Gravitational microlensing and vacuum fluctuations

Steve Carlip
U.C. Davis

Probing the Mystery:
Theory and Experiment in Quantum Gravity
Galiano Island, BC, Canada
August 2015

Setting up the question

Raychaudhuri equation:

$$\frac{d\theta}{d\lambda} = \ell^a \nabla_a \theta = -\frac{1}{2}\theta^2 - \sigma_{ab}\sigma^{ab} + \omega_{ab}\omega^{ab} - 8\pi G T_{ab}\ell^a \ell^b$$

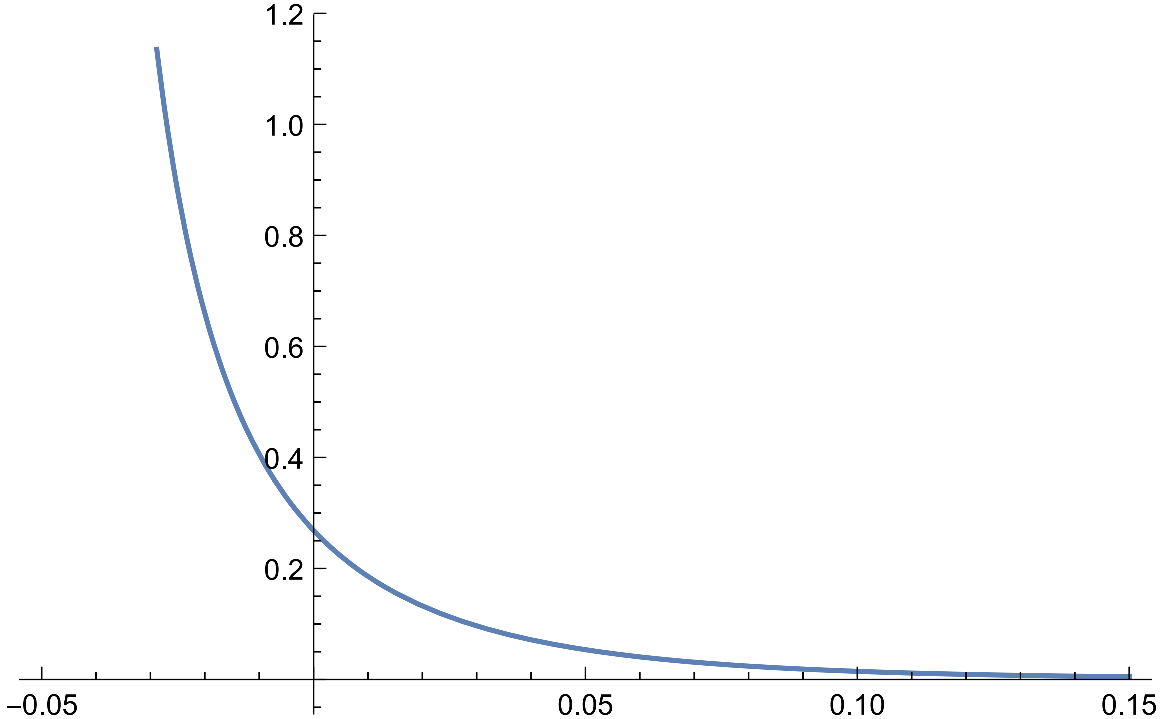
Vacuum fluctuations of the stress-energy tensor
induce fluctuations of expansion ⇒ (micro)lensing

Can we observe this effect?

First guess: obviously not

Gravitational fluctuations occur at the Planck scale – much too small

But: distribution of fluctuations is *very* non-Gaussian



Fewster, Ford & Roman

with a strict lower bound and a long subexponential tail

$\Rightarrow$ naive intuition is unreliable

More precisely:

Smear stress-energy tensor over length scale L [more later...]

Let $x = \frac{16\pi^2}{\hbar} L^4 T_{00} \; = 16\pi^2 \frac{L^4}{{\ell_p}^2} G T_{00}$

"typical" quantum fluctuation has $x \sim 1$

Fewster, Ford, Roman:

Compute $\langle T\,T\,T\,\ldots\rangle$

Are these the moments of any probability distribution?

Two dimensions: yes, unique, exactly calculable

Four dimensions: yes, not quite unique, approximately calculable

For electromagnetism:

Strict lower bound $x_0 \approx -.0472$

$$Prob(x) \sim c_0 (x - x_0)^{-2} e^{-a(x-x_0)^{1/3}}$$

with $c_0 \approx .955,\; a \approx .963$

Vacuum fluctuations and gambler's ruin

Raychaudhuri: $$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - 8\pi G T_{ab}\ell^a\ell^b + \ldots$$

Vacuum fluctuations of $T_{ab}\ell^a\ell^b$ are usually negative (defocusing)

But lower bound, long positive tail (focusing)

"Gambler's ruin"

Whatever the odds, if you bet long enough against a house with unlimited resources, you always lose in the end.

