Physics and Complexity

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Physics

Dictionary definition:

Branch of science concerned with the nature and properties of matter and energy

But today I want to use it as much as

a mind-set with valuable methodologies

and to show application

to many complex systems in many different arenas

Physics

as sometimes portrayed

Particle Physics

‘Fundamental’ particles

Cosmology

How it all began

Search for the

‘Theory of everything’

But not today
‘More is different’

Particle Physics ~~Cosmology~~

‘Fundamental’ particles How it all began

‘Theory of everything’

TOE is by no means the whole story

Many body systems often give new behaviour through co-operation

Both ‘fundamental’ and applicable

Examples of emergent phenomena

- Superconductivity
  - Flux quantization
- Magnetism
- Giant Magnetoresistance
  - Basis of modern high capacity data storage
- Quantum Hall Effect
  - Quantized conductivity plateaux

Highest accuracy measurement of e^2/h

Complexity/Complex Systems

- Many body systems
- Cooperative behaviour complex
  - non-trivial and new
    - not simply anticipatable from microscopics
      - occurs even with simple individual units and simple interaction rules
  - But with surprising conceptual similarities between superficially different systems

Typical approach

- Essentials?
  - Minimal models
  - Comparisons/checks: e.g. simulation
  - Analysis: maths & ansätze
- Important consequences?
- Transfers, similarities & differences?

Build → • Conceptualization

Lead to → • Generalization

• Application

Key ingredients

Frustration

Conflicts

Disorder

Frozen / self-induced / time-dependent

Emphasis

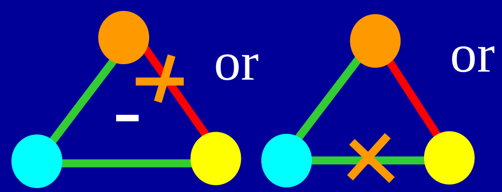
- Novel physics
- New concepts
- Minimalist models
- Interdisciplinary transfers
  - Much ubiquity, some differences
  - Relevance of noise and memory
  - Applicability

The Dean's Problem

- Dean to allocate *N* students to two dorms

- Some students like one another; prefer same dorm
- Others dislike like one another; prefer different dorms

- Cannot satisfy all → Frustration



- Best compromise for whole student body?

The Dean's Problem as combinatorial optimization

Maximise a Happiness function

$$H = +\sum_{(ij)} J_{ij} S_i S_j$$

Students

$S = +/-1$

Dorm A/B

$\sum$ means Sum

J: Inter-student friendship

Statistical Dean's problem

$$H = +\sum_{(ij)} J_{ij} S_i S_j$$

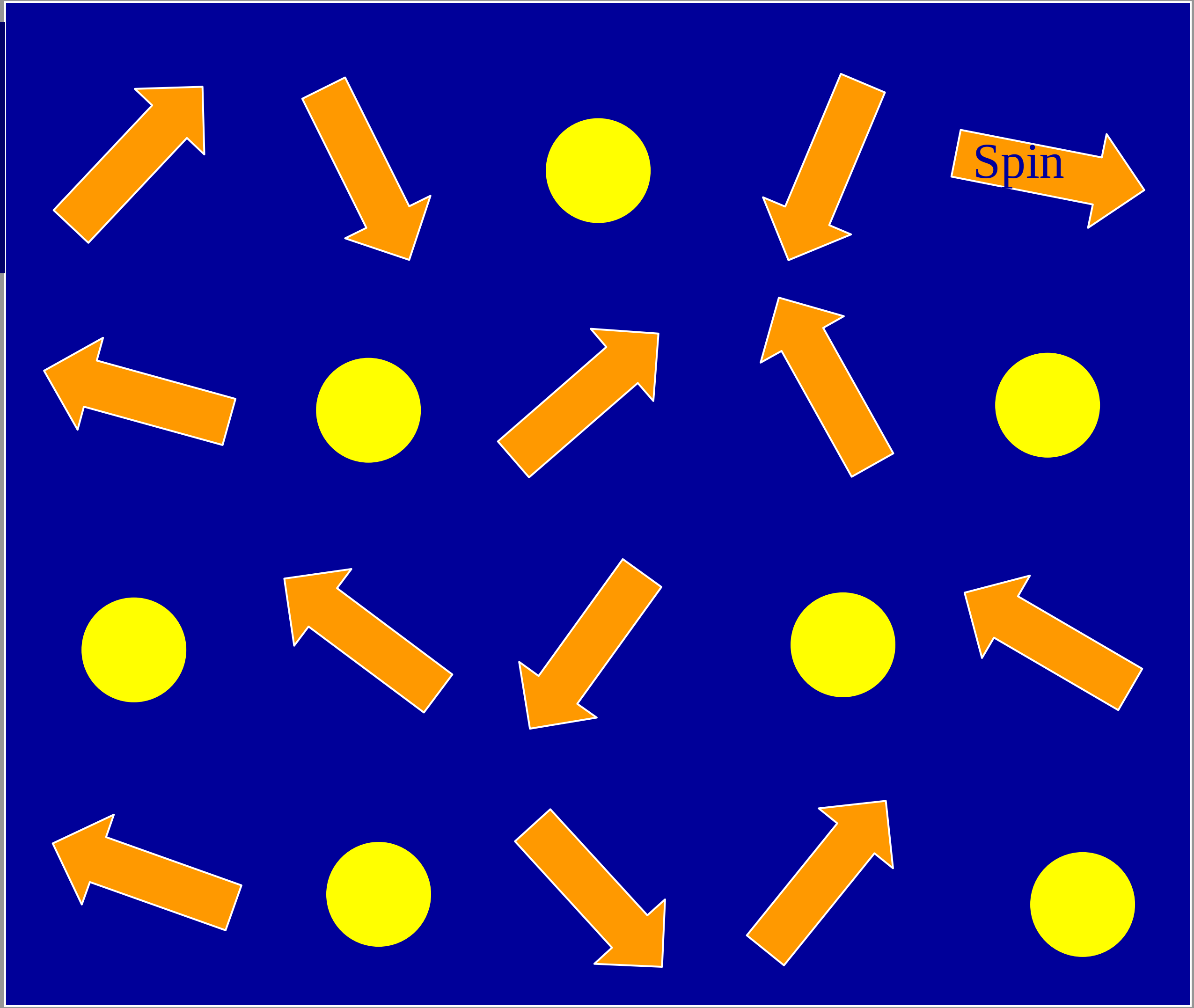
Chosen randomly ± from a distribution *P(J)*

Hard! 2^N choices: NP-complete: Clay Millenium prize problem # 4

Spin glass

Solid alloy:
Non-magnetic & magnetic atoms

e.g. AuFe
CuMn



Magnetic Moment ~ “spin”

Glass: Amorphous/ non-periodic

Amorphous Spin freezing

Hence “Spin glass”

Glass also ~ slow, aging, memory..

Mossbauer → freezing

Neutron diffraction → No periodicity

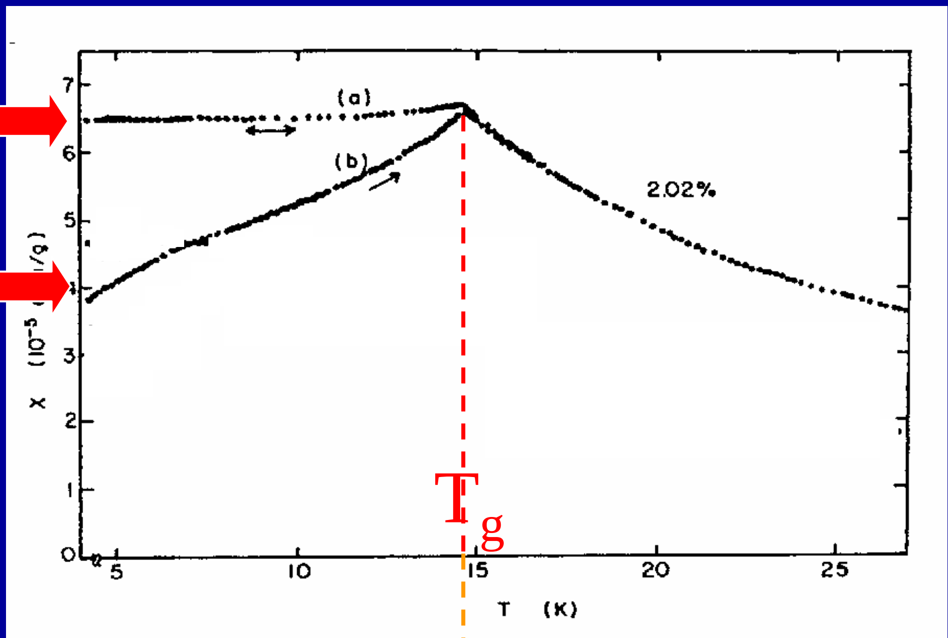
Spin glass

Phase transitions & preparation-dependence

Susceptibility (d*m*/d*h*)

Field-cooled

Zero-field cooled



AuFe

non-ergodic/non-equilibrating | ergodic/equilibrating

"Smoking gun" for complexity

Spin glass Hamiltonian/ Energy

Real: Site-disorder

$$H = -\sum_{\text{magnetic sites only}} J(\vec{R}_i - \vec{R}_j)\vec{S}_i . \vec{S}_j$$

Interaction

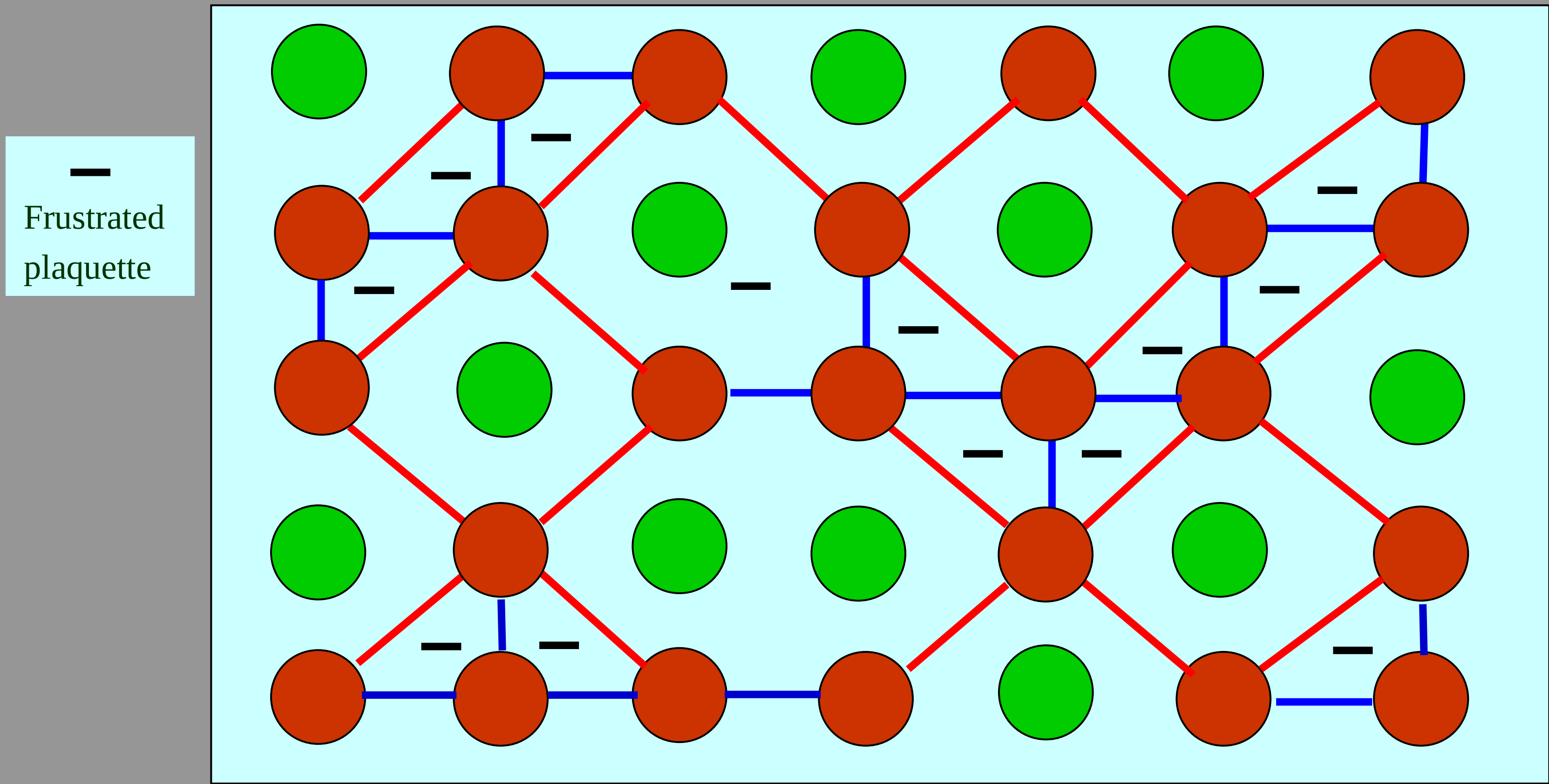
Varies in sign (and strength) as function of separation

