A Master Equation for addressing some Basic Issues of Gravitational Decoherence

- Based on QFT for matter and GR for gravity.

-- Structural Problems in some theories of gravitational / instrinsic decoherence

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Motivations to look into this issue:

"Innate conflict between the fundamental principles of general relativity and of quantum mechanics." *

1. **Macroscopic Quantum Phenomena**

2. **Emergent Quantum Mechanics**

- Are there fundamental changes as one traverses between the micro and the macro domains?
- New laws for meso-physics? (e.g., Leggett)

3. **Emergent Gravity: GR as Hydro, Gravity as Thermodynamics**

We assume the validity of QFT (for matter) and GR (for gravity)

- Questions the soundness of theories with spacetime fluctuation- induced decoherence

- Gravitational decoherence reveals the **textures of spacetime**, elemental or contains structure: May provide hint on whether gravity is **fundamental or emergent**

- L Diosi (84, 87, 89) //
- R. Penrose, *Phil. Trans. R. Soc. Lond.* A (1998) **356**, 1927-1939 / GRG (96)

G.C. Ghirardi, R. Grassi and A. Rimini, Phys. Rev. A42, 1057 (1990). GRW; Steve Adler's book

Part I: **Diosi-Penrose Schemes**:
(nonrelativistic) Schroedinger-Newton
and von-Neumann-Newton Eqs

Part II: Relativistic, covariant formulation based on known physics

QFT+GR: QFT in curved spacetime → semiclassical gravity → **stochastic gravity**
[*Hu .. 94-95, Verdaguer.. 96-99.* Hu and Verdaguer, Liv Rev Rel **11** (2008) 3] Einstein-Langevin Equation.
Noise from fluctuations of quantum matter field, **not** put in by hand.

For the present purpose, simpler: Einstein - Klein-Gordon Eq → Newton-Schrodinger Eq.

Part III:

- Quantum dynamics of N particles in a gravitational field (Anastopoulos PRD1996)
- Quantum Field description of particle moving in a grav. field (Anastopoulos and Hu 2013)
  -- derived a master equation, then take the nonrelativistic limit.

Quantum open system treatment of decoherence, cf Quantum Brownian Motion model.

Then compare with D-P and other modified quantum theories.

Many quantum alternatives cannot be found as limit of ordinary QFT+GR.

Penrose (1996) "**On gravity's role in quantum state reduction**". *Gen. Rel. Grav.* **28**, 581-600

addresses the question of the *stationarity* of a quantum system which consists of a linear superposition $|\psi\rangle = |\alpha\rangle + |\beta\rangle$ of two well-defined states $|\alpha\rangle$ and $|\beta\rangle$, each of which would be stationary on its own, and where we assume that each of the two individual states has the same energy E

$$\mathrm{i}\frac{\partial|\alpha\rangle}{\partial t} = E|\alpha\rangle, \quad \mathrm{i}\frac{\partial|\beta\rangle}{\partial t} = E|\beta\rangle.$$

If gravitation is ignored, then the quantum superposition $|\psi\rangle = a|\alpha\rangle + b\,|\beta\rangle$ would also be stationary,
with the same energy *E* and this is the normal supposition.

$$\mathrm{i}\frac{\partial|\psi\rangle}{\partial t} = E|\psi\rangle,$$

However, when the gravitational fields of the mass distributions of the states are taken into account, we must ask what the Schr¨odinger operator $\partial/\partial t$ actually *means* in such a situation.

Let us consider that each of the stationary states $|\alpha >$ and $|\beta >$ takes into account whatever the correct quantum description of its gravitational field might be, in accordance with Einstein's theory.

Then, to a good degree of approximation, there will be a classical spacetime associated with each of $|\alpha>$ and $|\beta>$, and the operator $\partial/\partial t$ would correspond to the action of the Killing vector representing the time displacement of stationarity, in each case.

Now, the problem that arises here is that these two Killing vectors are *different* from each other. They could hardly be the same, as they refer to time symmetries of two different spacetimes.

It could only be appropriate to identify the two Killing vectors with one another if it were appropriate to identify the two different spacetimes with each other point-by-point.

But such an identification would be at variance with the principle of general covariance, a principle which is fundamental to Einstein's theory. According to standard quantum theory, unitary evolution requires that there be a Schr¨odinger operator that applies to the superposition just as it applies to each state individually; and its action on that superposition is precisely the superposition of its action on each state individually.

There is thus a certain tension between the fundamental principles of these two great theories, and one needs to take a position on how this tension is to be resolved. *Jump to Diosi* →

Penrose's position is (provisionally) to take the view that an *approximate* pointwise identification may be made between the two spacetimes, and that this corresponds to a slight error in the identification of the Schr¨odinger operator for one spacetime with that for the other. This error corresponds, in effect, to a slight *uncertainty in the energy* of the superposition.

One can make a reasonable assessment as to what this energy uncertainty E_G might be, at least in the case when the amplitudes a and b are about equal in magnitude.

This estimate (in the Newtonian approximation) turns out to be the gravitational self-energy of the difference between the mass distributions of the two superposed states. This energy uncertainty E_G is taken to be a fundamental aspect of such a superposition and, in accordance with Heisenberg's uncertainty principle, the reciprocal hbar/E_G is taken to be a measure of the *lifetime* of the superposition (as with an unstable particle).

The two decay modes of the superposition $|\psi\rangle = a|\alpha\rangle + b|\beta\rangle$ would be the individual states $|\alpha\rangle$ and $|\beta\rangle$, with relative probabilities $|a|^2 : |b|^2$.

Diosi's scheme and difficulty, according to Penrose

This scheme has a number of points in common with that of Diosi (1987, 1989), particularly in that no additional fundamental constants are introduced other than the standard ones h, G and c (where, in fact, c does not enter, in this Newtonian approximation). However, Diosi's scheme encountered a certain severe difficulty, as pointed out by Ghirardi *et al* . (1990), who suggested a remedy that, unfortunately, required the reintroduction of an additional constant whose value is without fundamental motivation.

This difficulty is closely related to the fact that there has been no specifiicationof which particular quantum states are to be regarded as the (stable) \`basic' ones and which are to be regarded as the \`superpositions of basic states', the states which are to decay into basic states. If, for a single point particle, we considered the basic states to be position states, then the superpositions would involve an *infinite* gravitational energy uncertainty E_G so that state reduction to one of the basic position states would occur *instantaneously* on this scheme, which is clearly an unreasonable requirement.

It is for this reason that an additional parameter defining a fundamental length scale was introduced by Ghirardi *et al* . (1990), so that the state reduction would be to an entity of the size of this fundamental length.

Penrose's scheme:

Schr¨odinger-Newton (SN) *equation*

No additional parameters are required.

The *basic stationary states* into which a general superposition would decay by state reduction are to be stationary solutions of the *Schr¨odinger-Newton* (SN) *equation* in this Newtonian approximation, where velocities and gravitational potentials are small.

The SN equation is the Schr¨odinger equation for a wavefunction , where there is an additional term provided by a Newtonian potential Φ for the Newtonian matter distribution which is the expectation value of the mass distribution given by the Schr¨odinger wavefunction Ψ

The (stationary) solutions of the SN equation are obtained by solving this nonlinear pair of coupled differential equations.

According to the state reduction scheme of Penrose, *all* quantum measurements arise because of the instability of quantum superpositions involving significant mass displacements.

In various circumstances, where a piece of physical apparatus is involved in making the measurement, the mass movement would occur in the measuring apparatus itself.

An extreme situation might occur in an observer's retina or optic nerve, when it is the reception of an individual photon that is involved.

Very frequently, the major mass displacement would take place in the (random) *environment* when this environment becomes entangled with the quantum system under consideration.

Spontaneous state reduction in the environment would necessarily be accompanied by the simultaneous reduction of any quantum system with which it is entangled.

In this way, contact is made with the standard `decoherence' viewpoint of quantum state reduction, the essential distinction being that in the present scheme the state reduction is taken as *actual* rather than merely FAPP.

Diosi's modified QM

[arXiv:qp/060711]

Two inter-related elements of classical behaviour of a rigid macro-object:

1) precise center of mass localisation
2) decoherence (decay) of superposition between separate positions.

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[2] L. Diósi, Phys. Lett. **105A**, 199-202 (1984).

[3] L. Diósi and B. Lukács, Annln. Phys. **44**, 488-492 (1987).

[4] L. Diósi, Phys. Lett. **120A**, 377-381 (1987).

[5] L. Diósi, Phys. Rev. **A40**, 1165-1174 (1989).

Localization: Schr¨odinger-Newton Eq.

In both localisation and decoherence mechanisms, resp., the relevant quantity is the Newtonian interaction

$$U(X,X') = -G\int \frac{f(\mathbf{r}|X)f(\mathbf{r}'|X')}{|\mathbf{r}'-\mathbf{r}|}d\mathbf{r}d\mathbf{r}'$$

between two mass densities corresponding to two configurations X,X′ of the macro-objects that form our quantum system.

Typically for rigid objects, position X contains the center of mass coordinates x1, x2, and the rotation angles θ1, θ2, ...

For simplicity, we shall consider spherically symmetric or point-like objects, to discuss their translational degrees of freedom.

Hence X stands for x1, x2, . . . only.

With the help of the interaction potential (1), we construct the Schr¨odinger- Newton equation for the wave function (X) of the massive objects

$$i\hbar \frac{d\psi(X)}{dt} = \text{standard q.m. terms} + \int U(X,X')|\psi(X')|^2 dX' \, \psi(X) \, .$$

The second term on the rhs leads to stationary solitary solutions. The Schr¨odinger- Newton eq. ensures the stationary localisation of the objects.

Yet, the equation can not account for the expected decoherence of macroscopic superpositions like

|X> + |Y>.

Decoherence: von Neumann -Newton Eq.

The von Neumann equation which is equivalent to the standard Schr¨odinger equation evolves the density matrix ρ (X, Y) rather than the wave function ψ (X).

The von-Neumann-Newton equation reads

$$\frac{d\rho(X,Y)}{dt} = \text{standard q.m. terms} + \frac{U(X,X)+U(Y,Y)-2U(X,Y)}{2\hbar}\rho(X,Y)$$

The second term on the rhs contributes to an exponential decay of the superposition |X> + |Y>, with decoherence time:

$$\frac{2\hbar}{2U(X,Y)-U(X,X)-U(Y,Y)}$$

To avoid misunderstandings, we emphasize that the Schr¨odinger-Newton eq. (2) and the von-Neumann-Newton eq. (3) are two alternative equations to modify the standard quantum mechanics for macro-objects. We shall treat these two separate equations parallel to each other because the gravitational terms depend on the same Newton interaction (1) in both equations.

[The desired two effects, localisation plus decoherence, have been realised through a single stochastic Schr¨odinger/von-Neumann-Newton equation based invariably on the structure U(X,X').]

Overall: Diosi -Penrose theory

- Effect of gravity on a quantum system is to induce some fuzziness in space and/or time.
- This effect is captured by a noise term added phenomenologically, not in standard QM.
- D-P is a modification of quantum theory described by a stochastic Schroedinger equation,
- It is similar in nature to the GRW collapse models, but with no external parameter.

