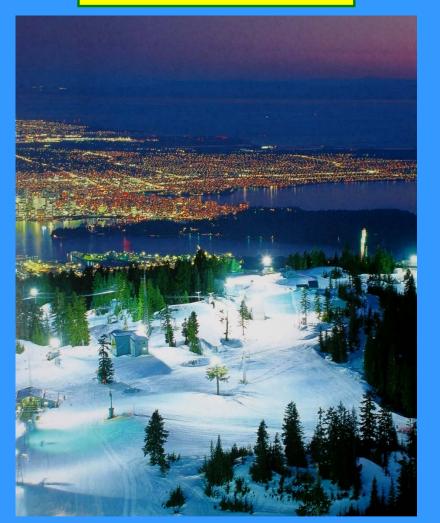
CORRELATED WORLDLINE THEORY OF QUANTUM GRAVITY: LOW-ENERGY CONSEQUENCES & TABLE-TOP TESTS

P.C.E. STAMP

Galiano, Aug 17th 2015



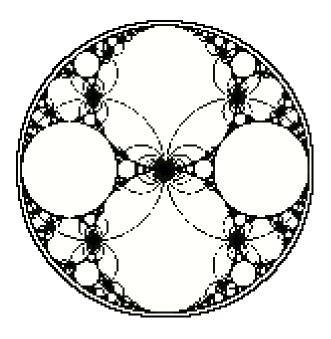
Physics & Astronomy UBC Vancouver





Pacific Institute for Theoretical Physics The talk will address the following themes:

- (i) The Correlated Worldline Theory: basic structure
- (ii) Thought experiments & Real Experiments

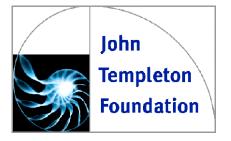


FURTHER INFORMATION:

Email: stamp@physics.ubc.ca Web: http://www.physics.ubc.ca/~berciu/PHILIP/index.html







CONFLICT BETWEEN GENERAL RELATIVITY & GRAVITY

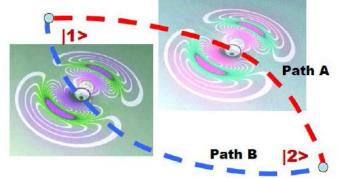
OLD ARGUMENTS: Feynman 1957, Karolhazy 1966, Eppley-Hannah 1977, Kibble 1978-82, Page 1981, Unruh 1984, Penrose 1996, argued that there is a basic conflict between the superposition principle & GR at ordinary 'table-top' energies.

Consider a 2-slit experiment with a mass M. Suppose we assume a 'wave-fn':

 $|\Psi
angle \ = \ a_1 |\Phi_1; \tilde{g}^{\mu
u}_{(1)}(x)
angle + a_1 |\Phi_2; \tilde{g}^{\mu
u}_{(2)}(x)
angle$

In a non-relativistic treatment we write

$$\begin{split} \Phi(\mathbf{r},t) &\equiv \langle \mathbf{r} | \Phi(t) \rangle = a_1 \Phi_1(\mathbf{r},t) + a_2 \Phi(\mathbf{r},t) \\ \text{and then: } \langle \Phi_1(t) | \Phi_2(t) \rangle = \int d^3 r \ \langle \Phi_1^*(\mathbf{r},t) | \Phi_2(\mathbf{r},t) \rangle \end{split}$$

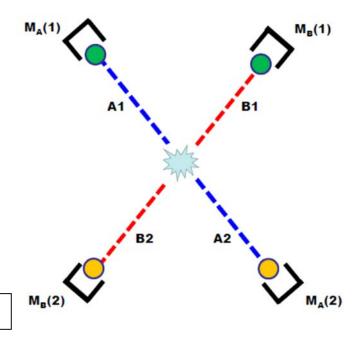


But now we have both a formal and a physical problem.

- (i) FORMAL PROBLEM: There are 2 different coordinate systems. (\mathbf{r}_i, t_j) , defined by the 2 different metrics $\tilde{g}_{(j)}^{\mu\nu}(x)$, & in general we cannot relate these.
- (ii) <u>PHYSICAL PROBLEM</u>: A "wave-function collapse" causes non-local changes, which if linked to the metric cause drastically unphysical changes in the metric.

