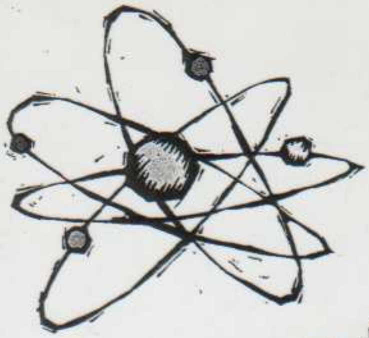
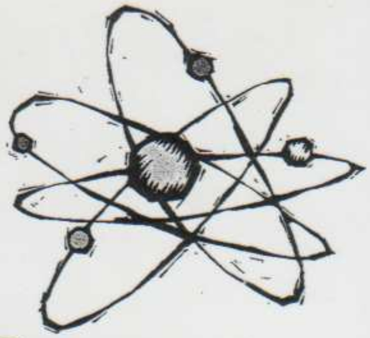




Lecture Summary

In the world about us the past is distinctly different from the future. Milk spills but doesn't unspill, eggs splatter but do not unsplatter, waves break but do not unbreak, we always grow older never younger. These processes all move in one direction in time–they are called "time–irreversible" and define the arrow of time. It is therefore very surprising that the relevant fundamental laws of nature make no such distinction between the past and the future. These laws permit all processes to be run backwards in time. This leads to a great puzzle–if the laws of nature permit it why don't we observe the above mentioned processes run backwards? Why does a video of an egg splattering run backwards look ridiculous? Put another way: how can the time-reversible motions of atoms and molecules, the microscopic components of material systems, give rise to the observed time–irreversible behavior of our everyday world? The resolution of this apparent paradox is the subject of my talk.

Time's Arrow and Boltzmann's entropy

Joel L. Lebowitz

0. What is Time?

I. Qualitative Aspects of Macroscopic Behavior

The Problem of Macroscopic Irreversibility

Boltzmann's Solution (also Maxwell, Thompson,...)

Typical vs. Average Behavior: The Law of Large Numbers

II. Cosmological Initial Conditions and the Origin of Low Entropy States

III. Velocity Reversal and Macro Stability

IV. Quantitative Aspects of Macroscopic Behavior

Entropy Increase in the Evolution of Macroscopic Systems

Boltzmann's H-function

Microscopic Derivation of Hydrodynamical Eqs.

What is time? If nobody asks me, I know; but if I were desirous to explain it to one that should ask me, plainly I know not.

Saint Augustine (354–430)

Some Hindu Concepts of Time

"Time is the Lord of all things
Time was the father of Prajapates."
Athana Veda XXI, 53, 1-8

" ... time exists in a latent stage during the dissolution of the world, and is awakened by the god at the moment of recreation."

From The Bhagavata Purana

... Philosophers tend to be divided into two camps. On one side there are those who regard the passage of time as an objective feature of reality, and interpret the present moment as the marker or leading edge of this advance. Some members of this camp give the present ontological priority, as well, sharing Augustine's view that the past and the future are unreal. Others take the view that the past is real in a way that the future is not, so that the present consists in something like the coming into being of determinate reality.

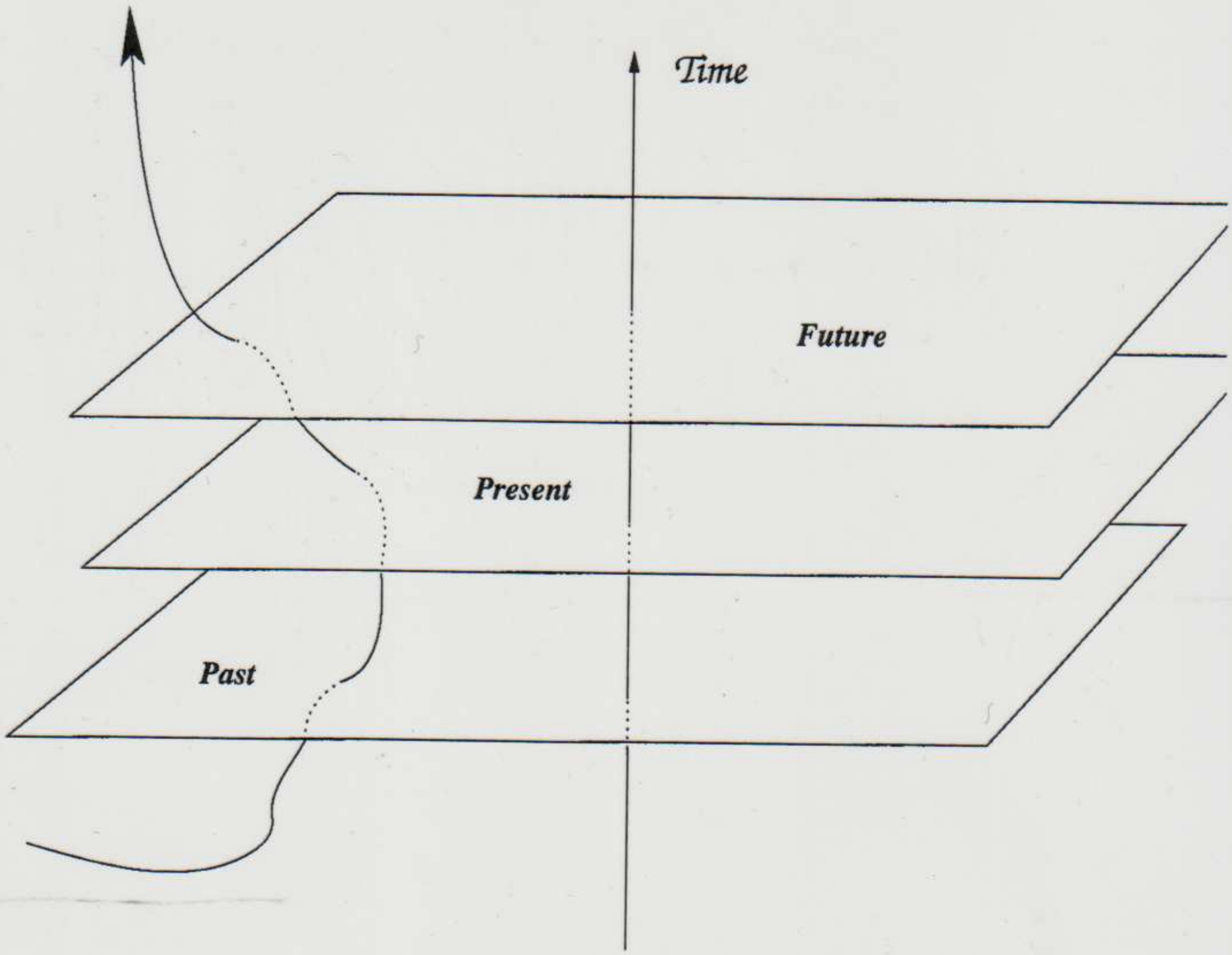
.... Philosophers in the opposing camp regard the present as a subjective notion, often claiming that **now** *is dependent on one's viewpoint in much the same way that* **here** *is. In this view there is no more an objective division of the world into the past, the present, and the future than there is an objective division of a region of space into here and there.*

