A new setting for spontaneous gauge symmetry breaking?[1]

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With the help of spontaneous symmetry breaking, unification and understanding have been successfully achieved for all forces save gravity. With the desire to include gravity in this framework, and in keeping with its presumed geometric nature, we seek a geometric scenario that exhibits spontaneous symmetry breaking. To this end we posit a tensor/scalar ($g_{\mu\nu}/\varphi$) model, which made its appearance in the derivation of the "new, improved" traceless energy momentum tensor $\theta^{CCJ}_{\mu\nu}$.

$$I_J = -\int d^4x\,\sqrt{|g|}\left(\frac{1}{12}\,R\,\varphi^2 + \frac{1}{2}\,g^{\mu\nu}\,\partial_\mu\,\varphi\,\partial_\nu\,\varphi\right) \equiv I_{Jordan}$$

$$\theta^{CCJ}_{\mu\nu} = \frac{2}{\sqrt{-g}}\,\frac{\delta I_J}{\delta g^{\mu\nu}}\,|_{g_{\mu\nu}\rightarrow \text{flat}}$$

In addition to diffeomorphism invariance, I_J possesses Weyl gauge invariance (local conformal invariance)

$$g^{\mu\nu} \;\rightarrow\; e^{2\theta} g^{\mu\nu}$$

$$\varphi \;\rightarrow\; e^{\theta}\varphi$$

where the “gauge” transformation parameter θ is a local function on space-time. Obviously such a symmetry is not seen in nature, so it must be spontaneously broken if I_J is to describe reality. Consequently many have observed that one may use the Weyl gauge symmetry to fix φ to unity in I_J, thereby arriving at the Einstein-Hilbert action. In this way one is led to the idea that Einstein-Hilbert theory is nothing but Weyl gauge theory in the “unitary gauge.”

$$I_J\Big|_{\varphi=1} \to I_{\text{Einstein-Hilbert}}$$

This viewpoint satisfies the long-standing desire in the physics community to incorporate conformal symmetry (local or global) into physical theory, as an extension of the Poincaré group.

But So-Young Pi and I have questioned the above with the following observations:

1. No gauge potential (connection) is present; in what sense does I_J define a "gauge theory" ?

2. By inverting the order of presentation, we recognized that φ is a spurion variable: upon replacing $g_{\mu\nu}$ in the Einstein-Hilbert action by $g_{\mu\nu}\,\varphi^2$, one arrives at the Weyl action.

$$I_{\text{Einstein-Hilbert}}\Big|_{g_{\mu\nu}\to g_{\mu\nu}\,\varphi^2} \to I_J$$

3. There is no dynamical/energetic reason for choosing the "unitary gauge."

4. The Weyl symmetry current vanishes identically. The computation is performed according to Noether's first theorem (applicable when transformation parameters are constant) and her second theorem (applicable when transformation parameters depend on space-time coordinates).

The former is a special case of the latter; both give the same result: no current.

Comment on this result: The identical vanishing of the Weyl current can not be exclusively attributed to the local nature of the transformation. For example in QED, the source current $j^\mu = \partial_\nu F^{\mu\nu}$ is obtained by the usual Noether first theorem procedure for the global symmetry:

$$\delta\ (\text{field}) = i\,\theta\ (\text{field}),$$

Application of the same Noether procedure to the local symmetry

$$\delta\, A_\mu(x) = \partial_\mu\, \theta(x)$$

gives the current $J^\mu(x) = \partial_\nu \Big(F^{\mu\nu}(x)\theta(x) \Big)$, which is identically conserved (super potential). J^μ reduces to j^μ when the gauge parameter is fixed to a global value. All this is guaranteed by Noether's second theorem, and never does any current vanish.

Comment on gauge potential: It appears that the true reason for vanishing Weyl current is that no derivations arise in Weyl transformations; *i.e.* as noted above there are no potentials undergoing inhomogenous local transformations. In fact potentials can be introduced in the following way. Observe that the $R\,\varphi^2$ term in I_J ensures that the Weyl non-invariance of the kinetic term $g^{\mu\nu}\,\partial_\mu\varphi\,\partial_\nu\,\varphi$ is canceled by the non-invariance of $R\,\varphi^2$. However, Weyl proposed a different mechanism for the construction of a Weyl invariant kinetic term: Rather than using non-minimal $R\,\varphi^2$ interaction, he introduced a "gauge potential" W_μ to absorb the non-invariance. One verifies that

$$I^{\text{Weyl}} = -\frac{1}{2}\int d^4\,x\sqrt{|g|}\,\left(g^{\mu\nu}[\partial_\mu\varphi + W_\mu\,\varphi][\partial_\nu\,\varphi + W_\nu\,\varphi]\right)$$

is Weyl invariant , provided W_μ transforms as

$$W_\mu \underset{\text{Weyl}}{\longrightarrow} W_\mu - \partial_\mu\,\theta.$$

We now demand that the W terms in I^{Weyl} reproduce the $R\,\varphi^2$ term in I_J. This is achieved when the following holds.

$$\frac{R}{12} = -D^\mu\,W_\mu + \frac{g^{\mu\nu}}{2}\,W_\mu\,W_\nu$$

As yet we do not know what is the significance of the above observations for the program of using spontaneously broken Weyl symmetry for physics. Clearly it is interesting to understand the similarities to and differences from the analogous structures in conventional gauge theory.