T

Many-body quantum spin models with quenched randomness. D.H. Les Houches '06

Strong interactions
+ Strong randomness.

1) (newer) + many excitations

Dynamics at high T.

Diffusion or localization?

Do these models generally have a localization transition at $T > 0$?

Vadim Oganesyan

2) (older) $T=0$ + low T

Probability distributions, phases, phase transitions.

Quantum Griffiths-McCoy singularities.

Strong-randomness renormalization group.

Various models with ∞-randomness low-E behavior.

O. Motrunich, K. Damle, D. Fisher, R. Bhatt

Absence of diffusion in certain random lattices?

P.W. Anderson 1958: Localization.

Was about interacting spins (e.g. nucleii).

He did single-particle localization only as a simplification to make progress on the questions.

Basic questions:

Many interacting degrees of freedom, quenched randomness, Hamiltonian dynamics.

Is it ergodic?
Does energy diffuse?
Is there dissipation + decoherence?

Not about low T. What about high T, with many excitations present?

Localization with interactions at high T.

Fleishman + Anderson 1980

- Gornyi, Mirlin, Polyakov 2004
- Basko, Aleiner, Altshuler 2005

Essential issue: Diffusion or localization of energy.

Can ask about this for any many-body Hamiltonian with quenched disorder.

¿Can it serve as its own "heat bath"?

A model we have been studying:

1D Spinless lattice fermions (hopping)

+ interactions

+ random potential

• No external bath. This many-body system must serve as it own "heat bath".

(No phonons, photons ....)

$$H = \sum_i \left(c_i^\dagger c_{i+1} + c_i^\dagger c_{i+2} + h.c. \right)$$

(we put 2nd neighbor hopping to break integrability.)

$$+ V \sum_i n_i n_{i+1}$$

interaction, often we focus on V=2.

$$+ \sum w_i n_i .$$

i.i.d. Gaussian random on-site potential

$\overline{w_i} = 0$

Is an XXZ spin chain with random z-field. $(s=\frac{1}{2})$

$$H = \sum_i \left[\left(c_i^\dagger c_{i+1} \cdots \right) + V n_i n_{i+1} + W_i n_i \right]$$

$\rightarrowtail$ with $T > 0$. (even $T = \infty$)

two well-understood limits:

- $V = 0$. non-interacting 1D fermions in a random potential. $(\overline{W_i^2} > 0)$

  All states are localized. No diffusion.

  Is this robust to "turning on" small $V \neq 0$?

- $W_i = 0$, $V \neq 0$. Interactions but no randomness.

  Particles + energy diffuse.

  Spectrum is quantum chaotic (GOE).

  - Many-body eigenstates are ergodic: fully extended + random in many-body Fock space (Hilbert space)