MORE RECENT ARGUMENTS: These all revolve around the continued success in producing larger and larger solid-state superpositions – both for single objects and for EPR systems.

All of these arguments related to Q Mmt problem



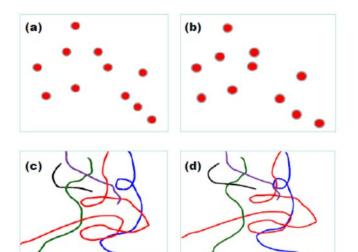
INGREDIENTS for a NEW THEORY

We would like to set up a theory which confronts the clash between GR & QM.

The difficulty is that both work really well, and have never been falsified. So we need to decide what to keep, & what to throw away.

Consider some arbitrary set of points and of lines and now <u>take away the spacetime receptacle, &</u> <u>the labels</u>. How do we COMPARE 2 configurations?

(i) The chronometry along the worldlines (uses QM, via E = mc²), to give us the connection





Note the key role played by indistinguishability in a quantum theory. NOW MAKE FOLLOWING ASSUMPTIONS

<u>ASSUMPTION 1</u>. The existence of world-lines in spacetime is fundamental. Spacetime is then defined by the world-lines of objects or fields.

<u>ASSUMPTION 2</u>. Superpositions and interference exist in Nature (along with entanglement); and the phase ϕ along world-lines is given by $\phi = hS$ where S is the worldline action.

These first two lead us to a path integral formulation

<u>ASSUMPTION 3</u>. The comparison/communication between different spacetimes in a superposition is achieved – indeed defined - by gravity itself. This is why it couples universally to matter. The comparison is one of accumulated phase along the worldlines (cf. assumptions 1 and 2).

What we wish now is to argue that this leads to a picture in which paths are correlated – so that the superposition principle is no longer valid

RULES of the GAME

First, the following question – basically a question about **DIETARY RESTRICTIONS**:

Q1: What is the most general modification we can make to QM/QFT, consistent with those features we wish to keep?

These features are:

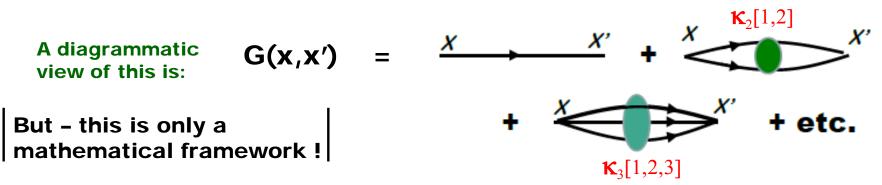
- (i) connection between phase (+ connection), and action on worldlines (paths)
- (ii) indistinguishability for multiple particles and/or fields
- (iii) fully relativistic obeying the weak principle of equivalence, no violation of causal structure, well-defined metric.

(iv) gravity/spacetime is treated as a quantum field as well as matter

The answer goes as follows; we change the mathematics to:

$$G_{o}(2,1) = \int_{1}^{2} \mathcal{D}q(\tau) e^{\frac{i}{\hbar}S(2,1)} \longrightarrow \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int_{1}^{2} \mathcal{D}q_{k}(\tau) \kappa_{n}[\{q_{k}\}] e^{\frac{i}{\hbar}S[q_{k};2,1]}$$

In other words, we allow arbitrary correlations between any number of different paths. Since the paths are no longer independent, the superposition principle is no longer valid in general !



The answer to the 1st question gave us a framework with almost infinite freedom to choose different correlators – in this sense it is almost completely useless.

Now a 2nd question, which is about <u>CULINARY CHOICE</u>

Q2: If the correlation between paths is "gravitational", what does this imply for the correlators κ_n[q₁,....q_n] ?

For all situations we will ever face on earth (and in most astrophysical situations) the following works:

(1) Use the action:
$$S_G = \frac{1}{\lambda^2} \int d^4x \left[\tilde{g}^{\mu\nu} R_{\mu\nu} - \frac{1}{2\alpha} (\partial_\mu \tilde{g}^{\mu\nu})^2 \right]$$
 with gauge-fixing term

(2) Use the correlator:

$$\kappa_n = \int \mathcal{D}\tilde{\mathfrak{g}}^{\mu\nu}(x) \ e^{\frac{i}{\hbar}S_G} \Delta[\tilde{\mathfrak{g}}^{\mu\nu}(x)]$$

