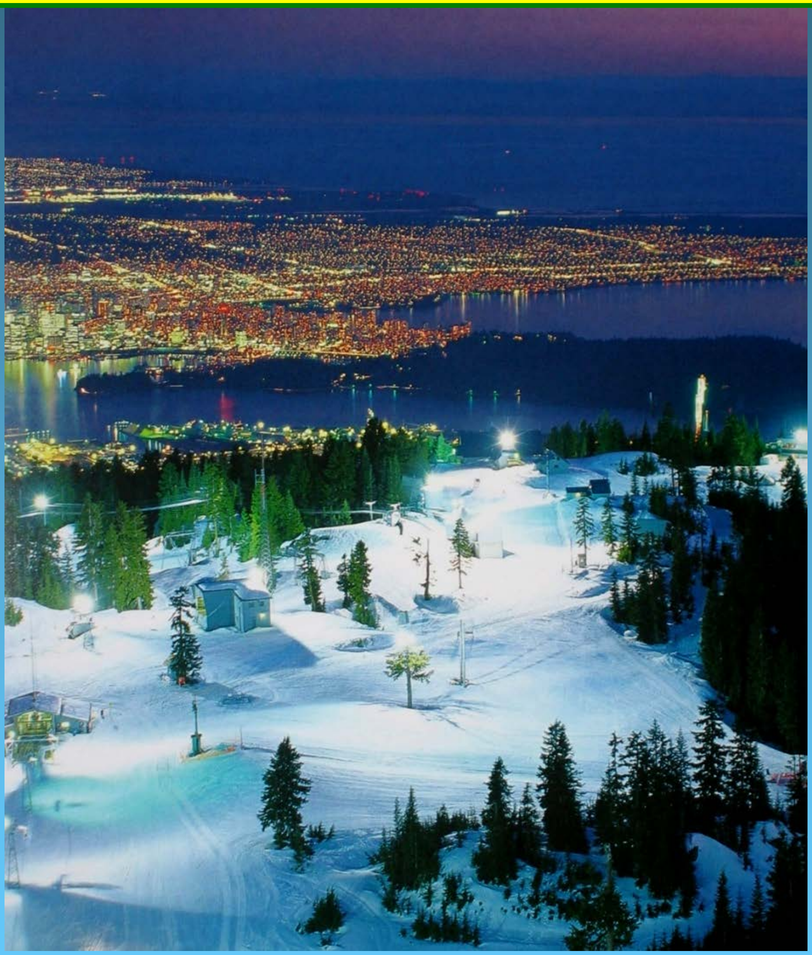
PCE STAMP

GRAVITATIONAL DECOHERENCE

Galiano Meeting, May 22nd 2013



Physics & Astronomy
UBC
Vancouver

Pacific Institute for Theoretical Physics

Currently at: Math Institute, Oxford Univ

The talk will address the following themes:

(i) Environmental decoherence – experimental tests
(ii) Intrinsic decoherence – a theoretical framework
(iii) Gravity vs Quantum Mechanics – theory, & possible experiments *

* Some of this work is part of a current collaboration with Bill Unruh

FURTHER INFORMATION:

Email: stamp@physics.ubc.ca
Web: http://www.physics.ubc.ca/~berciu/PHILIP/index.html

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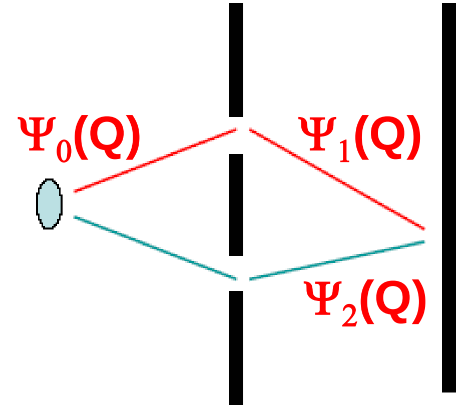
PART 1
INTRODUCTION

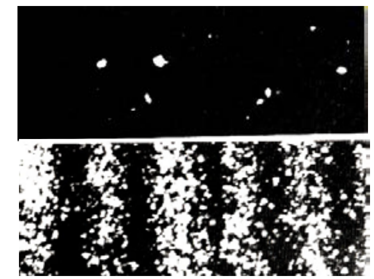
The MYSTERY of QUANTUM MECHANICS



According to Feynman (1965), ' the fundamental mystery of QM' is encapsulated in the '2-slit' experiment:

$\Psi_0(q)$ evolves according to

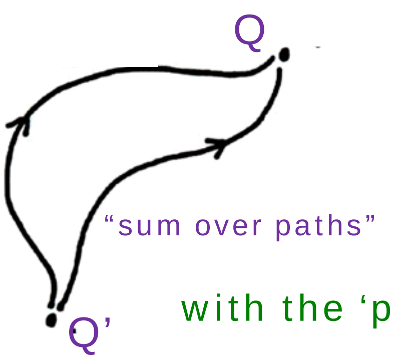
$$\Psi_o(q) \rightarrow [a_1 \Psi_1(q) + a_2 \Psi_2(q)]$$



The probability of seeing particle at position **Q** on screen:

$$P(Q) = |a_1 \Psi_1(Q) + a_2 \Psi_2(Q)|^2 = P_1 + P_2 + 2P_{12}$$

with cross-term $P_{12}(Q) = |a_1 a_2 \Psi_1(Q) \Psi_2(Q)|$



Feynman gave a beautiful formulation of QM that perfectly encapsulates this 'superposition'. He writes

$$\psi(Q,t) = \int dQ' \, G(Q,Q';t,t') \, \psi(Q',t')$$

with the 'path integral' sum:

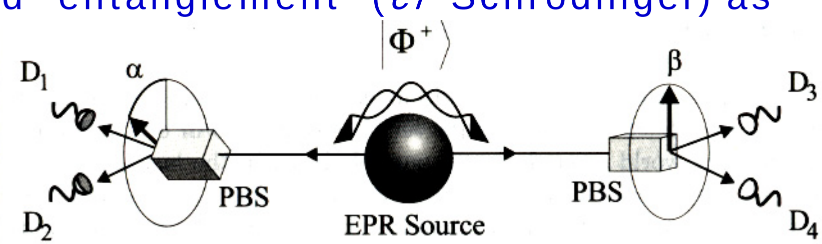
$$G(Q,Q';t,t') = \int_{q(t')=Q'}^{q(t)=Q} \mathcal{D}q(\tau) e^{\frac{i}{\hbar} S[q,\dot{q}]}$$

The MYSTERY of ENTANGLEMENT

However, long before this, Einstein (1935) fingered "entanglement" (*cf* Schrodinger) as the real mystery – embodied in states like

$$\Psi = [\phi_+(A)\phi_-(B) + \phi_-(A)\phi_+(B)]$$

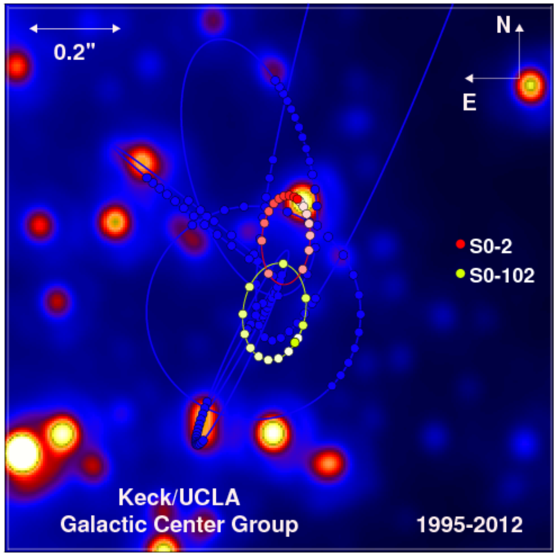
for which the quantum state of either individual system is literally meaningless!

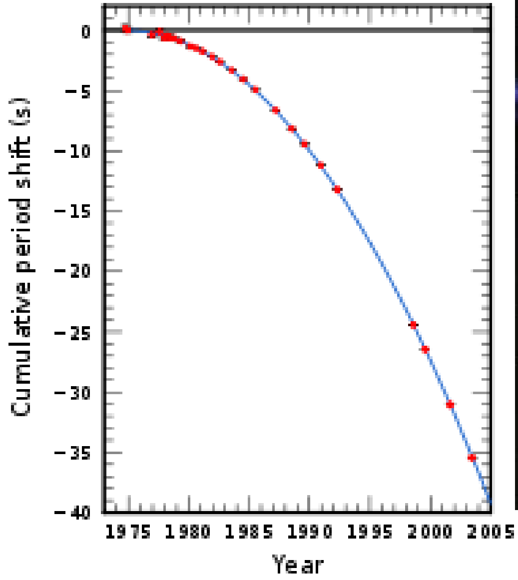


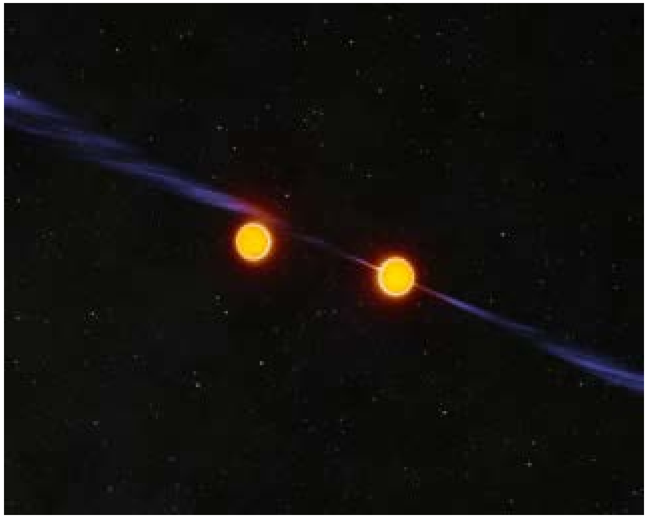
NB: QUANTUM MECHANICS WORKS REALLY WELL!

GENERAL RELATIVITY also works REALLY WELL

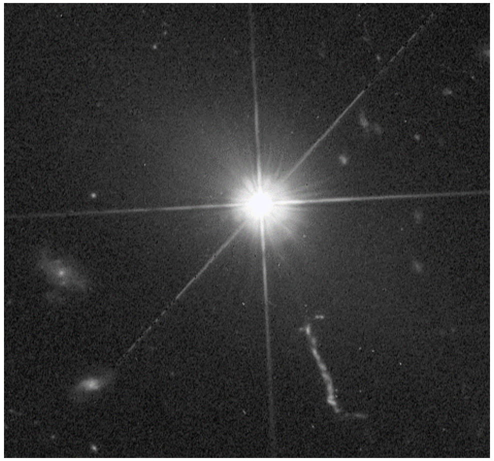
SUPERMASSIVE
BLACK HOLES & AGN





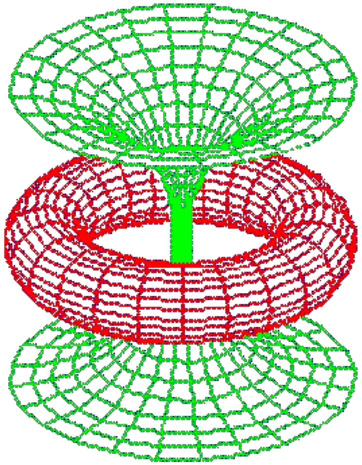


BINARY PULSAR



Largest so far:
2 x 10^{10} solar masses

Tests GR in many detailed ways – notably theory of rotating Black Holes, accretion discs, etc.