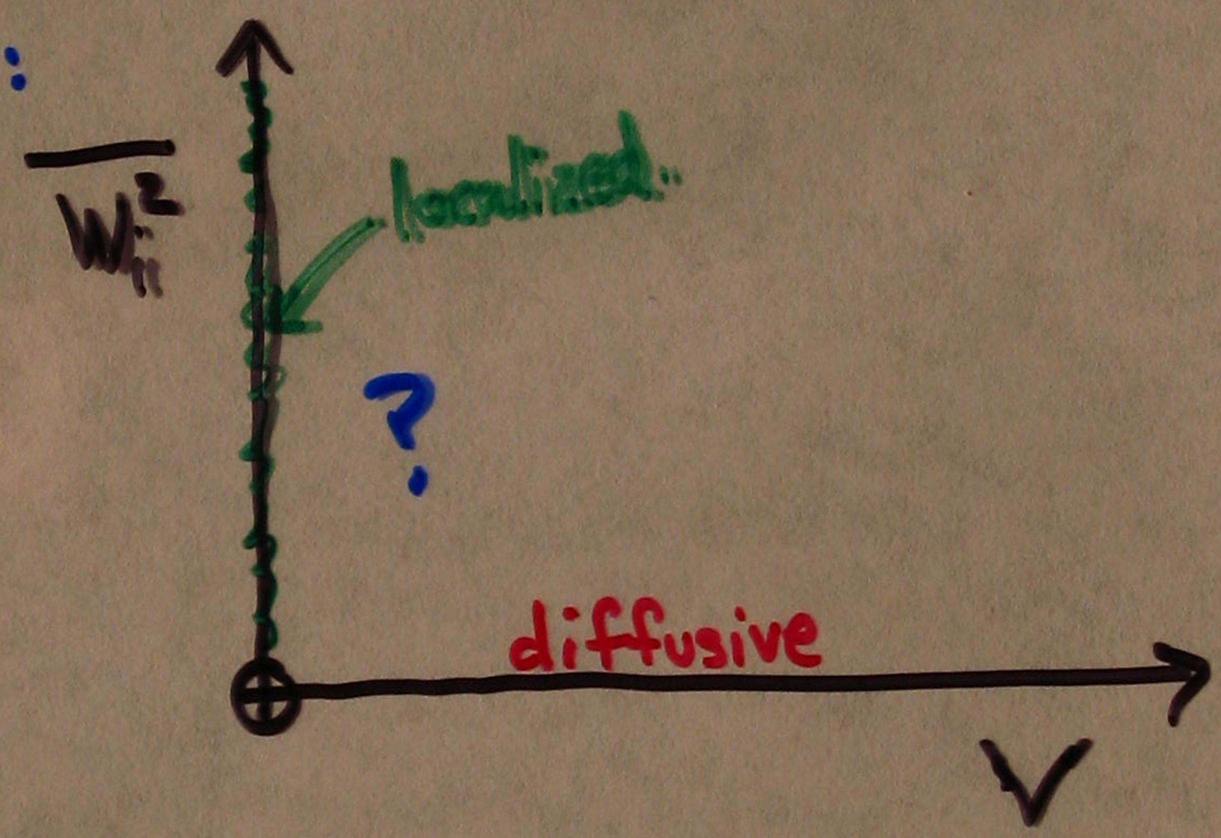
    stable to turning on weak randomness.

Mukerjee, Oganesyan, Huse PRB 73, 035113 (2006).

$$H = \sum_i \left[(c_i^\dagger c_{i+1} + \cdots) + V n_i n_{i+1} + W_i n_i \right]$$

What happens between $V=0$ and $W_i=0$ limits?

Phase diagram:



high T.

Does localized phase, with strictly zero diffusion in thermodynamic limit at $T>0$, survive to $V \neq 0$?

If yes, there is a $T>0$ localized/diffusive phase transition.
(insulator/metal)