II. Decoherence in Open Quantum Systems, with known QFT+ GR

- **Quantum Mechanics**: Path integral representation
  Nonequilibrium Statistical Mechanics: **Open Systems**
  - Classical: Projection Operator Formalism *Zwangzig-Mori (57,61)*
  - Quantum: Influence Functional Formalism *Feynman-Vernon (63)*

- **Quantum Open Systems:** Environment induced Decoherence

- **Models:** QBM, spin-boson, atom-field, Heisenberg chain **...**

First Class of Models : Quantum Brownian Motion: one harmonic osc interacting with n HOs *Caldeira-Leggett (83), Hu Paz Zhang (92)*

Classical field as environment: weak gravitational field
= 2 modes of minimally coupled massless scalar field

easy generalization from 1HO interacting with a scalar field *Unruh-Zurek (89)*

Closed System: **Quantum deterministic Dynamics**

= Subsystems A + B

System / reversible unitary evolution — many dof: **Environment**

- particle
- spacetime

(macro)

- field *
- fluctuations

* **COARSE GRAINING** (micro)

$S(x)$ $E(q)$

→ **Open System**

(Effective Theory)

$\mathcal{F}[x, x']$ after integrating out q, q'

nonunitary, dissipative irreversible

- **Influence Functional / action** $\mathcal{F} = e^{iS_{IF}}$

(Closed-time-path coarse-grained effective action)

S_{IF}: $\mu + i\nu$

Backreaction

DISSIPATION kernel μ ← **NOISE / Fluctuation** kernel $\nu(t,t')$ $= \frac{1}{\hbar}\langle \xi(t)\, \xi(t') \rangle$

renormalization

UV divergences

Decoherence

issues in Quantum Field Theory ◁ M (maco) — Q / meso / C — Mico ▷ Issues in NEq. Stat. Mechanics

- **Influence Functional** $\mathcal{F} = e^{iS_{IF}}$ / action

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Decoherence

renormalization

UV divergences

issues in Quantum Field Theory ◀

▶ Issues in NEq. Stat. Mechanics

Quantum → Classical via (environment-induced) decoherence

↓

Master Eqn., Fokker-Planck Eqn.

Langevin Eqn. | Classical Stochastic Dynamics |

e.g. $$M\frac{d^2}{ds^2}X_c(s) + 2M\int_0^s ds'\,\gamma(s-s')\frac{d}{ds'}X_c(s') + M\omega_0^2 X_c(s) = F_\xi(s)$$ Noise

X_c: CM of classical paths; $\frac{d\gamma}{ds} = \mu$ Dissipation; ω_0^2: Ω^2; F_ξ: ξ for linear cplg

A little bit of non-equil. stat. mechanics:

Closed Systems:

density matrix $\rho(x, q; x', q', t) = \psi \; \psi'$

propagator $\rho(x_f, q_f; x_f', q_f', t) = J(t \leftarrow 0)\, \rho(x_i q_i; x_i' q_i', 0)$

Open System:

integrate out q's

reduced density matrix $\rho_r(x_f, x_f', t) = \int dq_f \int dq_f' \, \delta(q_f - q_f') \, \rho(f, t)$

propagator $\rho_r(x_f, x_f', t) = \int dx_i \int dx_i' \, J_r(x_f, x_f', t \mid x_i, x_i', 0)\, \rho_r(x_i, x_i', 0)$

$$J_r(x_f, x_f', t \mid x_i x_i', 0) = \int_{x_i}^{x_f} \mathcal{D}x_i \int_{x_i'}^{x_f'} \mathcal{D}x_i' \; e^{\frac{i}{\hbar}\{S_s[x] - S_s[x']\}} \mathcal{F}_r[x, x']$$

Influence Functional:

Feynman Vernon (63)
Caldeira Leggett (83)

$$\mathcal{F}_r[x, x'] = \int dq_f \int dq_i \int dq_i' \, \rho_E(q_i, q_i', 0) \int \mathcal{D}q \int \mathcal{D}q'$$

$$e^{\frac{i}{\hbar}\{S_E[q] - S_E[q'] + S_I[x, q] - S_I[x', q']\}} = e^{\frac{i}{\hbar} S_{IF}}$$

influence action (S_{IF})

Entropy $S = -\rho_{red} \ln \rho_{red}$: measures information loss in environment

Quantum Brownian Motion

by coarse-graining, manifest in dissip.

Grabert et al (89), Hu, Paz & Zhang (92)

System: Single Oscillator: $S_s[x] = \int_0^t ds \left(\frac{1}{2} M \dot{x}^2 - V(x) \right)$

Environment: n-oscillators: $S_E[\{q_n\}] = \int_0^t ds \sum_n \left\{ \frac{1}{2} m_n \dot{q}_n^2 - \frac{1}{2} m_n \omega_n^2 q_n^2 \right\}$

Coupling: bilinear: $S_I[x, \{q_n\}] = \int_0^t ds \sum_n \{ -c_n x q_n \}$

Assume bath in thermal equilibrium at temp $T = \beta^{-1}$

$$\rho_E(\{q_{ni}\}, \{q'_{ni}\}, 0) = \prod_n \rho_n(q_{ni}, q'_{ni}, 0) = \prod_n \langle q_{ni} | e^{-\beta \hat{H}_n} | q'_{ni} \rangle$$

Since n osc. are uncoupled, $\mathcal{F}$ factorizes: $\mathcal{F}[x, x'] = \prod \mathcal{F}_n[x, x']$

QBM

$$S[x, q] = S[x] + S_E[q] + S_{int}[x, q]$$

(system) (environment) (interaction)

$$= \int_0^t ds \left\{ \frac{1}{2} M (\dot{x}^2 - \Omega^2 x^2) + \sum_n \left[\frac{1}{2} m_n (\dot{q}_n^2 - \omega_n^2 q_n^2) \right] + \sum_n (-c_n x q_n) \right\}$$

- Bath : Spectral density $\quad I(\omega) = \sum_n \delta(\omega - \omega_n) \frac{C_n^2}{2 m_n \omega_n}$

Temp T

introduce cutoff Λ : $I(\omega) \to 0$ for $\omega \gg \Lambda$

$$I(\omega) = \frac{2}{\pi} M \gamma_0 \omega \left(\frac{\omega}{\Lambda}\right)^{n-1} e^{-\omega^2/\Lambda^2} \quad \sim \omega^n$$

$n > 1$ supraohmic
$= 1$ ohmic
< 1 subohmic

- Initial factorization condn. ← creates (artificial) initial jolt

- ρ_r reduced density matrix : integrate out q

HPZ Master Equ: PRD 45, 2843 (1992)

Streaming — freq. ren.

$$i\hbar\frac{\partial}{\partial t}\rho_r(x,x',t) = \Big\{\Big[-\frac{\hbar^2}{2M}\Big(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial x'^2}\Big)+\frac{1}{2}M\Omega^2(x^2-x'^2)\Big]+\frac{1}{2}M\,\delta\Omega^2(t)(x^2-x'^2)$$

Dissipation — Diffusion

$$-i\hbar\Gamma(t)(x-x')\Big(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\Big)-iM\Gamma(t)\,h(t)(x-x')^2$$

D_1 (normal)

$$+\hbar\Gamma(t)f(t)(x-x')\Big(\frac{\partial}{\partial x}+\frac{\partial}{\partial x'}\Big)\Big\}\rho_r(x,x',t)$$

D_2 (anomalous)

E.g. Ohmic, hi T: $D_1 \propto T$, $D_2 \propto T^{-1}$

Decoherence Studies:

- Decoherence Time very short at high temperatures (e.g. Zurek '91)

  but could be very long at zero / low temperature

Temp, Bath, Type of Coupling → Contributing factors

see e.g. Paz, Habib & Zurek (93)

Quantum Open System

Closed System: **Density Matrix** $\hat{\rho}(t) = \mathcal{J}(t, t_i)\hat{\rho}(t_i).$

$\mathcal{J}(x, \mathbf{q}, x', \mathbf{q}', t \mid x_i, \mathbf{q}_i, x'_i, \mathbf{q}'_i, t_i)$ is the (**unitary) evolutionary operator** of the system from initial time t_I to time t.

OPEN SYSTEM: System (s) interacting with an Environment (e) or Bath (b): Integrate out (coarse-graining) the bath dof renders the system open. Its evolution is described by the Reduced Density Matrix

$$\rho_r(x, x') = \int_{-\infty}^{+\infty} dq \int_{-\infty}^{+\infty} dq' \rho(x, \mathbf{q}; x', \mathbf{q}')\delta(\mathbf{q} - \mathbf{q}')$$

$$\rho_r(x, x', t) = \int_{-\infty}^{+\infty} dx_i \int_{-\infty}^{+\infty} dx'_i \; \mathcal{J}_r(x, x', t \mid x_i, x'_i, t_i) \; \rho_r(x_i, x'_i, t_i \;).$$

Quantum Brownian Motion

Q Harmonic Oscillator + Bath (HOB) Feynman-Vernon (63) and many others
- time-independent frequency: QBM1 Caldeira-Leggett (83) Hu- Paz- Zhang (92)
- time-dependent frequency: QBM2 connects to Q Field Theory Hu Matacz (94)

System (S): quantum oscillator with time dependent natural frequency
Environment (E) : n-quantum oscillators
with time-dependent natural frequencies = Scalar Field
Coupling: c_n F (x) q_n.

$$S[x,\mathbf{q}] = S[x] + S_E[\mathbf{q}] + S_{\rm int}[x,\mathbf{q}]$$

$$= \int_0^t ds \left[\frac{1}{2} M(s)[\dot{x}^2 + B(s)x\dot{x} - \Omega^2(s)x^2] \right.$$

$$\left. + \sum_n \left\{ \frac{1}{2} m_n(s)[\dot{q}_n^2 + b_n(s)q_n\dot{q}_n - \omega_n^2(s)q_n^2] \right\} \quad + \sum_n \left(-c_n(s)F(x)q_n \right) \right]$$

Influence Functional

Assume **factorizable condition** between the system (s) and the bath (b) initially

$$\hat{\rho}(t = t_i) = \hat{\rho}_s(t_i) \times \hat{\rho}_b(t_i),$$

:

Evolutionary operator for the reduced density matrix is

$$\mathcal{J}_r(x_f, x_f', t \mid x_i, x_i', t_i) = \int_{x_i}^{x_f} Dx \int_{x_i'}^{x_f'} Dx' \, \exp\left(\frac{i}{\hbar}\left\{S[x] - S[x']\right\}\right) \, \mathcal{F}[x, x']$$

Influence Functional

$$\mathcal{F}[x, x'] = \int_{-\infty}^{+\infty} d\mathbf{q}_f \int_{-\infty}^{+\infty} d\mathbf{q}_i \int_{-\infty}^{+\infty} d\mathbf{q}_i' \int_{\mathbf{q}_i}^{\mathbf{q}_f} D\mathbf{q} \int_{\mathbf{q}_i'}^{\mathbf{q}_f} D\mathbf{q}'$$

$$\exp\left(\frac{i}{\hbar}\left\{S_b[\mathbf{q}] + S_{\text{int}}[x, \mathbf{q}] - S_b[\mathbf{q}'] - S_{\text{int}}[x', \mathbf{q}']\right\}\right) \times \rho_b(\mathbf{q}_i, \mathbf{q}_i', t_i)$$