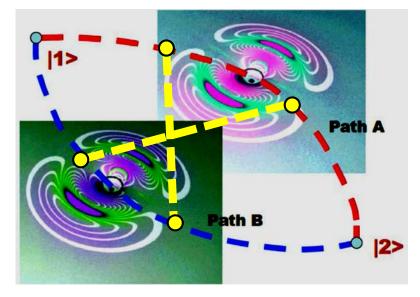
ie., integrate over different spacetimes with a weighting factor

metric gravita density action

gravitational Faddeev-Popov action determinant

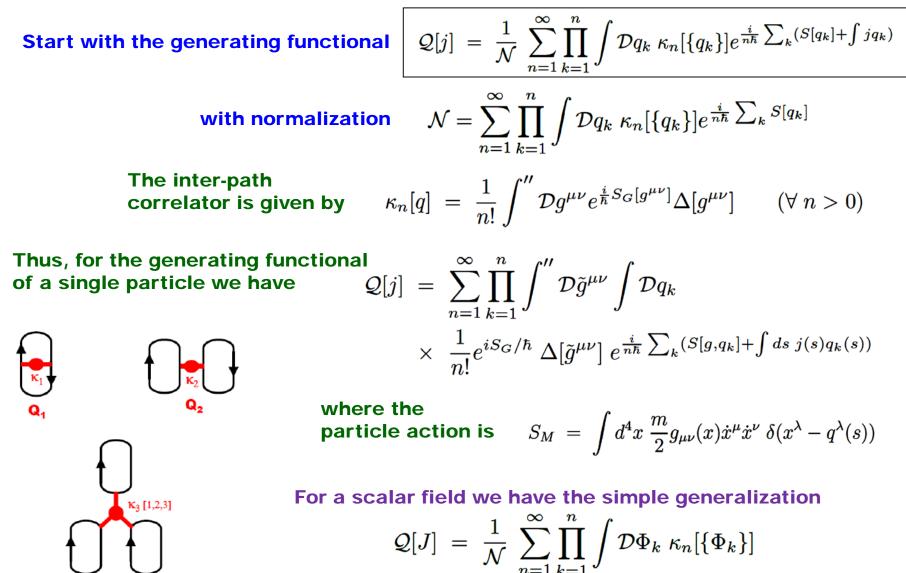
Now what this does is **COMMUNICATE BETWEEN PATHS** the information about each path's spacetime status (and what the object is doing to spacetime).

We then get a PREDICTIVE THEORY with NO ADJUSTABLE PARAMETERS



CWL THEORY: FORMAL STRUCTURE

I. GENERATING FUNCTIONAL



 $\times e^{\frac{i}{n\hbar}(S[\Phi_k] + \int d^4x J(x)\Phi_k(x))}$

II. CORRELATION FUNCTIONS

=

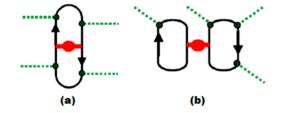
For a single particle we define the CWL correlator

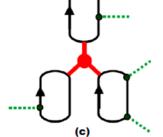
$$\begin{aligned} \mathcal{G}_{n}^{\sigma_{1},..\sigma_{n}}(s_{1},..s_{n}) &= \left(\frac{-i}{\hbar}\right)^{n} \lim_{j(s)\to 0} \left[\frac{\delta^{n}\mathcal{Q}[j]}{\delta j(s_{1}\sigma_{1})..\delta j(s_{n}\sigma_{n})}\right] \\ &= \sum_{r=1}^{\infty} \prod_{\alpha=1}^{r} \int \mathcal{D}q^{\alpha}(\tau) \; e^{\frac{i}{r\hbar}\sum_{\alpha} S_{o}[q^{\alpha}]} \; \kappa_{r}(\{q^{\alpha}\}) \; \prod_{j=1}^{n} \left(\sum_{\alpha=1}^{r} q^{\alpha}(s_{j},\sigma_{j})\right) \\ &\sum_{r=1}^{\infty} \frac{1}{r!} \int^{\prime\prime} \mathcal{D}g^{\mu\nu} e^{\frac{i}{\hbar}S_{G}[g^{\mu\nu}]} \Delta[g^{\mu\nu}] \prod_{\alpha=1}^{r} \int \mathcal{D}q^{\alpha}(\tau) \; e^{\frac{i}{r\hbar}\sum_{\alpha} S_{o}[q^{\alpha},g^{\mu\nu}]} \; \prod_{j=1}^{n} \left(\sum_{\alpha=1}^{r} q^{\alpha}(s_{j},\sigma_{j})\right) \end{aligned}$$

We can represent this messy formula diagrammatically by the sum shown at right, where the green hashed lines represent current insertions – we sum over all combinatoric possibilities.

