Electrodynamic Effects of Inflationary Gravitons

S. P. Miao

National Cheng Kung University, Taiwan

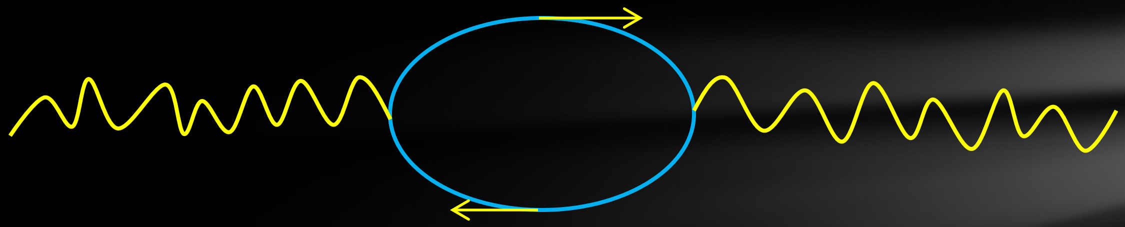
Collaborate with D. Glavan, T. Prokopec and R.P. Woodard

FRW Geometry

- Homogeneous, isotropic and spatial flat:
  $dS^2 = -dt^2 + a^2(t)d\vec{x} \cdot d\vec{x}$
- Physical distance: $a(t)d\vec{x}$
- Physical momentum: $\frac{\vec{k}}{a(t)}$
- $\varepsilon \to 0$ ➔ Locally de Sitter Background
- $H(t) = H =$ const. ➔ $a(t) \sim e^{Ht}$ ➔ Max. accelerated
- $H < 10^{14}$ Gev ➔ Particles almost effectively massless
- $GH^2 < 5 \times 10^{-11}$
  - QG still perturbative, not negligible

Spacetime Expansion Strengthens Loop Effects

- How to think about QFT Effects?
  - Classical response to virtual particles



- Maximum IR Enhancements:
  - PERSISTENCE TIME ➔ m = 0 + inflation
  - EMERGENCE RATE ➔ no conformal invariance
- Realized by
  - Massless, minimally coupled scalars
  - Gravitons

Fossilized quantum fluctuations

- Dynamical gravitons ~ MMC scalar (1946: Liftshits)

  $\mathcal{L} = -\frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g}$ in co-moving coordinates

- $L = \int d^3x\, \mathcal{L} \rightarrow \int \frac{d^3k}{(2\pi)^3}\frac{a^3}{2}\left[\,\left|\frac{d\tilde{\varphi}}{dt}\right|^2 - \left(\frac{kc}{a}\right)^2 |\tilde{\varphi}|^2\,\right]$
- Each mode k is S.H.O. with $m(t) \sim a^3(t)$ & $\omega(t) = \frac{kc}{a(t)}$
- $\mathrm{L} = \frac{m}{2}\dot{q}^2 - \frac{m\omega^2}{2}q^2$

  $\rightarrow \dot{q}^2 + 3H\dot{q} + \omega^2 q^2 = 0$

  $\rightarrow q(t) = u(t,k)\alpha + u^*(t,k)\alpha^+,\ \ [\alpha, \alpha^+] = 1$
- $u(t,k) = \frac{H}{\sqrt{2k^3}}[1 - ix]e^{ix}$, $x = \frac{k}{Ha(t)}$
- $k/a(t) \ll H \rightarrow x \ll 1 \rightarrow u(t,k) \sim$ constant
- $< \Omega|E(t)|\Omega > = [N + \frac{1}{2}]\hbar\omega$, $N(t,k) = [\frac{1}{2x}]^2$