Usually short-range ferro (+), then antiferro (-), …

Try to minimize H

Conflicts / Frustration

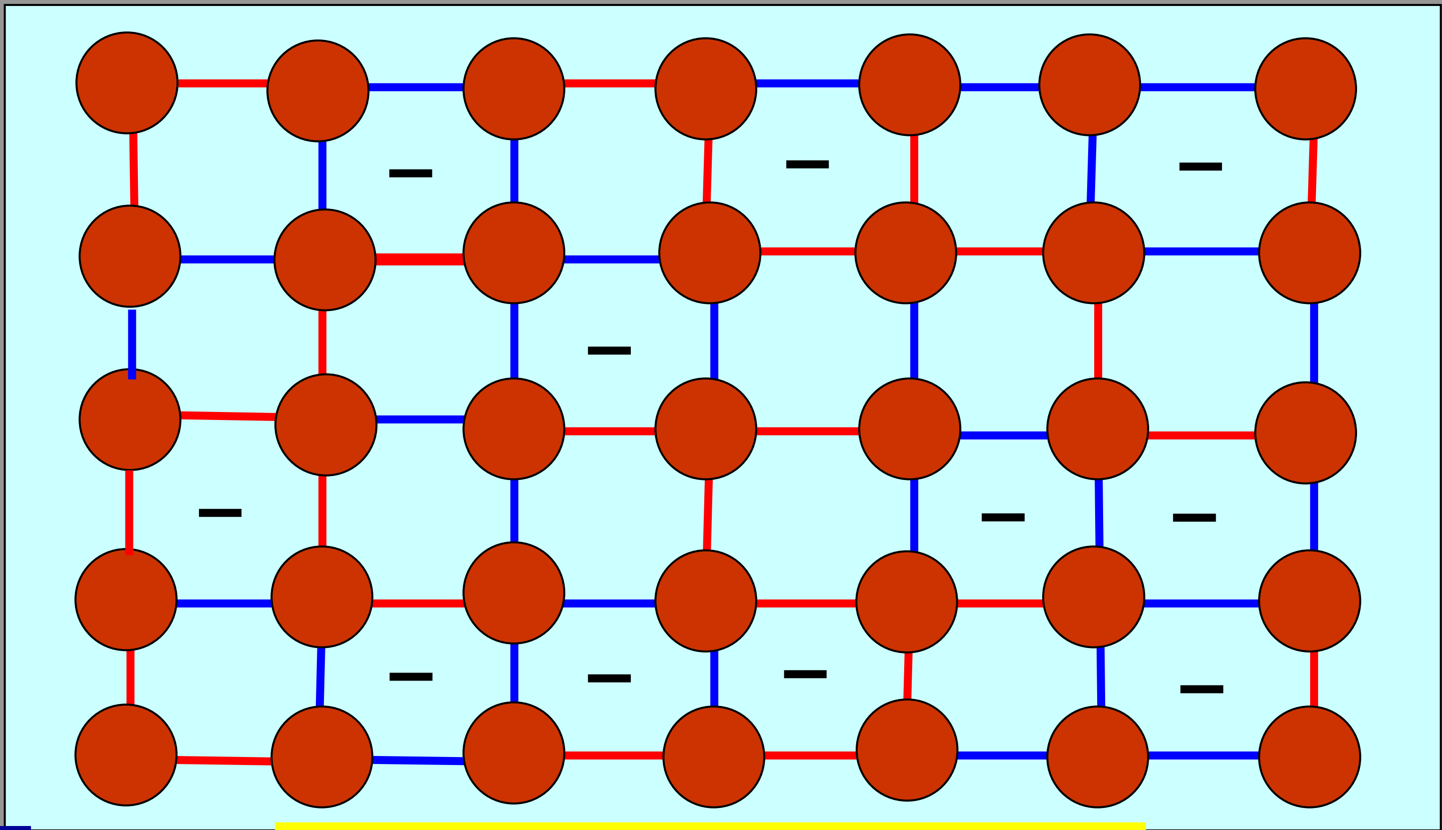
Random site frustration



n.n. ferromagnetic ↑↑ n.n.n. antiferromagnetic ↑↓

Convenient alternative model

Random bond



Simulations
qualitatively
~ experiment

$$H_{EA} = -\sum J_{ij}\,\vec{S}_i.\vec{S}_j;\ P(J_{ij})$$

quenched random

n.n. not exactly soluble
∞-range soluble but subtle

Dean's model ≡ Range-free Spin Glass Model

Dean's	Unhappiness	Friendship	Dorm allocation
Spin glass	Hamiltonian	Exchange interactions	Spins ~ magnetic moments

$$H = -\sum_{(ij)} J_{ij} S_i S_j$$

$$S_i = \pm 1;$$ A / B

Statistical physics: Typical properties

Soluble+ but very subtle$

\+ Not quite Clay P=NP?

$ Complex

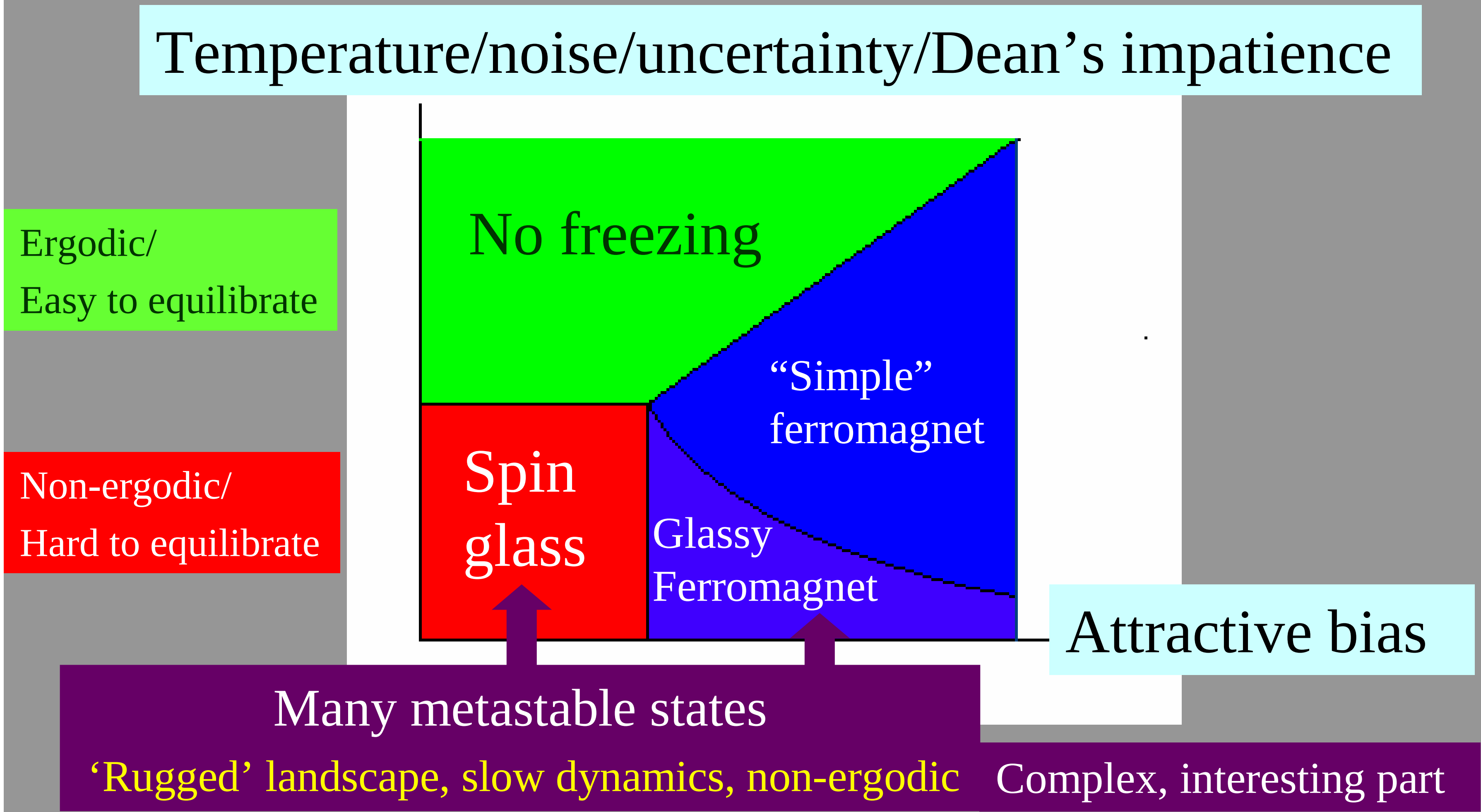
Methodologies to study

Symbiosis

of

- Analytic
- Simulational
- Experimental

Phase diagrams



Rugged Landscape Paradigm

Two-dimensional cartoon of high dimensional concept

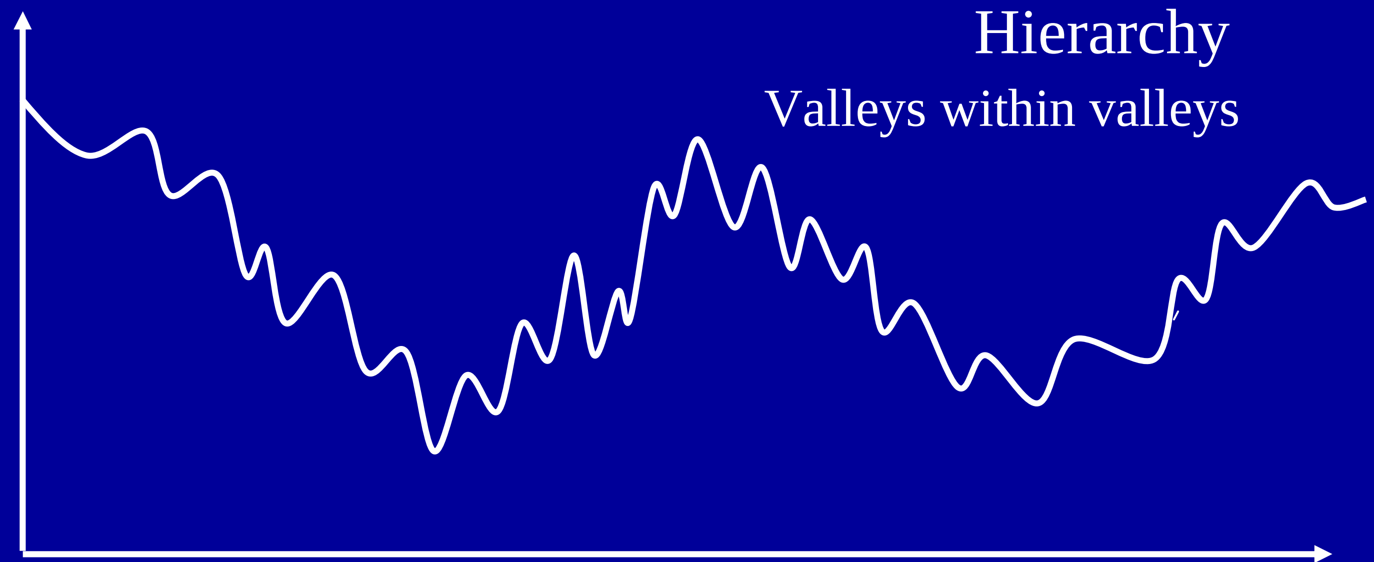
Many metastable states

Hierarchy

Valleys within valleys

Cost

to minimise



Coordinates

Dynamics

Mostly downhill motion

Hard to minimise: sticks: glassiness

Understanding?

- Nature and relationship of metastates
  - And hierarchies
- Macroscopic dynamics
  - Non-equilbration, aging and memory
- Origins and necessary ingredients

Methodologies

- Analytic
- Simulational
- Experimental

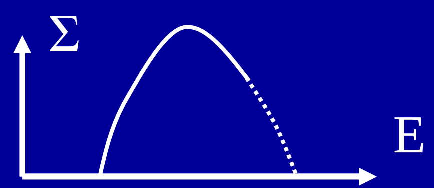
Symbiosis

↓

Extension

How do we know?