Often this is called the **block universe view**, *the point being that it regards reality as a single entity of which time is an ingredient, rather than as a changeable entity set* **in** *time.*

Huw Price, in Time's Arrow and Archimedes' Point

The Tralfamadorians can look at all different moments just the way we can look at a stretch of the Rocky Mountains, for instance. They can see how permanent all the moments are, and they can look at any moment that interests them. It is just an illusion we have here on earth that one moment follows another like beads on a string...

Kurt Vonnegut, Slaughter-House-Five (1969)


Time
Future
Present
Past

when someone asked Yogi Berra

"What time is it?"

he replied

"Do you mean now?"

The laughter evoked by this anecdote shows how strongly we hold a common notion of the present.

Jim Hartle in The Physics of Now

American Journal of Physics, 2004

Time present and time past

Are both perhaps present in time future

And time future contained in time past.

If all time is eternally present

All time is unredeemable.

T.S. Eliot, Four Quartets

Michaelle has left this strange world just before me. This is of no importance. For us convinced physicists the distinction between past, present and future is an illusion, although a persistent one.

Albert Einstein, Letter to Besso's Sister

Do you believe time is a truly basic concept that must appear in the foundations of any theory of the world, or is it an effective concept that can be derived from more primitive notions in the same way that a notion of temperature can be recovered in statistical mechanics?

The results were as follows: 20 said there was no time at a fundamental level, 12 declared themselves to be undecided or wished to abstain, and 10 believed time did exist at the most basic level. However, among the 12 in the undecided/abstain column, 5 were sympathetic to or inclined to the belief that time should not appear at the most basic level of theory.

Julian Barbour in The End of Time:
The Next Revolution in Physics
Oxford University Press, 2000

The "Arrow of Time" and Quantum Mechanics

A.J. Leggett

I do strongly agree that if in the year 2075 physicists look back on us poor quantum-mechanics-besotted idiots of the twentieth century with pity and head-shaking, an essential ingredient in their new picture of the universe will be a quite new and to us unforeseeable approach to the concept of time: and that to them our current idea about the asymmetry of nature with respect to time will appear as naive as do to us the notions of nineteenth-century physics about simultaneity.

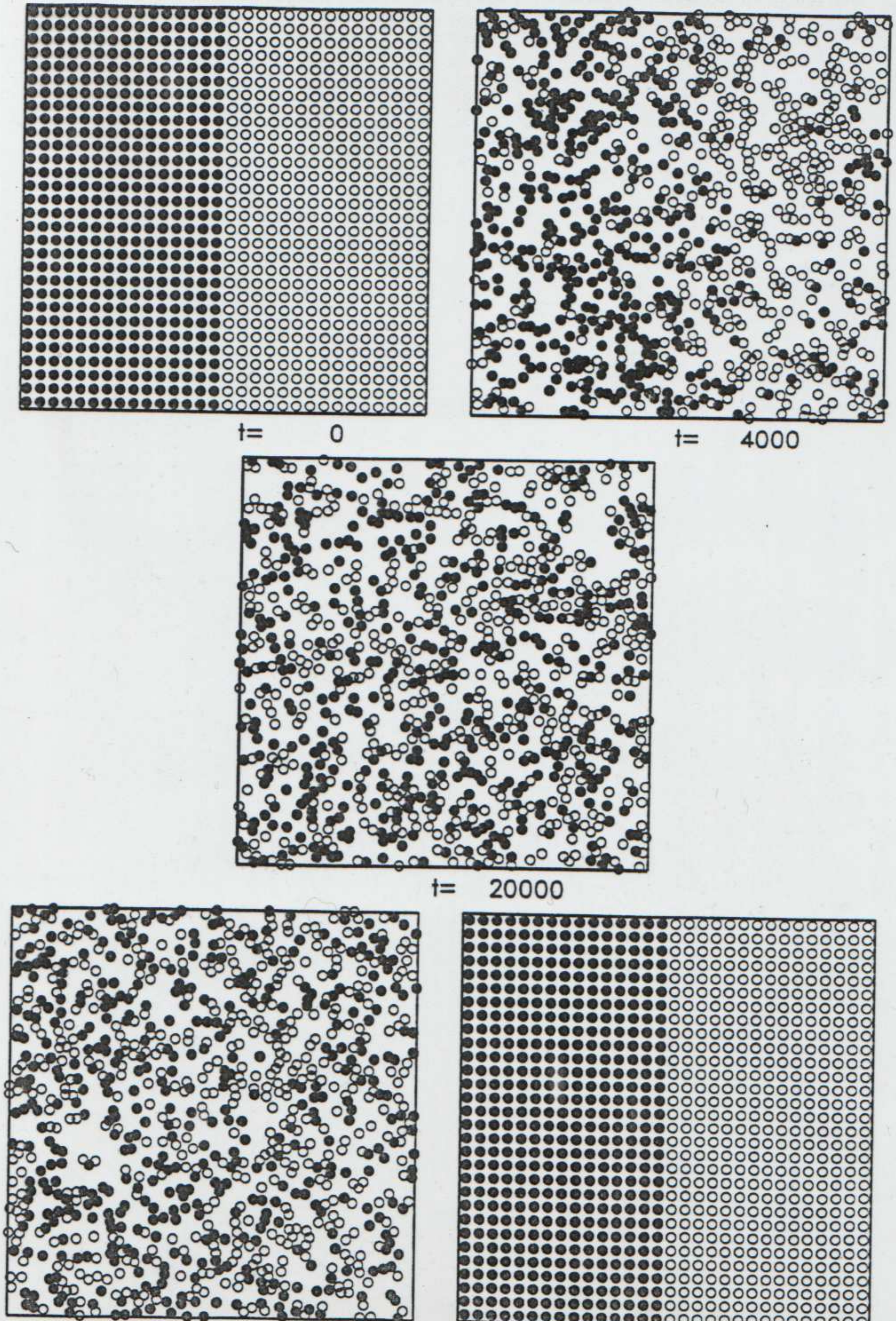
I. Introduction

"One of the perenially challenging problems of theoretical physics is that of the "arrow of time". Everyday experience teaches us that the future is qualitatively different from the past, that our practical powers of prediction differ vastly from those of memory, and that complex physical systems tend to develop in the course of time in patterns distinct from those of their antecedents. On the other hand, "all" the "microscopic" laws of physics ever seriously propounded and widely accepted are entirely symmetric with respect to the direction of time; they are form-invariant with respect to time reversal."