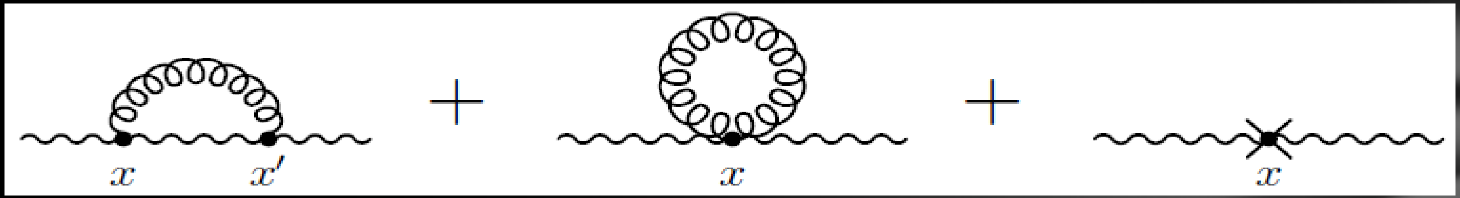
Motivation

- Gravitons + Massless Fermion
  - 1st IR log using dimensional regularization in de Sitter
  - Fermion mode fun. $\sim \# G H^2 \ln[a(t)]$ at 1-loop
- No $\# G H^2 \ln[a(t)]$ in ``QG + MMC'' (Kahya & Woodard)
- IR gravitons only couple to mmc through red-shift K.E.
- But fermions has extra SPIN in addition (0803.2377)
- secular effects from spin-spin interactions
- simple rules for catching leading log. has derived
- It differs from Starobinsky's IR truncation
- Photon also has spin

What We Did

$\mathcal{L} = \frac{(R-2\Lambda)\sqrt{-g}}{16\pi G} - \frac{1}{4} F_{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\sigma\nu} \sqrt{-g}$ +c-terms

1. Compute $i[^{\mu}\Pi^{\nu}](x;x')$ from gravitons at 1 loop in dim. reg.



2. Renormalize with BPHZ counter terms (cf. LEFT)

$$\Delta\mathcal{L} = C_4 D_\alpha F_{\mu\nu} D_\beta F_{\rho\sigma} g^{\alpha\beta} g^{\mu\rho} g^{\nu\sigma} \sqrt{-g} + \bar{C} H^2 F^{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma} \sqrt{-g}$$
$$+ \Delta C H^2 F_{ij} F_{kl} g^{ik} g^{jl} \sqrt{-g}$$

3. Quantum-correct Maxwell's equations

$$\partial_\nu\left[\sqrt{-g}\, g^{\nu\rho} g^{\mu\sigma} F_{\rho\sigma}(x)\right] + \int d^4x' [^{\mu}\Pi^{\nu}](x;x') A_\nu(x') = J^{\mu}(x)$$

- $J^{\mu} = 0$ ➔ dynamical photons
- $J^{\mu} \neq 0$ ➔ electromagnetic forces

Why there should be an effect

Neither photons nor gravitons are charged

- So how can they polarize the vacuum?

But they DO carry momentum

- Dynamical photons have $\vec{p} = \hbar c \vec{k}$
- Virtual photons carry $\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$

Virtual gravitons can add or subtract to this

- de Sitter gravitons $\left(\vec{k}, \lambda\right)$ have $N(t, k) = \left[\frac{Ha(t)}{2ck}\right]^2$
- Gravitons with $k_{phys} \sim H \gg k_{photon} = \frac{k}{a(t)}$

What happens on a flat background

arXiv:1202.5800 by Leonard & Woodard

- $\kappa^2 \equiv 16\pi G$ $\quad\quad$ $\Delta x^2 \equiv \|\vec{x} - \vec{x}'\|^2 - (|t - t'| - i\epsilon)^2$
- $i[^\mu \Pi^\nu](x; x') = (\eta^{\mu\nu}\partial' \cdot \partial - \partial'^\mu \partial^\nu)\left[\frac{\#\kappa^2}{\Delta x^{2D-2}}\right]$
- $\Delta\mathcal{L} = C_4 D_\alpha F_{\mu\nu} D_\beta F_{\rho\sigma} g^{\alpha\beta} g^{\mu\rho} g^{\nu\sigma}\sqrt{-g}$
- $i[^\mu \Pi^\nu_{ren}](x; x') = (\eta^{\mu\nu}\partial' \cdot \partial - \partial'^\mu \partial^\nu)\partial^4\left[\frac{\kappa^2}{192\pi^2}\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2}\right]$
- No change in dynamical photons
- Coulomb: $\Phi(r) = \frac{q}{4\pi r}\left[1 + \frac{2G}{3\pi r^2} + O(G^2)\right]$
  - Radkowski (1970)
  - 0th order field distorts virtual gravitons nearby & these add momentum to virtual photons carrying force