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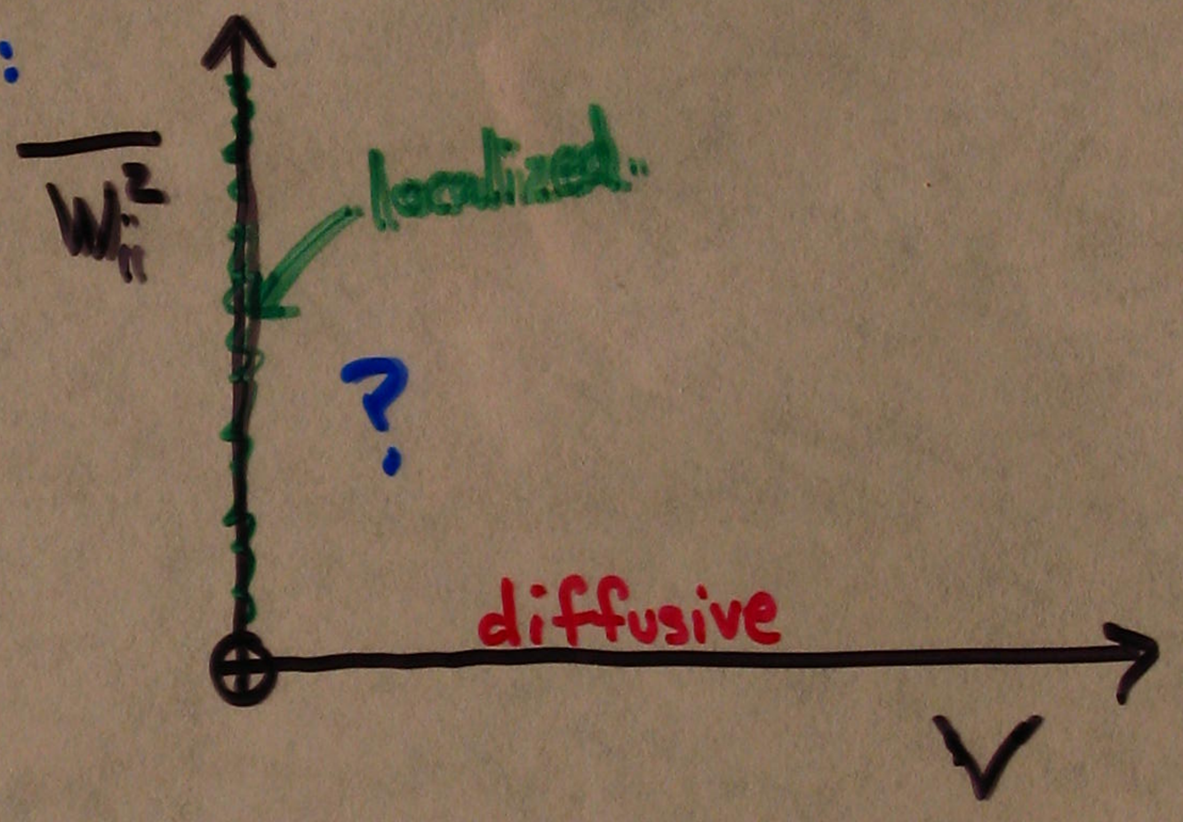

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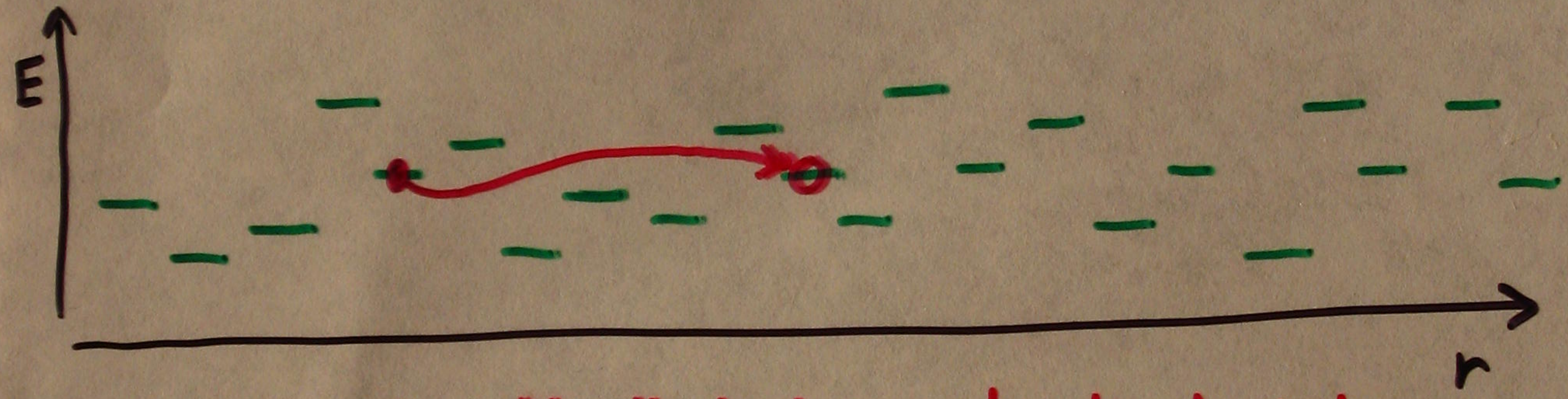
"Traditional" picture of localized "regime" at $T > 0$:

Variable-range hopping conduction.
(Mott, Efros-Shklovskii)



particles "hop" between localized states.

Energy differences are given to or taken from heat bath.

