Black Holes

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Black Holes

Black Holes: A *black hole* is a region of spacetime where gravity is so strong that nothing—not even light—that enters that region can ever escape from it.

Michell (1784); Laplace (1798):

Escape velocity:

$$\frac{1}{2}mv_e^2 = G\frac{mM}{R}$$

so $v_e > c$ if

$$R < R_S \equiv \frac{2GM}{c^2} \approx 3\frac{M}{M_\odot}\mathrm{km}$$

Michell and Laplace predicted that stars with $R < R_S$ would appear to be black.

General relativity Nothing can travel faster than light, so if light is "pulled back", then so is everything else. A body with $R < \frac{2GM}{c^2}$ cannot exist in equilibrium; it must undergo complete gravitational collapse to a singularity. There is considerable (but mainly indirect!) evidence in favor of the "cosmic censorship conjecture": The end product of this collapse is always a black hole, with the singularity hidden within the black hole.

Formation of Black Holes in Astrophysics

1. Stellar collapse: After exhaustion of its thermonuclear fuel, a star can support itself against collapse under its own weight only if it is able to generate pressure without high temperature:

- For $M < 1.4 M_\odot$ support by electron degeneracy pressure is possible: white dwarfs
- For $M <\sim 2 M_\odot$ support by neutron degeneracy pressure/nuclear forces is possible: neutron stars
- However, if $M >\sim 2 M_\odot$ and the excess mass is not shed (e.g., in a supernova explosion), complete gravitational collapse is unavoidable: black holes

Mass range of black holes formed by stellar collapse: $\sim 2M_\odot < M <\sim 100M_\odot$. About 20 very strong candidates are known from binary X-ray systems.

2. Collapse of the central part of a galactic nucleus or star cluster: A variety of processes can plausibly lead to the formation of massive black holes at the center of galactic nuclei or dense clusters of stars. Black holes are believed to be the "central engine" of quasars. Massive black holes are believed to be present at the centers of most galaxies.

Mass range: $\sim 10^5 M_\odot < M <\sim 10^{10} M_\odot$

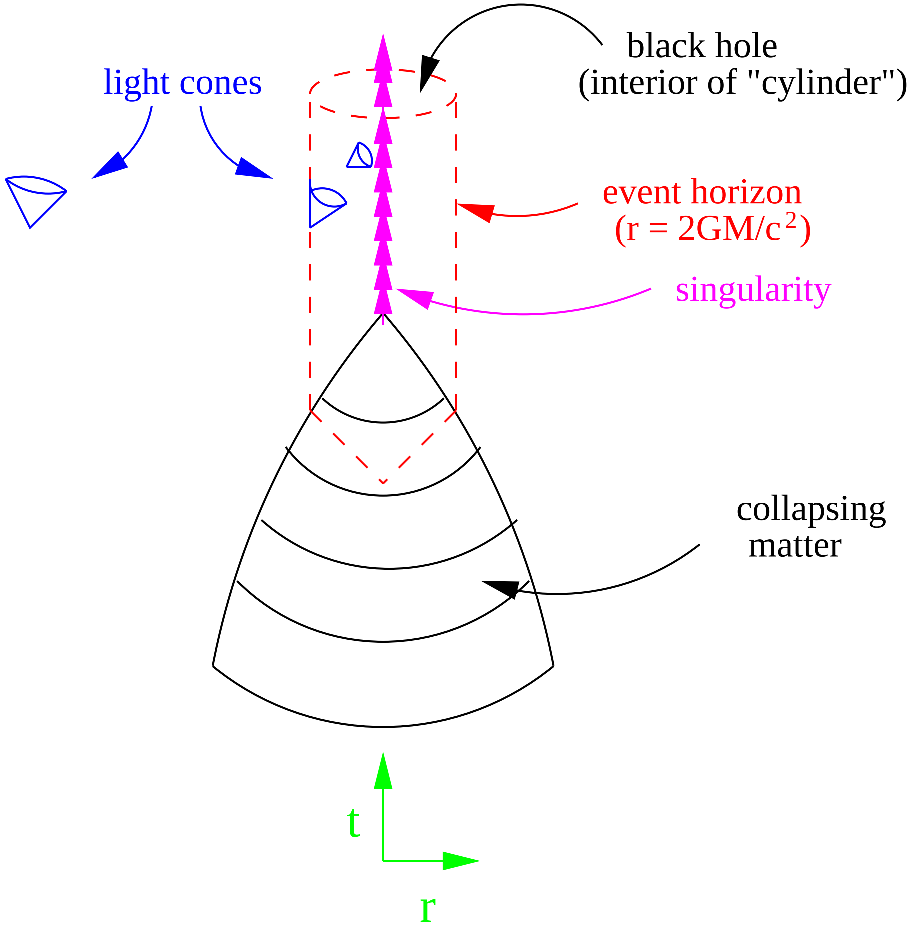
Almost all nearby galaxies show evidence for the presence of a massive black hole, and about a dozen show very strong evidence. There is convincing evidence for the presence of a black hole of mass $\sim 4 \times 10^6 M_\odot$ at the center of our own galaxy.