- Computer simulations
- Analytic calculations
  - e.g. metastable minima



$$\Sigma = \ln N_{meta}$$

  - Overlap: $$q^{SS'} = N^{-1}\sum_i \langle \sigma_i \rangle^S \langle \sigma_i \rangle^{S'}$$

S ~ macrostate

Overlap distribution: $$P(q) = \sum_{SS'} W_S W_{S'} \delta(q - q^{SS'})$$

Prob of S

Conventional: $P(q)$ has single δ-function

Complex: $P(q)$ has structure

→ many non-equivalent macrostates

Recall

Very simple microscopic entities

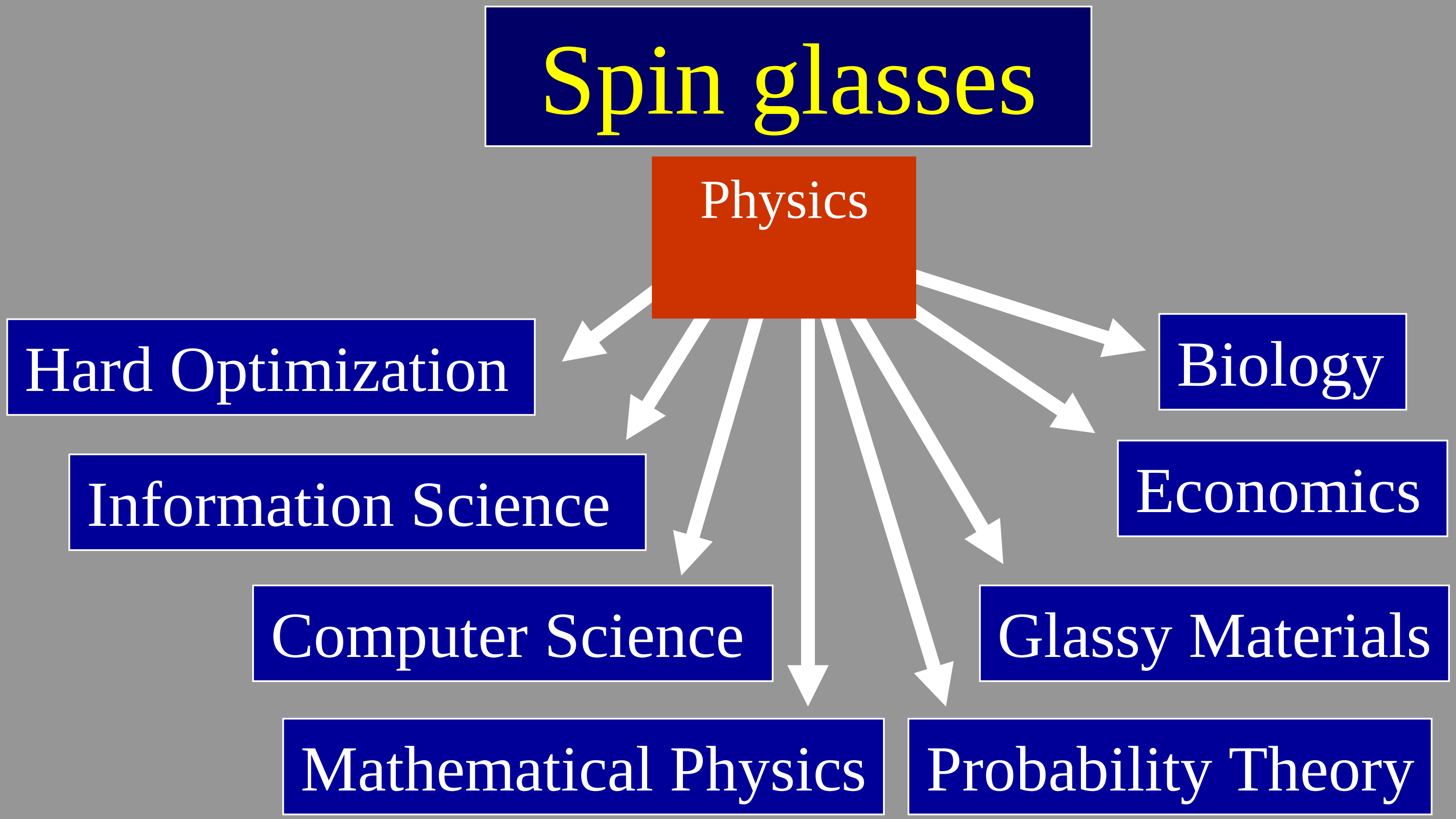
Very simple pairwise interactions

Rich complexity in collective behaviour

due to frustration and disorder

`Complex' is different from `complicated'

Extensions


Spin glasses
Physics
Hard Optimization
Information Science
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Mathematical Physics
Probability Theory
Glassy Materials
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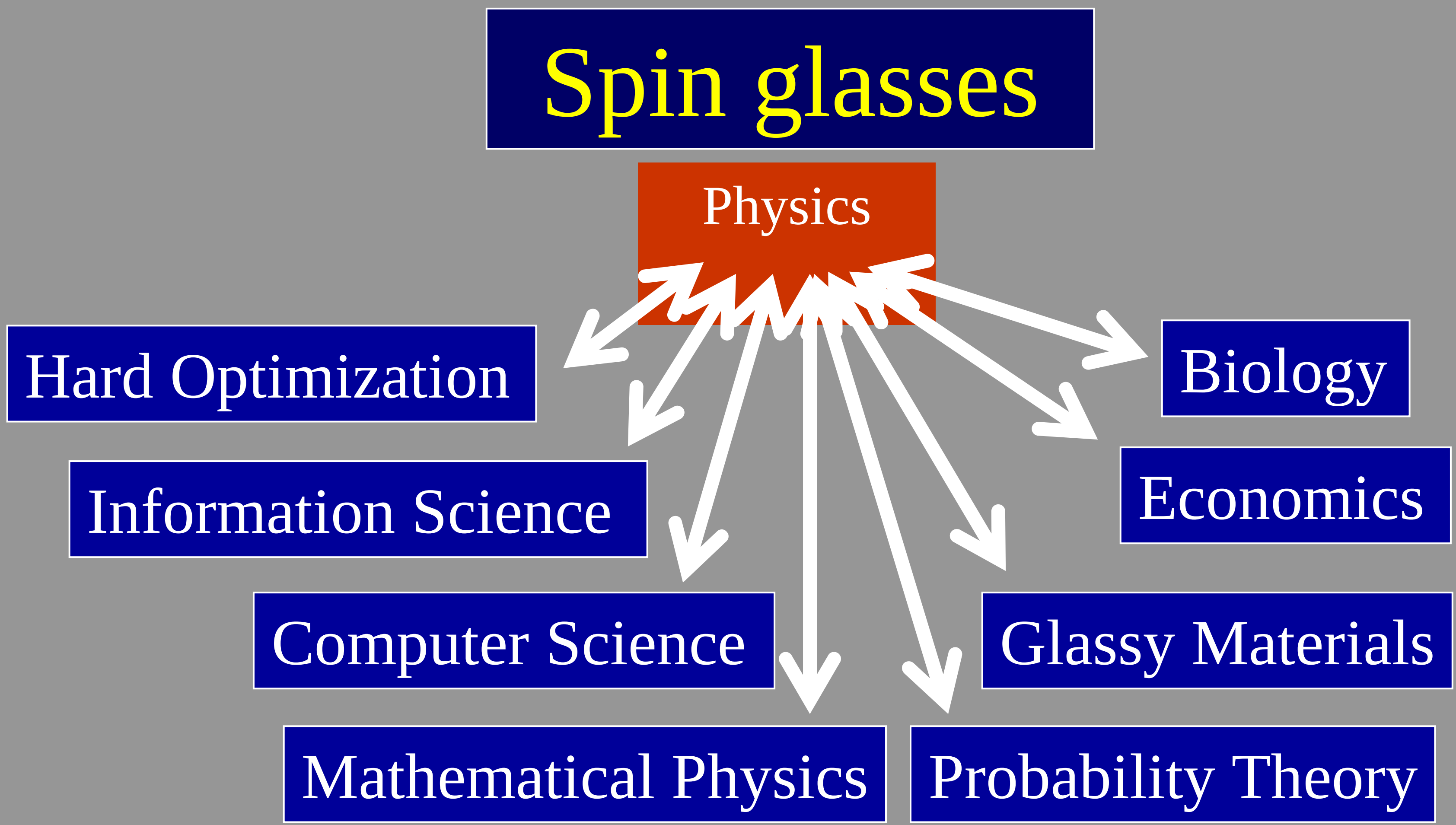
Biology

Two-way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General theoretical structure

Control functions

$$F(\{J_{ij....k}\},\{S_{ij..}\},\{T\})$$

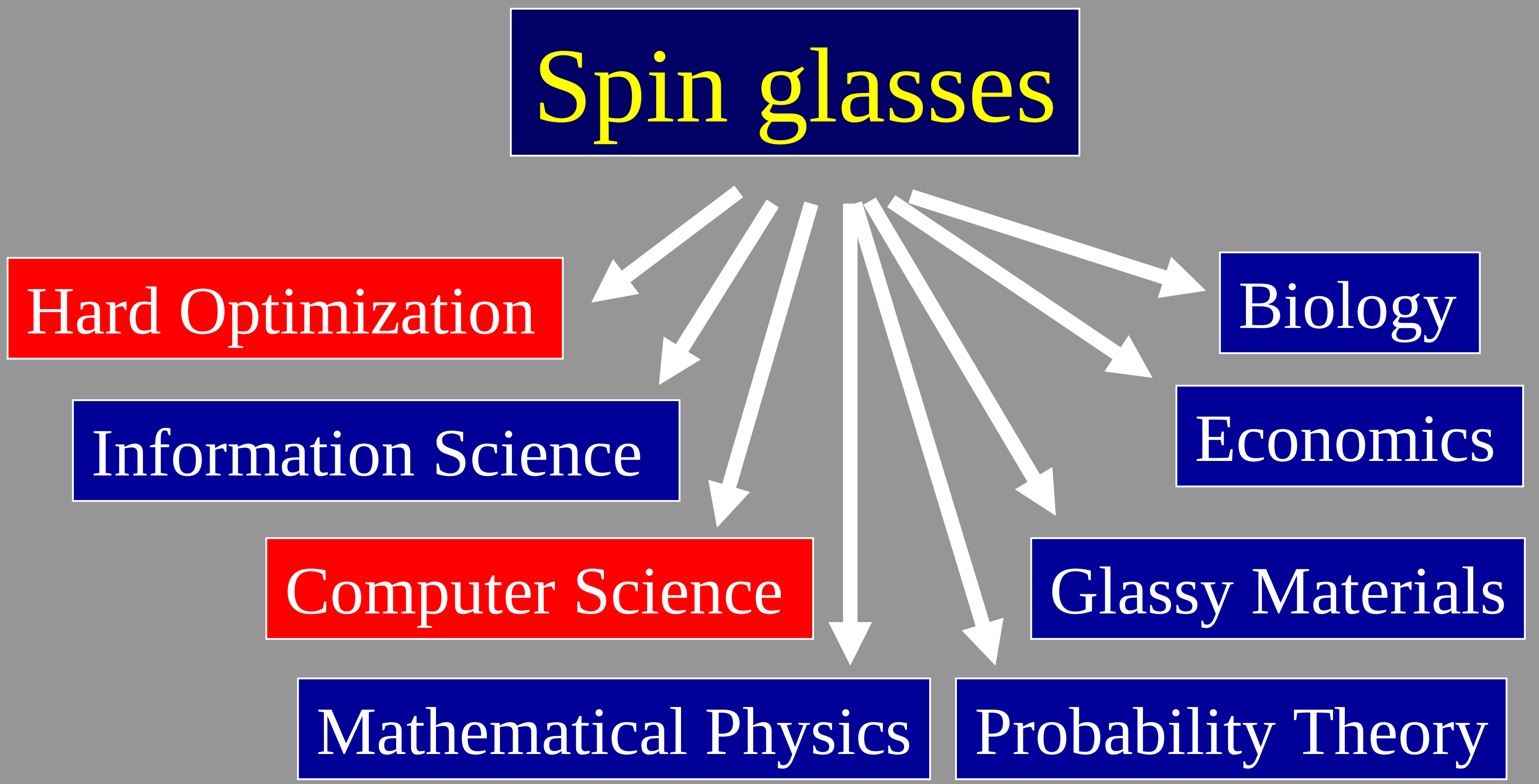
	$\{J_{ij....k}\}$	$\{S_{ij..}\}$
Statics:	Fixed	Variable
Dynamics:	Slow	Fast

$\{T\}$: External influences

Control functions, but who controls?

- Physics: nature/physical laws
- Biology: nature but not necess. equilibrium
- Hard optimization: we choose algorithms
- Information science: we have choice
- Markets: supervisors, government bodies
- Society: governments can change rules

Examples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Examples

- Minimizing a cost
  - *e.g.* distribution of tasks
- Satisfiability
  - Simultaneous satisfaction of ‘clauses’
- Error correcting codes
  - Capacity and accuracy

Two issues

- What is achievable in principle?
  - Analogue in stat. physics:
  - thermodynamics (“statics”)/equilibrium
- How to achieve it?
  - Needs algorithms ~ (computational) dynamics

Two issues

- What is achievable in principle?
  - Analogue in stat. physics:
  - thermodynamics (“statics”)/equilibrium
    - May be still be hard to find
- How to achieve it?
  - Needs algorithms ~ dynamics
    - But glassiness can badly hinder efficacy
    - Equilibrium may not be practically achievable

K-satisfiability

simultaneous satisfiability

of many 'clauses' of length K

$$(x_{i_1} \text{ or } x_{i_2} \text{ or.. } \overline{x_{i_K}}) \text{ and } (x_{j_1} \text{ or } \overline{x_{j_2}} \text{ or.. } x_{j_K}) \text{ and ...}$$

