations up to 500 m/s<sup>2</sup> (about 50 times the local gravitational acceleration). The facility permits up to three drops per day. The capsule contains

all of the components necessary to prepare and observe a BEC, such as the laser systems for cooling the atoms, the ultrahigh-vacuum chamber with the atom chip, the current drivers and power supplies, a charge-coupled device (CCD) camera, and a control computer. The vacuum chamber is surrounded by two magnetic shields and allows us to include an atom interferometer in future experiments. Moreover, the catapult underneath the movable polystyrene pool offers the possibility of extending the time of free fall to 9 s.

week ending

1 MARCH 2013



FIG. 2 (color). Mach-Zehnder interferometry of a BEC in microgravity as realized in the ZARM drop tower in Bremen (a) where absorption imaging (b) brings out the interference fringes (c). The preparatory experimental sequence (a) includes capturing cold atoms in a magneto-optical trap (MOT), loading an Ioffe-Pritchard trap, creating a BEC, and applying the DKC followed by the adiabatic rapid passage (ARP). The remaining time before the capture of the capsule at the bottom of the tower is used for AI and imaging of the atoms. The AMZI below the atom chip [top plane of (b)] is formed by scattering the BEC off moving Bragg gratings generated by two counterpropagating laser beams (red arrows directed along the y axis),



## OUTLINE

 Quantum systems as "test masses" a brief (very incomplete) survey on table-top quantum experiments that probe gravity

 Quantum systems as "source masses"? ,what prevents this from becoming a practical experiment?'

Quantum control of levitated massive systems towards a "quantum Cavendish" experiment



## An ultimate experiment? Entanglement by gravity...



FEYNMAN: "Therefore, there must be an amplitude for the gravitational field, provided that the amplification necessary to reach a mass which can produce a gravitational field big enough to serve as a link in the chain does not destroy the possibility of keeping quantum mechanics all the way. There is a *bare* possibility (which I shouldn't mention!) that quantum mechanics fails and becomes classical again when the amplification gets far enough, because of some minimum amplification which you can get across such a chain. But aside from that possibility, if you believe in quantum mechanics up to any level then you have to believe in gravitational quantization in order to describe this experiment."

Chapel Hill Conference 1957 (29)

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WITTEN: "What prevents this from becoming a practical experiment?"

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Chapel Hill Conference 1957 (29)

Example: For 2 lead spheres of diameter 500 µm, an initial superposition size for sphere 1 of  $\Delta r = 5 \times 10^{-7}$  m and preparation of sphere 2 in a motional ground state (100 Hz trap frequency) with  $\Delta x_0 = 10^{-15}$  m, we obtain  $\Gamma_{ent} = 1.5$  Hz, i.e. gravitational entanglement is established on a second time scale.

$$\Gamma_{ent} = \left(\frac{GM}{\hbar}\right) \Delta r \rho \Delta x_0$$

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#### COM superposition states of massive systems: where do we stand?





Arndt group (Vienna): S. Gerlich, S. Eibenberger et al., Nature Communications 2, 263 (2011)

#### COM superposition states of massive systems: where do we stand?



Micromechanics, 2×10<sup>13</sup> atoms m ~ 10<sup>-12</sup> kg = 7×10<sup>14</sup> AMU Δx ~ 10<sup>-16</sup> m (~10<sup>-10</sup>x its diameter)



# Quantum ground state and single-phonon control of a mechanical resonator

A. D. O'Connell<sup>1</sup>, M. Hofheinz<sup>1</sup>, M. Ansmann<sup>1</sup>, Radoslaw C. Bialczak<sup>1</sup>, M. Lenander<sup>1</sup>, Erik Lucero<sup>1</sup>, M. Neeley<sup>1</sup>, D. Sank<sup>1</sup>, H. Wang<sup>1</sup>, M. Weides<sup>1</sup>, J. Wenner<sup>1</sup>, John M. Martinis<sup>1</sup> & A. N. Cleland<sup>1</sup>



## Decoherence

Joos & Zeh, Caldeira & Leggett, Unruh & Zurek Paz & Zurek, Hu & Paz & Zhang, Milburn, ...



 $\wedge$ 

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + \mathcal{L}[\rho]$$
Master equation approach
$$\begin{aligned} \dot{\kappa} |\mathcal{L}_{g}[\rho]|x'\rangle &= \begin{cases} -[\Delta x]^{2} \Delta_{g} \langle x | \rho | x' \rangle, & \Delta x < \frac{h}{n \bar{\nu}} \\ -\Gamma'_{g} \langle x | \rho | x' \rangle, & \Delta x < \frac{h}{n \bar{\nu}} \\ \Delta x > \frac{h}{n \bar{\nu}} \\ \overline{\mu} \\ \overline{\nu} \\ \end{array}$$
Gas scattering
$$\begin{aligned} \text{Gas scattering} \\ \text{See also} \\ \text{O. Romero-Isart et al.,} \\ \text{PR 107, 020405} \\ (2011) \\ \text{O. Romero-Isart et al.,} \\ \text{PR 107, 020405} \\ (2011) \\ \text{O. Romero-Isart, PRA} \\ \text{A}_{0} &= \frac{20\rho^{2}R^{3}}{h}, & \Delta x \ll R \\ \Lambda_{P} &= \Lambda_{D} = \frac{20\rho^{2}R^{3}}{h}, & \Delta x \ll R \\ \Lambda_{P} &= \frac{20\rho^{2}R^{3}}{h}, & \Delta x \ll R \\ \Lambda_{P} &= \frac{20\rho^{2}R^{5}}{h}, & \Delta x \ge R. \end{aligned}$$