Time Symmetry in the Quantum Process of Measurement

Aharonov, Bergmann, Lebowitz

The Physical Review, Vol. 134, No. 6B, B1410–B1416, 22 June 1964



t= 0 | t= 4000 | t= 20000 | t= 30000 | t= 40000

Levesque + Verlet: (Taxwand)

Microscopic Reversibility Classical Mechanics

("Similar" in Quantum Mechanics)

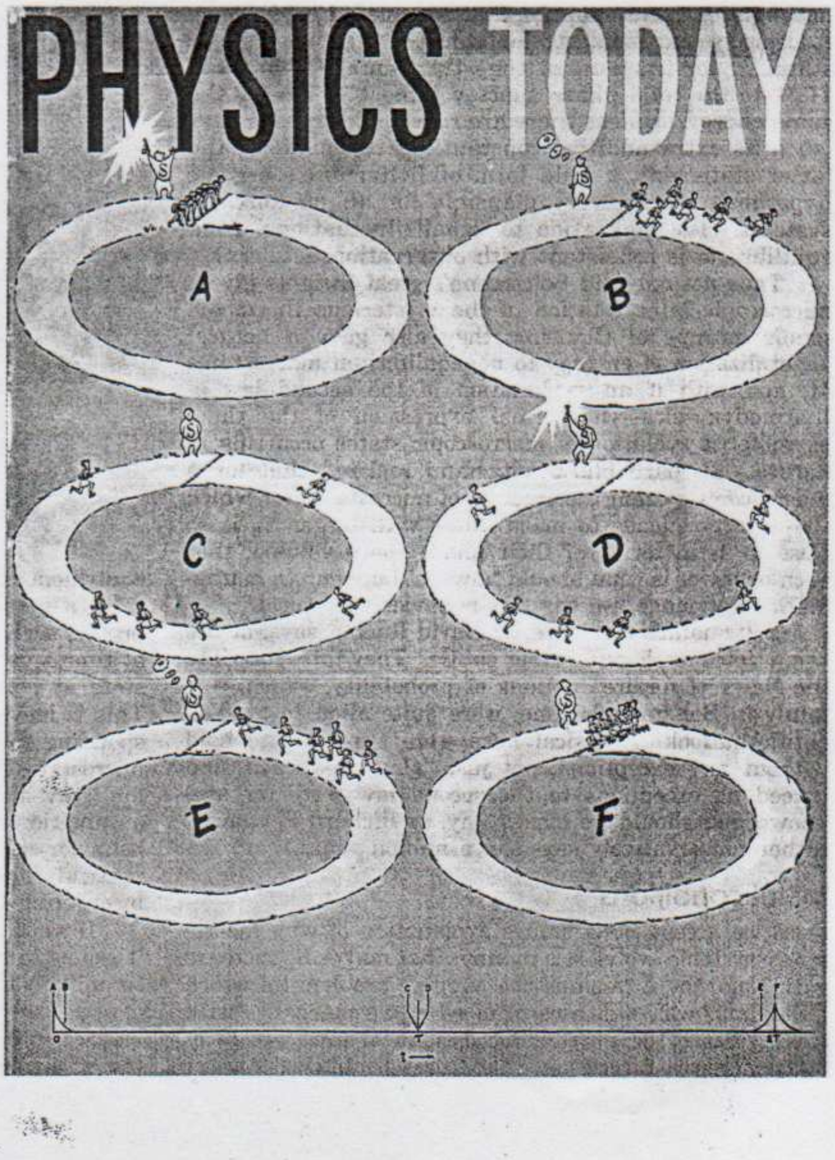
Microstate $X = (\mathbf{r}_1, \mathbf{v}_1, \ldots, \mathbf{r}_N, \mathbf{v}_N) \in \Gamma$

$RX = (\mathbf{r}_1, -\mathbf{v}_1, \ldots, \mathbf{r}_N, -\mathbf{v}_N) \in \Gamma$

Evolution: $X \to T_t X = X(t), \qquad t \in (-\infty, \infty)$

$$T_\tau X(t) = X(t + \tau)$$

$$\underline{T_\tau R T_\tau X(t) = RX(t)}$$



Reversing time. PHYSICS TODAY cover from November 1953 shows athletes on a racetrack. At the first gunshot, they start running; at the second, they reverse and run back, ending up again in a line. The drawing, by Kay Kaszas, refers to an article by Erwin L. Hahn on the spin echo effect on page 4 of that issue. **Figure 4**

If, then, the motion of every particle of matter in the universe were precisely reversed at any instant, the course of nature would be simply reversed for ever after. The bursting bubble of foam at the foot of a waterfall would reunite and descend into the water; the thermal motions would reconcentrate their energy, and throw the mass up the fall in drops re-forming into a close column of ascending water. ... And if also the materialistic hypothesis of life were true, living creatures would grow backwards, with conscious knowledge of the future, but no memory of the past, and would become again unborn. But the real phenomena of life infinitely transcend human science; and speculation regarding consequences of their imagined reversal is utterly unprofitable.

W. Thomson, in The Kinetic Theory of the Dissipation of Energy, Proc. of the Royal Soc. of Edinburgh, **8** 325 (1874)

Classical Mechanics of Isolated Systems

Hamiltonian $H(X)$; $X \to T_t X$, all t

Microstate: $X = (q_1, \dots, q_N, p_1, \dots, p_N) \in \Gamma$

Macrostates

Let M describe the macrostate of a system of N atoms in a box V, say $N \gtrsim 10^{20}$. As an example we may take M to consist of the specification, to within a given accuracy, of the energy and number of particles in each half of the box V.