$i[^{\mu}\Pi^{\nu}](x;x')$ for de Sitter

arXiv:1304.7265 by Leonard &Woodard

de Sitter breaking ➔ 2 structure functions

- $i[^{\mu}\Pi^{\nu}] = (\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho})\partial_{\rho}\partial'_{\sigma}F(x;x')$
  $+(\bar{\eta}^{\mu\nu}\bar{\eta}^{\rho\sigma} - \bar{\eta}^{\mu\sigma}\bar{\eta}^{\nu\rho})\partial_{\rho}\partial'_{\sigma}G(x;x')$
- $\bar{\eta}^{\mu\nu} \equiv \eta^{\mu\nu} + \delta^{\mu}_{0}\delta^{\nu}_{0}$

de Sitter breaking ➔ noninvariant counterterm

- Regulation tech. + gauge fixing
- It's easy to criticize when you never compute anything

But simple EFE in conformal coordinates

- $\partial_{\nu}F^{\nu\mu}(x) + \partial_{\nu}\int d^4x'[iF(x;x')F^{\nu\mu}(x') + iG(x;x')\bar{F}^{\nu\mu}(x')] = J^{\mu}(x)$
- Solve perturbatively using Schwinger-Keldysh

1-loop structure functions

$$F(x;x') = \frac{\kappa^2}{8\pi^2}\left\{H^2\left[\ln(a)+\alpha\right] + \frac{1}{a^2}\left[-\frac{1}{3}\ln(a)+\beta\right]\left(\partial^2 + 2Ha\partial_0\right) + \frac{H}{3a}\partial_0\right\} i\delta^4(x-x')$$
$$-\frac{\kappa^2}{1536\pi^4}\frac{1}{a}\partial^6\left\{\frac{1}{a'}\left[\ln^2\left(\frac{1}{4}H^2\Delta x^2\right) - 2\ln\left(\frac{1}{4}H^2\Delta x^2\right)\right]\right\}$$
$$+\frac{\kappa^2 H^2}{128\pi^4}\left\{\left[\frac{1}{4}\partial^4 + \partial^2\partial_0^2\right]\ln^2\left(\frac{1}{4}H^2\Delta x^2\right) + \left[-\frac{1}{2}\partial^4 + 2\partial^2\partial_0^2\right]\ln\left(\frac{1}{4}H^2\Delta x^2\right)\right\},$$

$$G(x;x') = \frac{\kappa^2 H^2}{6\pi^2}\left[-\ln(a)+\gamma\right] i\delta^4(x-x')$$
$$-\frac{\kappa^2 H^2}{384\pi^4}\partial^4\left\{\ln^2\left(\frac{1}{4}H^2\Delta x^2\right) - 2\ln\left(\frac{1}{4}H^2\Delta x^2\right)\right\}.$$

Quantum Maxwell for de Sitter photons

arXiv:1408.1448 with Wang & Woodard

0th order photons on de Sitter same as flat

- $F^{0i}_{(0)} = ik\varepsilon^i e^{-ik\eta + i\vec{k}\cdot\vec{x}}$, $F^{ij}_{(0)} = i(k^i\varepsilon^j - k^j\varepsilon^i)\, e^{-ik\eta + i\vec{k}\cdot\vec{x}}$
- $\varepsilon^i(\vec{k},\lambda)$ same as in flat space

EFE's only reliable at late times

- Missing perturbative initial state corrections
- Counterterms have arbitrary finite parts

1 loop field strengths: E grows and B falls

- $F^{0i}_{(1)} \rightarrow F^{0i}_{(0)} \times \frac{\kappa^2 H^2 \ln(a)}{8\pi^2}$, $F^{ij}_{(1)} \rightarrow F^{ij}_{(0)} \times \frac{\kappa^2 H^2}{8\pi^2} \frac{ik\ln(a)}{Ha}$
- Inflationary gravitons jiggle the photon

Perturbation theory eventually breaks down

- Needs $\ln(a) = Ht \sim \frac{8\pi^2}{\kappa^2 H^2} > 10^{10}$ e-foldings!