Kerr geometry

GRAVITATIONAL
LENSING

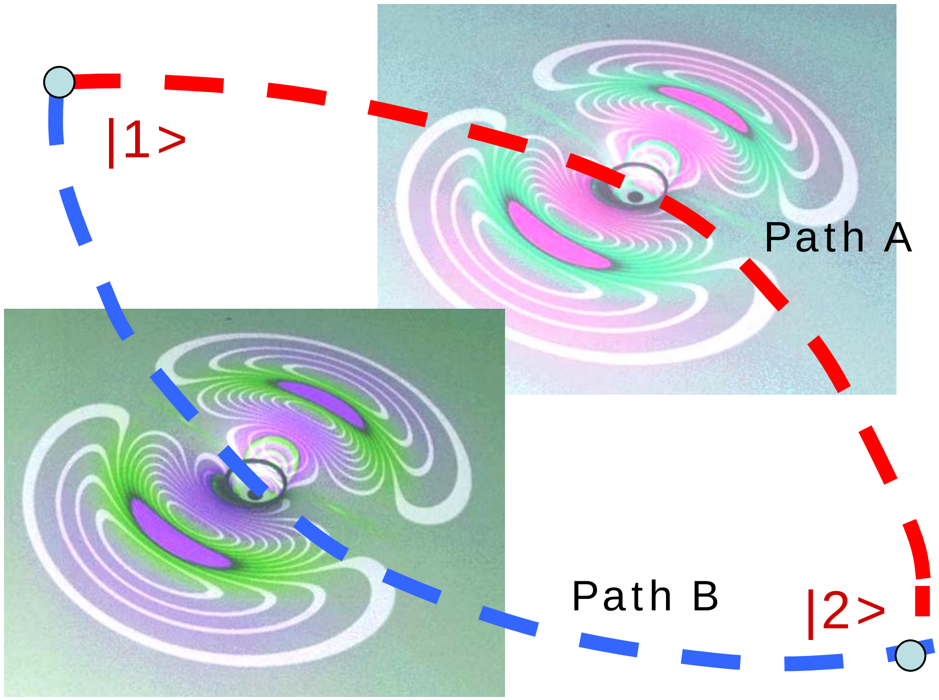
PROBLEMS with QM + GR

1. TWO-PATH EXPERIMENT

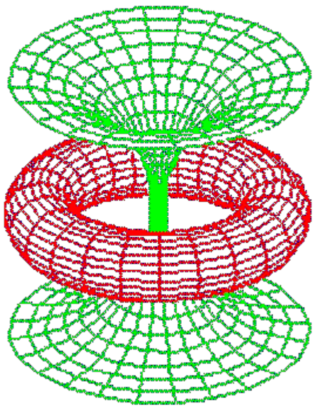
A mass M is constrained to move along 2 paths between states |1> and |2>

A superposition of stress tensors $T_{\mu\nu}(x)$ generates a superposition of 2 different spacetimes.

This creates problems in GR – the states exist in different manifolds, & viewed as Q objects they have different vacua. Superposing spacetime topologies gives huge problems.



Indeed, the mere existence of 'Macroscopic Quantum Superpositions' creates intractable problems of principle



2. SPACETIME NEAR A SPINNING PARTICLE

Consider a very light QM particle with spin (eg., a neutrino). Then GR gives spacetime a Kerr (or Kerr-Newman) structure near the particle. The radius of the 'singular ring' in the Kerr geometry is $a = L/mc$

The mass of the neutrino puts bounds on this – we find that $a > 200$ Angstroms (cf. Matt Visser)!

An ARGUMENT against the breakdown of QM

There is an argument which has become remarkably influential in the Q Gravity and string community against modifications of QM. It goes as follows:

If there is some info being 'hidden' from the universe (this would happen with any kind of intrinsic decoherence), then the density matrix of any system involved in such intrinsic decoherence would have to look like

$$\mathrm{tr}\,\dot{\rho} = 0 = -\mathrm{tr}\left\{ h_{00}\rho + \sum_{\alpha \neq 0} (h_{0\alpha} + h_{\alpha 0}) Q^{\alpha}\rho + \sum_{\alpha,\beta \neq 0} h_{\alpha\beta} Q^{\beta} Q^{\alpha} \rho \right\}$$

where the Q-matrices are a complete orthogonal set for the system. Such an eqtn of motion would arise from a 'random force' on the system, of simplest form: $H(t) = H_0 + \sum_{\alpha} j_{\alpha}(t) Q^{\alpha}$, with correlator $\langle j_{\alpha}(t)\, j_{\beta}(t')\rangle = h_{\alpha\beta}\delta(t-t')$.

A field theory would have

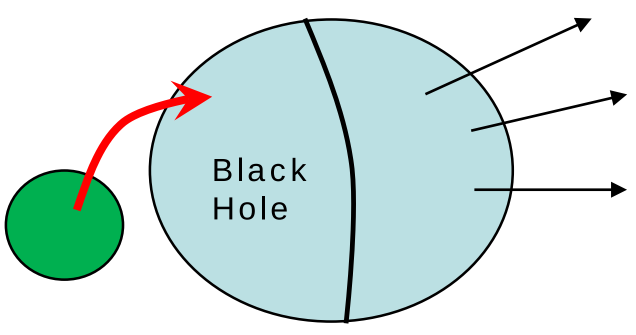
$$\dot{\rho} = -i\left[\int \mathrm{d}^3x\, H(x), \rho\right] - \tfrac{1}{2}\int \mathrm{d}^3x\, \mathrm{d}^2y\, h_{\alpha\beta}(x-y)\left(\{Q^{\beta}(y)Q^{\alpha}(x), \rho\} - 2Q^{\alpha}(x)\rho Q^{\beta}(y)\right)$$

But this violates energy-momentum conservation – so is not legitimate

T Banks, L Susskind, M Peskin Nucl Phys B244, 125 (1984)

Actually this argument is wrong, as we shall see below; see also

WG Unruh, Phil Trans Roy Soc A370, 4454 (2012)



An ARGUMENT for a modification of QM

Consider the following argument, due to Penrose: The proper time elapsed in 2 branches of a superposition cannot be directly compared, & there is a time uncertainty involved in this comparison, which can be related to an energy uncertainty given in the weak-field regime by

$$\Delta E = 2E_{1,2} - E_{1,1} - E_{2,2}.$$

where

$$E_{i,j} = -G \int \int d\vec{r}_1 d\vec{r}_2 \frac{\rho_i(\vec{r}_1)\rho_j(\vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|}$$

R Penrose Gen Rel Grav 28, 581 (1996)

The problem here is quantitative. Estimates of the decoherence time depend on how one models the mass distribution. Here are 2 estimates provided by these authors, for a superposition of 2 different mass states:

$$\Delta E = \frac{Gmm_1}{x_0}\left(\frac{24}{5} - \frac{1}{\sqrt{2\kappa}}\right)$$

"Zero point" estimate

$$\Delta E = 2Gmm_1\left(\frac{6}{5a} - \frac{1}{\Delta x}\right)$$

"nuclear radius" estimate

These numbers differ by roughly 1000

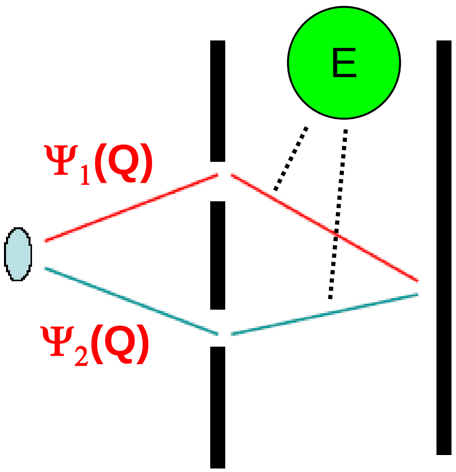
W Marshall et al., PRL 91, 130401 (2003)
D Kleckner et al., NJ Phys 10, 095020 (2008)

PART 2

SOME REMARKS on

ENVIRONMENTAL DECOHERENCE

ENVIRONMENTAL DECOHERENCE 100



Some quantum system with coordinate Q interacts with any other system (with coordinate x) ; typically they then form an entangled state

Example: In a 2-slit expt., the particle coordinate Q couples to photon coordinates, so that:

$$\Psi_o(Q)\ \ \Pi_q \phi_q^{in} \ \rightarrow\ [a_1 \Psi_1(Q)\, \Pi_q\, \phi_q^{(1)} \ +\ a_2 \Psi_2(Q)\, \Pi_q\, \phi_q^{(2)}\ ]$$

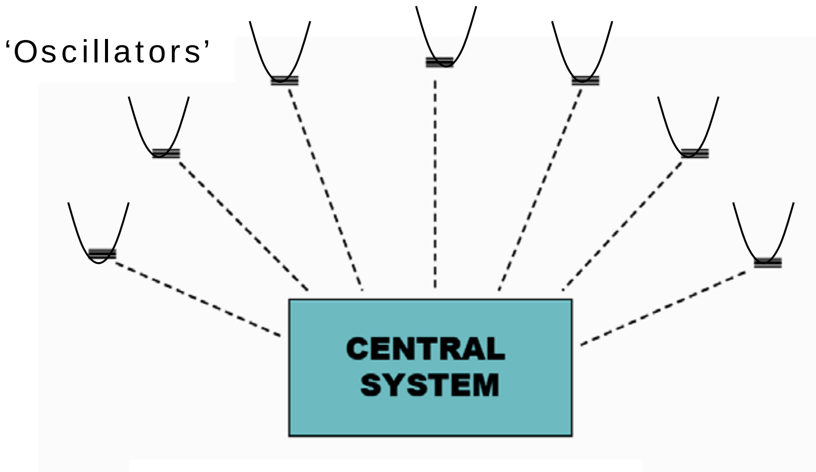
We see that the environmental photons are ENTANGLED with the particle – and the evolution of the photons is thus contingent upon that of the particle

Now suppose we have no knowledge of / control over, the photon states – we then average over these states, consistent with the experimental constraints. In the extreme case this means we lose all information about the PHASES of the coefficients a_1 & a_2 **(and in particular the relative phase between them). This process is called DECOHERENCE**

NB 1: No requirement for energy to be exchanged between the system and the environment – only a communication of phase information.

NB 2: Nor does phase interference between the 2 paths have to be associated with a noise coming from the environment- what matters is entanglement - that the state of the environment be CHANGED according to the what is the state of the system.

CURRENT MODELS of ENVIRONMENTAL DECOHERENCE



$$H_{\rm eff}^{\rm osc} = H_0 + H_{\rm int} + H_{\rm env}^{\rm osc}$$

Bath: $$H_{osc} = \sum_{q=1}^{N_o} \left(\frac{p_q^2}{m_q} + m_q\omega_q^2 x_q^2\right)$$

Int: $$H_{\rm int}^{\rm osc} = \sum_{q=1}^{N}[F_q(Q)x_q + G_q(P)p_q]$$

Very SMALL (~ O($1/N^{1/2}$))

Phonons, photons, magnons, spinons,
Holons, Electron-hole pairs, gravitons,..