Clearly M is determined by X (we will thus write $M(X)$) but there are many X's (in fact a continuum) which correspond to the same M. Let Γ_M be the region in Γ consisting of all microstates X corresponding to a given macrostate M and denote by $|\Gamma_M| = (N!h^{3N})^{-1} \int_{\Gamma_M} \Pi_{i=1}^{N} d\mathbf{r}_i d\mathbf{p}_i$, its symmetrized $6N$ dimensional Liouville volume (in units of h^{3N}).

In particular $\Gamma_{M_{eq}}$ is the region on the energy surface (or shell) correspond to equilibrium macrostate, e.g. the fraction of particles in each half of the box $\frac{1}{2} \pm 10^{-8}$

Central insight of Maxwell, Thomson (Kelvin) and (particularly) Boltzmann is that for a "Macroscopic" system, i.e. $N \gg 1$, say $N \gtrsim 10^{20}$ the energy shell looks like this

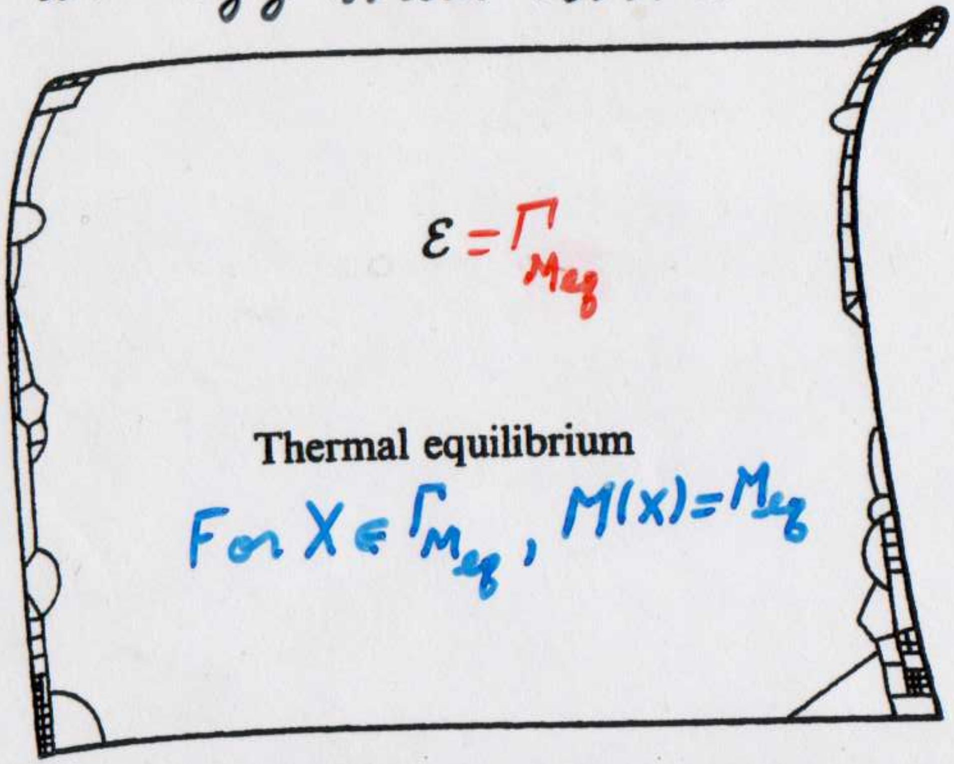


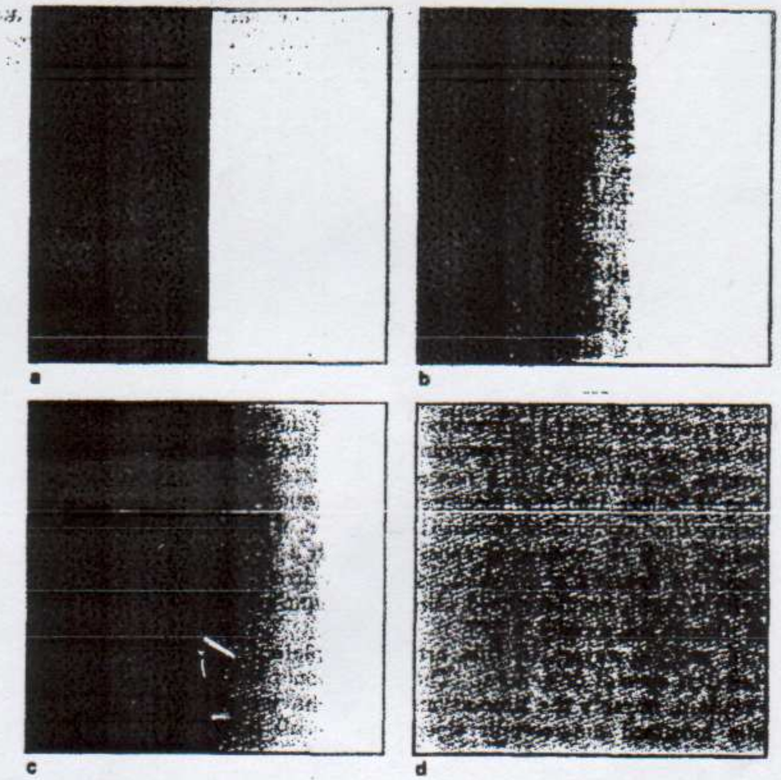
Fig. 27.4 The particular box $\mathcal{E}$, representing thermal equilibrium, has a volume [redacted] that is normally practically equal to the volume [redacted] of the entire energy shell [redacted], and therefore far exceeds the volumes of all the other boxes together.

From "The Road to Reality" by R. Penrose

The smaller regions represent nonequilibrium macrostates M_α, e.g. .49 % in the right half of V.

$$H(X) = \Sigma \tfrac{1}{2} m v_i^2 + \Sigma \phi(r_i - r_j)$$

Snapshots giving four macrostates M of system; ink with two colors.



How would you order this sequence of "snapshots" in time? Each represents a macroscopic state of a system containing, for example, two fluids. **Figure 1**

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Macrostate $M(X)$, $X \subset \Gamma_M$, $N >> 1$.

Fig. 1 shows M_a, M_b, M_c, M_d.