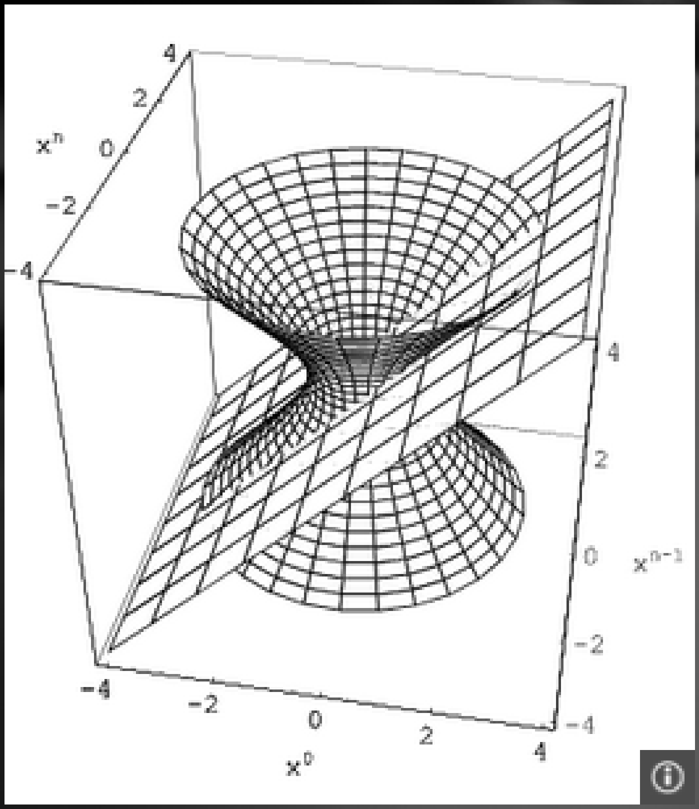
Co-Moving versus Static Observers

FLAT SLICING

- $ds^2 = a^2[-d\eta^2 + d\vec{x}\cdot d\vec{x}]$
- $a = \frac{-1}{H\eta} \qquad (-\infty < \eta < 0)$
- $D(0, x; \eta) = ax$ GROWS

STATIC COORDINATES

- $ds^2 = -[1 - H^2r^2]dt^2 + \frac{dr^2}{1-H^2r^2} + r^2 d\Omega^2$
- $0 \le r < H^{-1}$ (STUFF LEAVES)
- $D(0, r; t) = H^{-1}\arcsin(Hr)$ CONSTANT
- $a = e^{Ht}\sqrt{1 - H^2r^2} \quad , \qquad x = \frac{r}{a}$



Co-moving charge & magnetic dipole

arXiv:1308.3453 by Glavan, Miao, Prokopec & Woodard

- $J^\mu(\eta, \vec{x}) = q\delta^\mu_0 \delta^3(\vec{x})$
  - $\Phi = \frac{q}{4\pi x}\left\{1 + \frac{\kappa^2 H^2}{8\pi^2}\left[\frac{1}{3a^2H^2x^2} + \ln(aHx)\right] + \cdots\right\}$
  - Inflationary gravitons have $p \sim H \gg \frac{k}{a} \sim \frac{1}{ax}$
- $J^\mu(\eta, \vec{x}) = \epsilon^{0\mu jk} m_j \partial_k \delta^3(\vec{x})$
  - $\vec{A} = \vec{m} \times \vec{\nabla}\left(\frac{1}{4\pi x}\left\{1 + \frac{\kappa^2 H^2}{8\pi^2}\left[\frac{1}{3a^2H^2x^2} - \frac{2}{3}\ln(Hx)\right] + \cdots\right\}\right)$
  - Not quite flat space but no secular enhancement