Influence Action

$$= \exp\left(\frac{i}{\hbar}\delta\mathcal{A}[x, x']\right)$$

Influence functional for a Paramp

$$\mathcal{F}[x,x'] = \exp\Big\{-\frac{i}{\hbar}\int_{t_i}^{t} ds \int_{t_i}^{s} ds' \Big[F(x(s)) - F(x'(s))\Big]\mu(s,s')\Big[F(x(s')) + F(x'(s'))\Big]$$
$$-\frac{1}{\hbar}\int_{t_i}^{t} ds \int_{t_i}^{s} ds' \Big[F(x(s)) - F(x'(s))\Big]\nu(s,s')\Big[F(x(s')) - F(x'(s'))\Big]\Big\}$$

$$\Sigma(s) = \frac{1}{2}\Big(F(x(s)) + F(x'(s))\Big),$$

$$\Delta(s) = F(x(s)) - F(x'(s)),$$

Dissipation μ and Noise ν Kernels

$$\mathcal{F}[x,x'] = \exp\Big\{\frac{i}{\hbar}\int_{t_i}^{t} ds\Delta(s)\langle\xi(s - \frac{1}{\hbar^2}\int_{t_i}^{t} ds \int_{t_i}^{s} ds'\Delta(s)\Delta(s')C_2(s,s')\Big\}$$

$$\langle\bar{\xi}(t)\bar{\xi}(t')\rangle = C_2(s,s') \equiv \hbar\nu(s,s')$$

Langevin Equation:

$$\frac{\partial L}{\partial x} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - 2\frac{\partial F(x)}{\partial x}\int_{t_i}^{t}\mu(t,s)F(x(s))ds = -\frac{\partial F(x)}{\partial x}\bar{\xi}(t)$$

Noise and Dissipation Kernels

Equation of Motion for the amplitude function of a Parametric Oscillator

$b_n = 0$ and $m = 1$ $\quad \kappa_n = m_n(t_i)\omega_n(t_i)$ $\qquad \ddot{X}_n + \omega_n^2(t) X_n = 0,$

$$\mu(s,s') = \frac{i}{2}\int_0^\infty d\omega I(\omega,s,s')\Big[X_\omega^*(s)X_\omega(s') - X_\omega(s)X_\omega^*(s')\Big],$$

$$\nu(s,s') = \frac{1}{2}\int_0^\infty d\omega I(\omega,s,s')\coth\left(\frac{\hbar\omega(t_i)}{2k_BT}\right)\Big[\cosh 2r(\omega)\Big[X_\omega^*(s)X_\omega(s') + X_\omega(s)X_\omega^*(s')\Big]$$
$$-\sinh 2r(\omega)\Big[e^{-2i\phi(\omega)}X_\omega^*(s)X_\omega^*(s') + e^{2i\phi(\omega)}X_\omega(s)X_\omega(s')\Big]\Big].$$

$$I(\omega,s,s') = \sum_n \delta(\omega-\omega_n)\frac{c_n(s)c_n(s')}{2\kappa_n}$$

Spectral Density Function

$I(\omega) \sim \omega^n$ $\qquad$ n=1: Ohmic, n>1 Supra Ohmic; n<1 Subohmic

Squeezed and Rotation parameters: $\hat{\rho}_b(t_i) = \prod_n \hat{S}_n(r(n),\phi(n))\hat{\rho}_{\rm th}\hat{S}_n^\dagger(r(n),\phi(n))$

e.g., for an initial squeezed thermal bath

Stochastic Equations

Non- Markovian
(Hu-Paz-Zhang 92)
Master Equation:

$$i\hbar\frac{\partial}{\partial t}\hat{\rho}_r(t) = [\hat{H}_{\mathrm{ren}}, \hat{\rho}] + iD_{pp}[\hat{x}, [\hat{x}, \hat{\rho}]] + iD_{xx}[\hat{p}, [\hat{p}, \hat{\rho}]]$$

$$+ iD_{xp}[\hat{x}, [\hat{p}, \hat{\rho}]] + iD_{px}[\hat{p}, [\hat{x}, \hat{\rho}]] + \Gamma[\hat{x}, \{\hat{p}, \hat{\rho}\}],$$

Nonlocal dissipation
Nonlocal fluctuations
(Colored noise)

$$\hat{H}_{\mathrm{ren}} = \frac{\hat{p}^2}{2M(t)} - \frac{B(t)}{4}(\hat{p}\hat{x} + \hat{x}\hat{p}) + \frac{M(t)}{2}\Omega_{\mathrm{ren}}(t)\hat{x}^2.$$

Wigner Function:

$$F_W(\Sigma, p, t) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} e^{ip\Delta/\hbar}\left\langle \Sigma - \frac{\Delta}{2}\middle|\hat{\rho}\middle|\Sigma + \frac{\Delta}{2}\right\rangle d\Delta,$$

Fokker-Planck or Wigner Equation: (Non-Markovian)

$$\frac{\partial}{\partial t}F_W(\Sigma, p, t) = \left[-\frac{p}{M(t)}\frac{\partial}{\partial \Sigma} + \frac{1}{2}M(t)\Omega^2_{\mathrm{ren}}(t)\ \Sigma\frac{\partial}{\partial p} + \Gamma(t)\frac{\partial}{\partial p}p - 2D_{pp}(t)\frac{\partial^2}{\partial p^2}\right.$$
$$\left. -\hbar D_{xx}(t)\frac{\partial^2}{\partial \Sigma^2} + 2\Big(D_{xp}(t) + D_{px}(t)\Big)\frac{\partial^2}{\partial \Sigma \partial p}\right]F_W(\Sigma, p, t).$$

Decoherence in QBM models:
1 HO System– nHO bath

$$S[x,q_n]=\int_0^t ds\left[\frac{1}{2}M(\dot{x}^2-\Omega_0^2x^2)+\sum_n \frac{1}{2}m_n(\dot{q}_n^2-\omega_n^2q_n^2)-\sum_n C_n x q_n\right], \qquad (3)$$

$$\Psi(x,t=0)=\Psi_1(x)+\Psi_2(x),$$

Hu Paz Zhang PRD 92
Paz Habib Zurek PRD 93

where

$$\Psi_{1,2}(x)=N\exp\left[-\frac{(x\mp L_0)^2}{2\delta^2}\right]\exp(\pm iP_0x),$$

$$N^2\equiv\frac{\bar{N}^2}{\pi\delta^2}=\frac{1}{2\pi^2\delta^2}\left[1+\exp\left[-\frac{L_0^2}{\delta^2}-\delta^2P_0^2\right]\right]^{-1}$$

$$W(x,p)=\int_{-\infty}^{+\infty}\frac{dz}{2\pi}e^{ipz}\rho(x-z/2,x+z/2)$$

$$W(x,p,t)=W_1(x,p,t)+W_2(x,p,t)+W_{\text{int}}(x,p,t)\ , \tag{18}$$

where

$$W_{1,2}(x,p,t)=\frac{\bar{N}^2}{\pi}\frac{\delta_2}{\delta_1}\exp\left[-\frac{(x\mp x_c)^2}{\delta_1^2}\right]\times\exp[-\delta_2^2(p\mp p_c-\beta(x\mp x_c))^2]\ , \tag{19}$$

$$W_{\text{int}}(x,p,t)=2\frac{\bar{N}^2}{\pi}\frac{\delta_2}{\delta_1}\exp(-A_{\text{int}})\times\exp\left[-\frac{x^2}{\delta_1^2}-\delta_2^2(p-\beta x)^2\right]\times\cos[2\kappa_p p+2(\kappa_x-\beta\kappa_p)x]\ , \tag{20}$$

Pointer Basis: Interaction Hamiltonian

left: xq
right: pp

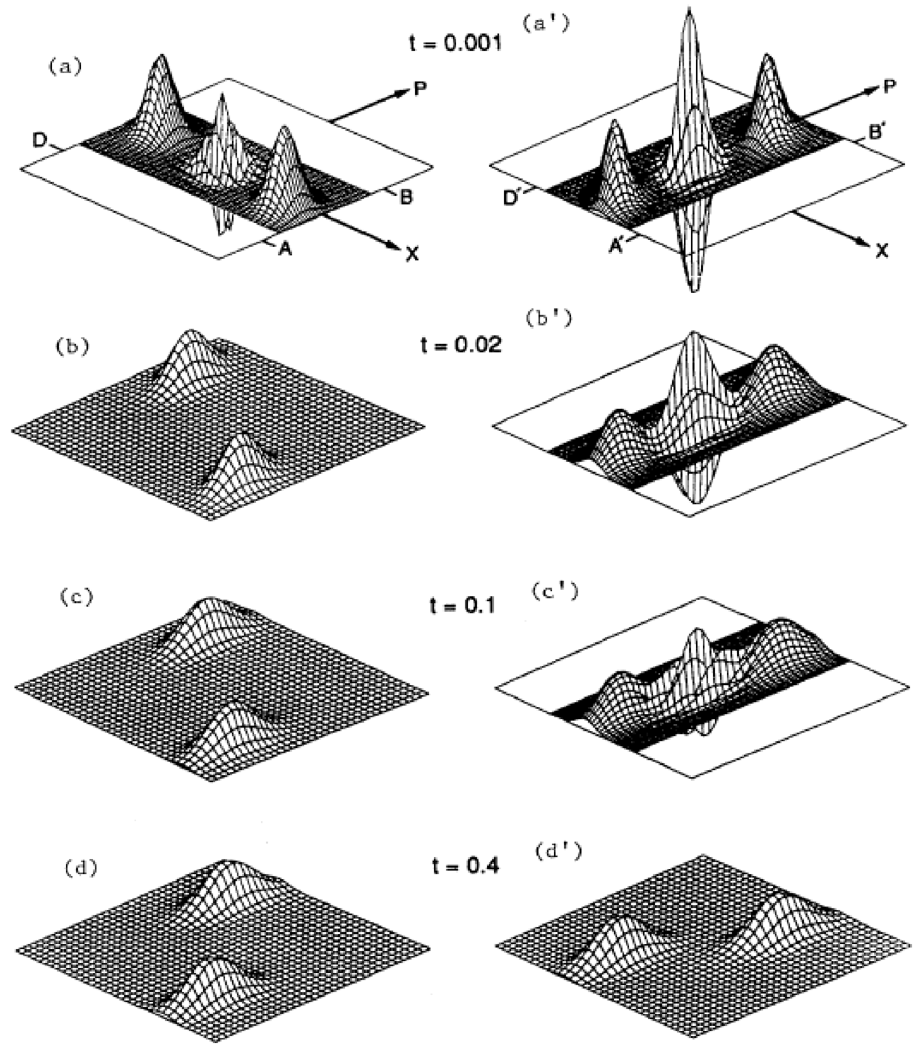


FIG. 2. The time evolution of initial conditions A and A'. The oscillations disappear faster in the first case since the environment can distinguish between the two peaks. In the second case, the interference is damped over a dynamical time scale.

II. Relation of Master Equations in QOS to stochastic Schroedinger Eq

Quantum State diffusion / reduction is usually discussed with a **stochastic Schroedinger equation.**
Similar for q trajectory, q jump

Distinction from **master equation description**:
Same Schroedinger picture.

From Q state diffusion (single run) easy to derive density matrix.
From density matrix, need to "unravel" to q trajectory.