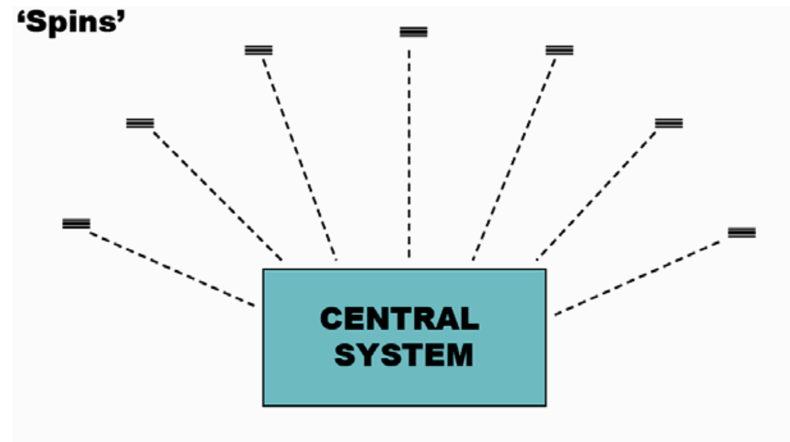
Feynman & Vernon, Ann. Phys. 24, 118 (1963)

Caldeira & Leggett, Ann. Phys. 149, 374 (1983)
AJ Leggett et al, Rev Mod Phys 59, 1 (1987

DELOCALIZED
BATH MODES

↓

OSCILLATOR
BATH



$$H_{\rm eff}^{\rm sp}(\Omega_0) = H_0 + H_{\rm int}^{\rm sp} + H_{\rm env}^{\rm sp}$$

Bath: $$H_{\rm env}^{\rm sp} = \sum_{k}^{N_s} \mathbf{h}_k \cdot \boldsymbol{\sigma}_k + \sum_{k,k'}^{N_s} V_{kk'}^{\alpha\beta}\sigma_k^{\alpha}\sigma_{k'}^{\beta}$$

Interaction: $$H_{\rm int}^{\rm sp} = \sum_{k}^{N_s} \boldsymbol{F}_k(P,Q)\cdot\boldsymbol{\sigma}_k$$

NOT SMALL !

Defects, dislocation modes, vibrons,
Localized electrons, spin impurities,
nuclear spins, ...

LOCALIZED
BATH MODES

↓

SPIN BATH

(1) P.C.E. Stamp, PRL 61, 2905 (1988)
(2) NV Prokof’ev, PCE Stamp, J Phys CM5, L663 (1993)
(3) NV Prokof’ev, PCE Stamp, Rep Prog Phys 63, 669 (2000)

FORMAL ASPECTS of ENVIRONMENTAL DECOHERENCE

density matrix propagator:

$$K(Q_2, Q_2'; Q_1, Q_1'; t, t') = \int_{Q_1}^{Q_2} \mathscr{D}q \int_{Q_1'}^{Q_2'} \mathscr{D}q' \, e^{-i/\hbar(S_0[q]-S_0[q'])} \mathscr{F}[q, q'],$$

with $$\mathcal{F}[Q, Q'] = \prod \langle \hat{U}_k(Q, t) \hat{U}_k^\dagger(Q', t) \rangle$$

Here the unitary operator $\hat{U}_k(Q, t)$ describes the evolution of the kth environmental mode, given that the central system follows the path $Q(t)$ on its 'outward' voyage, and $Q'(t)$ on its 'return' voyage; and $\mathcal{F}[Q, Q']$ acts as a weighting function, over different possible paths $(Q(t), Q'(t'))$.

Easy for oscillator baths (it is how Feynman set up quantum field theory); we integrate out a set of driven harmonic oscillators, with Lagrangians:

$$L = \frac{M}{2}\dot{x}^2 - \frac{M\omega^2}{2}x^2 - \gamma(t)x$$

Thus:

$$\mathcal{F}[Q,Q'] = \prod_q^{N_o} \int \mathcal{D}x_q(\tau) \int \mathcal{D}x_q(\tau') \exp\left[\frac{i}{\hbar}\int d\tau \frac{m_q}{2}[\dot{x}_q^2 - \dot{x}_q'^2 + \omega_q^2(x_q^2 - x_q'^2)] + [F_q(Q)x_q - F_q(Q')x_q']\right]$$

Bilinear coupling

$$F[q,q'] = \exp\left[-\frac{1}{\hbar}\int_{t_o}^{t} d\tau_1 \int_{t_o}^{\tau_1} d\tau_2 [q(\tau_1) - q'(\tau_2)][\mathcal{D}(\tau_1 - \tau_2)q(\tau_2) - \mathcal{D}^*((\tau_1 - \tau_2)q'(\tau_2)]\right]$$

Bath propagator

For spin baths it is more subtle:

$$\mathcal{F}[Q,Q'] = \prod_k^{N_s} \int \mathcal{D}\boldsymbol{\sigma}_k(\tau) \int \mathcal{D}\boldsymbol{\sigma}_k(\tau') \exp\left[\frac{i}{\hbar}(S_{int}[Q,\boldsymbol{\sigma}_k] - S_{int}[Q',\boldsymbol{\sigma}_k'] + S_E[\boldsymbol{\sigma}_k] - S_E[\boldsymbol{\sigma}_k'])\right]$$

$$S_{int}^{sp}(Q,\boldsymbol{\sigma}_k) = -\int d\tau \sum_k^{N_s} \boldsymbol{F}_k(P,Q)\cdot\boldsymbol{\sigma}_k$$

$$S_{env}^{sp} = \int d\tau \left[\sum_k^{N_s}(\mathcal{A}_k \cdot \frac{d\boldsymbol{\sigma}_k}{dt} - \mathbf{h}_k\cdot\boldsymbol{\sigma}_k) - \sum_{k,k'}^{N_s} V_{kk'}^{\alpha\beta}\sigma_k^\alpha\sigma_{k'}^\beta\right]$$

Vector coupling

Berry phase coupling

MECHANISMS of ENVIRONMENTAL DECOHERENCE: a SIMPLE PICTURE

Easiest to visualize this in path integral theory:

(1) OSCILLATOR BATH   Oscillator Lagrangian: $L_q(x_q, \dot{x}_q; t) = \frac{m_q \dot{x}_q^2}{2} - \Upsilon_q(t) x_q$

Each oscillator is subject to a force $\Upsilon_q(t) = m_q \omega_q^2 x_q - F_q(Q(t))$

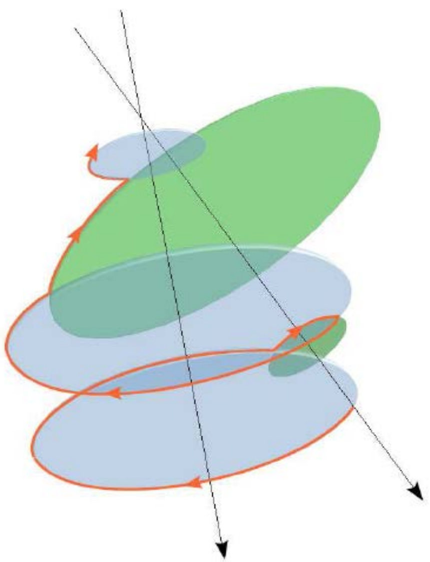
Problem exactly solvable (Feynman). Each oscillator very weakly coupled to system, & slowly entangles with it...weak oscillator excitation, DISSIPATION

(2) SPIN BATH   Each bath spin has the Lagrangian

$$L(\boldsymbol{\sigma}_k, \dot{\boldsymbol{\sigma}}_k; t) = \mathscr{A}_k \cdot \frac{d\boldsymbol{\sigma}_k}{d\tau} - \boldsymbol{\Upsilon}_k(t) \cdot \boldsymbol{\sigma}_k$$

with the force: $\boldsymbol{\Upsilon}_k(t) = \mathbf{h}_k + \mathbf{F}_k(t) + \xi_k(t)$

Entanglement with system via $F_k(P, Q)$.(not weak)
This problem is highly non-trivial (in general UNSOLVABLE even for spin-1/2 !).



Precessional path for bath spin

** Decoherence is precessional – NO DISSIPATION

Example:
Spin qubit

$$\hat{H}_{QB} = H_{QB}^0(\vec{\tau}) + \sum_k (\vec{\gamma}_k + \xi_k) \cdot \vec{\sigma}_k$$

field: $\gamma_k^\alpha = h_k^\alpha + \sum_\beta \omega_k^{\beta\alpha} \tau_\beta$

Enough now of generalities, bearing in mind that:

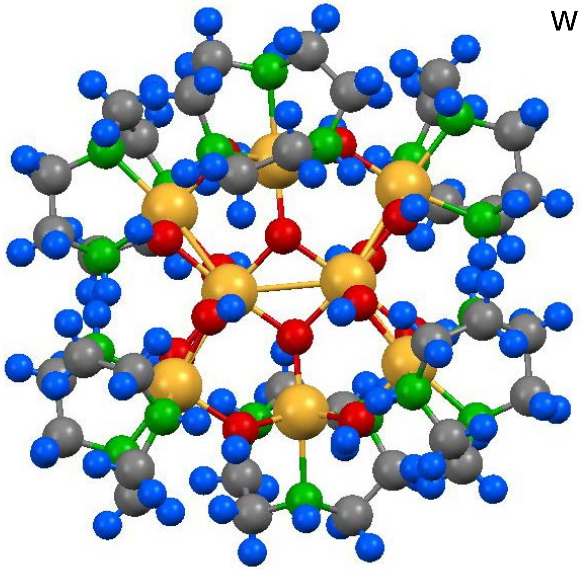
" *Only wimps specialize in the general case. Real scientists pursue examples.* "

MV Berry: Ann NY Acad Sci 755, 303 (1995)