The same structure exists for a set of fields. Thus, eg., for a single scalar field we have the explicit expansion, for the 4-point correlator, given by

$$\mathbb{G}_{4}^{\sigma_{1},..\sigma_{4}}(x_{1},..x_{4}) = \oint \mathcal{D}\phi(x) e^{\frac{i}{\hbar}S[\phi]} \prod_{j=1}^{4} \phi(x_{j},\sigma_{j}) + \int \mathcal{D}\phi(x) \oint \mathcal{D}\phi'(x) e^{\frac{i}{\hbar}(S[\phi]+S[\phi'])} \kappa_{2}[\phi,\phi'] \prod_{j=1}^{4} [\phi(x_{j},\sigma_{j}) + \phi'(x_{j},\sigma_{j})]$$

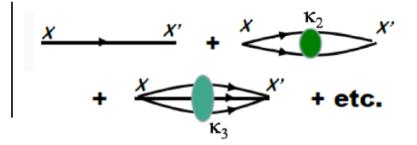




III. STRUCTURE of PROPAGATORS

Recall that in ordinary QM we have the 1-particle propagator:

$$K(x,x') = \int_{x'}^{x} \mathcal{D}q \; e^{\frac{i}{\hbar}S_M[q]}$$



In CWL theory we have the generalization:

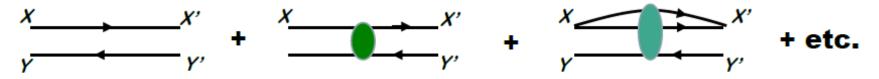
$$\mathcal{K}(x,x') = \int \mathcal{D}g^{\mu\nu} \Delta(g) \ e^{\frac{i}{\hbar}S_G[g^{\mu\nu}]} \sum_{n=1}^{\infty} \prod_{k=1}^n \int_{x'}^x \mathcal{D}q_k \ e^{\frac{i}{n\hbar}\sum_k S_M[q_k,g^{\mu\nu}]}$$

which is shown diagrammatically at top right.

For a many-body system we can define the N-particle propagator

$$\mathcal{K}_{N}(x_{1},...x_{N};x_{1}',...x_{N}') = \prod_{j=1}^{N} \int \mathcal{D}g^{\mu\nu} \Delta(g) \ e^{\frac{i}{\hbar}S_{G}[g^{\mu\nu}]} \sum_{n_{j}=1}^{\infty} \prod_{k_{j}=1}^{n_{j}} \int_{x_{j}'}^{x_{j}} \mathcal{D}q_{k_{j}}^{(j)} \ e^{\frac{i}{n_{j}\hbar}\sum_{k_{j}=1}^{n_{j}}S_{M}[q_{k_{j}}^{(j)},g^{\mu\nu}]}$$

Diagrammatically we have:



All of this has an obvious generalization to fields – for propagation between initial and final field configurations

IV. <u>CONDITIONAL / COMPOSITE PROPAGATORS</u>

Let's first recall that in conventional QFT we can define the composite propagator/correlator: $\chi_1^{(p)}(x, x'|\{q(t_{\alpha})\}) = \langle x|\hat{T}\{q(t_1), ...q(t_p)\}|x'\rangle$

 $\chi_1^{(x,x)}|\{q(t_{\alpha})\}\rangle = \langle x|I|\{q(t_{\alpha})\}\rangle$ which has the path integral representation: $\chi_1^{(p)}(x,x'|\{q(t_{\alpha})\}) = \int_x^x \mathcal{D}q \ e^{\frac{i}{\hbar}S[q]} \prod_{j=1}^p f_{j}^{(p)}$

$$\begin{aligned} x'|\{q(t_{\alpha})\}) &= \int_{x'}^{x} \mathcal{D}q \; e^{\frac{i}{\hbar}S[q]} \prod_{\alpha=1}^{p} q(t_{\alpha}) \\ &= (-i\hbar)^{p} \frac{\delta^{p}}{\delta j(t_{1})...\delta j(t_{p})} \; K_{1}(x,x'|j(t)) \Big|_{j=0} \end{aligned}$$

where we have defined the external current-dependent propagator:

$$K_1(x, x'|j) = \int_{x'}^x \mathcal{D}q \ e^{\frac{i}{\hbar}(S[q] + \int dt j(t)q(t))}$$