$x = 1,\ \textit{true}$

$\bar{x} = 0,\ \textit{false}$

Especially

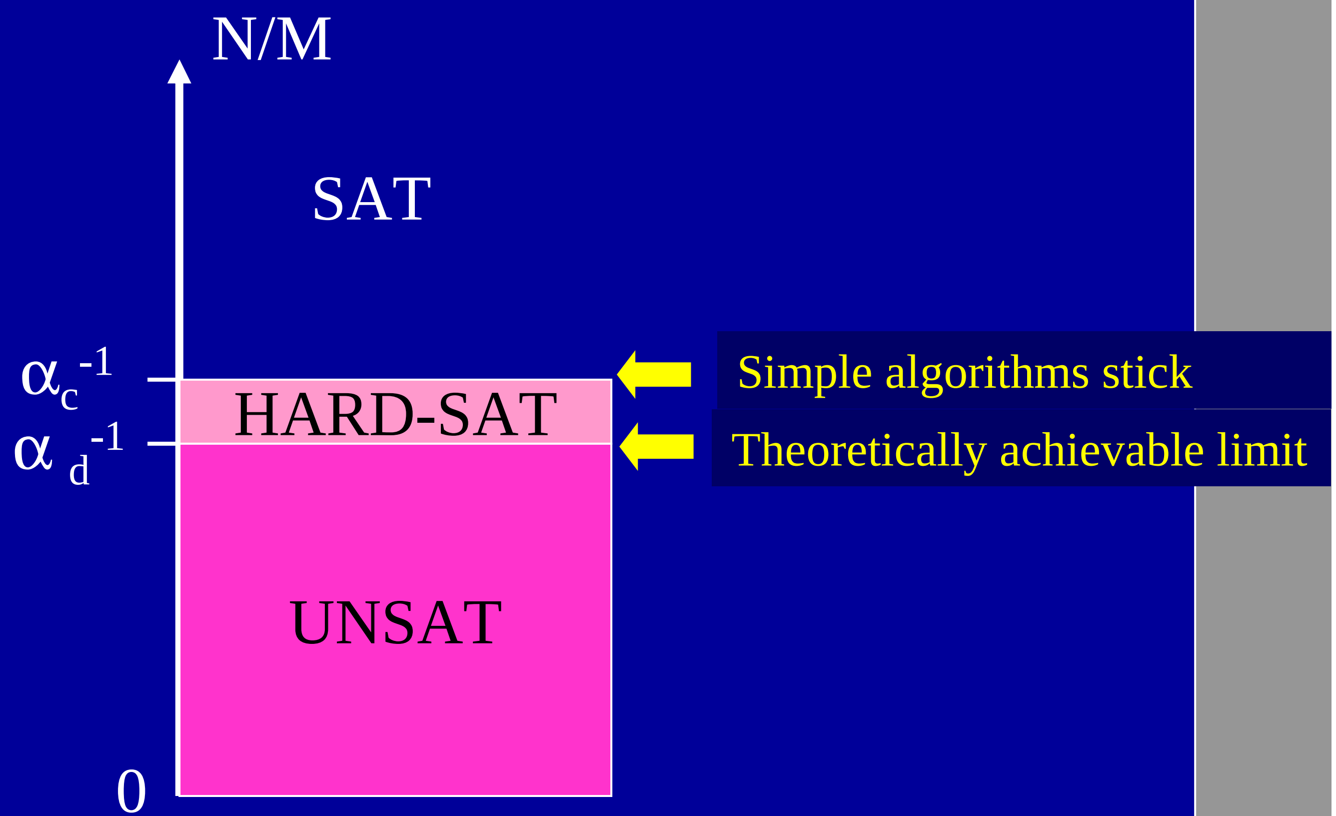
Random K-SAT

$$\alpha \equiv \frac{M}{N} = \left\{ \frac{\#\text{ of clauses}}{\#\text{ of variables}} \right\}$$

Phase transition(α): SAT / UNSAT

Random K-SAT

Phase transitions



Physicists recognised this subtlety through comparison with *K-spin glass*

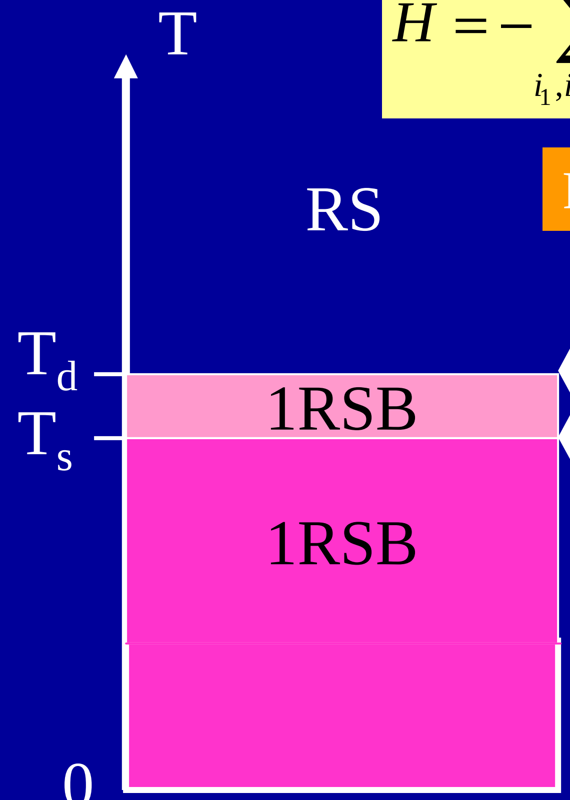
Where the idea came from

K (>2) -spin glass

$$H = -\sum_{i_1, i_2 .. i_K} J_{i_1 i_2 .. i_K} S_{i_1} S_{i_2} .. S_{i_K}$$

Random

2 transitions



Dynamical transition

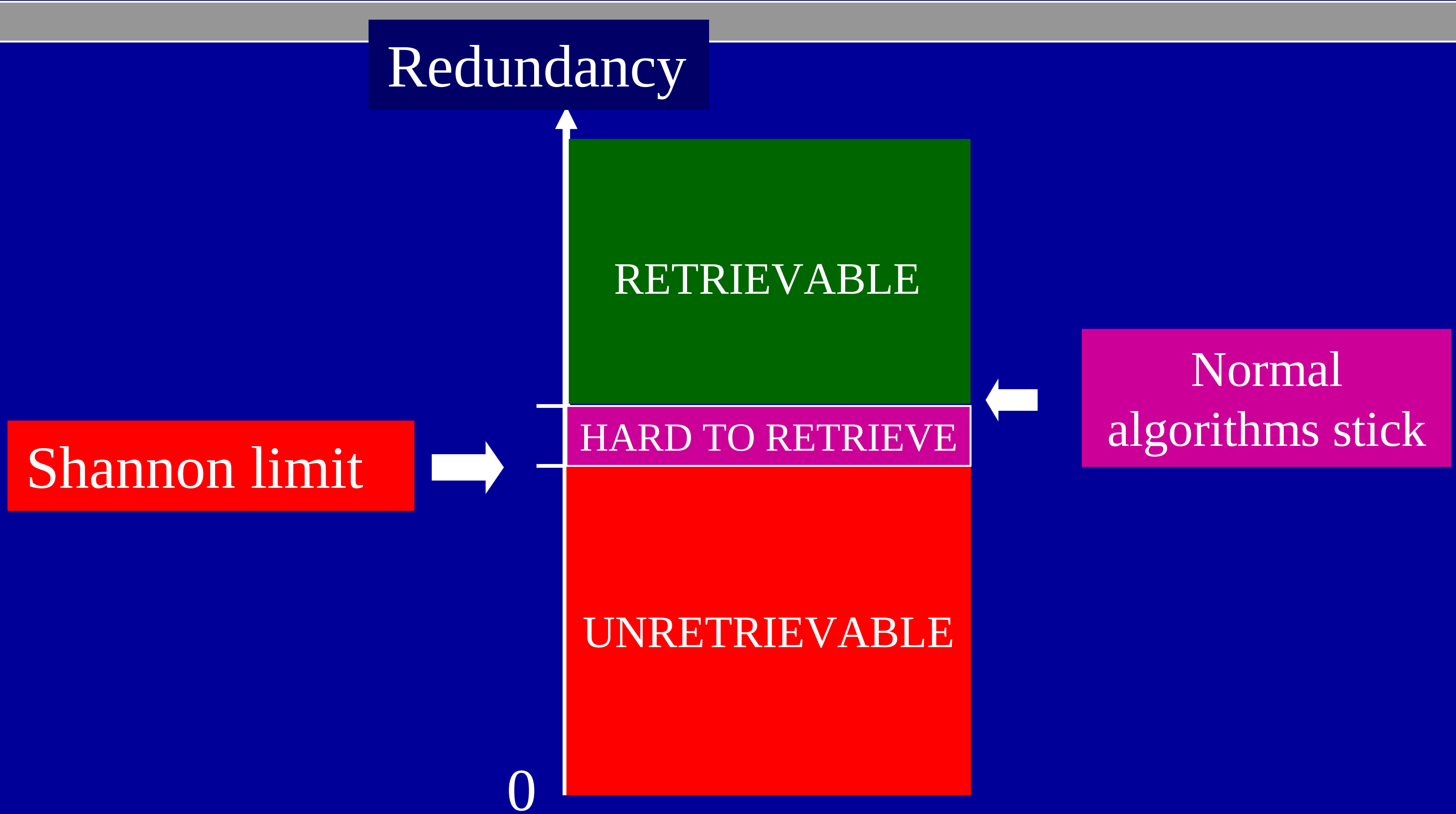
Thermodynamical transition

RSB=Glassy

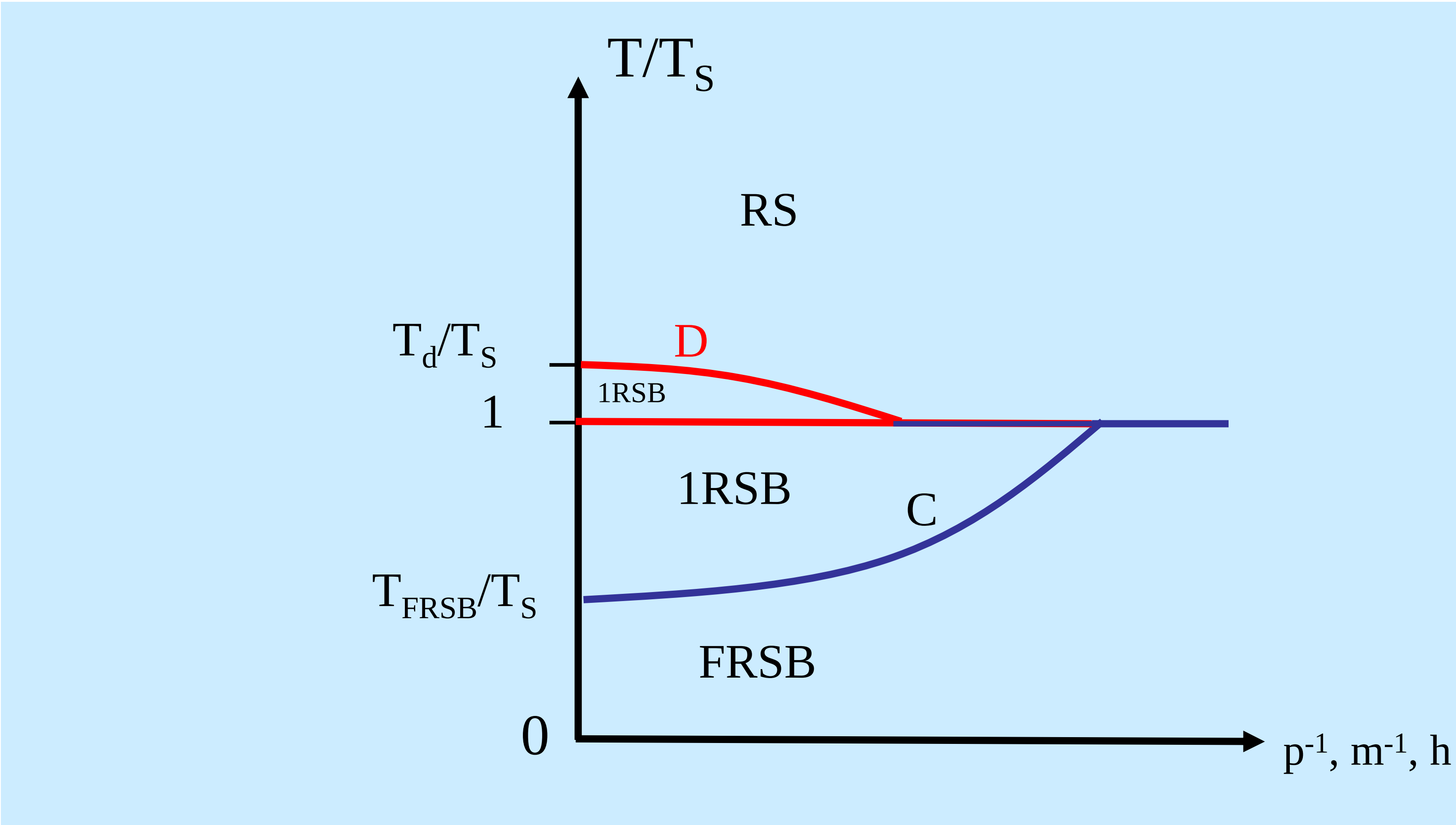
1RSB = all macrostates equally orthogonal

Originally looked at as a purely intellectually interesting extension

Similarly: error-correcting codes



Generic phase transitions



Potts, quadrupolar, p-spin in field

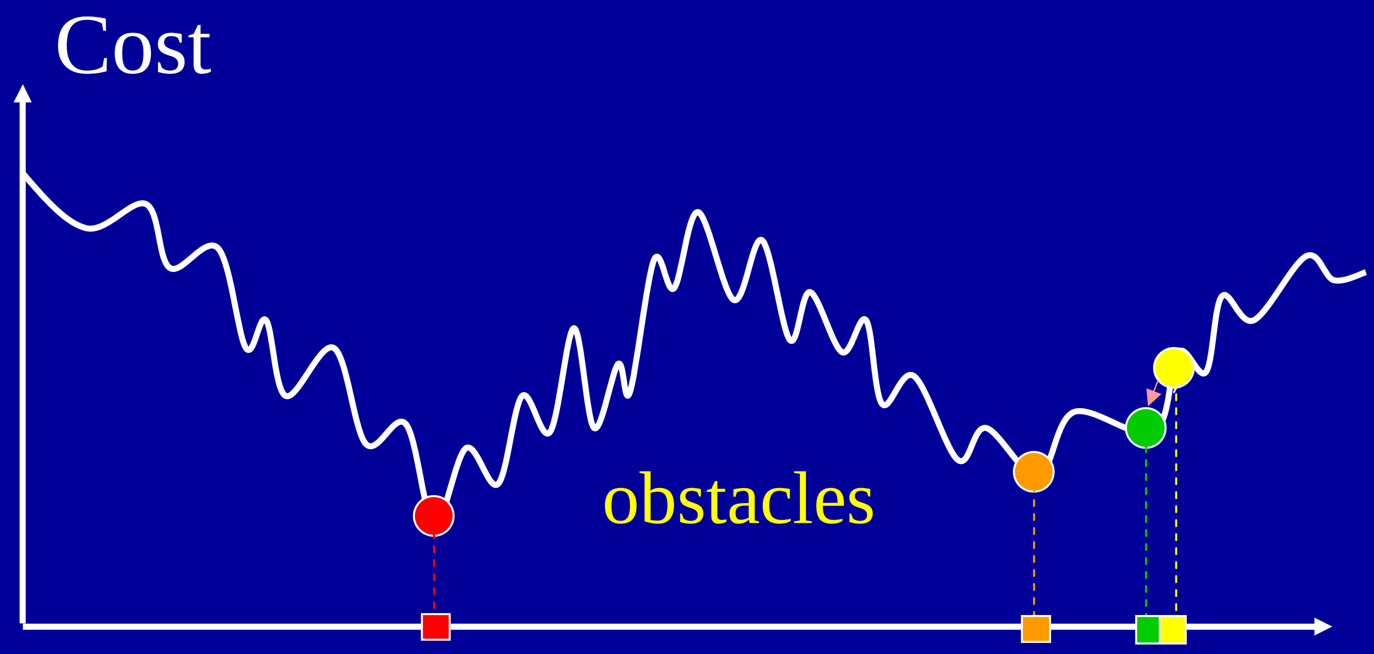
Understanding brings opportunities

- Normal physics
  - Nature gives dynamics
- Artificial and model systems
  - We can design dynamics
  - Computational algorithms & simulational expts.
    - Simulated annealling
    - Simulated tempering
    - Great advance: Survey propagation

