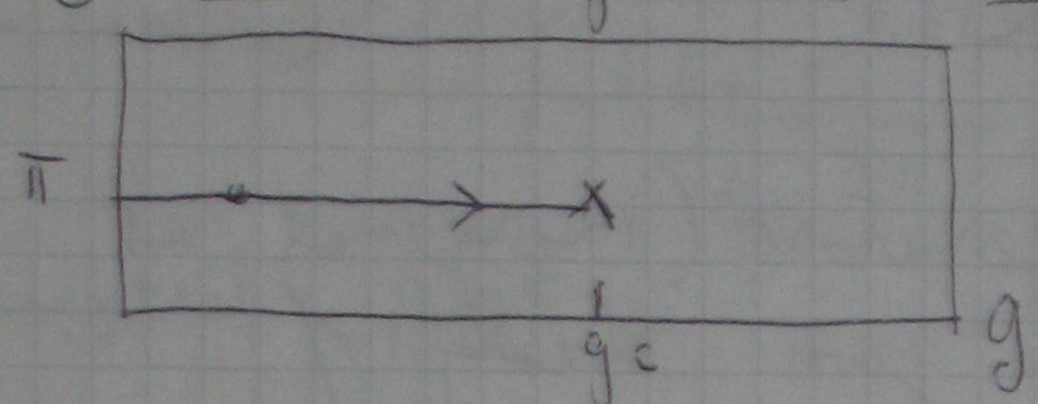
Les Houches Lecture 3

$\frac{1}{2}$ - Integer - S Chains



- $S = \frac{1}{2}$ case easiest to understand
- its extreme low energy behavior (at $\omega \to 0$) is believed to be universal to all $\frac{1}{2}$-integer S
- now get gapless excitations like in S.W.T. but no broken symmetry in ground state and power-law correlations

$$\langle \vec{S}_i \cdot \vec{S}_j \rangle \sim \frac{(-1)^{i-j}}{|i-j|^{\eta}}$$

true more generally for XXZ case:

$$H = J \sum_j \left[\tfrac{1}{2}\left(S_j^+ S_{j+1}^- + h.c. \right) + \Delta\, S_j^z S_{j+1}^z \right]$$

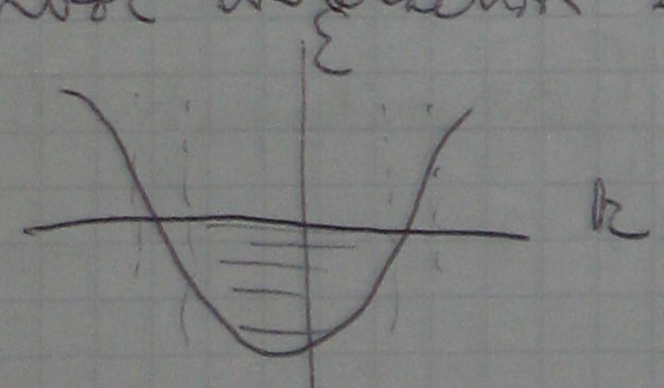
η (and velocity v) depend on Δ

- can proceed by an approach based on small Δ which can be extrapolated to $\Delta \to 1$.

JW transformation:

$$S_j^z = \psi_j^+ \psi_j - \tfrac{1}{2}, \quad S_j^+ = \psi_j^+ e^{i\pi \sum_{k=1}^{j-1} n_k}$$
$$= n_j - \tfrac{1}{2}$$

- gives spinless fermion model with nearest neighbor interaction $\Delta n_j n_{j+1}$



$\frac{1}{2}$-filling at zero field

$k_F = \pi/2$

- low energy theory derived by linearizing dispersion relation around $\pm k_F$
$\Rightarrow$ Lorentz invariant Q.F.T.
- now bosonize
- up to irrelevant operators (marginal at SU(2) point, $\Delta = 1$) this gives theory of non-interacting massless boson

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \varphi)^2$$ - only 1 component!

$$S^z_{(x)} = \frac{1}{2\pi R} \frac{\partial \varphi}{\partial x} + \text{const } (-1)^x \cos \varphi / R$$

$$S^-_{(x)} \sim \text{const } e^{i 2\pi R \tilde{\varphi}} \left[(-1)^x + \text{const } \cos \varphi / R \right]$$

$$R = R(\Delta) = \frac{1}{\sqrt{2\pi}} \quad \text{at } \Delta = 1 \Rightarrow SU(2) \text{ symmetry}$$

here $\varphi = \varphi_L(t+x) + \varphi_R(t-x)$, $\tilde{\varphi} = \varphi_L - \varphi_R$

- apologies for <u>many</u> different notations

from free boson Green's function

$$\langle \varphi_{L/R}(t \pm x)\, \varphi_{L/R}(0) \rangle = -\frac{1}{4\pi} \ln (t \pm x)$$

we get Green's functions of spin operators

- $$\langle S^z_x S^z_0 \rangle \sim \left[\frac{1}{(t-x)^2} + \frac{1}{(t+x)^2} \right] + \frac{(-1)^x}{(x^2 - t^2)^{\frac{1}{4\pi R^2}}}$$