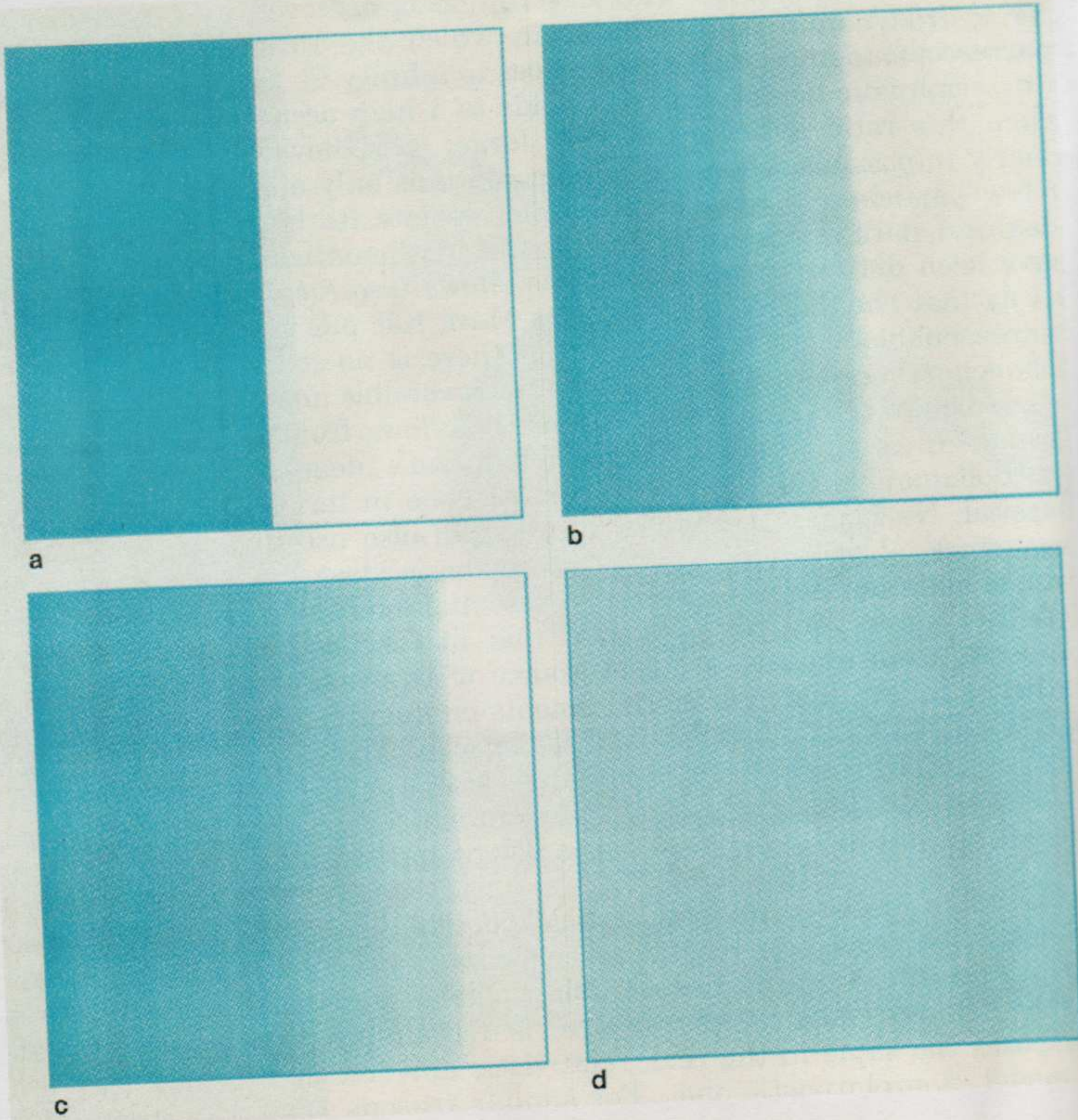
If $X \in \Gamma_{M_b}$ so is $RX \in \Gamma_{M_b}$.

Hence if $M_a \rightarrow M_b \rightarrow M_c \rightarrow M_d$ possible

so is $M_d \rightarrow M_c \rightarrow M_b \rightarrow M_a$

SO WHY IS ONE DIRECTION COMMON

AND THE OTHER NEVER SEEN?



How would you order this sequence of “snapshots” in time? Each represents a macroscopic state of a system containing, for example, two fluids.

or different temperatures

Boltzmann's Answer

Consider phase space region consisting of all microstates X giving rise to a macrostate

$$M : \Gamma_M = \{X : M(X) = M\}$$

[consistent with total energy E, and other "relevant" constants of motion, within some tolerance]. Then for Fig. 1

$$\Gamma_{M_a} \cdot \quad , \quad \Gamma_{M_b} \cdot \quad , \quad \Gamma_{M_c} \cdot \quad \Gamma_{M_d} \sim \Gamma_{M_{eq}}$$

$$|\Gamma_{M_a}| << |\Gamma_{M_b}| << |\Gamma_{M_c}| << |\Gamma_{M_d}|$$

for typical size of macrosystem: 1 Mole

$$|\Gamma_{M_a}|/|\Gamma_{M_{eq}}| \sim 10^{-10^{23}}$$

Boltzmann (then) argued that it is "very very likely" that if the system starts in a microstate $X \in \Gamma_{M_a}$ and then is let go (say by lifting some constraint like a partition) then it will go towards macrostates for which Γ_M is larger—especially when the discrepancy between Γ_{M_a}, $\Gamma_{M_b}, \ldots, \Gamma_{M_{eq}}$ is so *large*.

(Here is where ~ "equal a priori probability" comes in)

To connect argument with second law Boltzmann defined for each M, and thus for each X giving rise to M, a number, the Boltzmann entropy of a macrostate,

$$S_B(M(X)) = \log |\Gamma_M|$$

$$|\Gamma_M| \sim \int_{\Gamma_M} \Pi d^3\mathbf{r}_i d^3\mathbf{v}_i$$

He then showed that for $M = M_{eq}$

$$S_B(M_{eq}) \simeq V s_{eq}(n, e) = S_{eq}(E, N, V)$$

when $n = N/V$, $e = E/V$. $S_{eq}(E, V, N) \equiv$ thermodynamic entropy of Clausius defined entirely macroscopically $= V s(e, \rho)$

In particular $|\Gamma_M / \Gamma_{M_{eq}}| \sim e^{+V\Delta s(M)}$

$\Delta s = s(M | e, \rho) - s(e, \rho)$

"On the basis of the kinetic theory of gases Boltzmann had discovered that, aside from a constant factor, entropy is equivalent to the logarithm of the "probability" of the state under consideration. Through this insight he recognized the nature of the course of events which, in the sense of thermodynamics, are "irreversible." Seen from the molecular-mechanical point of vicw, however, all courses of events are reversible. If one calls a molecular-theoretically defined state a microscopically described one, or, more briefly, micro-state, then an immensely large number (Z) of states belong to a macroscopic condition. Z then is a measure of the probability of a chosen macro-state. This idea appears to be of outstanding importance also because of the fact that its usefulness is not limited to microscopic description on the basis of mechanics."