Static charge & magnetic dipole

arXiv:1308.3453 by Glavan, Miao, Prokopec & Woodard

Point charge

- $\widetilde{\Phi} = \frac{q}{4\pi r}\left\{1 + \frac{\kappa^2 H^2}{8\pi^2}\left[\frac{1}{3H^2 r^2} + \ln(Hr)\right] + \cdots\right\}$
- No secular effect but a logarithmic de Sitter tail

Point magnetic dipole

$$\widetilde{F}_{0i}(\tau,\vec{r}) = e^{H\tau}\sqrt{1-H^2r^2}\,\frac{H\epsilon^{ijk}m^j\widehat{r}^k}{4\pi r^2}\left\{1 + \frac{\kappa^2 H^2}{8\pi^2}\left[\frac{5}{H^2r^2} + \frac{2}{3}\left[2-\ln\left(\frac{Hr}{a}\right)\right] + \text{Irrelevant}\right] + O(\kappa^4)\right\},$$

$$\widetilde{F}_{ij}(\tau,\vec{r}) = \frac{e^{H\tau}}{\sqrt{1-H^2r^2}}\,\frac{\epsilon^{ijk}}{4\pi r^3}\left\{m^k - 3\vec{m}\cdot\widehat{r}\widehat{r}^k\left[1-\frac{2}{3}H^2r^2\right] + \frac{\kappa^2H^2}{8\pi^2}\left[\frac{3m^k - 5\vec{m}\cdot\widehat{r}\widehat{r}^k}{H^2r^2} -\frac{2}{3}m^k\left[1+\ln\left(\frac{Hr}{a}\right)\right] + 2\vec{m}\cdot\widehat{r}\widehat{r}^k\left[\frac{2}{3}+\frac{2}{3}H^2r^2+\left(1-\frac{2}{3}H^2r^2\right)\ln\left(\frac{Hr}{a}\right)\right] + \text{Irrel.}\right] + O(\kappa^4)\right\}.$$

The Gauge Issue:

Photon & gravitons require fixing!

- EM gauge irrelevant because coupling to $F_{\mu\nu}$
- But flat $\partial^\mu h_{\mu\nu} = (\frac{b}{2})\partial_\nu h$ gives

$$i\left[{}^\mu\Pi^\nu_{\rm flat}\right](x;x') = \frac{\kappa^2\Gamma(\frac{D}{2})\Gamma(\frac{D}{2}-1)}{16(D-1)\pi^D}\Big[C_2+C_0(b)\Big]\Big[\eta^{\mu\nu}\partial'\cdot\partial-\partial'^\mu\partial^\nu\Big]\frac{1}{\Delta x^{2D-2}}$$

$$C_2 = \frac{(D-4)(D-2)^2(D+1)(D+2)}{4(D-1)}$$

$$C_0(b) = -\frac{1}{4}(D-2)^2\left[\left(\frac{Db-2}{b-2}\right)^2-4\left(\frac{D-4}{D-2}\right)\left(\frac{Db-2}{b-2}\right)+\frac{2(D-4)^2}{(D-2)(D-1)}\right]$$

- The result of Bjerrum-Bohr (hep-th/0206236)
  - Effect in gauge independent S-matrix

Misconceptions about the gauge issue

1. Gauge fixed GF's are complete nonsense
   - We construct the flat space S-matrix from them!
   - Need to separate physical information from unphysical
2. Gauge invariant GF's are different from gauge fixed
   - Every gauge fixed GF represents some invariant

   $A_0(t,x) = 0 = A_1(0,x) \;\; \rightarrow \;\; A_1(t,x) = \int_0^t ds\, F_{01}(s,x)$

   - Gauge independent GF's are sums of gauge fixed

   E.g., Woodard's thesis work !
3. This is a trivial problem
   - No cosmological S-matrix → what is being measured?
   - Why is everyone ignoring this problem?