$\langle S^+ S^- \rangle$ ~~look~~ is quite different except at $\Delta = 1$

- for uniform part $e^{i 2\pi R \tilde{\varphi} \pm i \varphi / R} \sim e^{i\left(2\pi R \pm \frac{1}{R}\right)\varphi_L - i\left(2\pi R \mp \frac{1}{R}\right)\varphi_R}$

<u>not</u> a sum of chiral terms (depending on $t \pm x$ only) except at $R = \frac{1}{\sqrt{2\pi}}$ $(\Delta = 1)$

(3)

$t \to t - i\epsilon$, Fourier transform

- factorises $\int \frac{dt_+ \, e^{it_+(\omega-k)/2}}{t_+^{\,n_+}}$ and $(+ \to -)$

staggered part $\sim \frac{\theta(\omega-|k|)}{(\omega^2-k^2)^{1-\frac{1}{4\pi R^2}}} \sim \frac{1}{\sqrt{\omega^2-k^2}}$

$(\Delta=1)$

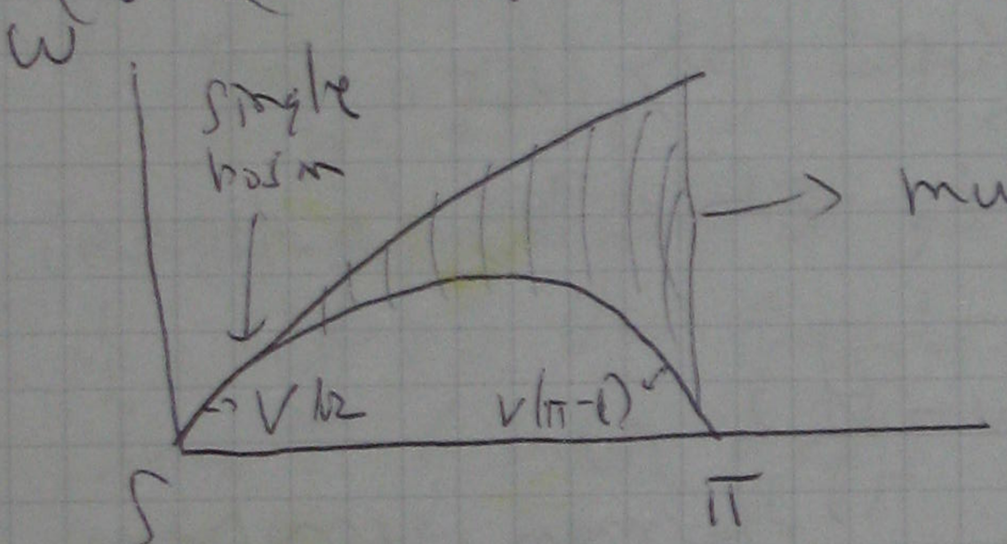
uniform part $\int dt_- \, e^{it_-\left(\frac{\omega+k}{2}\right)} \int \frac{dt_+}{t_+^2} e^{it_-(\omega-k)k}$

$\propto (\omega-k)\,\delta(\omega+k)$

$\sim |k|\,\delta(\omega-|k|)$

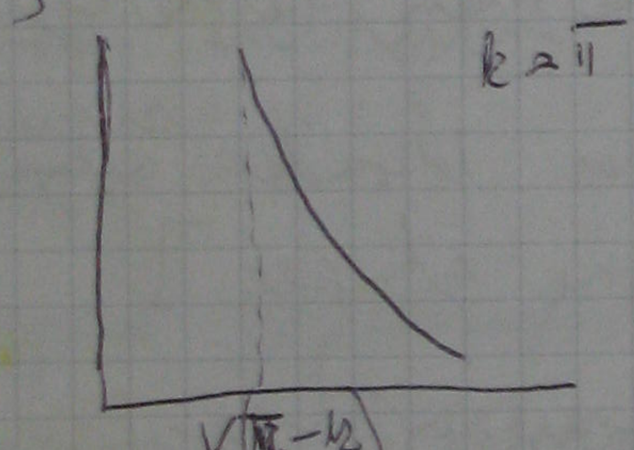
for purely chiral case $-\langle S^z S^z \rangle$ for all Δ

(and also $\langle S^+ S^- \rangle$ at $\Delta=1$)



$\to$ multiple bosons $e^{i\frac{\varphi}{R}} \simeq \sum \left(\frac{i\varphi}{R}\right)^n \frac{1}{n!}$

n boson continua all start at $\omega = V(\pi-k)$



at finite T $\quad (t \pm x) \to \frac{1}{\pi T} \sin \pi T (t \pm x)$

Green's functions can be expressed as Euler beta-functions (Schulz)

- still get a δ-fn in uniform part due to chiral structure

<u>Electron Spin Resonance</u> [see talk by D Zakharov]