A. Einstein, Autobiographical Notes

To Summarize:

"In other words, the impossibility of a decrease of entropy seems to be reduced to an improbability"

J. W. Gibbs quoted by L. Boltzmann

It is this probabilistic argument which Ruelle calls "simple but subtle" that apparently could not be understood by Zermelo and some other contemporaries of Boltzmann.

Of course "we" understand this—
yet there is still some problem

The applicability of probability theory to a particular case cannot of course be proved rigorously. ... Despite this, every insurance company relies on probability theory. ... It is completely incomprehensible to me how anyone can see a refutation of the applicability of probability theory in the fact that some other argument shows that exceptions must occur now and then over a period of eons of time; for probability theory itself teaches just the same thing. (Boltzmann) by Brian Greene

different numbers, there are a million ways you can win, so your chances of striking it rich are a million times higher.

Entropy is a concept that makes this idea precise by counting the number of ways, consistent with the laws of physics, in which any given physical situation can be realized. *High entropy means that there are many ways; low entropy means there are few ways.* If the pages of *War and Peace* are stacked in proper numerical order, that is a low-entropy configuration, because there is one and only one ordering that meets the criterion. If the pages are out of numerical order, that is a high-entropy situation, because a little calculation shows that there are
1245521984537783433660029353704988291633611012463890451368
8769126468689559185298450437739406929474395079418933875187
6527656714059286627151367074739129571382353800016108126465
3018234205620571473206172029382902912502131702278211913473
5826558815410713601431193221575341597338554284672986913981
5159925119085867260993481056143034134383056377136715110570
4786941333912934192440961051428879847790853609508954014012
5932850632906034109513149466389839052676761042780416673015
4945522818861025024633866260360150888664701014297085458481
5141598392546876231295293347829518681237077459652243214888
7351679284483403000787170636684623843536242451673622861091
9853939181503076046890466491297894062503326518685837322713
6370247390401891094064988139838026545111487686489581649140
3426444110871911844164280902757137738090672587084302157950
1589916232045813012950834386537908191823777738521437536312
2531641598589268105976528144801387748697026525462643937189
3927305921796747169166978155198569769269249467383642278227
3345776718073316240433636952771183674104284493472234779223
4027225630721193853912472880929072034271692377936207650190
4571097887744535443586803319160959249877443194986997700333
2494630732437553532290674481765795395621840329516814427104
2227608124289048716428664872403070364864934832509996672897
3446425310349300626622014604312051101093282396249251196897
8283306192150828270814393659987326849047994166839657747890
2124562796195600187060805768778947870098610692265944872693
4100008726998763399003025591685820639734851035629676461160
0225159200113722741273318074829547248192807653266407023083
2754286312646671501355905966429773337131834654748547607012
4233012872135321237328732721874825264039911049700172147564

The spontaneous transition from order to disorder is the quintessence of Boltzmann's theory . . . This theory really grants an understanding and does not . . . reason away the dissymetry of things by means of an a priori sense of direction of time . . . No one who has once understood Boltzmann's theory will ever again have recourse to such expedients. It would be a scientific regression beside which a repudiation of Copernicus in favor of Ptolemy would seem trifling. (Schrödinger)

Nevertheless, objections to the theory have been raised again and again in the course of past decades and not (only) by fools but (also) by fine thinkers. If we . . . eliminate the subtle misunderstandings . . . we . . . find . . . a significant residue . . . which needs exploring . . . (Schrödinger)

Initial Conditions

"First, my good friend, you state that the two directions of your time variables, from $-t$ to $+t$ and from $+t$ to $-t$ are a priori equivalent. Then by fine arguments appealing to common sense you show that disorder (or 'entropy') must with overwhelimg probability increase with time. Now, if you please, what do you mean by 'with time'? Do you mean in the direction $-t$ to $+t$? But if your interferences are sound, they are equally valid for the direction $+t$ to $-t$. If these two directions are equivalent a priori, then they remain so a posteriori. The conclusions can never invalidate the premise. Then you inference is valid for both directions of time, and that is a contradiction."

E. Schrödinger

In terms of our example

If we assumed that system in Fig. 1b was typical of Γ_{M_b} and compute its *antecedent* state, we would get $\sim M_c$ and *not* M_a. Why can we use statistical arguments of Boltzmann for prediction and not for retrodiction?

Fact: If state X_a was typical of Γ_{M_a} and $X_a \to X_b$ then X_b is *not* typical of Γ_{M_b}. Nevertheless its *future* macro behavior but not its *past* can be obtained by using typical points of Γ_{M_b}.

In Fig. 1 we tacitly assumed that initial state of low S_B, M_a, was prepared by some experimentalist who is herself in a state of low entropy–being born in such a state and maintained there by low entropy foods, Using low entropy radiation from sun,... and that such preparation generally produces a typical microstate. Fine, but ... and what about

But where does it all start? events in regions such as stars where there are no experimentalists?

Cosmological Initial State.

"Is the apparent irreversibility of all known natural processes consistent with the idea that all natural events are possible without restriction? ... That in nature the transition from a probable to an improbable state does not take place as often as the converse, can be explained by assuming a very improbable initial sate of the entire universe surrounding us. ... This is a reasonable assumption to make, since it enables us to explain the facts of experience, and one should not expect to be able to deduce it from anything more fundamental." Boltzmann also wrote

"One may speculate that the universe as a whole is in thermal equilibrium and therefore dead, but there will be local deviations from equilibrium which may last for the relatively short time of a few eons. For the universe as a whole, there is no distinction between the "backwards" and "forwards" directions of time, but for the worlds on which living beings exist, and which are therefore in relatively improbable states, the direction of time will be determined by the direction of increasing entropy, proceeding from less to more probable states."