Our Conjecture:

arXiv:1204.1784 by Miao & Woodard

- $i[^\mu\Pi^\nu]$ on de Sitter must inherit the gauge dependence of $i[^\mu\Pi^\nu]$ on flat background
- Maybe leading secular effects gauge independent
  - NB these effects aren't present in flat space
  - Cf. poles terms of gauge fixed GF's in flat space QFT
- How to check? ➔ re-compute in other gauges
  - EM gauge doesn't matter
  - General covariant GR gauge: $D^\mu h_{\mu\nu} = \frac{b}{2}\partial_\nu h^\mu_\mu$
  - arXiv:1205.4468 by Mora, Tsamis & Woodard

Old gauge noncovariant but SIMPLE

$g_{\mu\nu} \equiv a^2[\eta_{\mu\nu} + \kappa h_{\mu\nu}]$

$$i[_{\mu\nu}\Delta_{\rho\sigma}](x;x') = \sum_{I=A,B,C} [_{\mu\nu}T^I_{\mu\nu}] \times i\Delta_I(x;x')$$

- $[_{\mu\nu}T^I_{\rho\sigma}]$ are constants
- $i\Delta_I(x;x')$ are simple functions in D=4
  - $y(x;x') \equiv H^2 aa'[\|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\epsilon)^2]$
  - $i\Delta_A \sim \frac{1}{y} - \ln(y) + \ln(aa')$, $i\Delta_B \sim i\Delta_C \sim \frac{1}{y}$
- But our gauge does break de Sitter invariance
  - We need a noninvariant counterterm
  - The mathematical physicists don't like this

New gauge covariant but HARD

$$g_{\mu\nu} \equiv a^2\eta_{\mu\nu} + \kappa h_{\mu\nu}$$

- $i[{}_{\mu\nu}\Delta_{\rho\sigma}] = i[{}_{\mu\nu}\Delta^2_{\rho\sigma}] + i[{}_{\mu\nu}\Delta^0_{\rho\sigma}]$
- $i[{}_{\mu\nu}\Delta^2_{\rho\sigma}](x;x') = \boldsymbol{P}^{\alpha\beta}_{\mu\nu}(x)\, \boldsymbol{P}^{\gamma\delta}_{\rho\sigma}(x')\big[\partial_\alpha\partial'_\gamma y\ \partial_\beta\partial'_\delta y\ S^2(x;x')\big]$
  - $\boldsymbol{P}^{\alpha\beta}_{\mu\nu}$ 4^{th} order, transverse & traceless
  - $S^2(x;x')$ independent of b but breaks de Sitter
- $i[{}_{\mu\nu}\Delta^0_{\rho\sigma}](x;x') = P_{\mu\nu}(x)\, P_{\rho\sigma}(x')[S^0(x;x')]$
  - $P_{\mu\nu} = D_\mu D_\nu - \beta^{-1} g_{\mu\nu}[\square - (\beta - D)H^2]$ $\qquad \beta \equiv \frac{Db-2}{b-2}$
  - $S^0(x;x')$ depends on b but de Sitter invariant for $b > 2$
- Easier to enthuse over de Sitter invariance without computing!
  - Tensor factors no longer constant
  - Propagator expansions no longer terminate for $D = 4$

STILL need noninvariant counterterm!

$$i\left[{}^{\mu}\Pi^{\nu}_{3\mathrm{pt}}\right](x;x') = \partial_\kappa \partial'_\theta \Big\{ i\kappa a^{D-6} V^{\mu\rho\kappa\lambda\alpha\beta}\, i\left[{}_{\alpha\beta}\Delta_{\gamma\delta}\right](x;x') \times i\kappa a'^{D-6} V^{\nu\sigma\theta\phi\gamma\delta}\, \partial_\lambda \partial'_\phi i\left[{}_{\rho}\Delta_{\sigma}\right](x;x') \Big\}$$

- Three reasons:
  1. Time-ordered interactions
  2. QG interactions have two derivatives

     $\partial_\mu \partial'_\nu \, i\Delta(x;x') = \#\delta^0_\mu \delta^0_\nu \, i\delta^D(x-x')$ +naive
  3. Coincident graviton propagator diverges
- Analytic continuation from Euclidean fails
  - It also misses power law IR divergences
  - And sometimes results in negative-normed states