- add uniform field in Z-direction

$$H \sum_i S^z_i \to \frac{H}{2\pi R} \int dx \frac{\partial \varphi}{\partial x}$$

$$\varphi \to \varphi + \cancel{2\pi R} \frac{Hx}{2\pi R} \quad \text{eliminates it}$$

$$S^z_x \to \frac{H}{(2\pi)^2 R^2} + \frac{1}{2\pi R}\frac{\partial \varphi}{\partial x}$$

$$+ \cancel{(-1)^x} \cos\left[(\pi + 2\pi m)x + \frac{\varphi}{R}\right] \qquad \left[m = \frac{H}{(2\pi)^2 R^2}\right]$$

$k_c = k \pm 2\pi m$ (shift of fermi surface)

- likewise $S^-_x \sim e^{-i 2\pi R \tilde{\varphi}} \left[e^{i 2\pi m x} e^{i\varphi/R} + h.c. \} + \text{const } (-1)^x \right]$

- wave-vector of <u>uniform</u> part shifts

- absorption intensity $\propto \langle S^- S^+ \rangle (k=0, \omega)$

$$\sim \langle e^{+i 2\pi R \tilde{\varphi} \pm i\varphi/R} \; e^{-i 2\pi R\tilde{\varphi} \mp i\varphi/R} \rangle (k=H, \omega)$$

- isotropic case $\propto H \delta(\omega - H)$

$\hookrightarrow$ Zeeman energy

various types of small anisotropies broaden ~~single~~ signal [and shift it and give various line shapes]

- a simple example is symmetric exchange (or dipole) anisotropy

$$\delta H = \delta J \sum_i S^z_i S^z_{i+1} \quad [\text{or longer range}]$$

- adds a term $\propto \delta J \left(\frac{\partial \varphi}{\partial x}\right)^2$ to H in field theory

shifts R away from SU(2) value by $O\left(\frac{\delta J}{J}\right)$

$\sim O(1)$

now $\langle S^+S^-\rangle\ (k=H,\omega)$ is no longer a δ-function
- finite width

$$\langle S^+S^-\rangle \sim i\,\xi\,\Gamma\left(\tfrac{i}{4}\lambda^2 + \frac{i(\omega-H)}{2\pi T}\right) \cancel{\Gamma(\ldots)} \Big/ \Gamma\left(i(\omega+H)/2\pi T\right)$$

from Schulz $\sim \dfrac{2H}{4\pi T}\,\dfrac{\cancel{\ldots}}{i\lambda^2 + \xi\frac{(\omega-H)}{2\pi T}} \times \ldots$

~~$\mathrm{Im}\langle S^+S^-\rangle \sim$~~ $(H \ll T)$

$\Rightarrow$ Lorentzian line shape, width $\propto T\left(\frac{\delta J}{J}\right)^2$
(for $H \ll T$)

- scaling with T follows from simple scaling argument width $\propto$ energy, perturbation is marginal, width independent of H (for $H \ll T$)

(- result only valid at $T \ll J$)

- same scaling appears to be true for more general orientations of exchange anisotropy
(there are log corrections $-\left(\frac{\delta J}{J}\right)^2 T \ln^2 T$
  - for $\delta J < 0$)

- ~~also corrections of higher order in δJ~~

[if $\delta J > 0$ this result breaks down at exponentially low T of order gap]

- more interesting case is staggered field
$h \sum_j (-1)^j S^x_j$ - arises (with $h \propto H$) from staggered gyromagnetic tensor or Dzyaloshinski-Moriya interaction (staggered)

$\delta H \sim h \cos 2\pi R\tilde{\varphi}$ - sine-Gordon model.

assume $R = \dfrac{1}{\sqrt{2\pi}}$

dimension of $\cos\sqrt{2\pi}\,\tilde{\varphi}$ is $\frac{1}{2}$ $\Rightarrow h \sim \Delta^{3/2}$

$\Delta \sim h^{2/3}$

- we can calculate width (of Lorentzian line shape) using perturbation theory (and a Dyson expansion in terms of self-energy of boson field)

now width $\propto \frac{h^2}{T^2}$

- again follows from scaling $h \sim \Delta^{3/2}$, width ~ energy
$h \sim (\text{energy})^{3/2}$, no dependence on H explicitly
- but $h = cH$ for some small coefficient
- c-depends on field direction
width $\propto c^2 H^2 / T^2$

- this must break down at $T \sim \Delta \sim h^{2/3}$
at $T \to 0$ we recover a δ-function line shape
- creation of single massive particle in sine-Gordon model dominates [from calculation of form factors]
- at $0 < T \ll \Delta$ width $\sim e^{-\Delta/T}$

- resonance location is H + small correction at $T \gg \Delta$

$\sqrt{\Delta(H)^2 + H^2} = \sqrt{(cH)^{4/3} + H^2}$ at $T \ll \Delta$.

- i.e. energy of single-particle excitation at wave-vector H
- can test this formula by angular dependence