QBM models, work of Strunz Yu, (earlier with Diosi, Gisin), have proven the equivalence of q trajectory formulation with density matrix formulation for **convolutionless nonMarkovian** processes, such as described by the HPZ Eq.

Nonrelativistic Weak Field Limit

- Einstein eqn → Newton eq.
- Klein-Gordon eqn → Schrodinger eq.
  [rel. quantum field → 1-particle representation]

- **Einstein –KG eqn**: Self consistent soln
  Reduction in the nonrelativistic limit to the
  Newton-Schrodinger equation.

What is needed here is q field theory in curved spacetime:

- Where the **background spacetime is a perturbed Minkowski space: Graviton bath** acting as environment to

- **System**: massive scalar field → **Quantum Particle Motion**

- Specifically,

  Campos and Hu Phys. Rev. D 58, 125021 (1998) treated a scalar field in a weakly perturbed Minkowsky space at finite temperature which act as a model for thermal graviton bath.

  [See also D. Arteaga, R. Parentani, and E. Verdaguer, Phys. Rev. D 70, 044019 (2004).]

- The system of scalar field can be reduced to a particle trajectory by means of the worldline influcence functional formalism. [next slide]

Quantum Field → Particle Motion

> Blencowe (2012)[arXiv:1211.4751] used a coherent state representation to get the particle motion.

Worldline Influence Functional (WLIF)

Field→ Particle:

1) Particle Number (Fock Space) Representation: second quantization.

2) **Waves** (wave equation: first quantization)

a) **Phase**: Hamilton-Jacobi function, Eikonal Approx [Wheeler-Feynman 1949]

→ Action, saddle pt approx, q. corrections, trajectory of particle derived

b) **Ray** representation: Constructive interference of wavefronts

→ Geodesics eqn

Worldline is a ray representation of the quantum field

At the lowest order: classical geodesics.

Adding quantum corrections gives semiclassical theory.

WLIF or Influence Action contains quantum phase information and the the backreaction effects of the environment

~ Quantum Open Systems Approach: [Feynman-Vernon 1963]

Influence Functional Approach to Gravitational Decoherence

- **WLIF for N HOs interacting with a quantum field**

  Raval Hu and Anglin (1996) derived the coupled Langevin equations for this system, with Fluctuation-Dissipation relation manifest and a new Correlation-Propagation relation

- **WLIF for a charge /mass moving in a classical field:**

-- **Charge in EM field**: Phil Johnson and Hu (2000)

-- **Mass in Grav Field:** self force studies: Chad Galley & Hu (06)

- **Scalar field in a thermal graviton bath:** Campos and Hu (1998)

- **Graviton-induced decoherence of moving detectors**

  Anastopoulos, Hsiang and Hu, (in progress)

- **For the gravitational decoherence problem:**

  - **Gravitational self energy** plays a key role in the D-P scheme.

  - Need to account for that in whichever model deemed suitable.

III. N quantum particles (described by a scalar field) **in a gravitational field**

1. Hamiltonian for a massive scalar field interacting with a gravitational field
2. 3+1 decomposition. Perturbation off Minkowski space background .
3. Gauge choice, transverse-traceless components: physical degrees of freedom:
4. Hamiltonian -- Quantization → Hamiltonian operator
5. Tracing out the gravitational field. Technically possible for weak perturbuations → Master eq for reduced density matrix of matter field [similar to QBM model]
6. Projecting to one-particle subspace
7. Take non-relativistic limit.

   [It is similar to CA's 96 paper, but here the calculations are more rigorous and the effect of self-gravity is fully taken into account.]

We prefer to describe the quantum mater as a field because the **coupling is by the well-defined Laplace-Baltrami operator**. In contrast, a treatment that starts from particles coupled to the gravitational field, the interaction term would be of the form

$$\hat{H}_{int} = \int d^3x f(\mathbf{x} - \mathbf{q}) \hat{A}_{ij} \hat{h}^{ij}(\mathbf{x}),$$

where Aij is an operator on the particle's Hilbert space,

q represents the position of the particle, and

f is a phenomenological function that needs to be inserted in order to describe the localization of the interaction, taking into account the finite dimensions of the particle.

There is **no fixed rule that allows for the determination of the function f** from first principles, as is necessary in a treatment of gravitational decoherence. A quantum field theory treatment of particle-field interaction is more fundamental and **avoids the ambiguities in the choice of couplings.**

Action

The action for a classical scalar field theory describing the matter degrees of freedom ϕ interacting with the gravitational field is

$$S[g,\phi] = \frac{1}{\kappa}\int \sqrt{-g}d^4xR + \int d^4x\left(-\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}m^2\phi^2\right), \qquad (1)$$

where ∇_μ is the covariant derivative defined on a background spacetime with Lorentian metric $g_{\mu\nu}$, R is the spacetime Ricci scalar and m is the scalar-field's mass.

3+1 decomposition

$$S_{3+1}[h_{ij},\phi,N,N^i] = \frac{1}{\kappa}\int dtd^3xN\sqrt{h}\left[K_{ij}K^{ij} - K^2 + {}^{(3)}R \right. \qquad (2)$$

$$\left. + \frac{1}{2N^2}\dot{\phi}^2 - \frac{1}{2}(h^{ij} - \frac{N^iN^j}{N^2})\nabla_i\phi\nabla_j\phi - \frac{1}{N^2}\dot{\phi}N^i\nabla_i\phi\right],$$

where N is the lapse function, N^i the shift vector, and

$$K_{ij} = \frac{1}{2N}\left(\dot{h}_{ij} - \nabla_iN_j - \nabla_jN_i\right) \qquad (3)$$

is the extrinsic curvature on Σ. The dot denotes taking the Lie derivatives with respect to the vector field $\frac{\partial}{\partial t}$.

Weak gravitational perturbation off Minkowski background

We consider perturbations around the Minkowski spacetime ($N = 1, N^i = 0, h_{ij} = \delta_{ij}$) that are first-order with respect to κ. That is, we write

$$h_{ij} = \delta_{ij} + \kappa\gamma_{ij}, \qquad N = 1 + \kappa n, \qquad N^i = \kappa n^i, \tag{4}$$

and we keep in Eq. (3) only terms up to first order in κ. We obtain

$$S_{lin}[\gamma_{ij}, \phi, n, n^i] = \int dt d^3x \left(\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\partial^i\phi\partial_i\phi - \frac{1}{2}m^2\phi^2\right)$$
$$+\kappa \int dt d^3x \left[\frac{1}{4}(\dot{\gamma}_{ij} - 2\partial_{(i}n_{j)})(\dot{\gamma}^{ij} - 2\partial^{(i}n^{j)}) - \frac{1}{4}(\dot{\gamma} - 2\partial_i n^i)^2\right.$$
$$\left. -V[(\partial\gamma)^2] + n(\partial_i\partial_j\gamma - \partial^2\gamma)\right]$$
$$+\frac{\kappa}{2}\int dt d^3x \left[(\frac{1}{2}\gamma - n)\dot{\phi}^2 - 2n^i\dot{\phi}\partial_i\phi + \gamma^{ij}\partial_i\phi\partial_j\phi - (n + \frac{1}{2}\gamma)(\partial^i\phi\partial_i\phi + m^2\phi^2\right.$$

The indices in Eq. (5) are raised and lowered with the background 3-metric δ_{ij}. We have defined $\gamma = \delta^{ij}\gamma_{ij}$. The "potential" $V[(\partial\gamma)^2]$ corresponds to the second order terms in the expansion of $\sqrt{h}^3R$ with respect to γ. Explicitly,

$$V = -\frac{1}{2}\partial_k\gamma_{ij}\partial^i\gamma^{kj} - \frac{1}{2}\partial_k\gamma\partial^k\gamma + \partial_i\gamma\partial_k\gamma^{ik} + \frac{1}{4}\partial_k\gamma_{ij}\partial^k\gamma^{ij}. \tag{6}$$

The first term in Eq. (5) is the action for a free scalar field on Minkowski spacetime, the second term describes the self-dynamics of the perturbations and the third term describes the matter-gravity coupling. Note that the terms for the gravitational self-dynamics and the matter-gravity coupling are of the same order in κ.

2.2. The Hamiltonian

To obtain the Hamiltonian we perform the Legendre transform of the Lagrangian density $\mathcal{L}_{lin}$ associated to the action Eq. (5). The conjugate momenta Π^{ij} and π of γ_{ij} and ϕ respectively are

$$\Pi^{ij} := \frac{\partial \mathcal{L}_{lin}}{\partial \dot{\gamma}_{ij}} = \frac{\kappa}{2}\left(\dot{\gamma}_{ij} - \dot{\gamma}\delta^{ij} + \partial^i n^j + \partial^j n^i - 2\partial_k n^k \delta^{ij}\right), \tag{7}$$

$$\pi \quad := \frac{\partial \mathcal{L}_{lin}}{\partial \dot{\phi}} = \dot{\phi} + \kappa\left[(\frac{1}{2}\gamma - n)\dot{\phi} - n^i\partial_i\phi\right]. \tag{8}$$

The conjugate momenta $\Pi_n = \partial\mathcal{L}_{lin}/\partial\dot{n}$ and $\Pi^i_{\vec{n}} = \partial\mathcal{L}_{lin}/\partial\dot{n}_i$ vanish identically. Thus, the equations $\Pi_n = 0$ and $\Pi^i_{\vec{n}} = 0$ define primary constraints.

The Hamiltonian $H = \int d^3x(\Pi^{ij}\dot{\gamma}_{ij} + \pi\dot{\phi} - \mathcal{L}_{lin})$ is

$$\begin{aligned} H = \int d^3x \Bigg[\Bigg(&\frac{\Pi^{ij}\Pi_{ij} - \frac{1}{2}\Pi^2}{\kappa} + \kappa V[(\partial\gamma)^2]\Bigg) + \mathfrak{e}(\phi,\pi) \\ &-\frac{\kappa}{2}\left[\gamma\mathfrak{e}(\phi,\pi) + \gamma^{ij}\partial_i\phi\partial_j\phi - \gamma(\partial_k\phi\partial^k\phi + m^2\phi^2)\right] \\ &+ n\left[\partial^2\gamma - \partial_i\partial_j\gamma^{ij} + \mathfrak{e}(\phi,\pi)\right] + n_i\left[-2\partial_j\Pi^{ji} + \kappa\mathfrak{p}^i(\Pi,\phi)\right]\Bigg], \end{aligned} \tag{9}$$

where $\Pi = \Pi^{ij}\delta_{ij}$, and

$$\mathfrak{e}(\phi,\Pi) = \frac{1}{2}\pi^2 + \frac{1}{2}\partial_i\phi\partial^i\phi + \frac{1}{2}m^2\phi^2 \tag{10}$$

is the energy density of the scalar field, and

$$\mathfrak{p}^i(\phi,\pi) = \pi\partial^i\phi \tag{11}$$

is the momentum density (energy flux).

Eq. (9) can also be obtained from the full gravitational Hamiltonian

$$H = \int d^3x \left[N \left(\frac{\Pi^{ij}\Pi_{ij} - \frac{1}{2}\Pi^2}{\kappa\sqrt{h}} - \sqrt{h}\,{}^3R + \mathfrak{h}(\phi, \pi, h_{ij}) \right) \right.$$
$$\left. + N^i \left(-2\nabla_j \Pi^j{}_i + \mathfrak{h}_i(\phi, \pi, h_{ij}) \right) \right], \tag{12}$$

by expanding the metric variables around flat spacetime as in Eq. (4) and keeping terms to first order in κ. Eq. (12) applies to a larger class of field theories than the one we consider in this paper: any diffeomorphism-invariant action where matter fields do not couple to derivatives of the spacetime metric gives rise to a Hamiltonian of the form (12) (plus additional constraints reflecting other gauge symmetries).