$$\begin{aligned} \text{Are } &= \frac{20\rho^{2}R^{5}}{h}, & \Delta x \ge R. \end{aligned}$$

### How massive can we go?





## Pushing mechanical quantum control to the next level

# o the next level

Coupling to

gravity

#### **Q:** How to achieve large mass <u>AND</u> long coherence time in a quantum experiment?



#### A: Quantum control of levitated mechanical systems!

- Ref.
- Quantum control of a trapped massive object >> 10<sup>10</sup> atoms
- Long coherence times (up to seconds) through free fall dynamics
- Exceptional force sensitivity

#### universität Isolation of COM motion from the environment: wien Levitated nanospheres as high-Q mechanical oscillators Ashkin & Dziedzic, APL 28, 333 (1976) J. Gieseler, R. Quidant, C. (20um Si oil) Science and Technology Dellago, L. Novotny, $10^{10}$ Nature Nanotechnology MACRO NEMS MEMS 9,358 (2014) (70 nm SiO2) p = 10<sup>-6</sup> mbar $p = 10^{-6}$ mbar 10<sup>8</sup> High Tension Large Scale **Quality Factor** 10<sup>6</sup> $p = 10^{-3} mbar$ Q-V13 Low Temp. $10^{4}$ Diamond D. Grass (Vienna) Magnetomotive (350 nm SiO2 Diamond inside hollow core fibre) 10<sup>2</sup> CNT Capacitive Silicon 40 Graphene γ [kHz] 30 $10^{0}$ 20 10<sup>-26</sup> $10^{-20}$ 10<sup>-17</sup> 10<sup>-23</sup> 10<sup>-14</sup> 10<sup>-11</sup> 10<sup>-8</sup> 10<sup>-5</sup> $10^{-2}$ 10 Volume [m<sup>3</sup>]

5

10

20

p [mbar]

15

25

30

35

M. Imboden, P. Mohanty, Phys. Rep. 534, 89 (2014)

### Sensitivity to gravitational forces?



## {T, Q, ω, m}



### Measuring gravity between microscopic source masses ?



Jonas Schmöle, Mathias Dragosits, Hans Hepach

Smallest source mass to date: **120 g** W. Michaelis et al., Meterologia 32, 267–276 (1995)

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## **Big G: the open problem**





The search for Newton's constant Clive Speake and Terry Quinn

> The "G machine," now housed at the University of Birmingham in the UK, was used at the International Bureau of Weights and Measures in France to measure Newton's gravitational constant.

G-whizzes disagree over gravity

Recent measurements of gravitational constant increase uncertainty over accepted value.

Three decades of careful experimentation have painted a surprisingly hazy picture of the constant governing the most familiar force on Earth.

NEWS

#### Physics Today July 2014

NATURE/Vol 466/26 August 2010

Vienna Center for Quantum Science and Technology



Figure 1. Measurements of Newton's gravitational constant G have vielded conflicting results. Here, the results of torsion-balance (maroon), pendulum (blue), and beam-balance (green) experiments discussed in the text are shown, along with the location and year in which they were measured. Error bars correspond to one standard deviation; the shaded region indicates the assigned uncertainty of the value recommended by the Committee on Data for Science and Technology in 2010. (Adapted from T. J. Quinn et al., Phys. Rev. Lett. 111, 101102, 2013.)

## **Big G: the open problem**





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## **Big G: the open problem**



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#### >10mm diameter spheres Vienna Center for Quantum Science and Technology reach torsional-pendulum 10<sup>-18</sup> 10<sup>8</sup> Newton like precision $\Delta G/G < 10^{-5}$ Thermal @ T=300K 10<sup>7</sup> (at room temperature) 10<sup>-20</sup> $\beta$ (S/N) This is for a $\mathbf{Q} = 2 \times 10^4$ 10<sup>6</sup> *β*=1 oscillator! $\rightarrow$ levitation!! 10-22 10<sup>5</sup> Best published S/N<sup>1</sup> Newton/Thermal m<sup>2</sup>1 104 ц Ц 10<sup>-24</sup> Average S/N<sup>2</sup> < X^2 10<sup>3</sup> 10<sup>-26</sup> 10<sup>2</sup> 10<sup>1</sup> 10<sup>-28</sup> 10<sup>0</sup> 5.10<sup>-1</sup> 5.10<sup>-1</sup> 10<sup>-30</sup> 10<sup>-4</sup> 5.10-4 10<sup>-3</sup> r [m]