Spin 2 Structure Functions

$$\mathcal{L}(y) \equiv \mathrm{Li}_2\Big(\frac{y}{4}\Big) + \ln\Big(1-\frac{y}{4}\Big)\ln\Big(\frac{y}{4}\Big) - \frac{1}{2}\ln^2\Big(\frac{y}{4}\Big)$$

$$F_2(x;x') = \frac{85\kappa^2 H^2}{72\pi^2}\,\ln(a) i\delta^4(x-x') - \frac{\kappa^2 H^2}{16\pi^4}\Big[\ln(aa')+\frac{1}{3}-\gamma\Big]\nabla^2\Big(\frac{1}{\Delta x^2}\Big)$$

$$+\frac{5\kappa^2 H^2}{144\pi^4}\,\partial^2\Big(\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big) - \frac{5\kappa^2 H^6 (aa')^2}{144\pi^4}\Big\{\frac{\mathcal{L}(y)}{2} + \frac{2(2-y)\ln(\frac{y}{4})}{4y-y^2} + \frac{2}{y}\Big\}$$

$$G_2(x;x') = -\frac{5\kappa^2 H^2}{4\pi^2}\,\ln(a) i\delta^4(x-x') + \frac{\kappa^2 H^2}{24\pi^4}\Big[\ln(aa')+\frac{1}{3}-\gamma\Big]\nabla^2\Big(\frac{1}{\Delta x^2}\Big)$$

$$+\frac{\kappa^2 H^2 aa'}{96\pi^4}(\partial_0^2+\nabla^2)\Big(\frac{1}{\Delta x^2}\Big) + \frac{5\kappa^2 H^6 (aa')^2}{72\pi^4}\Big\{\frac{(1-y)\mathcal{L}(y)}{4} + \frac{(y-3)\ln(\frac{y}{4})}{4-y}\Big\}$$

Spin 0 Structure Functions

$$F^{\rm ren}_{0d}(y) = \Big(\frac{2b-1}{b-2}\Big)^2 \Big\{ \frac{\kappa^2}{48\pi^2} \frac{\ln(a)}{aa'} \partial^2 i\delta^4(x-x') - \Big(\frac{b-8}{b-2}\Big) \frac{\kappa^2 H^2}{72\pi^2} i\delta^4(x-x') - \frac{\kappa^2 H}{48\pi^2 a} \partial_0 i\delta^4(x-x') + \frac{\kappa^2}{384\pi^4} \frac{\partial^4}{aa'} \Big[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big] + \Big(\frac{b-8}{b-2}\Big) \frac{\kappa^2 H^2}{576\pi^4} \partial^2 \Big[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big] - \frac{\kappa^2 H^6}{6\pi^4} (aa')^2 \mathcal{N}_F(y) \Big\}$$

$$G^{\rm ren}_{0d}(y) = \Big(\frac{2b-1}{b-2}\Big)^2 \Big\{ \frac{\kappa^2 H^2}{24\pi^2} \Big[1 - \ln(a)\Big] i\delta^4(x-x') - \frac{\kappa^2 H^2}{192\pi^4} \partial^2 \Big[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big] + \frac{\kappa^2 H^6}{12\pi^4} (aa')^2 \mathcal{N}_G(y) \Big\}$$

Conclusions

- Some terms same but others different
- Must re-solve EFE to check conjecture
- Tedious but straightforward integrations
- Intuition says the effect is real
    - Inflation really does produce huge ensemble of IR
    - Should scatter photons more the further they go
    - This would not even be doubted in flat space
- But proving it IS an important step in cosmo QFT

One more thought…

- $i[^{\mu}\pi^{\nu}](x;x') =< \Omega \left| \frac{i\delta S}{\delta A_{\mu}(x)} \frac{i\delta S}{\delta A_{\nu}(x')} + \frac{i\delta^2 S}{\delta A_{\mu}(x)\delta A_{\nu}(x')} \right| \Omega >$
- Gauge dependent because x^{μ} & x'^{μ}not geometrically fixed
- So fix it !

E. g., geodesic from origin in direction x^{μ}