It follows that the longitudinal part of the metric perturbation ${}^L\gamma_{ij}$ and the transverse trace ${}^T\Pi$ of the gravitational conjugate momentum are pure gauge, reflecting the freedom of space and time reparameterization in the evolution of the matter degrees of freedom. The associated symmetry is *not* that of spacetime diffeomorphisms, but of the spacetime diffeomorphisms that preserve the spacelike foliation introduced for the purpose of the 3+1 decomposition. The fact that time and space reparameterizations are not dynamical in general relativity is a very important criterion for all proposers of alternative models of gravitational decoherence to take notice. *Any postulate of dynamical or stochastic fluctuations that correspond to space and time reparameterizations conflicts with the fundamental symmetries of general relativity.*

Gauge Choice must preserve the Lorentz frame of foliation:

$q^i = 0$ and $\tau = 0$, or equivalently ${}^L\gamma_{ij} = 0$ and ${}^T\Pi = 0$.

Thus the true physical degrees of freedom in the system correspond to the transverse traceless components $\bar{\gamma}_{ij}$, $\bar{\Pi}^{ij}$ of the metric perturbations and conjugate momenta, and to the matter variables ϕ and π. The Hamiltonian (9) then becomes

$$H = \int d^3x \left(\frac{1}{\kappa} \tilde{\Pi}^{ij} \bar{\Pi}_{ij} + \frac{\kappa}{4} \partial_k \bar{\gamma}_{ij} \partial^k \bar{\gamma}^{ij} + \mathfrak{e} - \frac{\kappa}{2} \int d^3x \bar{\gamma}^{ij} \mathfrak{t}_{ij} \right)$$
$$+ \frac{\kappa}{2} \int d^3x d^3x' \left(\frac{\mathfrak{e}(x)[\mathfrak{e}(x') - p(x') - \frac{1}{2} g(x)]}{2\pi |\mathbf{x} - \mathbf{x}'|} - \mathfrak{p}^i(x) \mathfrak{p}^j(x') \Delta_{ij}(x - x') \right) \quad (21)$$

where

$$\Delta_{ij}(x) = \int \frac{d^3k}{(2\pi)^3 k^2} e^{-i\mathbf{k}\cdot\mathbf{x}} \left(\delta_{ij} - \frac{3k_i k_j}{4k^2} \right), \quad (22)$$

and we wrote $p(x) = \frac{1}{3} \partial_i \phi \partial^i \phi$ and $g(x) = m^2 \phi^2$.

Quantization:

We next proceed to the quantization of the physical degrees of freedom appearing in the Hamiltonian Eq. (21). We write the quantum operator representing the free part $\int d^3x \mathfrak{e}$ of the Hamiltonian as $\hat{H}_0$ and the operator representing the gravitational self-interaction as $\kappa \hat{V}_g$. Both operators act on the matter degrees of freedom. At the moment, we do

$$\hat{h}_{ij}(x) = \sqrt{\frac{2}{\kappa}} \sum_r \int \frac{d^3k}{(2\pi)^3\sqrt{2\omega_{\mathbf{k}}}} L^r_{ij}(\mathbf{k}) \left(\hat{b}_r(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} + b^\dagger_r(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{x}} \right), \tag{23}$$

$$\hat{\Pi}_{ij}(x) = -i\sqrt{\frac{\kappa}{2}} \sum_r \int \frac{d^3k}{(2\pi)^3} \sqrt{\frac{\omega_{\mathbf{k}}}{2}} L^r_{ij}(\mathbf{k}) \left(\hat{b}_r(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} - b^\dagger_r(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{x}} \right), \tag{24}$$

where $r = 1, 2$ denotes the two polarizations, and $\omega_{\mathbf{k}} = \sqrt{k_i k^i}$. The matrices L^r_{ij} are transverse-traceless, and normalized to satisfy the conditions $\sum_r L^r_{ij}(\mathbf{k}) L^r_{kl}(\mathbf{k}) = \frac{1}{2}(P_{ik}P_{jl} + P_{il}P_{jk})$, where $P_{ij} = \delta_{ij} - k_i k_j / k^2$ is the projector onto the transverse direction.

The operator representing the Hamiltonian Eq. (21) is

$$\begin{aligned} \hat{H} = \hat{H}_0 + \kappa\hat{V} + \sum_r \int \frac{d^3k}{(2\pi)^3} \omega_{\mathbf{k}} \hat{b}^\dagger_r(\mathbf{k}) \hat{b}_r(\mathbf{k}) \\ -\sqrt{\frac{\kappa}{2}} \sum_r \int \frac{d^3k}{(2\pi)^3\sqrt{2\omega_{\mathbf{k}}}} \left[\hat{b}_r(\mathbf{k}) \hat{J}^\dagger_r(\mathbf{k}) + b^\dagger_r(\mathbf{k}) \hat{J}_r(\mathbf{k}) \right], \end{aligned} \tag{25}$$

where $[\hat{b}_r(\mathbf{k}), \hat{b}_s(\mathbf{k}')] = [\hat{b}^\dagger_r(\mathbf{k}), \hat{b}_s(\mathbf{k}')] = 0$, $[\hat{b}_r(\mathbf{k}), \hat{b}^\dagger_s(\mathbf{k}')] = \delta(\mathbf{k} - \mathbf{k}')\delta_{rs}$. We defined the operators $\hat{J}_r(\mathbf{k}) = \hat{J}^\dagger_r(-\mathbf{k})$ as'

$$\hat{J}_r(\mathbf{k}) = L^r_{ij}(\mathbf{k}) \int d^3x e^{-i\mathbf{k}\cdot\mathbf{x}} \hat{\mathfrak{t}}^{ij}(x), \tag{26}$$

where $\hat{\mathfrak{t}}^{ij}(x)$ is the (normal-ordered) quantum operator representing the stress-tensor in Eq. (21).

Relation to QBM models:

Eq. (25) shows that the environment consists of a collection of harmonic oscillators coupled to the matter degrees of freedom. The coupling is linear with respect to the creation and annihilation operators of the environment oscillators. The system is formally similar to a quantum Brownian motion (QBM) model, with the transverse traceless degrees of freedom playing the role of the bath oscillators. In order to compare with the standard QBM models, we note that for the system consisting of a single-particle—a case considered in Ref. [29]— the interaction Hamiltonian between system and environment is proportional to $\hat{p}^2\hat{q}_i$, where $\hat{q}_i$ is the position operator of the environment oscillators (the gravitational perturbations) and $\hat{p}^2$ is the particle's momentum.

But only for quadratic order of perturbations

Initial condition for gravitational field

- Two situations:

1) Minkowski spacetime viewed as the ground state of a quantum gravity theory. Gravitons weakly coupled, decoherence insignificant

2) Spacetime as the thermodynamic / hydrodynamic limit of a more basic theory:

   Classical stochastic fluctuations, decoherence effect could be big.

We want to choose a state $\hat{\rho}_B$ that interpolates between the two alternatives. The state should be stationary, reflecting the time-translation symmetry of Minkowski spacetime.

Assuming that it is also a Gaussian state, the only choice is a thermal state at a “temperature” Θ: Not viewed as a temperature of the graviton environment, but as a phenomenological parameter interpolating between the fully quantum and the classical/stochastic regime of gravitational fluctuations.

Noise temperature, i.e., a parameter characterizing the power spectral density of the noise
in stochastic systems, that is not related to a thermodynamic temperature.

In fact, the specific form of the initial state may not affect significantly the physical predictions in certain regimes.

For a single non-relativistic particle only the behavior of the state in the deep infrared sector of the environment ($\omega \rightarrow 0$) contributes to the non-unitary part of the dynamics

A potential problem in the choice of the state above for the gravitational perturbations is that a thermal state is not Lorentz invariant. The ensuing open system dynamics would then lead to a breaking of Lorentz invariance of the field. However, in the present context, Lorentz invariance has been broken by gauge-fixing prior to quantization. There is no physical representation of the Lorentz group in the Hilbert space of the quantized gravitational perturbations, so we do not know *a priori* the rule under which a thermal state transforms with the changes of coordinate systems. We can *postulate* that the chosen initial state remains unchanged when transforming from one frame to another, or, more plausibly, that the thermal state is an approximation to a state that remains invariant under the, yet unknown, physical representation of the Lorentz group. In this perspective, the correct rule for Lorentz transformations can be obtained only if we have a gauge-invariant prescription for quantization of the matter-gravity system.

Master equation for the matter field

Tracing out the gravitational degrees of freedom yields to second-order in $\sqrt{\kappa}$ the master equation for the reduced density matrix $\hat{\rho}_t$ of the matter fields [3, 4]

$$\begin{aligned}\frac{\partial \hat{\rho}_t}{\partial t} &= -i[\hat{H}_0 + \frac{\kappa}{2}\hat{V}, \hat{\rho}_t] \\ &-\frac{\kappa}{4}\sum_a \frac{\coth\left(\frac{\omega_a}{2\Theta}\right)}{\omega_a}\left([\hat{J}_a^\dagger, [\hat{\tilde{J}}_a(\omega_a), \hat{\rho}_t]] + [\hat{J}_a, [\hat{\tilde{J}}_a^\dagger(\omega_a), \hat{\rho}_t]]\right) \\ &-\frac{\kappa}{4}\sum_a \frac{1}{\omega_a}\left(\{\hat{J}_a^\dagger, [\hat{\tilde{J}}_a(\omega_a), \hat{\rho}_t]\} - \{\hat{J}_a, [\hat{\tilde{J}}_a^\dagger(\omega_a), \hat{\rho}_t]\}\right), \end{aligned} \tag{27}$$

where we used the combined index a to denote the pair $(\mathbf{k}, r)$ such that $\sum_r \int \frac{d^3k}{(2\pi)^3} \to \sum_a$, $\hat{J}_r(\mathbf{k}) \to \hat{J}_a$ and so on. The operator $\hat{\tilde{J}}$ is defined as

$$\hat{\tilde{J}}_a(\omega) = \int_0^\infty ds e^{-i\omega s} e^{-i\hat{H}_0 s} \hat{J}_a e^{i\hat{H}_0 s}. \tag{28}$$

The master equation (27) has constant coefficients and it is of the Lindblad type [31]. Its derivation (to second order in $\sqrt{\kappa}$ does not require the Born and the Markov approximation, only the condition that the coupling is very small [3]. It does not hold for times much larger than the relaxation time, but this is not a problem for the study of decoherence.

Proceeding to the computation of the operators $\hat{J}_r(\mathbf{k})$, we decompose the quantum operator $\hat{\phi}(x)$ for the field in terms of creation and annihilation operators

$$\hat{\phi}(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2\omega_{\mathbf{p}}}} \left(\hat{a}_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{x}} + \hat{a}^\dagger_{\mathbf{p}} e^{-i\mathbf{p}\cdot\mathbf{x}} \right), \tag{30}$$

where $[\hat{a}_{\mathbf{p}}, \hat{a}_{\mathbf{p}'}] = [\hat{a}^\dagger_{\mathbf{p}}, \hat{a}^\dagger_{\mathbf{p}'}] = 0$, $[\hat{a}_{\mathbf{p}}, \hat{a}^\dagger_{\mathbf{p}'}] = \delta_{\mathbf{p}\mathbf{p}'}$, and $\omega_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$.