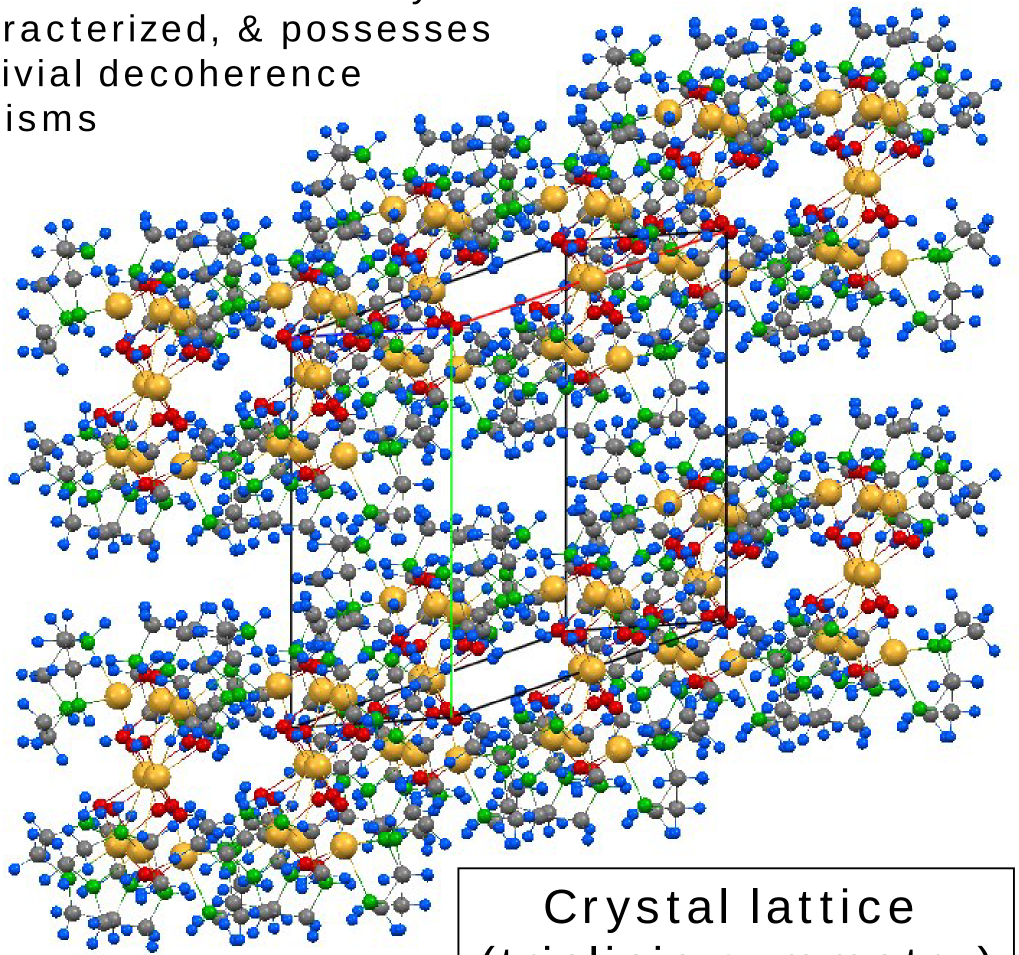
The Fe_8 MOLECULE: a TEST CASE for DECOHERENCE

"A theory is not a theory until it produces a number" R.P. Feynman (Lectures on Physics, 1965)

I now discuss a bona fide test for environmental decoherence theory which worked (many have NOT!). The system is the Fe_8 molecule – which is extremely well-characterized, & possesses 3 non-trivial decoherence mechanisms



Single molecule

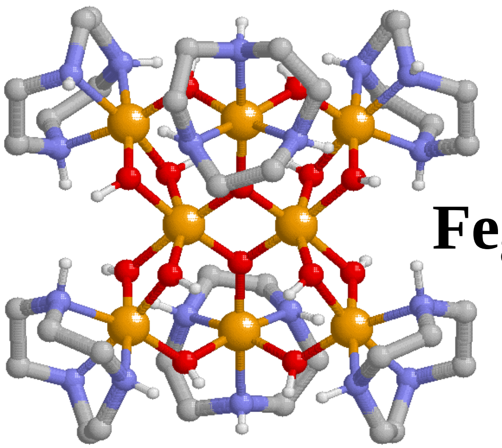


Crystal lattice (triclinic symmetry)

We have:

(i) long-range dipolar decoherence
(ii) Nuclear spin bath decoherence
(iii) Phonon oscillator bath decoherence

QUANTUM DYNAMICS of a single Fe-8 MOLECULE



Fe_8 $S = 10$

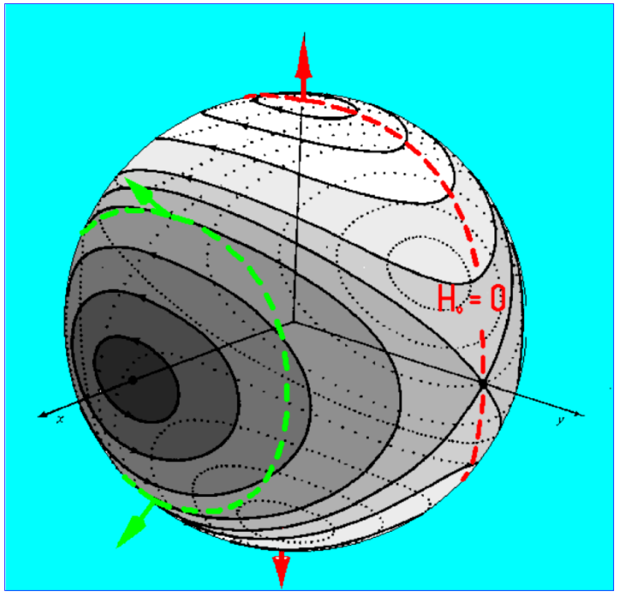
Low-T Quantum regime- effective Hamiltonian ($T < 0.36$ K):

$$\mathcal{H}_o(\hat{\tau}) = (\Delta_o \hat{\tau}_x + \epsilon_o \hat{\tau}_z)$$

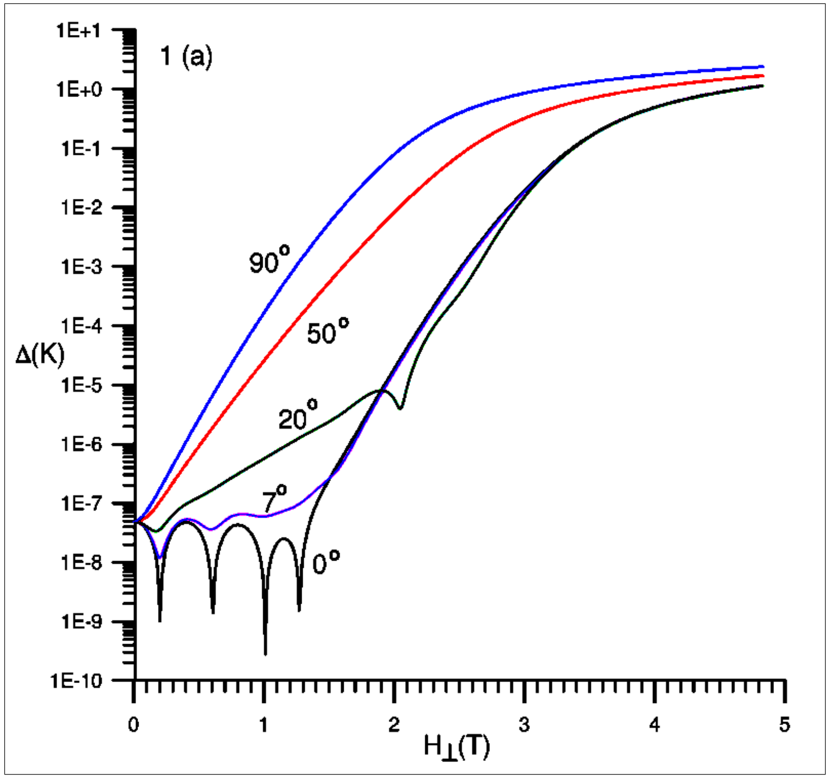
Longitudinal bias: $\epsilon_o = g\mu_B S_z H_o^z$

Eigenstates: $|\pm\rangle = [|\uparrow\rangle \pm |\downarrow\rangle]/\sqrt{2}$

This also defines orthonormal states: $|\uparrow\rangle, |\downarrow\rangle$



Feynman Paths on the spin sphere for a biaxial potential. Application of a field pulls the paths towards the field

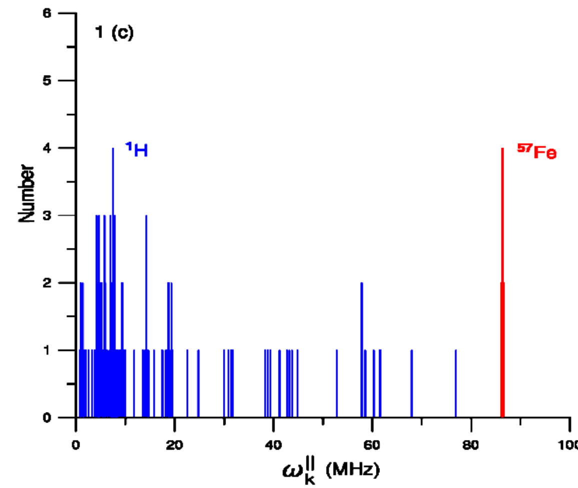


QUANTUM COHERENCE REGIME: *here quantitative predictions were made long before any experiments were done.*

DECOHERENCE IN Fe-8 SYSTEM

(A) Nuclear Spin Bath

Hyperfine couplings of all 213 nuclear spins are well known



$$H_{eff}^{CS} = [\Delta_o \hat{\tau}_+ e^{-i\sum_k \boldsymbol{\alpha}_k \cdot \boldsymbol{\sigma}_k} + H.c.] + \hat{\tau}^z(\epsilon_o + \sum_k \boldsymbol{\omega}_k \cdot \boldsymbol{\sigma}_k) + H_{env}^{sp}([\boldsymbol{\sigma}_k])$$

Nuclear spin decoherence rate

$$\gamma_\phi^{NS} = E_0^2/2\Delta_0^2 \quad \text{where} \quad E_o^2 = \sum_k \frac{I_k+1}{3I_k}(\omega_k^{\|} I_k)^2$$

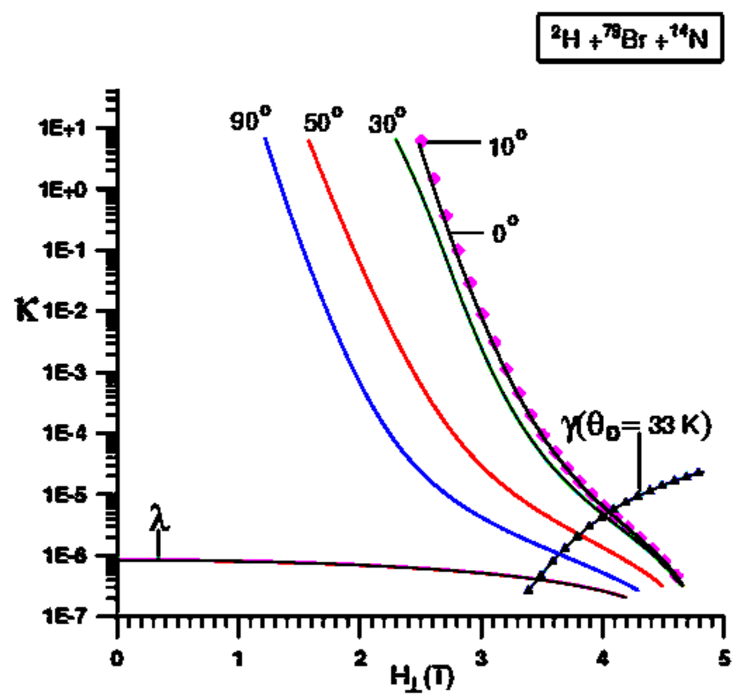
(b) Phonon Bath

Phonon spectrum and spin-phonon couplings are known. Phonon decoherence rate is:

$$\gamma_\phi^{ph} = \frac{\mathcal{M}_{\mathcal{AS}}^2 \Delta_0^2}{\pi \rho c_s^5 \hbar^3} \coth\left(\frac{\Delta_0}{k_B T}\right)$$

$$\mathcal{M}_{\mathcal{AS}}^2(H_y) \approx \tfrac{4}{3} D^2 |\langle \mathcal{A}|S_y S_z + S_z S_y|\mathcal{S}\rangle|^2$$

Total SINGLE QUBIT decoherence rate shown in Figure at right:



(c) Dipolar Decoherence

This is an example of "correlated errors" caused by inter-qubit interactions. It turns out to be very serious.