3. Primordial Black Holes: Could have been produced by the collapse of regions of enhanced density in the very early universe.

Mass range: anything

(There is no observational evidence for primordial black holes.)

Spacetime Diagram of Gravitational Collapse



light cones

black hole
(interior of "cylinder")

event horizon
(r = $2GM/c^2$)

singularity

collapsing
matter

Black Holes and Thermodynamics

Stationary black hole ↔ Body in thermal equilibrium

Just as bodies in thermal equilibrium are normally characterized by a small number of “state parameters” (such as E and V) a stationary black hole is uniquely characterized by M, J, Q.

0th Law

Black holes: The surface gravity, κ, is constant over the horizon of a stationary black hole.

Thermodynamics: The temperature, T, is constant over a body in thermal equilibrium.

1st Law

Black holes:

$$\delta M = \frac{1}{8\pi}\kappa\delta A + \Omega_H \delta J + \Phi_H \delta Q$$

Thermodynamics:

$$\delta E = T\delta S - P\delta V$$

2nd Law

Black holes:

$$\delta A \geq 0$$

Thermodynamics:

$$\delta S \geq 0$$

Analogous Quantities

$M \leftrightarrow E \leftarrow$ But M really is E!

$\frac{1}{2\pi}\kappa \leftrightarrow T$

$\frac{1}{4}A \leftrightarrow S$

Particle Creation by Black Holes

Black holes are perfect black bodies! As a result of particle creation effects in quantum field theory, a distant observer will see an exactly thermal flux of all species of particles appearing to emanate from the black hole. The temperature of this radiation is

$$kT = \frac{\hbar\kappa}{2\pi}\,.$$

For a Schwarzshild black hole $(J = Q = 0)$ we have $\kappa = c^3/4GM$, so

$$T \sim 10^{-7}\frac{M}{M_\odot}\,.$$

The mass loss of a black hole due to this process is

$$\frac{dM}{dt} \sim AT^4 \propto M^2 \frac{1}{M^4} = \frac{1}{M^2} \, .$$

Thus, an isolated black hole should "evaporate" completely in a time

$$\tau \sim 10^{73} (\frac{M}{M_\odot})^3 \text{sec} \, .$$

Analogous Quantities

$M \leftrightarrow E \leftarrow$ But M really is E!

$\frac{1}{2\pi}\kappa \leftrightarrow \; T \leftarrow$ But $\kappa/2\pi$ really is the (Hawking) temperature of a black hole!

$\frac{1}{4}A \leftrightarrow S$

The Generalized Second Law

Ordinary 2nd law: $\delta S \geq 0$

Classical black hole area theorem: $\delta A \geq 0$

However, when a black hole is present, it really is physically meaningful to consider only the matter outside the black hole. But then, can decrease S by dropping matter into the black hole. So, can get $\delta S < 0$.

Although classically A never decreases, it *does* decrease during the quantum particle creation process. So, can get $\delta A < 0$.

However, as first suggested by Bekenstein, perhaps have

$$\delta S' \geq 0$$

where

$$S' \equiv S + \frac{1}{4}\frac{c^3}{G\hbar}A$$

where S = entropy of matter outside black holes and A = black hole area.

A careful analysis of gedanken experiments strongly suggests that the generalized 2nd law is valid!

Analogous Quantities

$M \leftrightarrow E \leftarrow$ But M really is E!

$\frac{1}{2\pi}\kappa \leftrightarrow \ T \leftarrow$ But $\kappa/2\pi$ really is the (Hawking) temperature of a black hole!

$\frac{1}{4}A \leftrightarrow S \leftarrow$ Apparent validity of the generalized 2nd law strongly suggests that $A/4$ really is the physical entropy of a black hole!

Conclusions

The study of black holes has led to the discovery of a remarkable and deep connection between gravitation, quantum theory, and thermodynamics. It is my hope and expectation that further investigations of black holes will lead to additional fundamental insights.