Back-of-the-envelope estimate:

Let $\bar{\theta} = \sqrt{\frac{|x_0|}{\pi}\frac{\ell_p}{L^2}}$

- If $\theta > \bar{\theta}$, RHS of Raychaudhuri eqn < 0
  $\Rightarrow \theta$ is forced down to $\bar{\theta}$
- If $\theta < -\bar{\theta}$, RHS of Raychaudhuri eqn < 0
  $\Rightarrow \theta$ is forced down to $-\infty$
  $\Rightarrow$ focusing, microlensing

To force runaway focusing, need

$$\Delta\theta = \frac{d\theta}{d\lambda}L < -2\bar{\theta}$$

$$\Rightarrow x > 4\sqrt{\pi|x_0|}\,\frac{L}{\ell_p} = x_1$$

$$Prob(x > x_1) \sim \left(L/\ell_p\right)^{-4/3} e^{-\alpha(L/\ell_p)^{1/3}} \qquad \big(\alpha = \mathcal{O}(1)\big)$$

Significant probability of observation for L as large as $5 \times 10^5 \ell_p$

Time scale for focusing

Solve Raychaudhuri equation after $\theta < -\bar{\theta}$

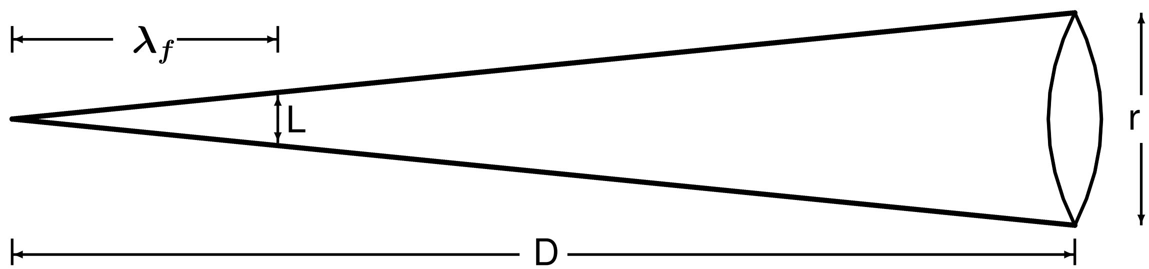
Find time to focusing

$$\lambda_f \sim \beta \frac{L^2}{\ell_p} \qquad \big(\beta = \mathcal{O}(1)\big)$$

Two scales:

- L sets scale of fluctuations
- λ_f sets scale at which these have potential observable effect

Some numbers



For significant magnification, need

$$\frac{r}{D} \sim \frac{L}{\lambda_f} \sim \frac{\ell_p}{L} \;\Rightarrow\; L \sim \frac{D}{r}\ell_p,\; \lambda_f \sim \left(\frac{D}{r}\right)^2 \ell_p$$

For large L to be relevant, need D/r large — small, distant sources

Unfortunately . . .

For observability, need $L/\ell_p \lesssim 5 \times 10^5$

$\Rightarrow \ \lambda_f \lesssim 10^{-21}$ cm

$E \sim 10^{10}$ TeV

Conversely . . .

Largest realistic L—white dwarfs in Large Magellanic Cloud

$L \sim 10^{-21}$ m

$\Rightarrow \ Prob(x) \sim 10^{-20000}$

$t_{fluc} \sim 10^{-29}$ s

What about multiple fluctuations?

Say N fluctuations of size $\delta(GT) \gtrsim \pm {\ell_p}^2/L^4$ along path of length D

random walk $\Rightarrow \langle\theta\rangle \sim \sqrt{N}{\ell_p}^2/L^3$

$\Rightarrow$ nonlinear regime when $L^3 \sim D{\ell_p}^2$

For cosmological source: significant for $L \sim 10^{-15}$ m: 1 GeV photon

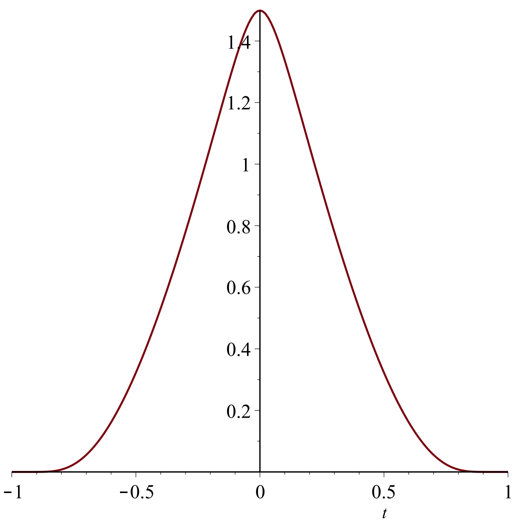
Maybe better: not really random walk (work in progress)

Is any further amplification possible? Maybe…

Dependence on smearing

Results so far based on Lorentzian smearing of T
(Fewster, Ford, and Roman, 2012)

New results: probability distribution depends on smearing function;
"fatter" tail for function with sharp cut-off
(Fewster and Ford, 2015)



$$Prob(x > x_1) \sim (L/\ell_p)^{-2/3} \, e^{-\alpha (L/\ell_p)^{1/6}}$$

Fewster & Ford

Now plausible probabilities for $L \sim 10^{-23}$ m

Need to better understand role of smearing