Now in CWL theory we have

$$\chi_1^{(p)}(x, x' | \{q(t_{\alpha})\}) = (-i\hbar)^p \frac{\delta^p}{\delta j(t_1) \dots \delta j(t_p)} \mathcal{K}_1(x, x' | j(t)) \Big|_{j=0}$$

where now the propagator involves the CWL sum:

$$\mathcal{K}_1(x,x'|j) = \int \mathcal{D}g^{\mu\nu} \Delta(g) \ e^{\frac{i}{\hbar}S_G[g^{\mu\nu}]} \sum_{n=1}^{\infty} \prod_{k=1}^n \int_{x'}^x \mathcal{D}q_k \ e^{\frac{i}{n\hbar}\sum_k \left(S_M[q_k,g^{\mu\nu}] + \int dt j(t)q_k(t)\right)}$$

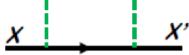
Working this out we get:

$$\chi_{1}^{(p)}(x,x'|\{q(t_{\alpha})\}) = \int \mathcal{D}g^{\mu\nu}\Delta(g) \ e^{\frac{i}{\hbar}S_{G}[g^{\mu\nu}]} \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int_{x'}^{x} \mathcal{D}q_{k} \ e^{\frac{i}{n\hbar}\sum_{k} S_{M}[q_{k},g^{\mu\nu}]} \prod_{\alpha=1}^{p} \left(\sum_{k'=1}^{n} q_{k}'(t_{\alpha})\right)$$

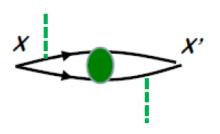
which has the diagrammatic interpretation shown on the next page

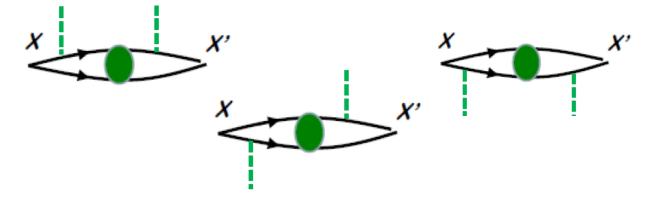
DIAGRAMMATIC INTERPRETATION

Consider for example a 1-particle propagator with 2 current insertions. Then the conventional QFT result is

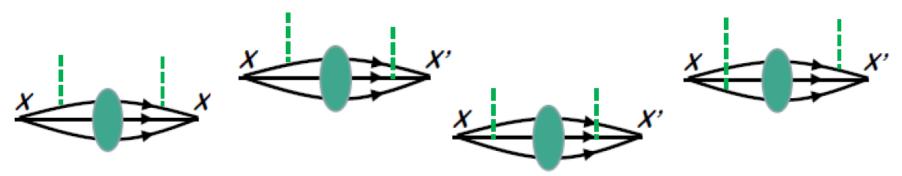


The first set of CWL corrections looks like:





The next set of CWL corrections looks like:



and so on....

HIGHER CONDITIONAL / COMPOSITE PROPAGATORS

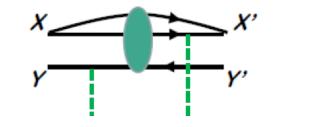
Consider, eg., the 2-particle propagator. Without writing down the formulas, it is obvious what we will get.

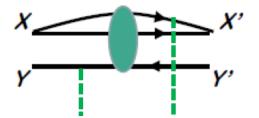
Thus, eg., if we have 2 external insertions, and the 2 particles are distinguishable, we have



(b) CWL corrections:

The lowest-order terms are:





It is fairly obvious where one goes on from here.