From Eq. (26), we obtain

$$\hat{J}_r(\mathbf{k}) = L^r_{ij}(\mathbf{k}) \int \frac{d^3p}{(2\pi)^3} \frac{p^i p^j}{\sqrt{2\omega_{\mathbf{p}}}} \left(\frac{\hat{a}_{\mathbf{p}} \hat{a}_{\mathbf{k}-\mathbf{p}}}{\sqrt{2\omega_{\mathbf{k}-\mathbf{p}}}} + \frac{\hat{a}^\dagger_{\mathbf{p}} \hat{a}^\dagger_{-\mathbf{k}-\mathbf{p}}}{\sqrt{2\omega_{\mathbf{k}+\mathbf{p}}}} + 2\frac{\hat{a}^\dagger_{\mathbf{p}} \hat{a}_{\mathbf{k}+\mathbf{p}}}{\sqrt{2\omega_{\mathbf{k}+\mathbf{p}}}} \right). \tag{31}$$

In the derivation of Eq. (31) we have used the normal ordered form of the operator $\hat{t}_{ij}(x) = \partial_i \hat{\phi}(x) \partial_j \hat{\phi}(x)$.

In order to compute the operator $\hat{\tilde{J}}_a(\omega)$ we write

$$f(\omega) = \int_0^\infty ds e^{-i\omega s} = \pi\delta(\omega) - iPV(\frac{1}{\omega}), \tag{32}$$

where PV denotes the Cauchy principal part. When evaluating $\hat{\tilde{J}}_a(\omega)$ according to Eq. (28), the terms in Eq. (31) involving two creation or two annihilation operators are multiplied by $f(\omega_{\mathbf{p}} + \omega_{\mathbf{p}'} \pm \omega)$. Since $\omega_{\mathbf{p}} >> m$, and the frequencies of the environment are bounded by a cut-off $\Lambda << m$, their contribution is suppressed in comparison to the other terms, which are multiplied by $f(\omega_{\mathbf{p}} - \omega_{\mathbf{p}'} \pm \omega)$. Hence,

$$\hat{\tilde{J}}_r(\mathbf{k}, \omega) \simeq L^r_{ij}(\mathbf{k}) \int \frac{d^3p}{(2\pi)^3} \frac{p^i p^j}{\sqrt{2\omega_{\mathbf{p}}}} \frac{\hat{a}^\dagger_{\mathbf{p}} \hat{a}_{\mathbf{k}+\mathbf{p}}}{\sqrt{\omega_{\mathbf{k}+\mathbf{p}}}} f(\omega_{\mathbf{k}+\mathbf{p}} - \omega_{\mathbf{p}} + \omega). \tag{33}$$

Gravitational Self-Interaction

The term $\hat{V}$ describing gravitational self-interaction is

$$\hat{V} = -\int \frac{d^3k}{(2\pi)^3}\left(\frac{2\hat{\mathfrak{e}}^\dagger_{\mathbf{k}}\mathfrak{e}_{\mathbf{k}}}{k^2} - \frac{2\hat{\mathfrak{e}}^\dagger_{\mathbf{k}}\hat{p}_{\mathbf{k}}}{k^2} + (\delta_{ij} - \frac{3k_ik_j}{4k^2})\frac{\hat{\mathfrak{p}}^{i\dagger}_{\mathbf{k}}\hat{\mathfrak{p}}^j_{\mathbf{k}}}{k^2} - \frac{\hat{\mathfrak{e}}^\dagger_{\mathbf{k}}\hat{g}_k}{k^2}\right),, \tag{34}$$

expressed in terms of the normal-ordered operators

$$\hat{\mathfrak{e}}_{\mathbf{k}} = \int \frac{d^3p}{(2\pi)^3}\left(\frac{\omega^2_{\mathbf{p}} - \omega_{\mathbf{p}}\omega_{\mathbf{k+p}} + \mathbf{p}\cdot\mathbf{k}}{4\sqrt{\omega_{\mathbf{p}}\omega_{\mathbf{k+p}}}}\hat{a}_{\mathbf{p}}\hat{a}_{-\mathbf{p}-\mathbf{k}} + \frac{\omega^2_{\mathbf{p}} - \omega_{\mathbf{p}}\omega_{\mathbf{p-k}} - \mathbf{p}\cdot\mathbf{k}}{4\sqrt{\omega_{\mathbf{p}}\omega_{\mathbf{p-k}}}}\hat{a}^\dagger_{\mathbf{p}}\hat{a}^\dagger_{\mathbf{k-p}} + \frac{\omega^2_{\mathbf{p}} + \omega_{\mathbf{p}}\omega_{\mathbf{p-k}} - \mathbf{p}\cdot\mathbf{k}}{2\sqrt{\omega_{\mathbf{p}}\omega_{\mathbf{p-k}}}}\hat{a}^\dagger_{\mathbf{p}}\hat{a}_{\mathbf{p-k}}\right) \tag{35}$$

$$\hat{p}_{\mathbf{k}} = \frac{1}{6}\int \frac{d^3p}{(2\pi)^3}\left(\frac{\mathbf{p}^2 + \mathbf{p}\cdot\mathbf{k}}{\sqrt{\omega_{\mathbf{p}}\omega_{p+k}}}\hat{a}_{\mathbf{p}}\hat{a}_{-\mathbf{p}-\mathbf{k}}\frac{\mathbf{p}^2 - \mathbf{p}\cdot\mathbf{k}}{\sqrt{\omega_{\mathbf{p}}\omega_{p-k}}}\hat{a}^\dagger_{\mathbf{p}}\hat{a}^\dagger_{\mathbf{k-p}} + +2\frac{\mathbf{p}^2 - \mathbf{p}\cdot\mathbf{k}}{\sqrt{\omega_{\mathbf{p}}\omega_{p-k}}}\hat{a}^\dagger_{\mathbf{p}}\hat{a}_{\mathbf{p-k}}\right) \tag{36}$$

$$\hat{\mathfrak{p}}^i_{\mathbf{k}} = \frac{i}{2}\int \frac{d^3p}{(2\pi)^3}\left[\sqrt{\frac{\omega_{\mathbf{p}}}{\omega_{\mathbf{p+k}}}}(p^i + k^i)\hat{a}_{\mathbf{p}}\hat{a}_{-\mathbf{k}-\mathbf{p}} + \sqrt{\frac{\omega_{\mathbf{p}}}{\omega_{\mathbf{p-k}}}}(p^i - k^i)\hat{a}_{\mathbf{p}}\hat{a}_{\mathbf{k-p}} + \left(\sqrt{\frac{\omega_{\mathbf{p}}}{\omega_{\mathbf{p-k}}}}(p^i - k^i) + \sqrt{\frac{\omega_{\mathbf{p-k}}}{\omega_{\mathbf{p}}}}p^i\right)\hat{a}^\dagger_{\mathbf{p}}\hat{a}_{\mathbf{p-k}}\right] \tag{37}$$

$$\hat{g}_{\mathbf{k}} = m^2\int \frac{d^3p}{(2\pi)^3}\left[\frac{1}{2\sqrt{\omega_{\mathbf{p}}\omega_{\mathbf{k+p}}}}\hat{a}_{\mathbf{p}}\hat{a}_{-\mathbf{p}-\mathbf{k}} + \frac{1}{2\sqrt{\omega_{\mathbf{p}}\omega_{\mathbf{p-k}}}}\hat{a}^\dagger_{\mathbf{p}}\hat{a}^\dagger_{\mathbf{k-p}} + +\frac{1}{\sqrt{\omega_{\mathbf{p}}\omega_{\mathbf{p-k}}}}\hat{a}^\dagger_{\mathbf{p}}\hat{a}_{\mathbf{p-k}}\right]. \tag{38}$$

Energy density, isotropic pressure, momentum density

Projection of master equation to one particle subspace

we restrict the density matrix $\hat{\rho}$ into the single-particle subspace $\mathbf{H}_1$ of the Hilbert space $\mathcal{H}$ of the field [34, 35]. A single-particle state is expressed in the field Hilbert space as $\int \frac{d^3p}{(2\pi)^3}\psi(\mathbf{p})\hat{a}^\dagger_{\mathbf{p}}|0\rangle$, where $\psi(\mathbf{p})$ is the particle's wave-function in momentum space and $|0\rangle$ is the field vacuum. The density matrix for a single particle $\hat{\rho}_1$ is thus represented by the field density matrix

$$\hat{\rho} = \int \frac{d^3p}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3}\rho_1(\mathbf{p},\mathbf{p}')\hat{a}^\dagger_{\mathbf{p}}|0\rangle\langle 0|\hat{a}_{\mathbf{p}'}, \tag{39}$$

where $\rho_1(\mathbf{p},\mathbf{p}') = \langle \mathbf{p}|\hat{\rho}_1|\mathbf{p}'\rangle_{\mathcal{H}_1}$ is the single-particle density matrix in the momentum representation.

We then project the master equation (27) to $\mathcal{H}_1$. To this end, we substitute a density matrix of the form Eq. (39) into Eq. (27) and retain only the terms that preserve this form.

The von Neumann term $-i[\hat{H}_0,\hat{\rho}]$ for the free Hamiltonian preserves the single-particle subspace, giving rise to a term $-i[\sqrt{\hat{\mathbf{p}}^2+m^2},\hat{\rho}_1]$ on $\mathcal{H}_1$ representing the evolution of a free relativistic particle.