L. Boltzmann

"We would like to argue that this is not the case. Suppose we do not look at the whole box at once, but only a piece of the box. Then, at a certain moment, suppose we discover a certain amount of order. In this little piece, white and black are separate. What should be deduce about the condition in places where we have not yet looked? If we really believe that the order arose from complete disorder by a fluctuation, we must surely take the most likely fluctuation which could produce it, and the most likely condition is **not** that the rest of it has also become disentangled! Therefore, from the hypothesis that the world is a fluctuation, all of the predictions are that if we look at a part of the world we have never seen before, we will find it mixed up, and not like the piece we just looked at. If our order were due to a fluctuation, we would not expect order anywhere but where we have just noticed it."

****

"I think it necessary to add to the physical laws the hypothesis that in the past the universe was more ordered, in the technical sense, than it is today—I think this is the additional statement that is needed to make sense, and to make an understanding of the irreversibility."

Richard Feynman (BBC lectures)
The Character of Physical Law

Initial state "after" Big Bang
~ uniform energy density : Matter field in equilibrium, gravitational field totally out of equilibrium.

Why?

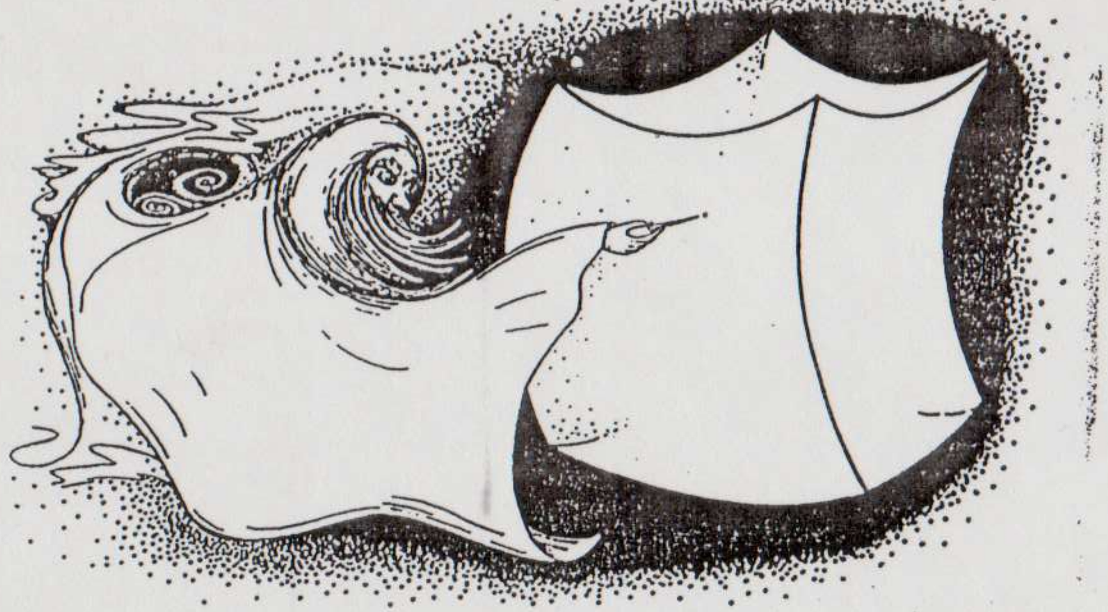


FIGURE 10. The Creator locating the tiny region of phase-space—one part in $10^{10^{123}}$—needed to produce a 10^{80}-baryon closed universe with a second law of thermodynamics in the form we know it.

$$|\Gamma_{M_0}| / |\Gamma_{M_{eq}}| \sim 10^{-10^{123}}$$

M_{eq} ~ Big Crunch.

Gravitational Black Hole Entropy

Clustering in Gravitational Systems at all Energies

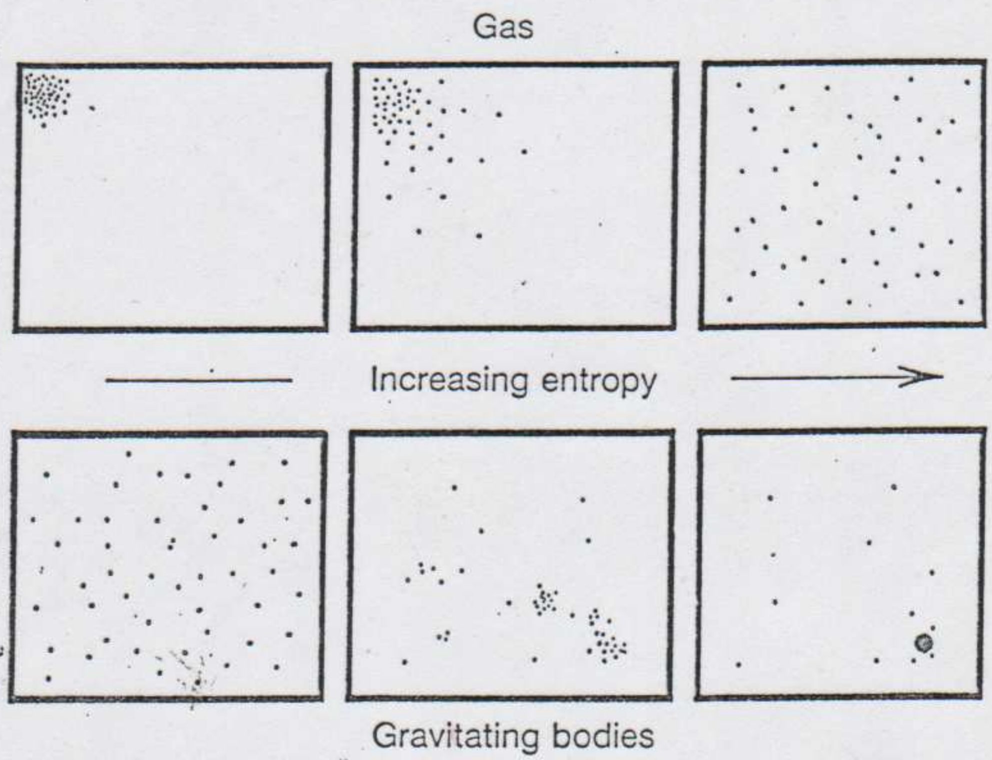


FIGURE 6. With a gas in a box, the maximum entropy state (thermal equilibrium) has the gas distributed uniformly; however, with a system of gravitating bodies, entropy can be increased from the uniform state by gravitational clumping to a black hole finally.

How compute the entropy for gravitating bodies?