V. GRAVITON EXPANSIONS



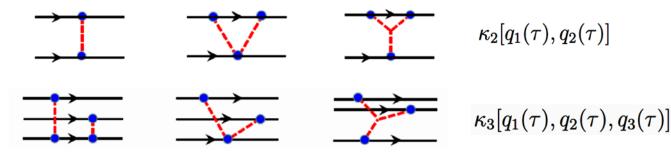
Suppose we make an expansion about a background spacetime – in this case flat space. Then:

$$\tilde{\mathfrak{g}}^{\mu
u}(x) = \eta^{\mu
u} + \lambda h^{\mu
u}(x)$$



The Lagrangian is written as a graviton expansion: $L_G = L_o - \int d^4 x U(h^{\mu\nu})$

The CWL generating functional then has the form shown at right, and the correlators have terms like those shown below.



EXAMPLE: DENSITY MATRIX PROPAGATOR

$$\begin{array}{lll} \text{Define } \hat{h}(x) = h^{\mu\nu}(x) \\ \text{and } \hat{D}(x) = D^{\mu\nu\lambda\rho}(x) \end{array} \quad \text{Then} \quad \begin{array}{lll} \mathcal{K}_{2,2';1,1'} &= \lim_{\hat{h}=0} \; \{e^{\frac{i}{2\hbar}(\delta_{\hat{h}}|\hat{D}|\delta_{\hat{h}'})} \\ &\times \; e^{\frac{-i}{\hbar}\int U(\hat{h})} \; \mathcal{K}_{2,1}[\hat{h}(x)] \; \mathcal{K}_{1',2'}[\hat{h}(x')]\} \end{array}$$

where
$$(\delta_{\hat{h}}|\hat{D}|\delta_{\hat{h}'}) = \int d^4x d^4x' \frac{\delta}{\delta \hat{h}(x)} \hat{D}(x,x') \frac{\delta}{\delta \hat{h}(x')}$$

and where $\mathcal{K}_{2,1}[\hat{h}(x)]$ is the CWL propagator in a field $\hat{h}(x)$

WEAK FIELD EXPANSON for an INTERFERENCE EXPERIMENT

We can calculate the 4-point correlator for the density matrix dynamics, but it is easier to just find the 2-point propagator. Again, recall the form this will take – after integrating over the field h(x) we have

~2

$$\mathcal{G}(2,1) = \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int_{1}^{2} \mathcal{D}q_{k}(\tau) \kappa_{n}[\{q_{k}\}] e^{\frac{i}{\hbar}S[q_{k};2,1]}$$

~2

The lowest correction to QM goes like:

$$\Delta \mathcal{G}(2,1) = \int_{1}^{z} \mathcal{D}q \int_{1}^{z} \mathcal{D}q' \,\kappa_{2}[q,q'] e^{\frac{i}{\hbar}(S[q]+S[q'])} + \dots$$

The lowest order irreducible diagrams for this first correction are at right. In de Donder gauge the graviton propagator is

Let's write this as $\kappa_2[q,q'] = e^{i\chi_2[q,q']} - 1$ and take the 'slow-moving' limit where $v \ll c$. Then $q \to (\mathbf{q},t)$; define the relative coordinate $\mathbf{r} = \mathbf{q} - \mathbf{q}'$

SLOW DYNAMICS

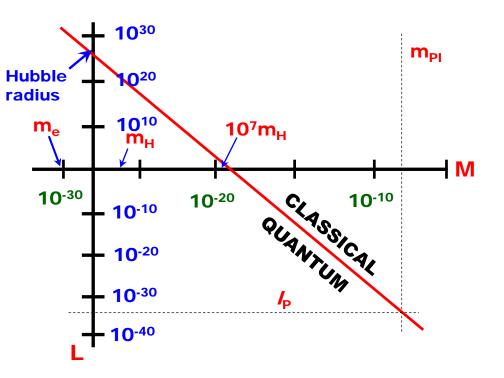
In any lab experiment involving massive objects, we will also be able to assume velocities << c. The correlator then

simplifies further, to

$$\kappa_2[\mathbf{q},\mathbf{q}'] = \exp{rac{i}{\hbar}\int^t d au} rac{4\pi Gm^2}{|\mathbf{q}(au)-\mathbf{q}'(au)|} - 1$$

so the path integral looks like that for a Coulomb attraction, with charges m. The key scales are

 $l_G(m) = \left(\frac{M_p}{m}\right)^3 L_p$ Newton radius (gravitational analogue of the Bohr radius) $\epsilon_G(m) = G^2 m^2 / l_G(m) \equiv E_p (m/M_p)^5$ Mutual binding energy for paths $R_s = 2Gm/c^2$ Schwarzchild radius for the particle (Classical)



The potential well created by this 'Coulomb-Newton' attraction causes a 'path-bunching'. 2 paths will bind if

 $\varepsilon_{G} > E_{Q}$

where E_Q is the energy scale associated with any other perturbations from impurities, phonons, photons, imperfections in any controlling potentials in the systems, and, worst of all, dynamics localized modes likes defects, dislocations, paramagnetic or nuclear spins, etc.