Controlled systems

New probes

Landscape paradigm for hard optimization



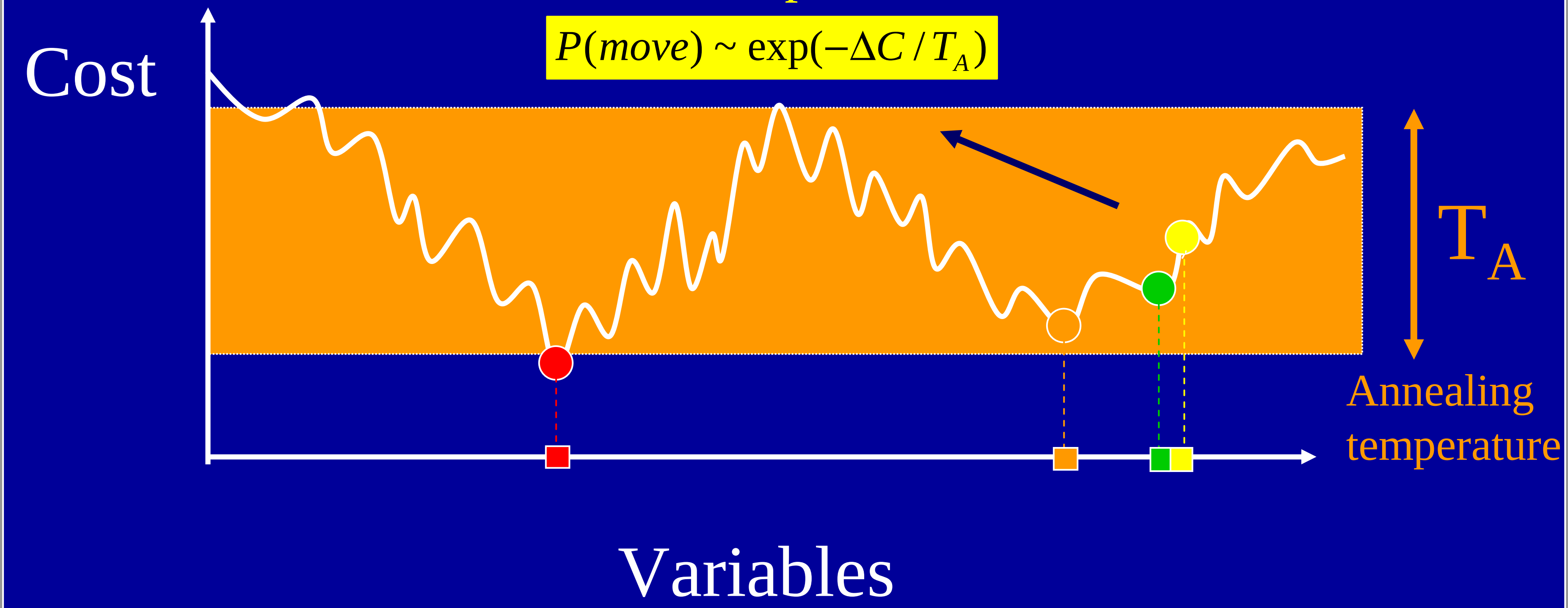
Steepest descent gets stuck

Simulated annealing

Probabilistic hill-climbing

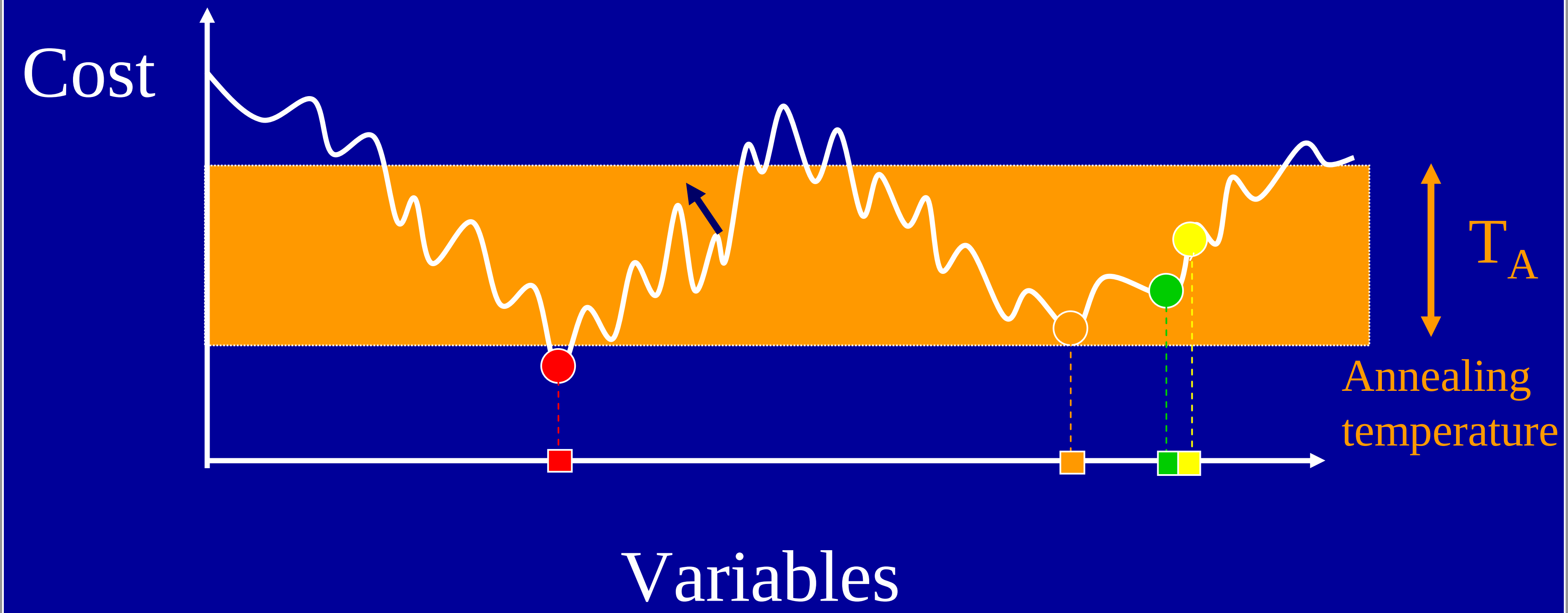
Add 'temperature'