The high-T (van Vleck) limiting form is

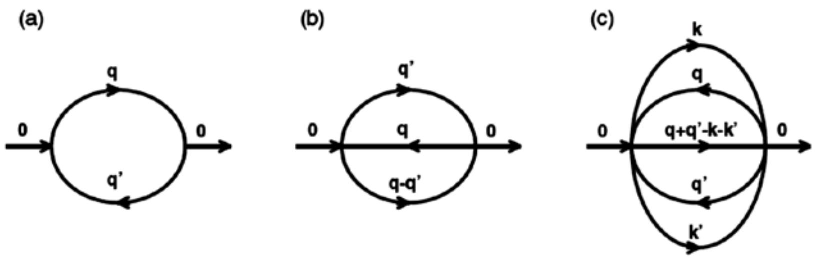
$$(\gamma_\phi^{\mathrm{vV}})^2 \approx \left[1 - \tanh^2\left(\frac{\Delta_0}{k_B T}\right)\right]\sum_{i\neq j}\left(\frac{\mathcal{A}_{yy}^{ij}}{\Delta_0}\right)^2,$$

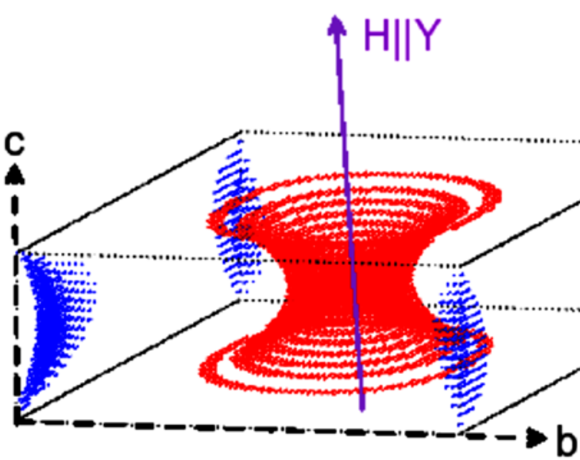
$$\mathcal{A}_{yy}^{ij} = \frac{U_d}{(2g_e S)^2}[(2\tilde{g}_y^2 + \tilde{g}_z^2)\mathcal{R}_{yy}^{ij} - (\tilde{g}_x^2 - \tilde{g}_z^2)\mathcal{R}_{xx}^{ij}],$$

$$\mathcal{R}_{\mu\nu}^{ij} = \mathcal{V}_c(|\mathbf{r}^{ij}|^2\delta_{\mu\nu} - 3r_\mu^{ij}r_\nu^{ij})/|\mathbf{r}^{ij}|^5$$

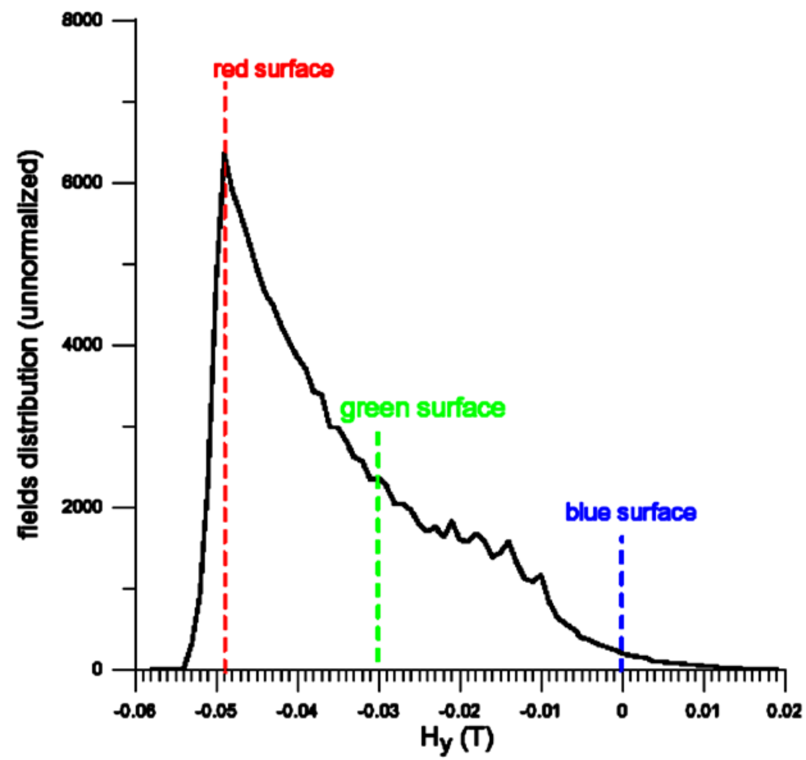
At low T one gets a quite different form

$$\gamma_\phi^m = \frac{2\pi}{\hbar\Lambda_c}\sum|\Gamma_{\mathbf{qq}'}^{(4)}|^2\mathcal{F}[\bar{n}_\mathbf{q}]\delta(\omega_0 + \omega_\mathbf{q} - \omega_{\mathbf{q}'} - \omega_{\mathbf{q}-\mathbf{q}'}).$$





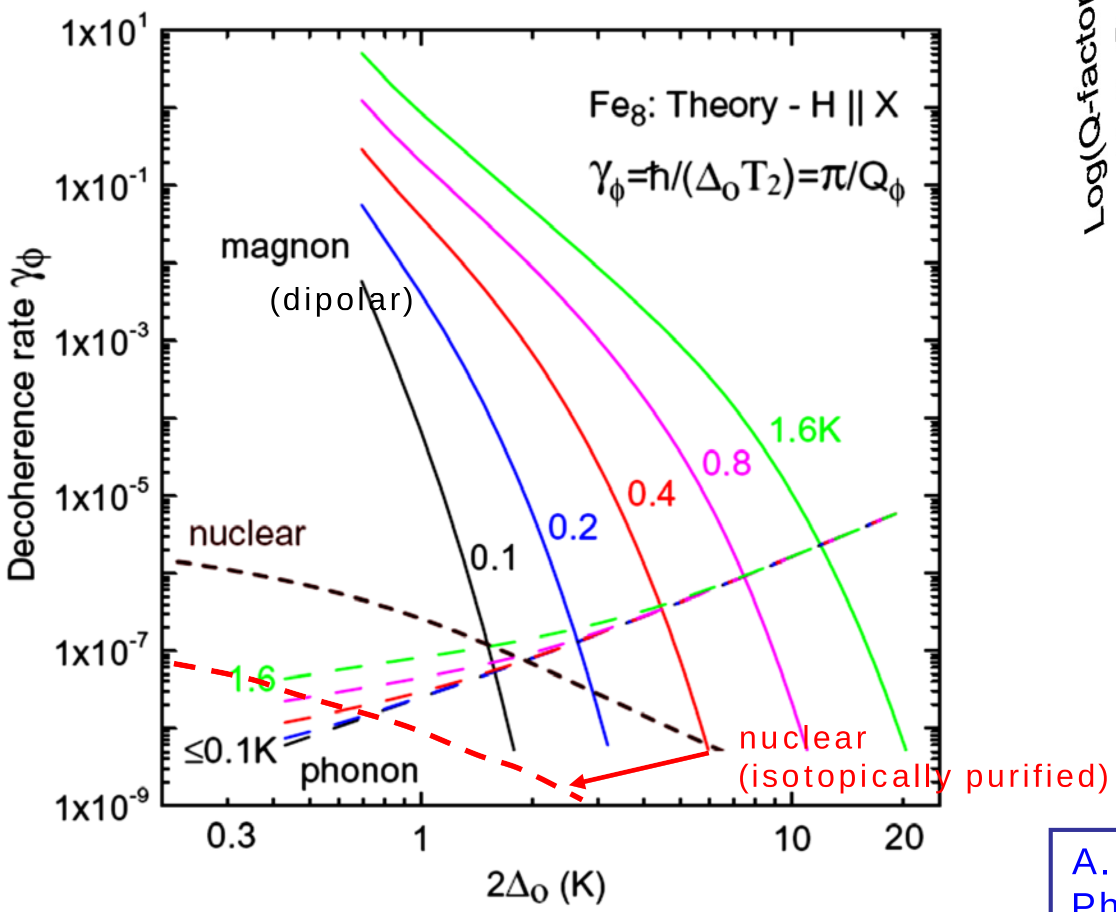
RESONANT SURFACES

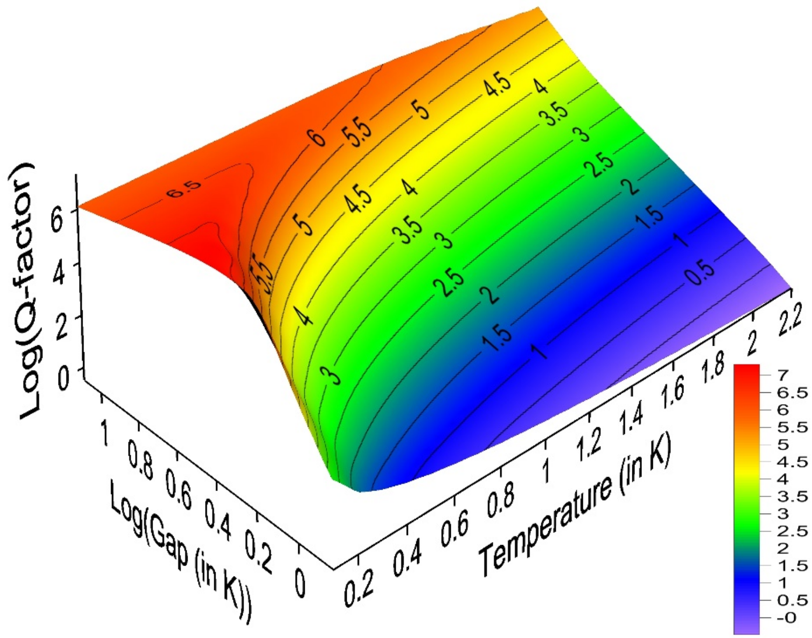


EXPERIMENTAL PREDICTIONS: the Fe-8 SYSTEM

NB: In any experimental test, we want to be able to vary different mechanisms INDEPENDENTLY

Suppose we now add all three forms of decoherence together; then we get the PREDICTIONS shown in Figs. below & at right:



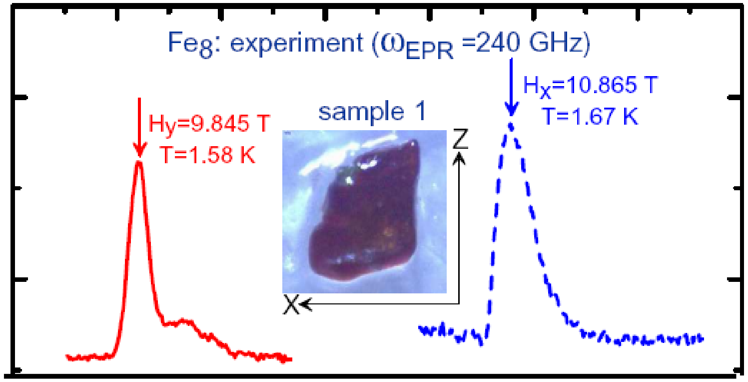


Decoherence Q-factor in Fe$_8$ crystal

Note the way in which these results allow us to optimize the design

A. Morello, P.C.E. Stamp, I.S. Tupitsyn, Phys Rev Lett 97, 207206 (2006)

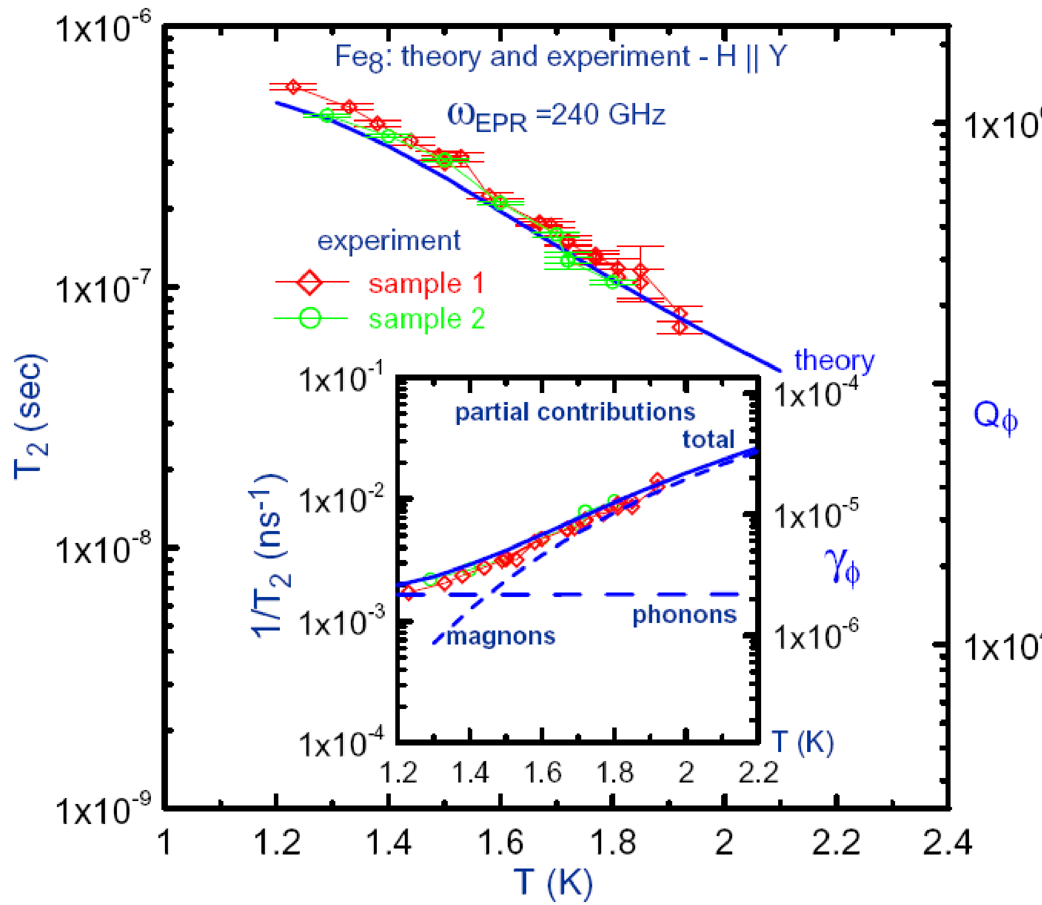
EXPERIMENTAL TEST: Fe_8



Using 'Hahn echo' ESR experiments, get good agreement with theory; no evidence for extrinsic decoherence sources.

S. Takahashi + al., Nature 476, 76 (2011)

Used 2 different crystals, and 2 field orientations



SOME FIRSTS
in this EXPERIMENT

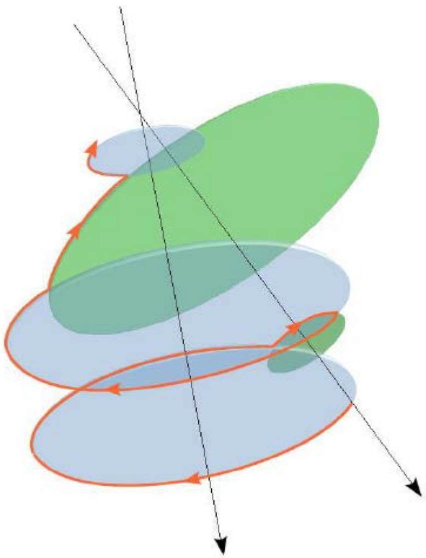
1. First detection of macroscopic spin precession of qubits
2. Lowest decoherence rate ever seen in molecular spin qubits.
3. First measurement of dipole decoherence in qubit array
4. First controlled measurement of decoherence rates from spin bath, oscillator bath, and dipolar interactions (with agreement with theory)

WHAT DO ENVIRONMENTAL DECOHERENCE EXPTS TEST?

1. Our understanding of many-particle Quantum Mechanics with interactions (the form of the effective Hamiltonians, techniques for calculating answers to physical questions.

2. Our understanding of decoherence mechanisms.

Notice that experiments like those described above confirm that <u>decoherence can occur without dissipation.</u>



PART 3

INTRINSIC DECOHERENCE

INTRINSIC DECOHERENCE: a THEORETICAL FORMULATION

There have been many suggestions for corrections to QM (Milburn, GRW, Pearle, Diosi, 't Hooft, Penrose, Weinberg, etc.).

Here we examine another kind of theory:

PCE Stamp, Phil Trans Roy Soc A370, 4429 (2012)

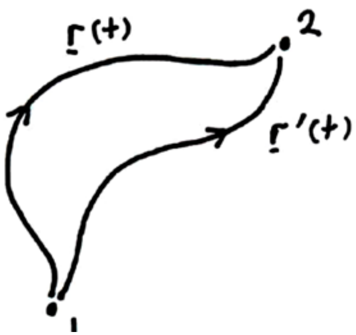
In standard QM: $G_o(\mathbf{r},\mathbf{r}';t,t') = \int_{\mathbf{r}'}^{\mathbf{r}} \mathcal{D}\mathbf{x}(\tau) \exp \frac{\mathrm{i}}{\hbar} \int_{t'}^{t} \mathrm{d}\tau L(\mathbf{x},\dot{\mathbf{x}};\tau)$ (path integral)

Let's now modify QM, as follows; let $\mathcal{G}(R,R') = G_o(R,R') + \Delta\mathcal{G}(R,R')$

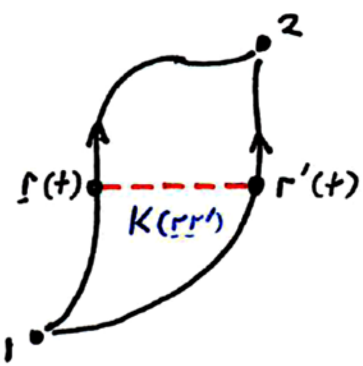
where

$$\Delta\mathcal{G}(\mathbf{r},\mathbf{r}';t,t') = \int_{\mathbf{r}'}^{\mathbf{r}} \mathcal{D}\mathbf{x}_1(\tau) \int_{\mathbf{r}'}^{\mathbf{r}} \mathcal{D}\mathbf{x}_2(\tau) \kappa[\mathbf{x}_1,\mathbf{x}_2] \exp \frac{\mathrm{i}}{2\hbar} \int_{t'}^{t} \mathrm{d}\tau [L(\mathbf{x}_1,\dot{\mathbf{x}}_1;\tau) + L(\mathbf{x}_2,\dot{\mathbf{x}}_2;\tau)]$$

The picture is then:



Standard QM



Modification to QM

This is viewed as merely the 2nd term in an infinite series. If the correction to QM is weak, then we can stop here (however for strong field gravitational decoherence this will not be enough).

This term merely renormalizes wave-functions & propagators.
Thus, for a free particle we have:

$$\Delta\mathcal{G}(X,X') \propto \int \mathcal{D}\mathbf{x}_1(\tau)\int \mathcal{D}\mathbf{x}_2(\tau)\,\kappa[\mathbf{x}_1,\mathbf{x}_2]\exp\frac{i}{2\hbar}\int d\tau\frac{m}{2}(\dot{\mathbf{x}}_1^2+\dot{\mathbf{x}}_2^2)$$

$$\propto \mathcal{A}(0,0;t,t')G_o(X,X')$$

However in a theory of this kind, the wave-function does not give us a direct description of the QM world. What we really want to know is how physical quantities evolve.

Let's write the time evolution of the probability density function as

$$\rho(2) = \int d1\mathcal{K}(2,1)\rho(1)$$

For the "density matrix propagator", we now have

$$\mathcal{K}(X,Y;X'Y') = \bar{K}(X,Y;X'Y') + \Delta\mathcal{K}(X,Y;X'Y')$$

and this causes intrinsic decoherence; in particular, there is a term:

$$\Delta\mathcal{K}(X,Y;X'Y') \sim \int_{\mathbf{X}'}^{\mathbf{X}}\mathcal{D}\mathbf{x}(\tau)\int_{\mathbf{Y}'}^{\mathbf{Y}}\mathcal{D}\mathbf{y}(\tau)\kappa[\mathbf{x},\mathbf{y}]\exp\frac{\mathbf{i}}{\hbar}\int_{t'}^{t}\mathrm{d}\tau[L(\mathbf{x},\dot{\mathbf{x}};\tau)-L(\mathbf{y},\mathbf{y};\tau)]$$

SLOW & FAST VARIABLES in this THEORY

To have a consistent framework, we need to be able to systematically integrate out high energy variables, and produce an unambiguous low-energy theory. Let's do this non-relativistically. Define

$$G_o(2,1) = \int_1^2 \mathcal{D}\mathbf{R}\; e^{\frac{i}{\hbar}\int dt L_o(\mathbf{R},t)} G_o^f(2,1)$$

such that $G_o^f(2,1) \;=\; G_o^f(\{x_k^{(2)}, x_k^{(1)}\}; t_2, t_1 | [\mathbf{R}(t)])$ defines the fast variables propagator in ordinary QM.