Non-unitary terms from gravitational backreaction

The non-unitary terms. To project the non-unitary terms of Eq. (27) into $\mathcal{H}_1$ we proceed as follows. The commutators with the $\hat{\hat{J}}_a$ operators of Eq. (33) preserve the single-particle subspace. The only terms that fail to preserve $\mathcal{H}_1$ are the components of $\hat{J}_a$ involving two creation or two annihilation operators in Eq. (31). Dropping these terms we find that the projection of the non-unitary terms correspond to a superoperator $\mathbf{L}$ on $\mathcal{H}_1$ defined by

$$\mathbf{L}[\hat{\rho}_1] = -\frac{\kappa}{4}\sum_a \left[\frac{\coth\left(\frac{\omega_a}{2\Theta}\right)}{\omega_a} \left([\hat{A}_a^\dagger, [\hat{B}_a, \hat{\rho}_1]] + [\hat{A}_a, [\hat{B}_a^\dagger, \hat{\rho}_1]] \right) \right.$$
$$\left. - \frac{1}{\omega_a} \left(\{\hat{A}_a^\dagger, [\hat{B}_a, \hat{\rho}_1]\} - \{\hat{A}_a, [\hat{B}_a^\dagger, \hat{\rho}_1]\} \right) \right], \tag{40}$$

where $\hat{A}_a \equiv \hat{A}_r(\mathbf{k})$ and $\hat{B}_a \equiv \hat{B}_r(\mathbf{k})$ are operators on $\mathcal{H}_1$ defined by their matrix elements in the momentum basis

$$\langle \mathbf{p}|\hat{A}_r(\mathbf{k})|\mathbf{p}'\rangle = L^r_{ij}(\mathbf{k}) \frac{p^i p^j}{\sqrt{\omega_{\mathbf{p}}\omega_{\mathbf{p}'}}} (2\pi)^3 \delta(\mathbf{p}' - \mathbf{p} - \mathbf{k}) \tag{41}$$

$$\langle \mathbf{p}|\hat{B}_r(\mathbf{k})|\mathbf{p}'\rangle = \langle \mathbf{p}|\hat{A}_r(\mathbf{k})|\mathbf{p}'\rangle f(\omega_{\mathbf{p}} - \omega_{\mathbf{p}'} + \omega_{\mathbf{k}}). \tag{42}$$

Gravitational Self-interaction

The gravitational self-interaction. The projection of the von Neumann term describing gravitational self-interaction onto $\mathcal{H}_1$ yields a term $-i\frac{\kappa}{2}[\hat{U}, \hat{\rho}_1]$, where

$$\hat{U} = -\int \frac{d^3k}{(2\pi)^3}\frac{1}{k^2}\left[F^1_{\mathbf{k}}(\hat{\mathbf{p}}) + F^2_{\mathbf{k}}(\hat{\mathbf{p}}) + F^3_{\mathbf{k}}(\hat{\mathbf{p}} + F^4_{\mathbf{k}}(\hat{\mathbf{p}})\right]. \tag{43}$$

The operators $F^i_{\mathbf{k}}(\hat{\mathbf{p}})$ are functions of the 3-momentum operator $\hat{p}^i$ for a relativistic particle. Each corresponds to one of the terms in the sum of Eq. (34), as they are projected in the single-particle Hilbert space. In particular, $F^1_{\mathbf{k}}$ corresponds to the term $2\hat{\mathfrak{e}}^\dagger_{\mathbf{k}}\mathfrak{e}_{\mathbf{k}}$, $F^2_{\mathbf{k}}$ corresponds to $-2\hat{\mathfrak{e}}^\dagger_{\mathbf{k}}\hat{p}_{\mathbf{k}}$, $F^3_{\mathbf{k}}$ corresponds to $(\delta_{ij} - \frac{k_ik_j}{2k^2})\hat{\mathfrak{p}}^{i\dagger}_{\mathbf{k}}\hat{\mathfrak{p}}^j_{\mathbf{k}}$, and $F^4_{\mathbf{k}}$ corresponds to $-\hat{\mathfrak{e}}^\dagger_{\mathbf{k}}\hat{g}_k$. Their explicit form is the following

$$F^1_{\mathbf{k}}(\hat{\mathbf{p}}) = 2\omega_{\mathbf{p}}\omega_{\mathbf{p}+\mathbf{k}} - \frac{\omega_{\mathbf{p}}}{\omega_{\mathbf{p}+\mathbf{k}}}(\mathbf{p}\cdot\mathbf{k} + \mathbf{k}^2) - \mathbf{p}\cdot\mathbf{k}, \tag{44}$$

$$F^2_{\mathbf{k}}(\hat{\mathbf{p}}) = -\frac{\mathbf{p}\cdot(\mathbf{p}+\mathbf{k})}{3\omega_{\mathbf{p}}\omega_{\mathbf{p}+\mathbf{k}}}(2\omega^2_{\mathbf{p}} - \mathbf{k}^2) \tag{45}$$

$$F^3_{\mathbf{k}}(\hat{\mathbf{p}}) = -\frac{1}{4}\left[\frac{\omega_{\mathbf{p}+\mathbf{k}}}{\omega_{\mathbf{p}}}\left(\mathbf{p}^2 - \frac{3(\mathbf{p}\cdot\mathbf{k})^2}{4k^2}\right) + \frac{\omega_{\mathbf{p}}}{\omega_{\mathbf{p}+\mathbf{k}}}\left((\mathbf{p}+\mathbf{k})^2 - \frac{3[(\mathbf{p}+\mathbf{k})\cdot\mathbf{k}]^2}{4k^2}\right)\right.$$
$$\left. + \; 2\mathbf{p}^2 + \frac{1}{2}\mathbf{k}\cdot\mathbf{p} - \frac{3(\mathbf{p}\cdot\mathbf{k})^2}{2k^2}\right] \tag{46}$$

$$F^4_{\mathbf{k}} = -\frac{m^2}{4}\left(\frac{3\omega_{\mathbf{p}}}{\omega_{\mathbf{p}+\mathbf{k}}} + \frac{\omega_{\mathbf{p}+\mathbf{k}}}{\omega_{\mathbf{p}}} + 2\mathbf{p}\cdot\mathbf{k} - \mathbf{k}^2\right). \tag{47}$$

Thus, the master equation for a single relativistic particle is

$$\frac{\partial\hat{\rho}_1}{\partial t} = -i[\sqrt{m^2 + \hat{\mathbf{p}}^2}, \hat{\rho}_1] - i\frac{\kappa}{2}[\hat{U}, \hat{\rho}_1] + \mathbf{L}[\hat{\rho}_1]. \tag{48}$$

Non-relativistic limit

3.5. The non-relativistic limit

The master equation (48) is still very complex. However, it simplifies significantly in the non-relativistic limit. For $|\mathbf{p}| << m$, the matrix elements of the operators $\mathbf{A}_a$ become

$$\langle \mathbf{p}|\hat{A}_r(\mathbf{k})|\mathbf{p}'\rangle \simeq L^r_{ij}(\mathbf{k})\frac{p^i p^j}{m}(2\pi)^3\delta(\mathbf{p}' - \mathbf{p} - \mathbf{k}), \tag{49}$$

and thus $\hat{A}_r$ can be expressed as

$$\hat{A}_r(\mathbf{k}) = L^r_{ij}(\mathbf{k})\frac{\hat{p}^i\hat{p}^j}{m}e^{ik_i\hat{X}^i}, \tag{50}$$

where $\hat{x}^i$ is the position and $\hat{p}_j$ the momentum operators of a non-relativistic particle.

The master equation for a non-relativistic particle interacting with gravity, valid to first order in κ.

$$\frac{\partial\hat{\rho}_1}{\partial t} = -\frac{i}{2m_R}[\hat{\mathbf{p}}^2, \hat{\rho}_1] - \frac{\kappa\Theta}{18m_R^2}(\delta^{ij}\delta^{kl} + \delta^{ik}\delta^{jl})[\hat{p}_i\hat{p}_j, [\hat{p}_k\hat{p}_l, \hat{\rho}_1]]$$

the renormalized mass $m_R = m(1 + \frac{9\kappa\Lambda}{2\pi^2}).$

Gravitational Decoherence Time

$$\frac{\partial \hat{\rho}}{\partial t} = -\frac{i}{2m_R}[\hat{p}^2, \hat{\rho}] - \frac{4\pi G\Theta}{9m_R^2}[\hat{p}^2, [\hat{p}^2, \hat{\rho}]]. \tag{61}$$

where we reinserted Newton's constant by setting $\kappa = 8\pi G$.

This master equation is exactly solvable in the momentum representation

$$\rho_t(p, p') = \exp\left[-\frac{i}{2m_R}(p^2 - p'^2)t - \frac{4\pi G\Theta}{9m_R^2}(p^2 - p'^2)^2 t\right]\rho_0(p, p') \tag{62}$$

It is evident that the master equation leads to decoherence in the momentum basis. Let us assume that the initial state is a superposition of two states, localized in momentum p_1 and p_2. Define the mean momentum $p = (p_1 + p_2)/2$ and $\Delta p = |p_2 - p_1|$. Then, after time of order of

$$t_{dec} = \frac{m_R^2}{G\Theta p^2 \Delta p^2} = \frac{1}{G\Theta m_R^2 v^2 \delta v^2}, \tag{63}$$

the momentum superpositions will have been destroyed; v and δv refer to the mean velocity and the velocity difference, respectively. Inserting back c and $\hbar$ the decoherence time is

$$t_{dec} = \frac{\hbar^2 c^5}{G\Theta m_R^2 v^2 \delta v^2}. \tag{64}$$

- Note dependence not only on G but another parameter, *Θ*. *A general feature of grav decoh*
- *The Newtonian force term*, that involves only Newton's constant, always appears in the Hamiltonian part of the evolution equation and it *does not lead to decoherence*.

- Decoherence is due to the transverse-traceless (TT) perturbations and the corresponding non-unitary term will involve parameters corresponding to the unequal-time correlation function that characterizes the perturbations.
- These parameters are, in principle, determined by the **detailed features of the environment**, like the **spectral density** of a harmonic oscillator bath

- With gravity as the environment it refers to the underlying constituents of Minkowski spacetime, what we referred to as **the 'textures'of spacetime..**

IV. Main Results

- Gravitational decoherence happens in the momentum basis, not (at least directly) in the position basis, as many env-ind decoh.

- Significance of space and time reparameterizations in the description of a quantum field interacting with linearized gravity. Require specific gauge choice.
- A gauge-invariant treatment of the associated constraints does not appear compatible with the structures of Poincar´e covariant QFT.

- Penrose's concern leading to his grav decoh proposal.

  Ours is a more formal characterization of the same point.

Critiques on STFI-Decoherence

- Many theories of gravitational, intrinsic or fundamental decoherence assume it is induced by temporal or spatial fluctuations (STFI) cast in terms of stochastic processes.

- Such fluctuations correspond to time or space reparameter-izations, which are pure gauge variables with no dynamical content.

- Being gauge variables, time and space reparameterizations are decoupled from the interaction at the level of the classical theory. Classical general relativity does not provide any information about the strength of such interactions. Newtonian's constant need not appear.

- The rationale for such theories of -decoherence come from sources which violate the fundamental symmetry of classical GR. STFI are not gravitational decoherence.

Broader Implications

- Gravitational decoherence depends strongly on assumptions about the nature of gravitational perturbations:

- The usual assumption that Minkowski spacetime is the ground state of quantum gravity would imply that gravitational perturbations are very weak and cannot lead to decoherence.

- However, if general relativity is a hydrodynamic theory and gravity is in the nature of thermodynamics, Minkowski spacetime should presumably be identified with a macrostate (i.e., a coarse-grained state of the micro-structures). In this case, the perturbations are expected to be much stronger acting as agents of decoherence.

- Thus, observation of the magnitude and features of gravitational decoherence may reveal the nature of gravity, whether it is elemental or emergent.

End

Discussions

- 1) Problems with modifying Quantum Mechanics, esp, *Decoherence due to space-time fluctuations*.

- 2) *Revealing the deeper textures of spacetime*. Gravity: fundamental or emergent ?

Comparison with Diosi-Penrose

- Our master equation derived from known physics -- quantum field theory and general relativity – looks very different from that of the Diosi-Penrose theories.

- The Lindblad operators are quadratic in momentum, so the environment can decohere momentum superpositions, but not position superpositions.

Our findings: [this slide made in 2011 is superseded by those in 2013]

- D-P master equation does not follow from any known physics, or at least from quantum theory and GR in their standard form.
- It is different from decoherence induced by gravity acting as an environment on quantum matter.