$$P(move) \sim \exp(-\Delta C / T_A)$$

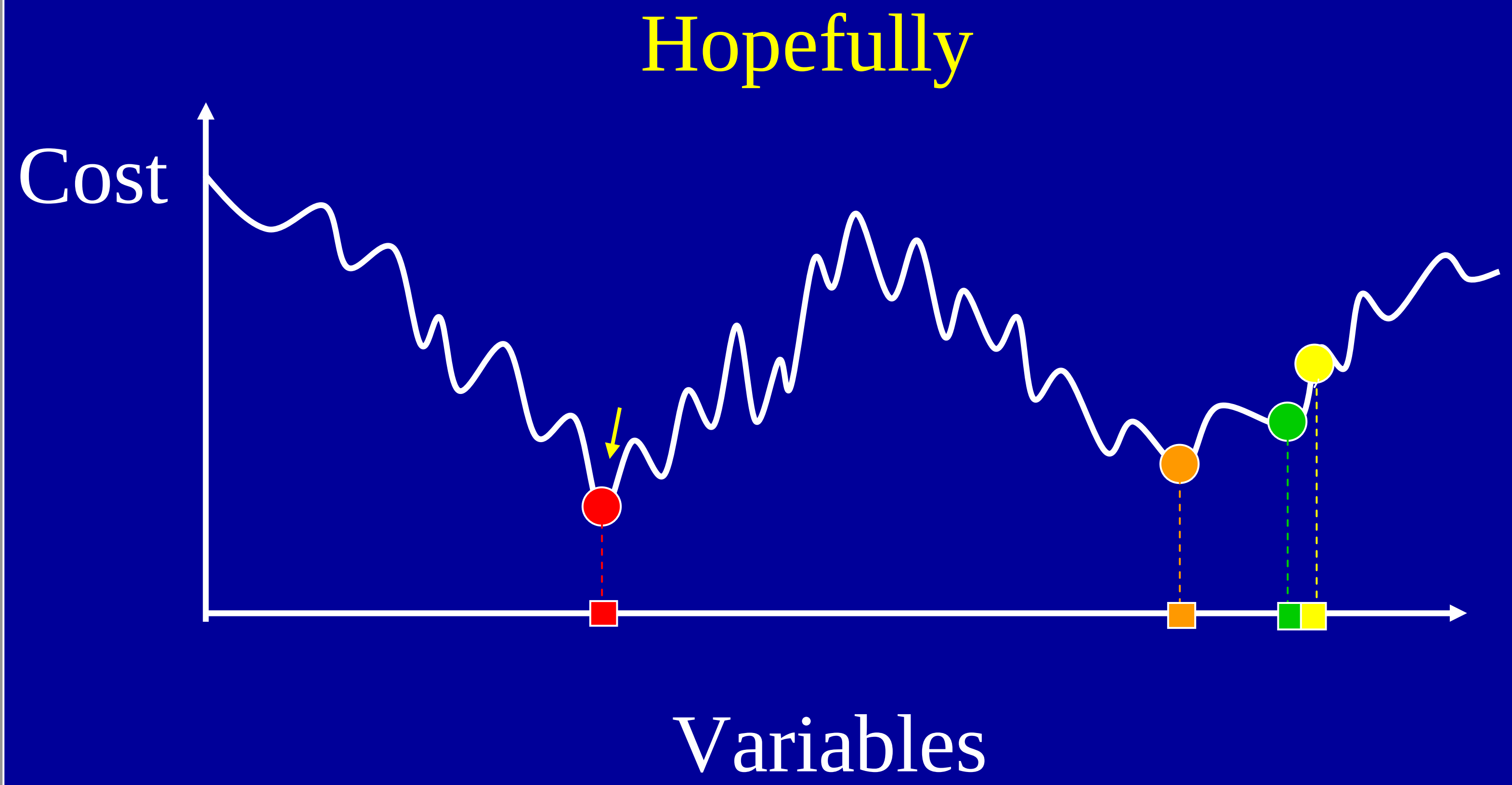


Simulated annealing

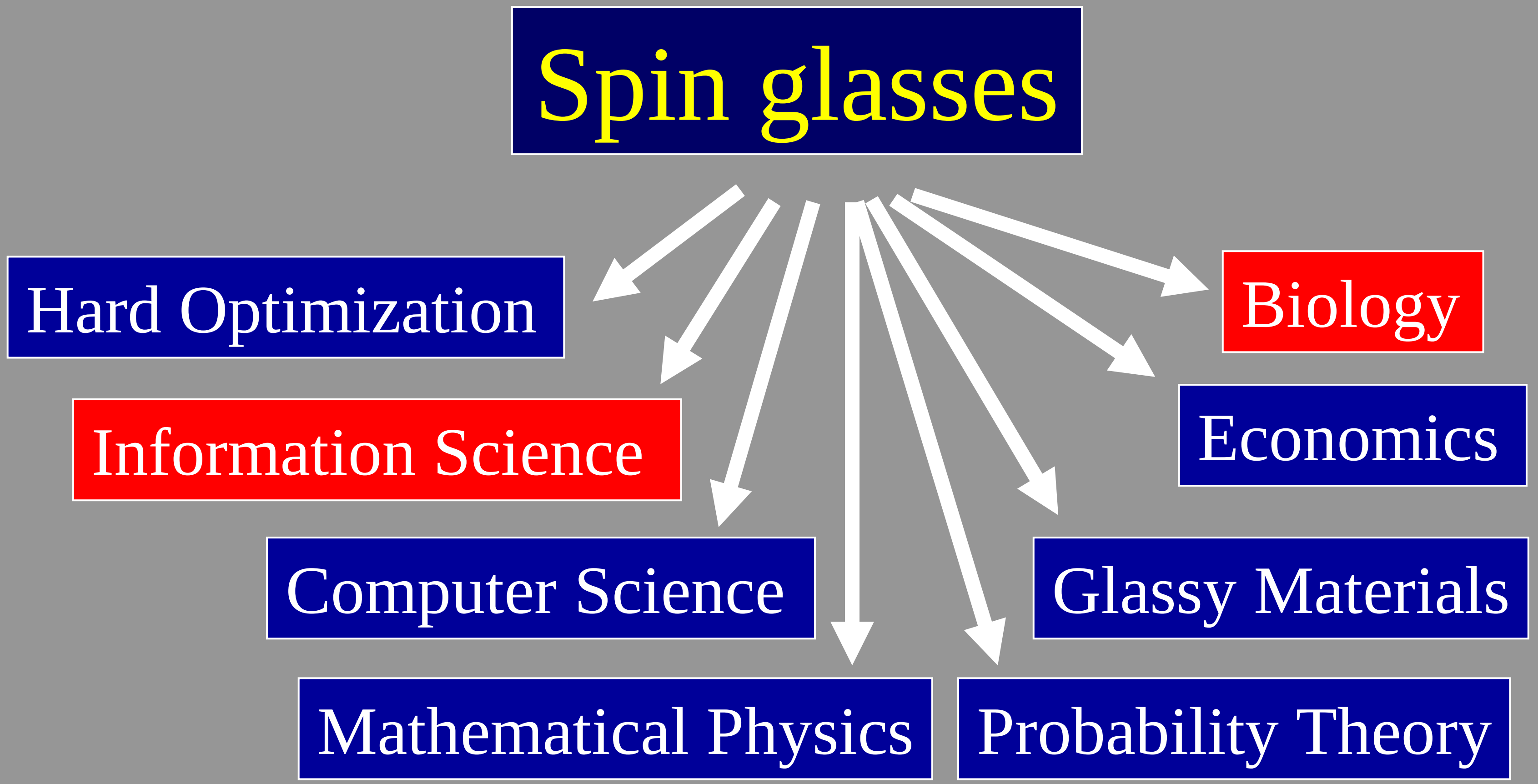
Gradually reduce T_A



Simulated annealing



Examples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Neural network: Hopfield

Quasi-spin statistical mechanics

Neurons, rate of firing

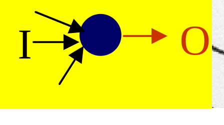
$$H = -\sum_{ij} J_{ij} S_i S_j$$

Synapses

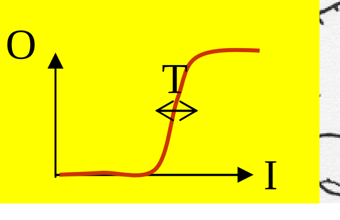
Quasi-random +/-

Store memories

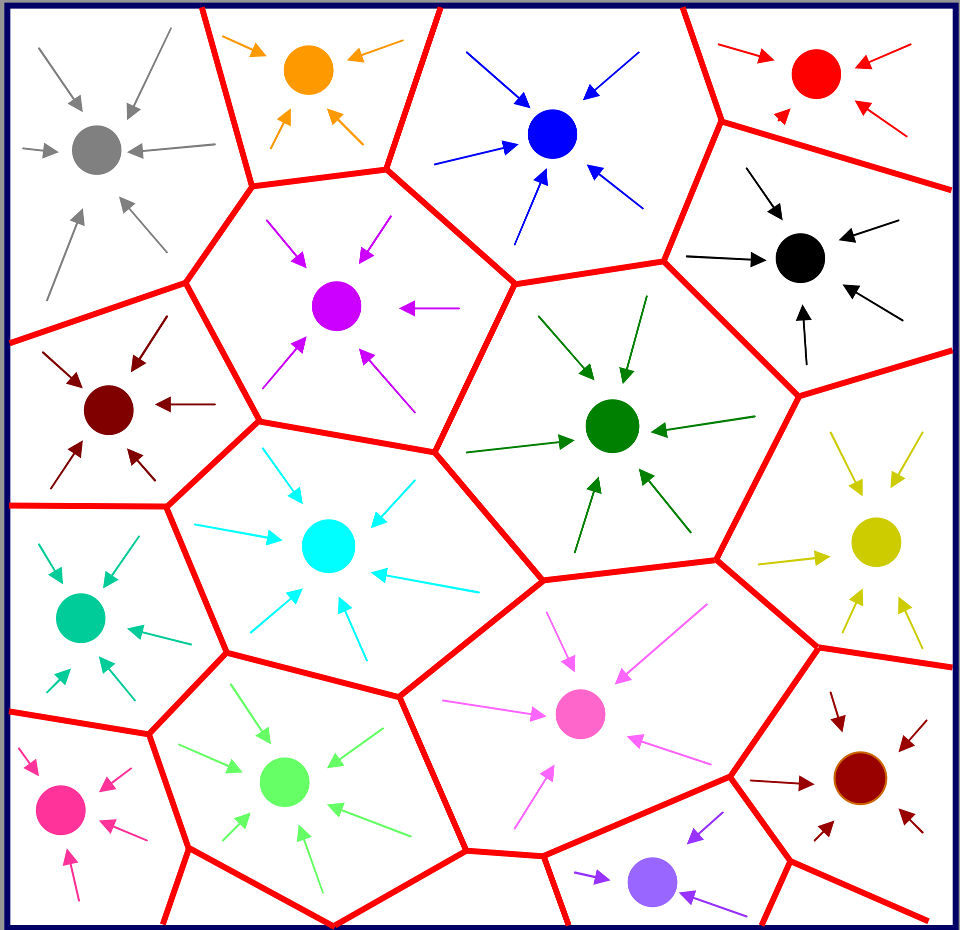
T ~ synaptic sigmoidal response rounding



$$I_i = \sum_j J_{ij} S_j$$



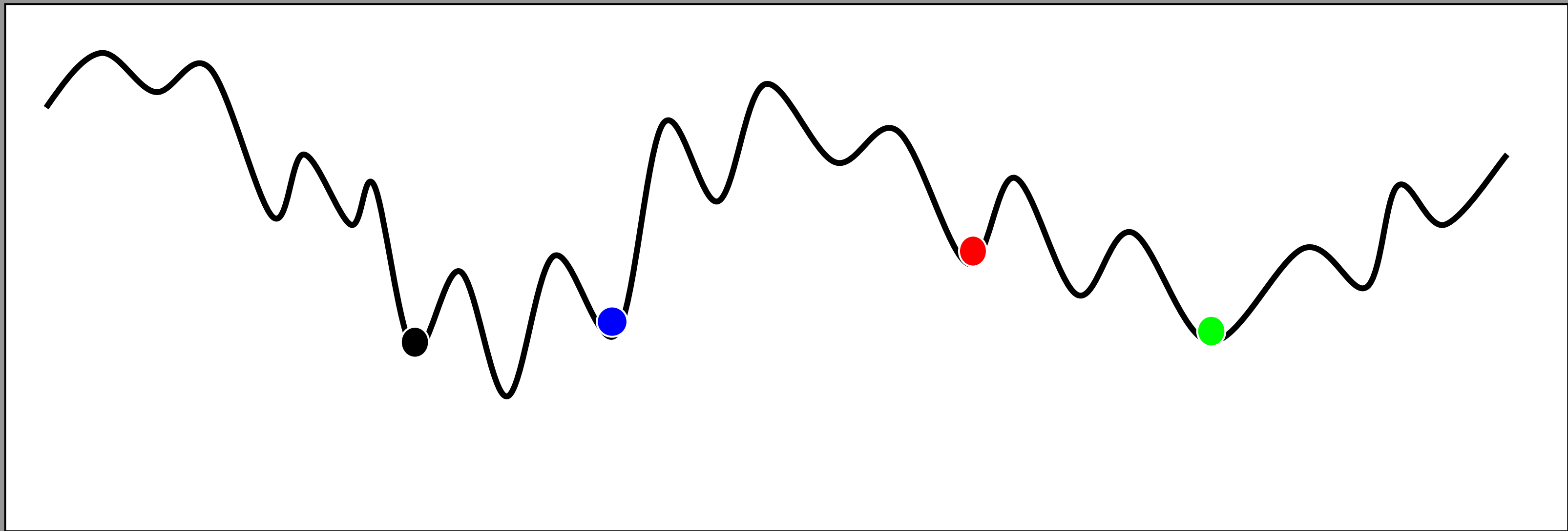
Attractors: tuned metastable states



Phase space

- Associative memory
  - 'attractors' 
  - ~ memorized patterns μ
- Retrieval basins
- Many memories
  - ~ many attractors
  - require frustration
  - Stored in {J}