N-PARTICLE SYSTEM (SLOW-MOVING)

We write positions around the centre of mass $\mathbf{R}_{o}(t) = \frac{1}{N} \sum_{j=1}^{N} \mathbf{q}_{j}(t)$ so that $\mathbf{q}_{i} = \mathbf{R}_{o} + \mathbf{r}_{j}$ The effective action is then $S_{o}[\mathbf{R}_{o}, \{\mathbf{r}_{j}\}] = \int d\tau \left[\frac{M_{o}}{2}\dot{\mathbf{R}}_{o}^{2} + \sum_{j=1}^{N} \frac{m_{j}}{2}\dot{\mathbf{r}}_{j}^{2} - \sum_{i < j}^{N} V(\mathbf{r}_{i} - \mathbf{r}_{j})\right]$

Then we have a propagator

$$\begin{split} \Delta \mathcal{G}(2,1) &= \int \mathcal{D} \mathbf{X}_o \int \mathcal{D} \mathbf{\Xi}_o \prod_j \int \mathcal{D} \mathbf{x}_j \int \mathcal{D} \boldsymbol{\xi}_j \\ &\times \kappa_2^N[\mathbf{\Xi}_o; \{\xi_j\}] \int d\mathbf{P} d\mathbf{K} \; e^{\frac{i}{\hbar N} (\mathbf{P} \cdot \mathbf{x}_j + \mathbf{K} \cdot \xi_j)} \; e^{i \Psi_2[\mathbf{\Xi}_o, \{\xi_j\}; \mathbf{X}_o, \{\mathbf{x}_j\}]} \end{split}$$

where the C.o.m. correlates gravitationally with the individual particles according to

$$\kappa_2^N[\mathbf{\Xi}_o, \{\xi_j\}] = \left(\exp\left[\frac{i\lambda^2}{4\pi\hbar} \int d\tau \sum_{j=1}^N \frac{m_j^2}{|\mathbf{\Xi}_o + \xi_j|} \right] - \delta_{\mathbf{\Xi}_o} \delta_{\xi_j} \right)$$

We now want to analyze this for a real solid

PHONON EFFECTS We can understand the main effect by looking at the displacement correlator

$$\langle u_i^{\alpha}(t_1)u_j^{\beta}(t_1)\rangle = \frac{1}{N}\sum_{\mathbf{Q}\mu}\frac{\hat{e}_{\mathbf{Q}\mu}^{\alpha}\hat{e}_{\mathbf{Q}\mu}^{\beta}}{2m\omega_{\mathbf{Q}\mu}} e^{i[\mathbf{Q}\cdot\mathbf{r}_{\mathbf{ij}}^{(\mathbf{o})}-\omega_{\mathbf{Q}\mu}(t_1-t_2)]}$$

Typical displacements: 10⁻¹² -- 6 x 10⁻¹² m

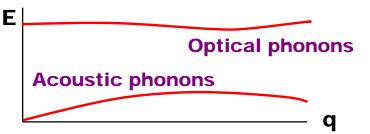


TABLE-TOP EXPERIMENTS

I. MECHANICAL OSCILLATOR

Now we add a term to the action: $S_U[\mathbf{R}_{\mathbf{o}}; \mathbf{F}_{\mathbf{o}}] = -\int d\tau \left[\frac{1}{2} U_o \mathbf{R}_{\mathbf{o}}^2(\tau) + \mathbf{F}_{\mathbf{o}}(\tau) \cdot \mathbf{R}_{\mathbf{o}}(\tau) \right]$