- (Penrose) "*In this way, contact is made with the standard `decoherence' viewpoint of quantum state reduction, the* ***essential distinction being that in the present scheme the state reduction is taken as actual rather than merely FAPP".***

Our Position:

"FAPP decoherence" of quantum matter by gravity acting as environment **is based on falsifiable assumptions in known physics, and thus is from the epistemological and logically views better than the**

"Actual" decoherence in the Diosi-Penrose scheme **put in by hand, argued phenomenologically to serve a stated purpose.**

3^{rd} class of working models:

Campos and Hu (98) considered a conformal massless scalar field at finite temperature in a weakly perturbed Minkowsky spacetime $g_{mn} = \eta_{mn} + h_{mn}$

Calculated the backreaction of the scalar field on the spacetime – meant to be useful for backreaction of Hawking radiation in (far-field) black hole dynamics

For the present problem: Matter field sector is the system, Gravity sector is the environment.

Need to convert scalar field to particle trajectory: WLIF

4^{th} class of models: O(N) ϕ^4 in CST

- Ramsey and Hu (1997):
  O(N) model in CST with ϕ^4 interaction

- Take the QM (0-dim q field theory) version.
  Calzetta & Hu (2002) QM O(N) model, large N expansion (semiclassical Limit)
  (CH worked out NLO- LN 2PI effective action for existence of H theorem considerations)

- Construct density matrix ρ (X,X').
  After integrating over gravitational field sector get equation of motion for reduced density matrix

Noise and fluctuations in quantum field *induced metric fluctuations* spacetime (foam) microstructure described by *Einstein-Langevin Eq*.

Stochastic Semiclassical Gravity

Main advantage: Minimal speculative assumptions

A natural extension of well known and tested theories:

1. Quantum field theory in curved spacetime , (e.g., Hawking effect)
2. Semiclassical gravity (e.g.,inflationary cosmology)

Semiclassical Gravity

Semiclassical Einstein Equation (schematically):

$$\tilde{G}_{\mu\nu}(g_{\alpha\beta}) = \kappa\langle \hat{T}_{\mu\nu}\rangle_q + \kappa\,(T_{\mu\nu})_c$$

$\tilde{G}_{\mu\nu}$ is the Einstein tensor (plus covariant terms associated with the renormalization of the quantum field)

$\kappa = 8\pi G_N$ and G_N is Newton's constant

Free massive scalar field

$$(\Box - m^2 - \xi R)\hat{\phi} = 0.$$

$\hat{T}_{\mu\nu}$ is the stress-energy tensor operator

$\langle\,\rangle_q$ denotes the expectation value

Stochastic Gravity

Einstein- Langevin Equation (schematically):

$$\tilde{G}_{\mu\nu}(g_{\alpha\beta}) = \kappa \left(T^{\mathrm{c}}_{\mu\nu} + T^{\mathrm{qs}}_{\mu\nu} \right)$$

$T^{c}_{\mu\nu}$ is due to classical matter or fields

$$T^{\mathrm{qs}}_{\mu\nu} \equiv \langle \hat{T}_{\mu\nu} \rangle_{\mathrm{q}} + T^{\mathrm{s}}_{\mu\nu}$$

$T^{\mathrm{qs}}_{\mu\nu}$ is a new stochastic term
related to the quantum fluctuations of $T_{\mu\nu}$

Einstein-Langevin Equation

- Consider a weak gravitational perturbation h off a background g $g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$, The ELE is given by (The ELE is Gauge invariant)

$$G_{ab}[g+h] + \Lambda(g_{ab} + h_{ab}) - 2(\alpha A_{ab} + \beta B_{ab})[g+h] = 8\pi G\left(\left\langle \hat{T}^{R}_{ab}[g+h]\right\rangle + \xi_{ab}[g]\right).$$

- Nonlocal dissipation and colored noise

Nonlocality manifests with **stochasticity**

because the gravitational sector is an open system

NOISE KERNEL

- Exp Value of 2-point correlations of stress tensor: bitensor

- Noise kernel measures quantum flucts of stress tensor

It can be represented by (shown via influence functional to be equivalent to) a classical stochastic tensor source $\xi_{ab}[g]$

$$\langle \xi_{ab} \rangle_s = 0 \qquad \langle \xi_{ab}(x)\xi_{cd}(y)\rangle_s = N_{abcd}(x,y)$$

- **Symmetric, traceless** (for conformal field), **divergenceless**

- In the stochastic gravity approach, the **source of noise is fully accountable**, coming from matter field’s q. fluctuations, not put in by hand.

- Einstein-Langevin equation describes the quantum matter- gravitational field **interaction in a self-consistent manner**.

Stochastic Gravity Program

- ***Review***

  B. L. Hu and E. Verdaguer, "Stochastic gravity: Theory and Applications", in ***Living Reviews in Relativity*** *7 (2004) 3. updated in* ***11*** *(2008) 3 [arXiv:0802.0658]*

- ***Recent work*** *(sample)*

  - Black hole fluctuations and Backreaction.

  B.L.Hu, A. Roura, Phys. Rev. D 76 (2007) 124018

  - Cosmological perturbations:

  A. Roura and E. Verdaguer, P*hys. Rev. D* (2008), (2009)

  - Current work by

  - L. Ford, JT Hsiang, SP Miao, R.Woodard, CH Wu,

  - Paul Anderson, Jason Bates, HT Cho and B L Hu

Conventional Theory: N Eq Quantum Statistical Mechanics

- Quantum Mechanics: Path integral representation
- Nonequilibrium Statistical Mechanics: Open Systems

  - Classical: Projection Operator Formalism Zwangzig-Mori (57,61)

  - Quantum: Influence Functional Formalism Feynman-Vernon (63)

- Q Brownian Model: Harmonic Oscillator + Bath (HOB)

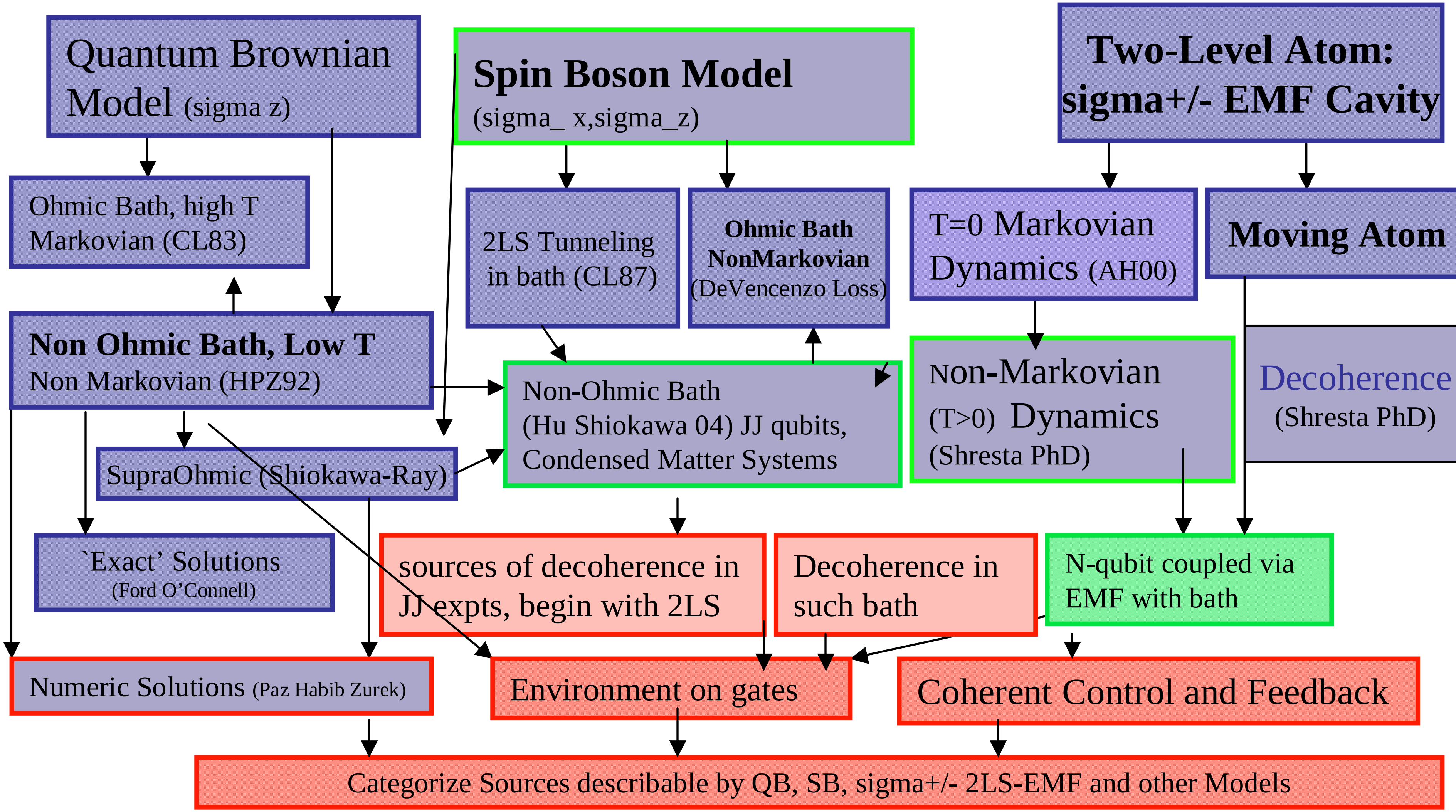
  - time-independent frequency: QBM1 Caldeira-Leggett (83) Hu- Paz- Zhang (92)

  - time-dependent frequency: QBM2 connects to Q Field Theory

Hu Matacz (94)

Decoherence Models: Non-Markovian Processes

B L Hu, University of Maryland 2003-04



QBM: Quantum Brownian Motion
HPZ: Hu Paz and Zhang (92,93)

SBM: Spin Boson Model
CL: Caldeira Leggett (83,87)
DL: Divencenzo Loss (03)

2LS: Two Level System
EMF: Electromagnetic Field
AH: Anastopoulos Hu 2000

Completed, submitted, or Published
In progress
Future Program

Influence Functional

$$\ln(F[x,y]) = \int_0^t ds \int_0^s ds'(x-y)(s) \times [-i\eta(s-s')(x+y)(s') - \nu(s-s')(x-y)(s')] , \qquad (12)$$

where $\nu(s)$ and $\eta(s)$ are the noise and dissipation kernels defined in terms of the spectral density:

$$\eta(s) = -\int_0^\infty d\omega\, I(\omega)\sin(\omega s) ,$$
$$\nu(s) = \int_0^\infty d\omega\, I(\omega)\coth\left[\frac{\omega}{2k_B T}\right]\cos(\omega s) . \qquad (13)$$

System: Quantum Brownian particle
Environment: n param HO → Scalar Field

$$i\hbar\frac{\partial}{\partial t}\hat{\rho}_r(t) = [\hat{H}_{ren}, \hat{\rho}] + iD_{pp}[\hat{x}, [\hat{x}, \hat{\rho}]] + iD_{xx}[\hat{p}, [\hat{p}, \hat{\rho}]]$$

$$+ iD_{xp}[\hat{x}, [\hat{p}, \hat{\rho}]] + iD_{px}[\hat{p}, [\hat{x}, \hat{\rho}]] + \Gamma[\hat{x}, \{\hat{p}, \hat{\rho}\}]$$

D • Diffusion terms in p^2, x^2, $xp+px$

→ Decoherence

Γ • Dissipation term