Rugged landscape



Valleys ~ attractors

$\{s_i\}$

Sculpture ~ learning

$\{J_{ij}\}$

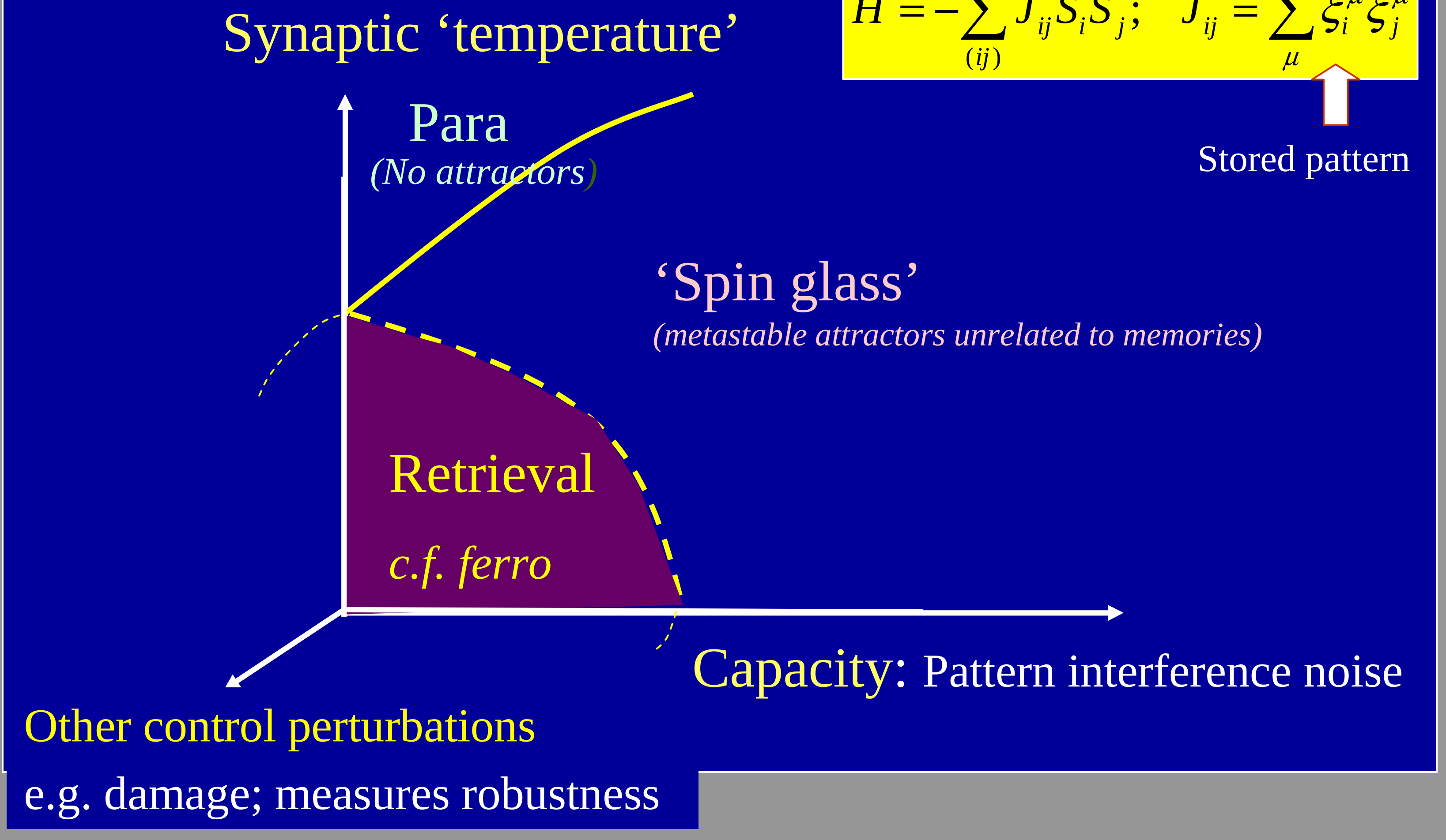
Different timescales

fast retrieval

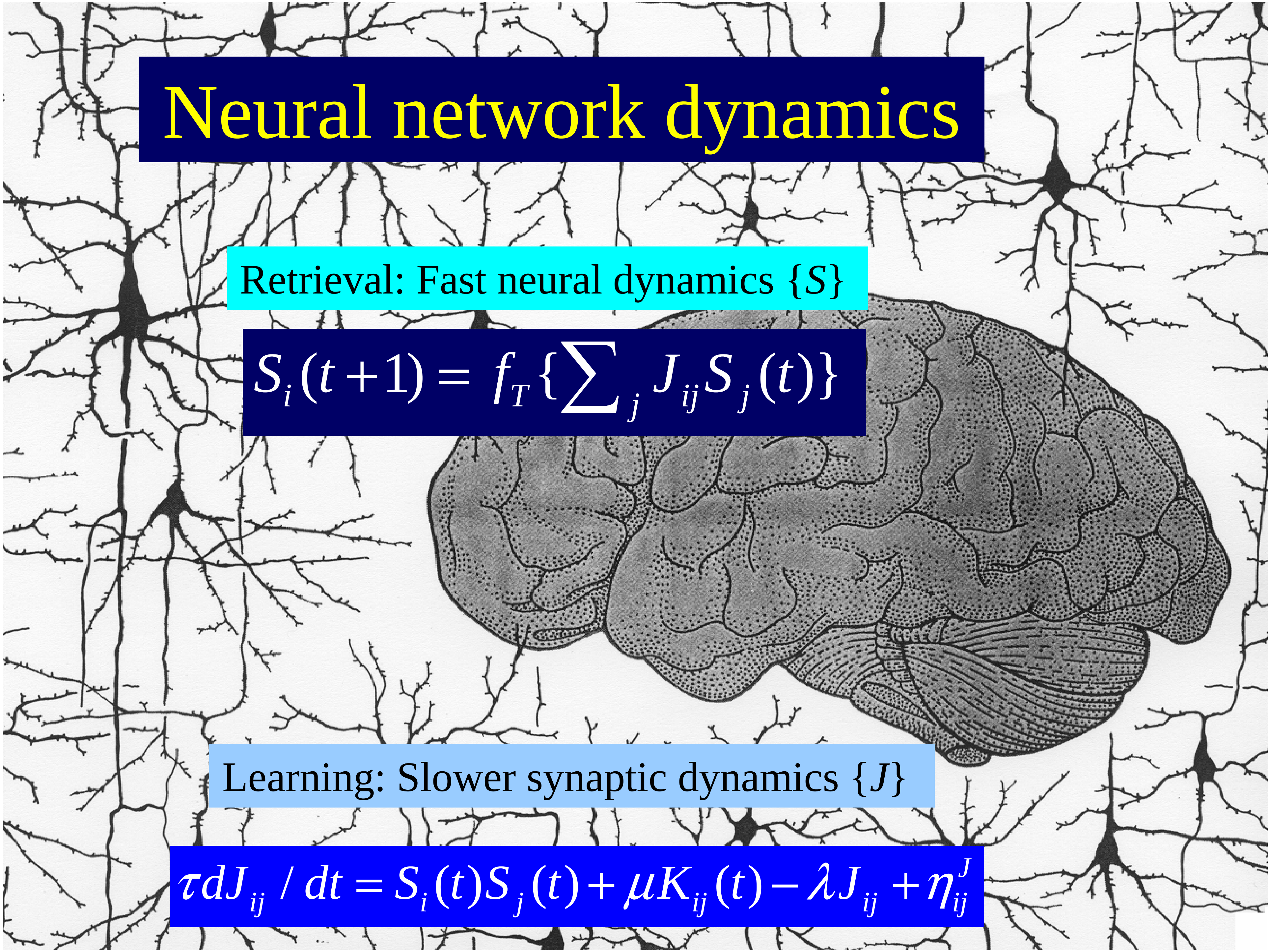
slow learning

‘Phase diagram’: Hopfield model

$$H = -\sum_{(ij)} J_{ij} S_i S_j ; \quad J_{ij} = \sum_{\mu} \xi_i^{\mu} \xi_j^{\mu}$$



Neural network dynamics



Retrieval: Fast neural dynamics $\{S\}$

$$S_i(t+1) = f_T\{\sum_j J_{ij} S_j(t)\}$$

Learning: Slower synaptic dynamics $\{J\}$

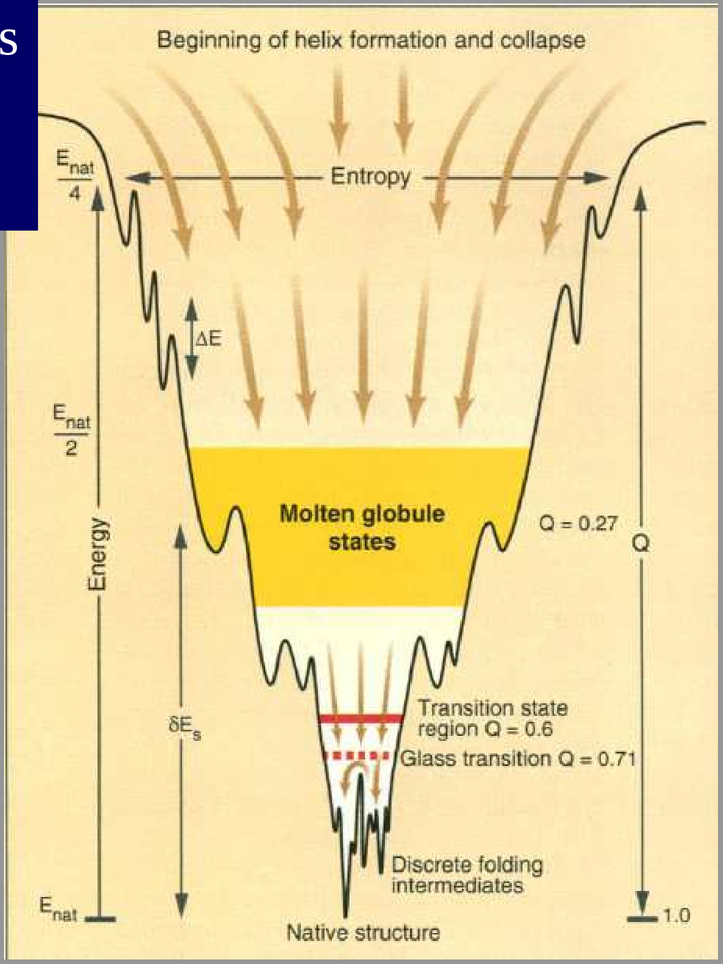
$$\tau dJ_{ij} / dt = S_i(t) S_j(t) + \mu K_{ij}(t) - \lambda J_{ij} + \eta_{ij}^J$$

Proteins

Proteins: Heteropolymrers
Many amino acids
Frustrated interactions

Must fold fairly easily
Minimal frustration

Folding funnel
Wolynes et. al.



Random heteropolymers
In general, very frustrated
Fold poorly, glassy

Evolution:
Initial random soup
Fast: try to fold
Slower time-scale:
Reproduction/mutation
Good folders selected

Analogies

Glassy/slow | More minimal frustration/faster

Spin glass | Neural network

SK | Hopfield

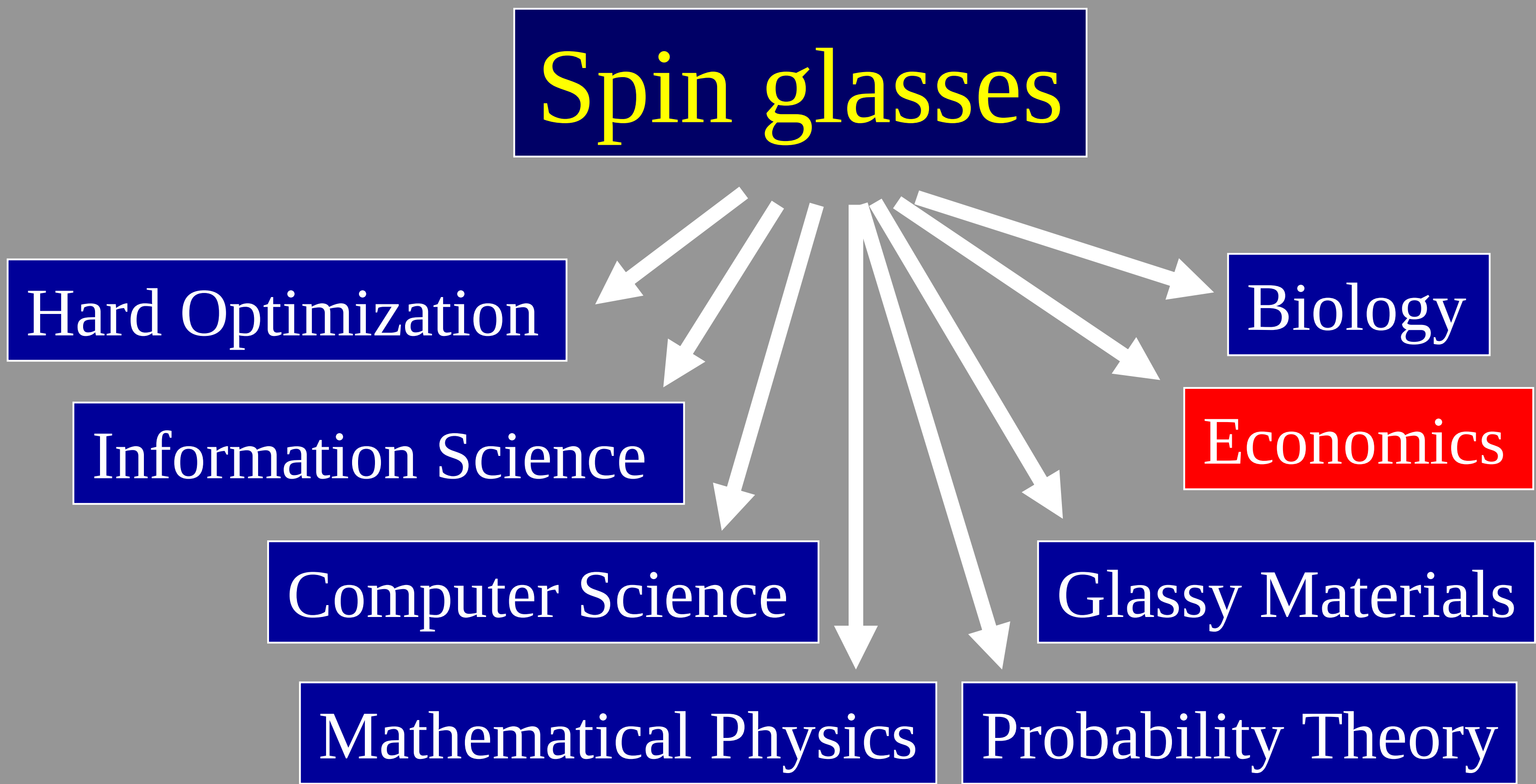
Random heteropolymer | Protein

Wolynes

Random Boolean network | Autocatalytic sets

Kauffman

Examples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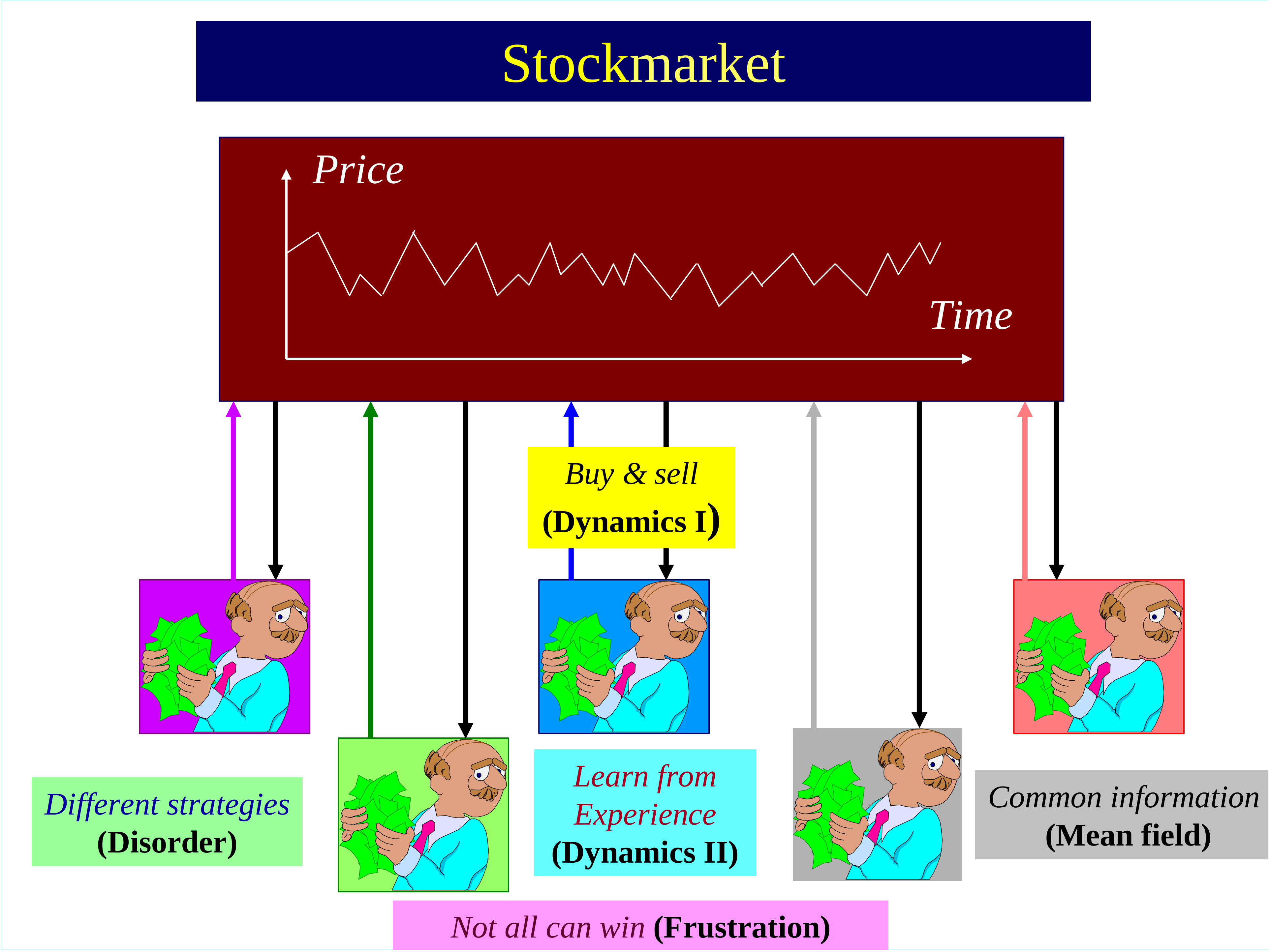
Stockmarket
Price
Time
Buy & sell
(Dynamics I)
Different strategies
(Disorder)
Learn from
Experience
(Dynamics II)
Common information
(Mean field)
Not all can win (Frustration)