In terms of the eigenfunctions of the bare Lagrangian, the new effective Hamiltonian of the low-energy variables becomes:

$$L_o(\mathbf{R}) - \epsilon_n(\mathbf{R}) - i\hbar\dot{\mathbf{R}} \cdot \langle n | \nabla_{\mathbf{R}} m \rangle$$

We can now write the correction to the density matrix of the system in the form

$$\Delta\mathcal{K}_{nm}(2,1) \;=\; \int_1^2 \mathcal{D}\mathbf{R} \int_1^2 \mathcal{D}\mathbf{R}' e^{i\Phi_{nm}[\mathbf{R},\mathbf{R}']} e^{\frac{i}{2\hbar}(L_o^{nm}(\mathbf{R}) + L_o^{nm}(\mathbf{R}))}$$

in which the phase of the correlator becomes

$$\Phi_{nm}[\mathbf{R},\mathbf{R}'] = \int \mathrm{d}t \dot{\mathbf{R}} \cdot \langle n(\mathbf{R}) | \nabla_{\mathbf{R}} | \alpha(\mathbf{R}) \rangle \chi_{\alpha\beta}[\mathbf{R},\mathbf{R}'] \langle \alpha(\mathbf{R}') | \nabla'_{\mathbf{R}} | m(\mathbf{R}') \rangle \cdot \dot{\mathbf{R}}'$$

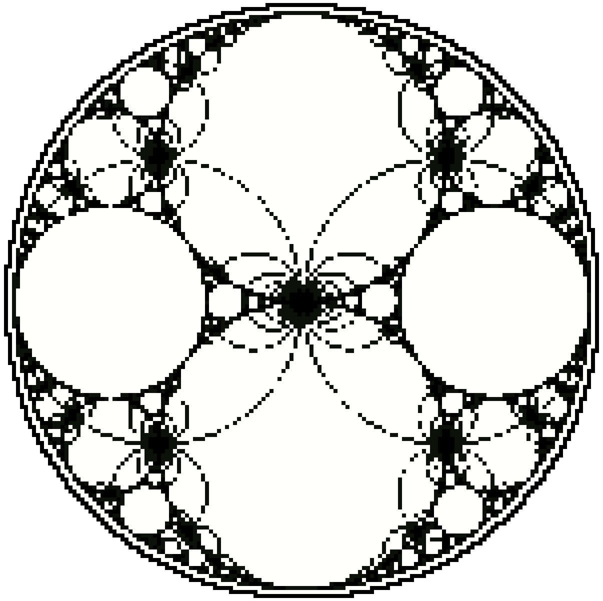
and where we have defined $\chi_{\alpha\beta}[\mathbf{R},\mathbf{R}'] = |\alpha(\mathbf{R})\rangle \kappa[\mathbf{R},\mathbf{R}'] \langle \beta(\mathbf{R}')|$

The standard Born-Oppenheimer approximation then consists in taking the diagonal elements of this. All this is easily generalized to a covariant relativistic form

So, all we need now is a physical mechanism...

PART 4

GRAVITATIONAL DISENTANGLEMENT



BASIC IDEA: GRAVITY MODIFIES QUANTUM MECHANICS

Suppose, in contrast to ideas in string theory &/or quantum gravity, we adopt the view that it is QM itself that has to be modified. Gravity will still be subject to QM (by the arguments given before), but we want to solve the problems by a modification of QM itself. We are strongly influenced here by the following argument; not only is GR very successful in explaining astrophysical phenomena, but also:

> The ***general theory of relativity*** was established by Einstein (and finally formulated by him in 1916), and represents probably the most beautiful of all existing physical theories.
>
> L.D. Landau, E.M. Lifshitz "The Classical Theory of Fields", sec.82

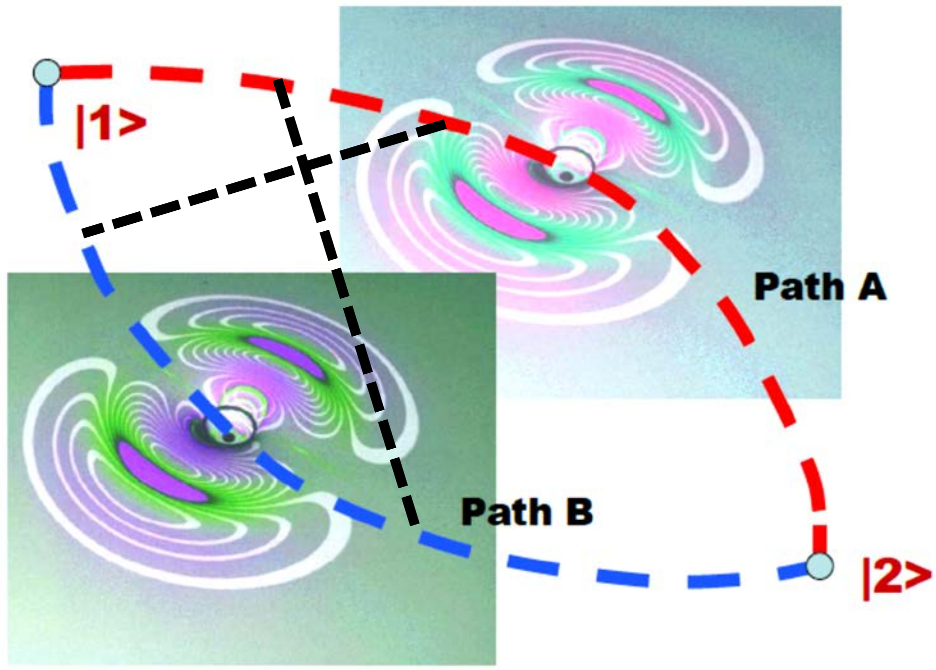
We therefore adopt the view that it is gravity itself that causes the breakdown of QM.

The formal theory for this idea is to be found in

PCE Stamp, Phil Trans Roy Soc A370, 4429 (2012) <basic idea>
PCE Stamp, to be published <formal theory>
F Suzuki, PCE Stamp, to be published <model calculations>

GRAVITATIONAL DECOHERENCE: the BASIC IDEA

We assume that we must begin by summing over amplitudes for different paths, with their attendant spacetime geometries.



However now these paths <u>COMMUNICATE</u> with each other.

This is not a communication between 'many universes', but rather unites all branches into one universe.

We will stick with the standard format for GR, with basic objects:

(i) <u>SPACETIME CURVATURE</u>: described by the Riemann tensor $\mathfrak{R}(x)$ dividing into 2 pieces, the traceless Weyl tensor $C(x)$, and the Ricci tensor $R(x)$

(ii) <u>MATTER</u>: described by the 'energy-momentum' tensor $T(x)$.

Thus we assume an action: $S = S_g + S_M$

with $S_g = \int d^4x \, g^{1/2}(x) g^{\mu\nu}(x) R_{\mu\nu}(x)$ and $T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}_M}{\delta g^{\mu\nu}}$,

FORM of DECOHERENCE CORRELATOR

We now assume that the connection between the different branches of the propagator is given by the gravitational field itself.

In the weak-field limit this apparently gives the correction to the propagator of an object as

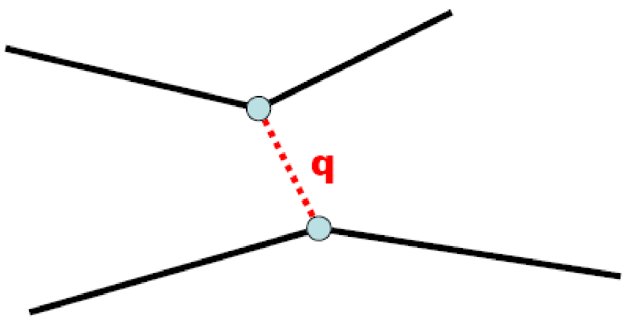
$$\Delta\mathcal{G}(x_2, x_1) \;=\; \int \mathcal{D}x \int \mathcal{D}x' \;\kappa[x, x'] \exp[\frac{i}{2\hbar}(S_o[x] + S_o[x'])]$$

where $S_o[x]$ is the action of the object concerned (matter, photons, etc.), and the correlator is now:

$$\kappa[x, x'] \;=\; \exp\left[\frac{i\lambda^2}{2\hbar}\int d^4x \int d^4x' \; T^{\mu\nu}(x)\mathcal{D}^o_{\mu\nu\lambda\rho}(x - x')T^{\lambda\rho}(x')\right] - 1$$

in which we have a graviton propagator given in momentum space by:

$$\mathcal{D}^o_{\mu\nu\lambda\rho}(q) \;=\; \frac{1}{2q^2}[\eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda} - \eta_{\mu\nu}\eta_{\lambda\rho}]$$



We discuss later how inevitable this form is in the full theory

NEWTONIAN LIMIT – PARTICLE PROPAGATION

We can take the limit of non-relativistic velocities in the previous formulas. Then we get a simple result – the correlator correction for a single particle of mass m becomes:

$$\kappa[\mathbf{r},\mathbf{r}'] = \exp \int^{t} \mathrm{d}\tau \frac{4\pi \mathrm{i} G m^2}{|\mathbf{r}(\tau) - \mathbf{r}'(\tau)|} - 1$$

Consider first the effect on the particle propagator – we have a correction

$$\Delta\mathcal{G}(X,X') \propto \int \mathcal{D}\mathbf{x}_1(\tau) \int \mathcal{D}\mathbf{x}_2(\tau) \kappa[\mathbf{x}_1,\mathbf{x}_2] \exp \frac{\mathrm{i}}{2\hbar} \int \mathrm{d}\tau \frac{m}{2}(\dot{\mathbf{x}}_1^2 + \dot{\mathbf{x}}_2^2)$$

But this is completely benign – it renormalizes the propagator to

$$\Delta\mathcal{G}(X,X') \propto \mathcal{A}(0,0;t,t')\, G_o(X,X')$$

Where the multiplicative term is just the 'return' propagator for a particle of charge m moving in a 'Coulomb field' of strength G.