In the absence of any coupling between the phonons and the centre of mass, we get

$$\begin{aligned} \mathcal{G}(2,1) &= G_{osc}(2,1) \ G_{c}(2,1) \\ &\equiv G_{osc}(\mathbf{X_{o}}^{(2)},\mathbf{X_{o}}^{(1)}|\mathbf{F_{o}}(t)) \ G_{c}(\mathbf{\Xi_{o}}^{(2)},\mathbf{\Xi_{o}}^{(1)};\{\xi_{j}^{(2)},\xi_{j}^{(1)}\}) \end{aligned}$$

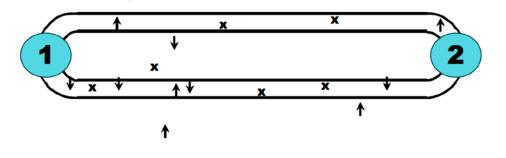
where the latter term incorporates the reduction of the path-bunching coming from individual ion dynamics.

The final result depends strongly on both the phonon dynamics and on the coupling of phonons to defects and spin impurities. One finds now that the onset of path bunching is pushed to mass scales M ~ 10^{18} m_H at which point the effective path-bunching length is ~ 10^{-16} m.

Such an experiment has many attractive features.

II. 2-SLIT EXPERIMENT

This is at first glance a very attractive experiment to analyse – but to realize it will be very difficult. For an extended mass the numbers come out similarly to those for the oscillator – but the influence of defects and impurities is much greater.



Such an experiment is likely impossible – even if one could do interference for such large objects.

CRUCIAL RESULT: The CWL CORRELATIONS & PATH BUNCHING MECHANISM DO <u>NOT</u> INVOLVE DECOHERENCE !!

COMPARISON with OTHER PREDICTIONS

<u>COMPARISON with PENROSE RESULT</u>: Penrose argues that the 2 proper times elapsed in a 2-branch superposition cannot be directly compared; there is a time uncertainty, related to an energy uncertainty given in weak field by

$$\Delta E = 2E_{1,2} - E_{1,1} - E_{2,2}$$

$$E_{i,j} = -G \int \int d\vec{r_1} d\vec{r_2} \frac{\rho_i(\vec{r_1})\rho_j(\vec{r_2})}{|\vec{r_1} - \vec{r_2}|}$$

There are 2 problems here:

(i) The density is fed in by hand – it should be calculated from the theory itself, and will depend on the UV cutoff

R Penrose Gen Rel Grav 28, 581 (1996)

W Marshall et al., PRL 91, 130401 (2003) D Kleckner et al., NJ Phys 10, 095020 (2008)

(ii) It is only the first term in an exponential.

To understand this, note that each individual term in our correlator is meaningless. It is not permissible to expand the exponential – if we do, each term gives a divergent contribution:

$$\begin{aligned} \kappa_{2}[\mathbf{r},\mathbf{r}'] &= \sum_{n=1}^{\infty} \prod_{i=1}^{n} \int^{t_{j}} d\tau_{j} \; \theta(\tau_{j} - \tau_{j-1}) \delta(t - \tau_{n}) \; \frac{(4\pi i Gm^{2})^{n}}{|\mathbf{r}(\tau_{j}) - \mathbf{r}'(\tau_{j})|} \\ &= \int^{t} d\tau \; \frac{4\pi i Gm^{2}}{|\mathbf{r}(\tau) - \mathbf{r}'(\tau)|} \; + \; \int^{t} d\tau \int^{\tau} d\tau' \; \frac{4\pi i Gm^{2}}{|\mathbf{r}(\tau) - \mathbf{r}'(\tau)|} \frac{4\pi i Gm^{2}}{|\mathbf{r}(\tau') - \mathbf{r}'(\tau)|} \; + \; \dots \end{aligned}$$

If we feed in the density by hand, the role of a UV cutoff is obvious from the results:

$$\Delta E = \frac{Gmm_1}{x_0} \left(\frac{24}{5} - \frac{1}{\sqrt{2}\kappa} \right) \quad \text{"Zero point"} \\ \text{estimate} \\ \Delta E = 2Gmm_1 \left(\frac{6}{5a} - \frac{1}{\Delta x} \right) \quad \text{"nuclear radius"} \\ \text{estimate}$$

These numbers differ by roughly 1000 !



WG UnruhH BrownR PenroseD CarneyM AspelmeyerA Gomez

PCE Stamp, Phil Trans Roy Soc A370, 4429 (2012) ", New J. Phys. 17, 06517 (2015)

G Semenoff RM Wald C Gooding