Simple minimalist model

Minority game

N agents 2 choices

Aim to be in minority

Individual strategies $\longrightarrow$ Collective consequence

- act on common information (e.g. minority choice for last m steps)
- preferences modified by experience (keep point-score)

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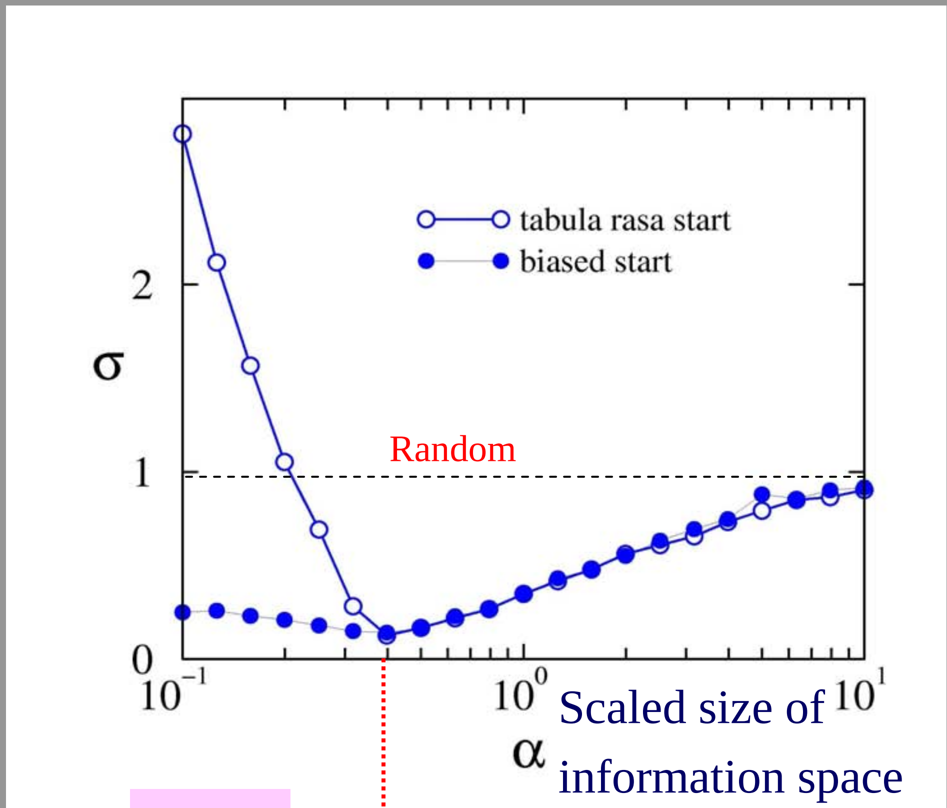
Correlated behaviour & phase transition

Phase transition
& ergodicity-breaking

Random strategies, random histories



Non-ergodic

Ergodic

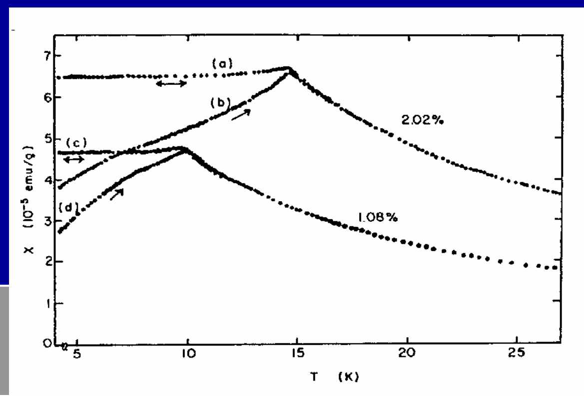
Phase transition: α_c

minimum in volatility

$\alpha < \alpha_c$ *non-ergodic*

$\alpha > \alpha_c$ *ergodic*

c.f. spin glass susc.



Coarse-grained time-average

Effective interaction between agents

$$H = \sum_{ij} J_{ij} s_i s_j + \sum_i h_i s_i$$

Quasi-random *J* and *h* related to agent strategies

c.f. spin glass or neural network

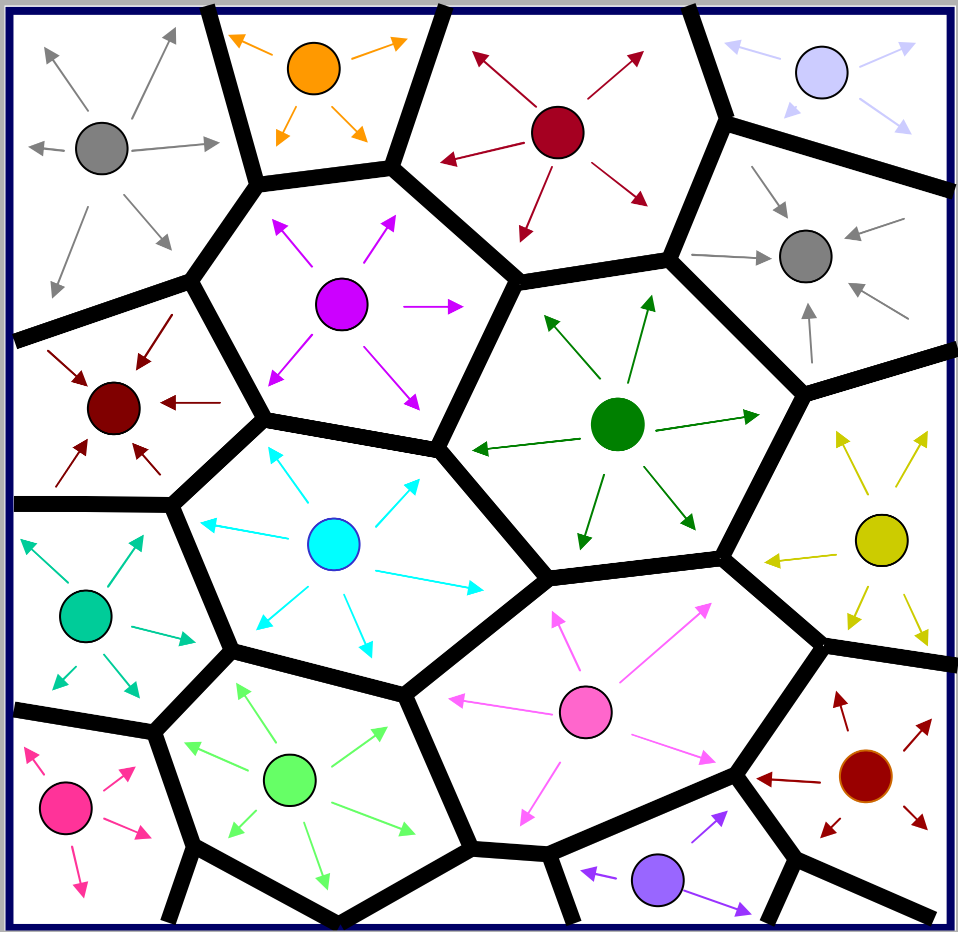
**

Strategy point-score dynamics for agents with 2 strategies

$$p_i(t+1) = p_i(t) - \partial H / \partial s_i \Big|_{\{s_i = \mathrm{sgn}\, p_i(t)\}}$$

Difference from Hopfield neural network

Minority game



$$H = +\sum_{ij} J_{ij} s_i s_j$$

$$J_{ij} = \sum_{\mu} \xi_i^{\mu} \xi_j^{\mu}$$

Many repellors

c.f. attractors in neural network

Theoretical methodology

<u>Starting points</u>

- Statics/thermodynamics:
  - Partition function

$$Z = Tr\{\exp[-\beta H]\}$$

- Dynamics:
  - Generating functional

$$Z = \int D\mathbf{S}(t)\,\delta(\text{microscopic eqn. of motion})$$

General approach

- Transform to *macro-variables*
- Average *physical observables* over disorder

→ typical behaviour

Details subtle:

Multi-replica/ multi-time correlation & response fns

Self-consistent memory and coloured noise

Finite-range: approx./mean-field or simulation

Infinite-range :extremal dominance ~ “soluble” but subtle

Magnitudes & ranges

- $N \to \infty$ units, p interactions per unit
- Solid: $N \sim 10^{23}$, $p \sim 10$, range often short
- Brain: $N \sim 10^{10}$, $p \sim 10^{5}$, range long
- Stockmarket: N large, information large

  → effective p large, range large

  but finiteness of N can matter:

  ergodic-nonergodic phase transition
  as N reduced at fixed p

Other types of complex systems

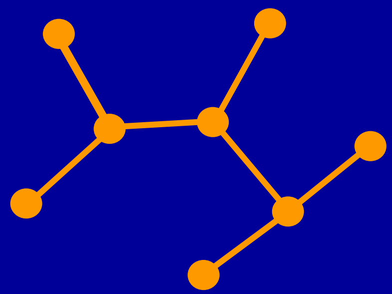
- Granular materials
  - e.g. compactivity: new statistical physics
- Rubbers and other polymeric materials
  - cross-links $\rightarrow$ topological constraints
- Other complex fluids and soft matter
- Structural glasses
  - Analogies with p-spin glasses, but also constrained dynamics
- Evolving networks

Network types

- Lattice — Solid
- Fully connected — Many information-driven
- Random graph: Erdös-Rényi — LDPC Codes
- Scale-free: $p(k) \sim k^{-\gamma}$ — Internet, protein-protein
- Growing — Internet
- Churn — Peer-to-peer

Random graphs

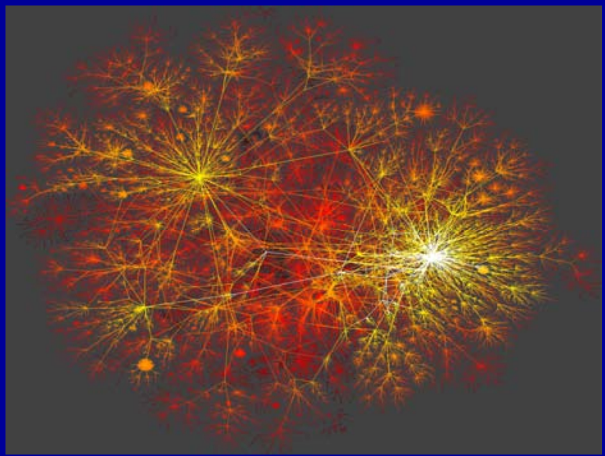
- Erdös-Rényi

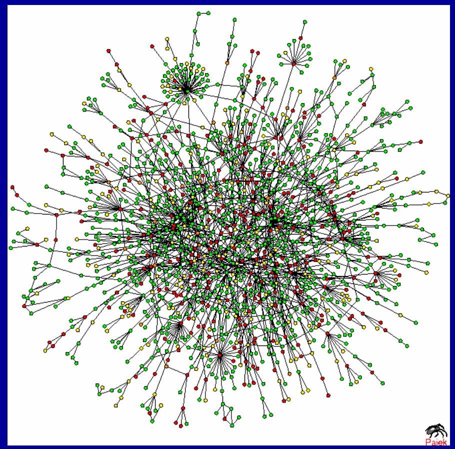


Poisson-distributed connectivity

Extend to fixed valence

- Scale-free

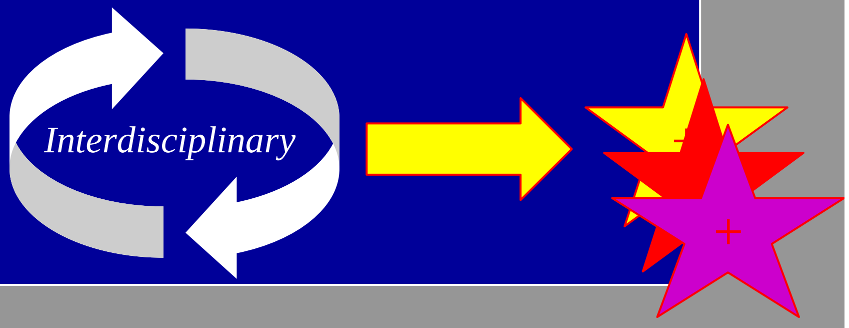




Internet (From Barabasi) Protein

Symbiosis of techniques and concepts

- Theoretical physics
  - – Minimalist modelling
  - – Sophisticated mathematical analysis
- Computer simulation
  - Both to check with more complicated real world
  - And to do experiments for which no real analogue
- Real experiment



Main conclusion

- Many examples of complex systems
  - Driven by frustrated interactions and disorder
    - Sometimes indirectly generated
    - Detailed balance or fundamentally out-of-equilibrium
    - Conceptual similarities despite different appearances
    - But also differences
  - Many opportunities for conceptual and mathematical transfer from physics
  - Offer the physicist challenges not present in conventional dictionary-definition "physics"

Concluding slogans

"More is different"

Many differents is complex

Fascinating Physics

Novel maths

Rosetta Stone to “read” other subjects

Collaborators
Teachers, colleagues, students, postdocs, friends
Phil Anderson
Sam Edwards
Walter Kohn
Tomaso Aste
Jay Banavar
Ludovic Berthier
Stefan Boettcher
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