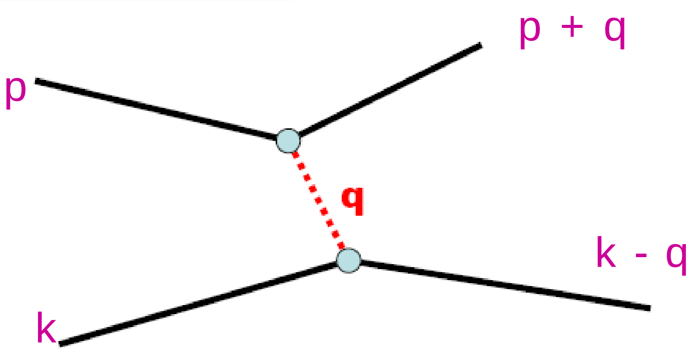
However, the density matrix for the system is not so simple – it contains a term which mimics decoherence, of form

$$\Delta\mathcal{K}(X,Y;X'Y') \sim \int_{\mathbf{X}'}^{\mathbf{X}} \mathcal{D}\mathbf{x}(\tau) \int_{\mathbf{Y}'}^{\mathbf{Y}} \mathcal{D}\mathbf{y}(\tau) \left[\exp \int \mathrm{d}\tau \frac{8 \mathrm{i} \pi G m^2}{|\mathbf{x}(\tau) - \mathbf{y}(\tau)|} - 1 \right] \exp \frac{\mathrm{i}}{2\hbar} \int \mathrm{d}\tau \frac{m}{2}(\dot{\mathbf{x}}^2 - \dot{\mathbf{y}}^2)$$

The key point to take from this result is that decoherence is always appearing directly in the phase

PHOTON PROPAGATION – a SURPRISING RESULT

Suppose we calculate the correlator for a photon., for which as usual



$$S_{EM} = -\frac{1}{4\mu_o} \int d^4x \; g^{1/2}(x) F_{\mu\nu}(x) F^{\mu\nu}(x)$$

The result is surprising. For the correlator itself we find

$$\kappa[x, x'] \;=\; \exp\left[\frac{i\lambda^2}{2\hbar} \sum_{k,p} \sum_q e^{iq(x-x')} [p^2 + k^2 + q_\mu (p^\mu - k^\mu)]\right]$$

and when this is substituted into our usual expression for the correction to the photon propagator, we find, up to higher quantum corrections, that

$$\Delta \mathcal{G} = 0 \quad !!$$

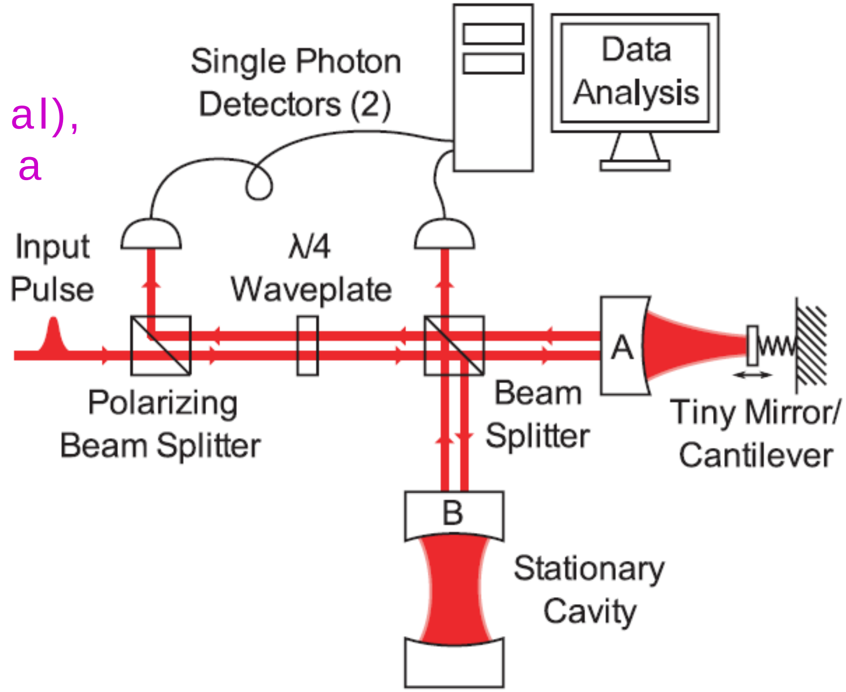
Likewise for the corrections to the density matrix. This result turns out to be related to a standard calculation in classical GR, by Tolman et al. (1931).

This is a remarkable result – to lowest approximation, the gravitational decoherence mechanism has no effect on photons at all.

There are inevitably corrections from higher-order quantum fluctuations to this result. It will be extremely interesting to bring such calculations into contact with observations of long-range photon propagation.

EXAMPLE of an EXPERIMENT

This planned experiment (Bouwmeester et al), has a system in which we have a photon in a superposition of cavity A and cavity B states, with an entanglement to a cantilever vibrational mode C, via the small mirror M on C. The Hamiltonian is taken to be



$$H = \hbar\omega_a\left[a^\dagger a + b^\dagger b\right] + \hbar\omega_c\left[c^\dagger c - \kappa a^\dagger a\left(c + c^\dagger\right)\right]$$

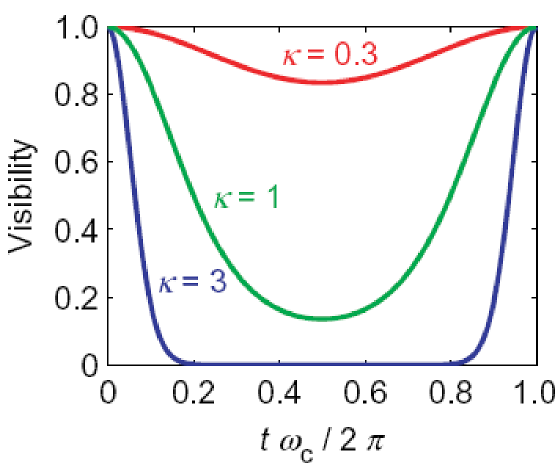
where $\kappa = \dfrac{\omega_a}{L\omega_c}\sqrt{\dfrac{\hbar}{2m\omega_c}} = \dfrac{\sqrt{2}Nx_0}{\lambda}$

Then if at $t = 0$ we are in the state $|\psi(0)\rangle = (1/\sqrt{2})(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B)|0\rangle_m$

the system evolves to time t to the state:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}e^{-i\omega_c t}[|0\rangle_A|1\rangle_B|0\rangle_C + e^{i\kappa^2(\omega_m t - \sin\omega_m t)}|1\rangle_A|0\rangle_B\ |\kappa(1 - e^{-i\omega_m t})\rangle_m]_C$$

with off-diagonal matrix element $v(t) = \mathrm{e}^{-\kappa^2(1-\cos(\omega_c t))}$



D Kleckner et al., N J Phys 10, 095020 (2008)

See also

I Pikowski et al., Nat Phys 8, 393 (2012)

SPECULATIONS on the FORM of a FULL THEORY

This is a really tough problem. In keeping with the informal spirit of this meeting, let me discuss briefly a few ideas on this. We want a form for $\mathcal{K}[x,x'] \;=\; e^{i\Phi[x,x']} - 1$

(1) First idea – try: $$\Phi[x,x'] \;=\; \frac{-1}{16\pi G}\int d^4x \int d^4x' \frac{1}{4\pi^2}\frac{g^{1/2}(x)g^{1/2}(x')}{\bar{\mathcal{D}}(x,x')[\Lambda^2 - 4\pi^2\mathcal{D}(x,x')]}$$
$$\times\,\{\Delta(x,x')C_{\mu\nu\alpha\beta}(x)C^{\mu\nu\alpha\beta}(x') + \Gamma(x,x')R(x)R(x')\}$$

This is no good – but we learn that we need to focus on the Weyl term.

(2) Try the following: $$\Phi[x,x'] = \int d^4x \int d^4x' J_{\alpha\mu\nu}(x)\mathcal{M}^{\alpha\mu\nu\beta\lambda\rho}(x-x')J_{\beta\lambda\rho}(x')$$

where we have defined the correlator of the Lanczos potential:

$$\mathcal{M}^{\alpha\mu\nu\beta\lambda\rho}(x-x') \;=\; \langle\Lambda_{\alpha\mu\nu}(x)\Lambda_{\beta\lambda\rho}(x')\rangle$$

which is coupled covariantly to the Schoutens-Cotton tensor:

$$J_{\alpha\beta\mu} = \nabla_\beta R_{\mu\alpha} - \nabla_\alpha R_{\mu\beta} \;+\; \frac{1}{6}(g_{\mu\beta}\nabla_\alpha R - g_{\mu\alpha}\nabla_\beta R)$$

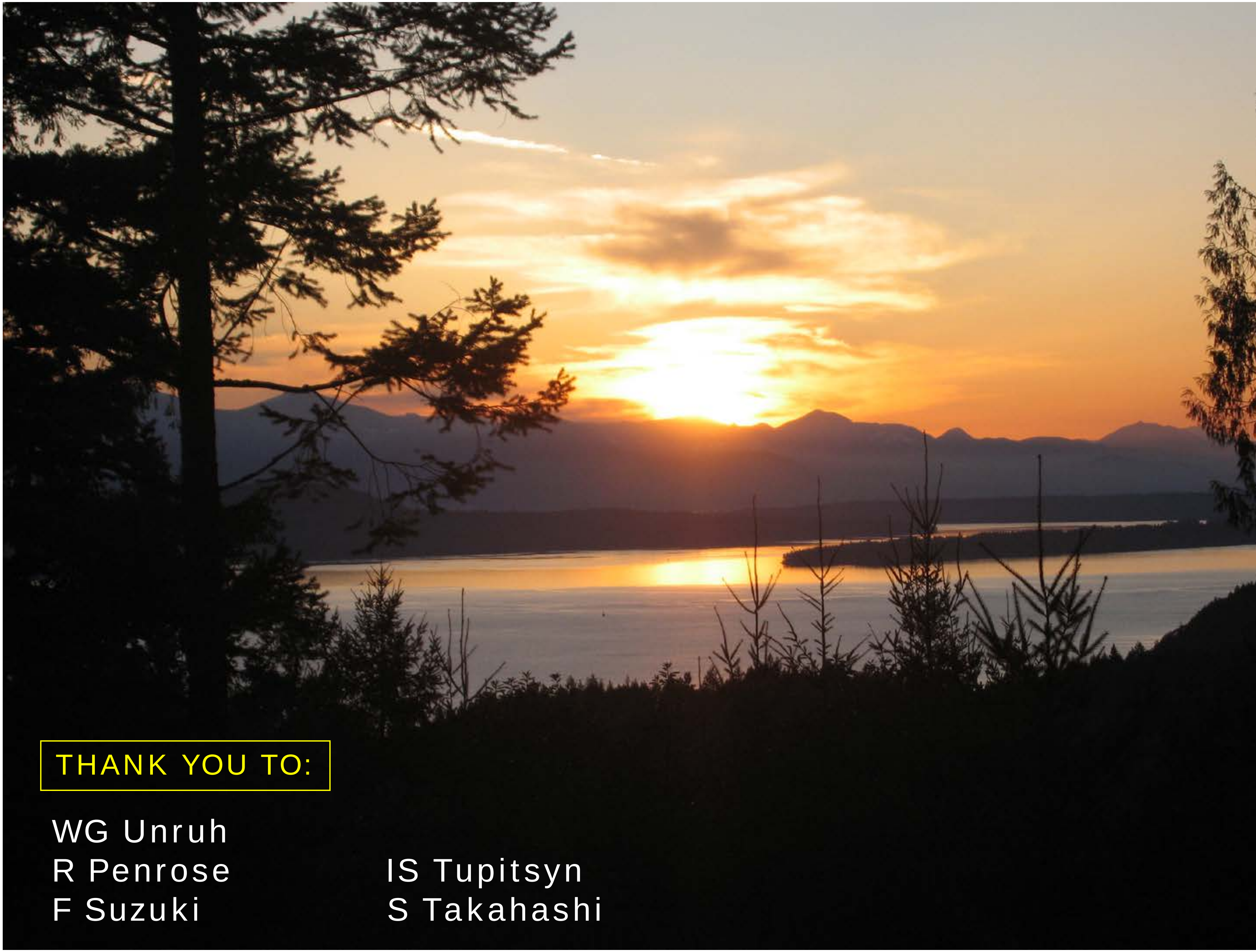
Still checking this one out.

Thus we do not have the full strong field form

So – no dessert just yet.....

This is still to come.....


ONGOING WORK
- Full calculation,
for massive superpositions
- Strong field theory ?


THANK YOU TO:
WG Unruh
R Penrose
F Suzuki
IS Tupitsyn
S Takahashi