2.5

Phase Transition as energy is lowered or volume decreased

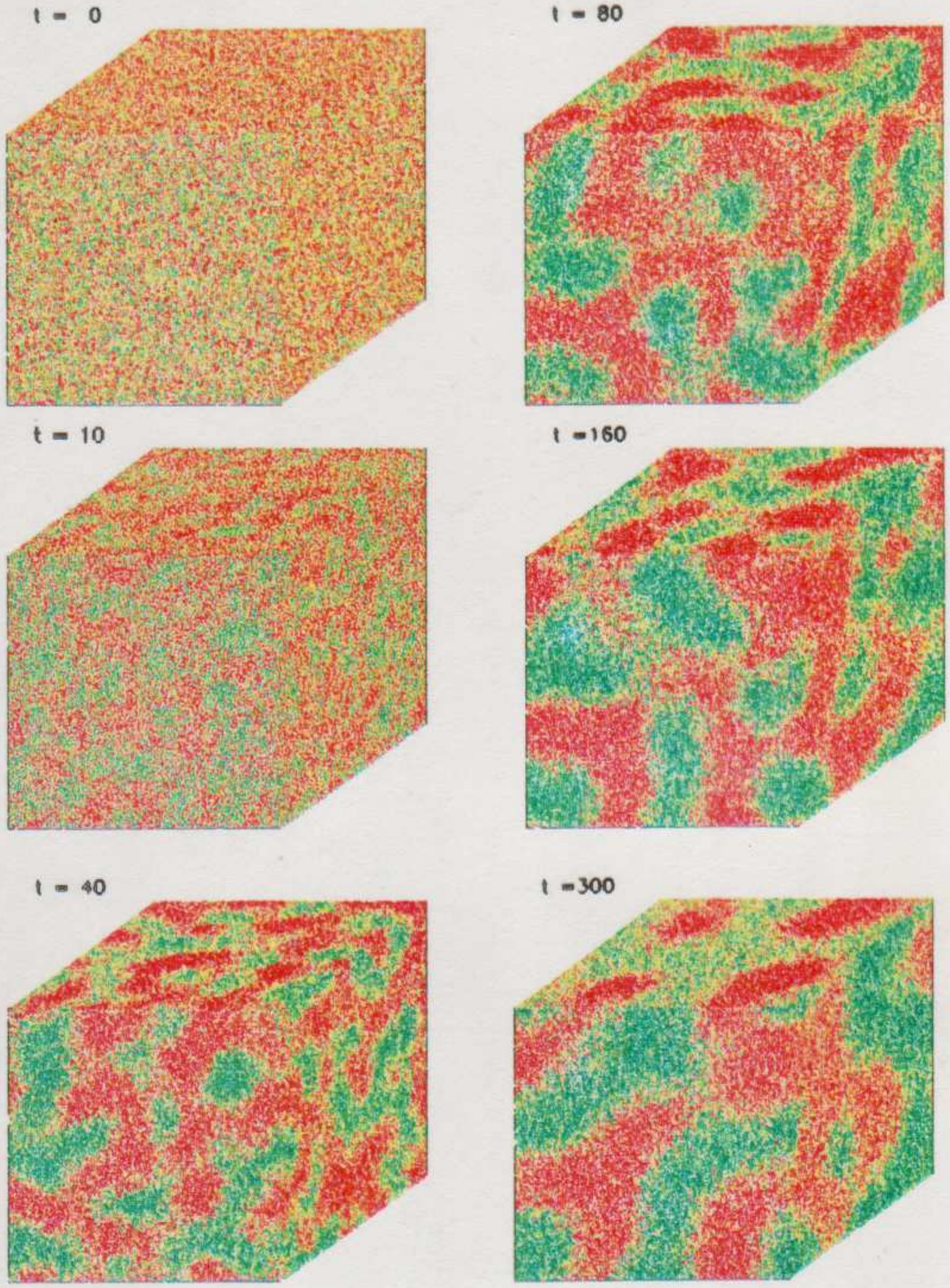


Figure 3.4: Snapshots of the system during phase segregation at critical quench; $\gamma^{-1} = 0.4$.

SORIN BASTEA, THESIS, 1997

The general struggle for existence of living beings is therefore not a fight for the elements- the elements of all organisms are available in abundance in air, water, and soil-, nor for energy, which is plentiful in the form of heat, unfortunately untransformably, in every body. Rather, it is a struggle for entropy [more accurately: negative entropy] that becomes available through the flow of energy from the hot Sun to the cold Earth. To make the fullest use of this energy, the plants spread out the immeasurable areas of their leaves and harness the Sun's energy by a process as yet unexplored, before it sinks down to the temperature level of our Earth, to drive chemical syntheses of which one has no inkling as yet in our laboratories. The products of this chemical kitchen are the object of the struggles in the animal world.

L. Boltzmann, The Second Law of the Mechanical Theory of Heat, (1886).

Translated by ?

Fig. 27.9 The Earth gives back the same amount of energy that it receives from the Sun, but what it receives from the Sun is in a much lower entropy form, owing to the fact that the Sun's yellow light has higher frequency than the infrared that the Earth returns. Accordingly, by Planck's $E = h\nu$, the Sun's photons carry more energy per photon than do those that Earth returns, so the energy from the Sun is carried by fewer photons than that returned by the Earth. Fewer photons means fewer degrees of freedom and therefore a smaller phase-space region and thus lower entropy than in the photons returned to space. Plants make use of this low entropy energy in photosynthesis, thereby reducing their own entropy, and we take advantage of the plants to reduce ours, by eating them, or eating something that eats them, and by breathing the oxygen that the plants release. This ultimately comes from the temperature imbalance in the sky that resulted from the gravitational clumping that produced the Sun.

Boltzmann's explanation of the time-reversal asymmetry of natural processes is that "this one-sidedness lies uniquely and solely in the initial conditions", by which he means "not ... that for each experiment one must specially assume just certain initial conditions" but rather that "it is sufficient to have a uniform basic assumption about the initial properties of the mechanical picture of the world". He proposes to "conceive of the world as an enormously large mechanical system ... which starts from a completely ordered initial state, and even at present is still in a substantially ordered state".

It is this fact that we are still in a state of low entropy that permits the existence of relatively stable neural connections, of marks of ink on paper, which retain over relatively long periods of time shapes related to their formation. Such nonequilibrium states are required for memories - in fact for the existence of living beings and of the earth itself.

We have no such records of the future and the best we can do is use statistical reasoning which leaves much room for uncertainty.

Equilibrium systems, in which the entropy has its maximal value, do not distinguish between past and future.