<sup>1</sup>J.H. Gundlach and S.M. Merkowitz, Phys. Rev. Lett. 85 2869 (2000) <sup>2</sup>G. T. Gillies and C. S. Unnikrishnan, Phil. Trans. R. Soc. A 2014 372 (2014)

## **Potential Application for "Big G" measurement**

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<sup>1</sup>J.H. Gundlach and S.M. Merkowitz, Phys. Rev. Lett. 85 2869 (2000) <sup>2</sup>G. T. Gillies and C. S. Unnikrishnan, Phil. Trans. R. Soc. A 2014 372 (2014)

## **Potential Application for "Big G" measurement**

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## Towards quantum state preparation of a free particle

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#### **Optically levitated nanospheres**

#### Magnetically levitated spheres

(Romero-Isart et al., 1112.5609 Cirio et al., 1112.5208)



Chang et al., quant-ph 0909.1548 (2009), PNAS 2010 Romero-Isart et al., quant-ph 0909.1469 (2009), NJP 2010 P. F. Barker et al., PRA 2010 early work: Hechenblaikner, Ritsch et al., PRA 58, 3030 (1998) Vuletic & Chu, PRL 84, 3787 (2000)

→ Harmonic oscillator in optical potential (negligible support loss, high Q)

→ Quantum control via cavity optomechanics (laser cooling, state transfer, etc.)

#### **Generation of quantum superposition states**

- single-photon quantum state transfer
- quantum state teleportation

• ..

• free fall . . .

- Akram, Kiesel, Aspelmeyer, Milburn, NJP 12, 083030 (2010)
- Khalili, Danilishin, Miao, Müller-Ebhardt, Yang, Chen, quant-ph 1001.3738 (2010)
- Romero-Isart, Pflanzer, Juan, Quidant, Kiesel, Aspelmeyer, Cirac, Phys. Rev. A 83, 013803 (2011)

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## **Towards quantum state preparation of a free particle**



#### **Generation of quantum superposition states**

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 Akram, Kiesel, Aspelmeyer, Milburn, NJP 12, 083030 (2010)

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- Romero-Isart, Pflanzer, Juan, Quidant, Kiesel, Aspelmeyer, Cirac, Phys. Rev. A 83, 013803 (2011)

### **Optically trapped nanospheres as mechanical resonators**





## Magnetically trapped superconductors as mechanical resonators



Trap frequencies ~ 20 Hz

0.03

0.025

0.02

0.01

0.005

9

Amplitude 0.012

T = 20 mK, p = 1e-6 mbar

S. Minter, R. Chiao, N. Prigge, M. Aspelmeyer

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# Magnetically trapped superconductors as mechanical resonators





J. Slater, J. Hofer, W. Wieczorek, M. Aspelmeyer





























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## FШF

Der Wissenschaftsfonds.



Vienna Science and Technology Fund



European Research Council

> Alexander von Humboldt Stiftung/Foundation





Quantum-"Mechanics" in Vienna: The Mirror Team 2015 Low-noise coatings & microfab *Garrett Cole → CMS* N.N. (cleanroom tech) Towards testing quantum gravity & QND measurements (with C. Brukner, M. Kim) *Sungkun Hong* Ralf Riedinger Philipp Köhler VLQ Vienna Center for Quantum Science and Technology

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Wissenschaftsfonds.



SEVENTH FRAMEWORK

PROGRAMME

pean

Quantum foundations and levitated resonators; precision measurements (with M. Arndt, O. Romero-Isart, M. Trupke, K. Schwab, Airbus/EADS) Nikolai Kiesel Rainer Kaltenbaek Josh Slater **Florian Blaser Uros** Delic **David Grass** Jonas Schmöle Mathias Dragosits Joachim Hofer Martin Siegele Hans Hepbach

**Christian Siegele** 

enna:

Quantum information interfaces (with K. Hammerer, J. Eisert, O. Painter) *Witlef Wieczorek* Jason Hölscher-Obermayer Sebastian Hofer Ramon Moghadas Nia Claus Gärtner Thomas Zauner



## **Quantum Controlling Levitated Massive Mechanical Systems**

#### GOAL

Establish quantum control of levitated massive mechanical systems

#### METHOD

- **Optical levitation** coupled to cavities
- Magnetic levitation coupled to superconducting circuits

#### MOTIVATION

Enable a new class of experiments at the interface between quantum physics and gravity



#### EXPECTED RESULTS

*Bottom-up*: Demonstrate **long-lived quantum coherence** of increasingly massive systems

*Top-down*: Measure **gravity** between **sub-mm source masses** 

Long-term: establish experiments that exploit the source mass character of the quantum system



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