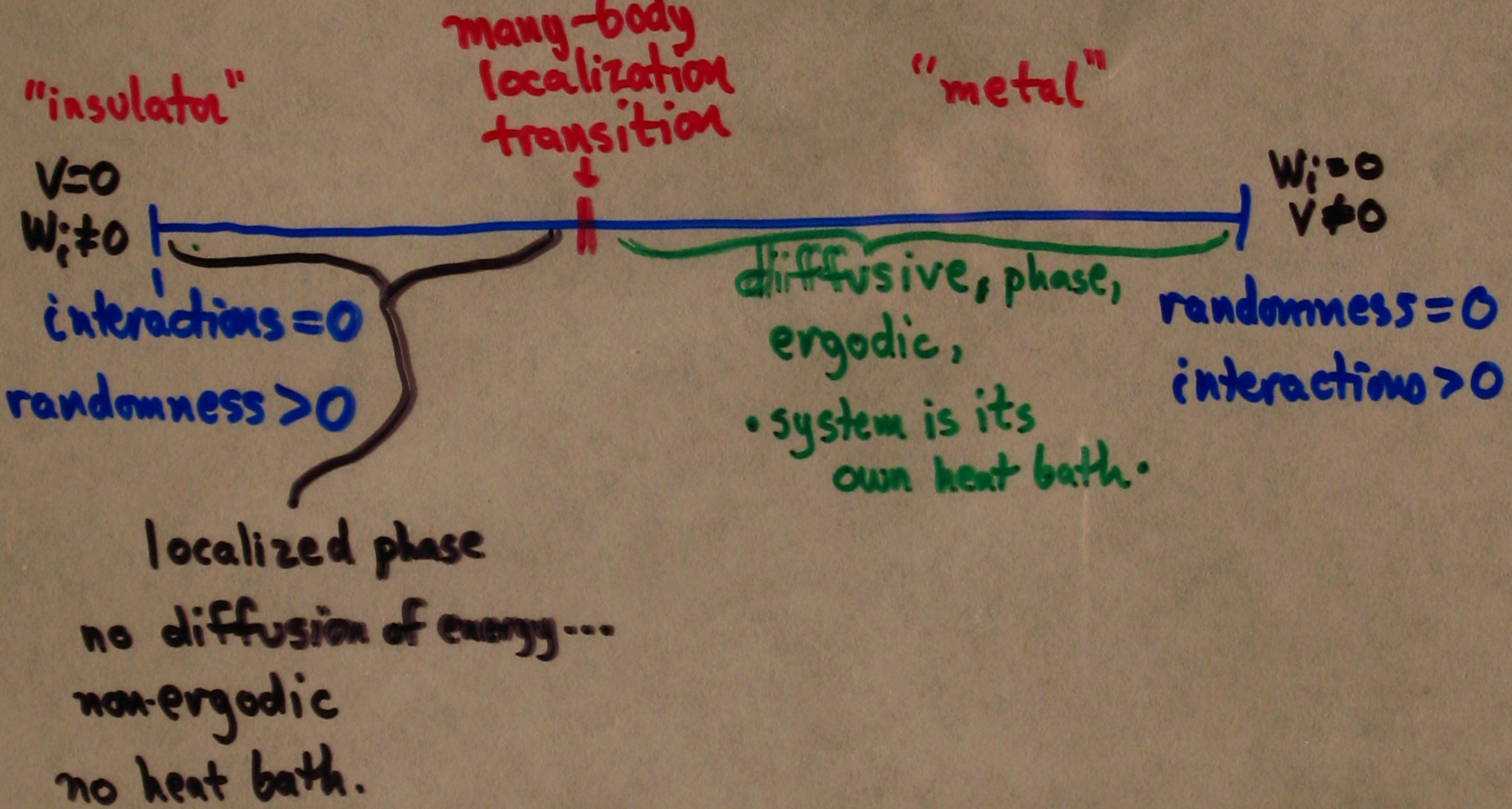
But is there a heat bath that can exchange energy at any place and in any amount?

Not at $V = 0$.

For $V \neq 0$ do particle-hole excitations + interactions make a functional (diffusive) heat bath?

Gornyi, et al., Basko, et al. argue for

nonergodic phase at $V>0$: **and** $T>0$:



They argue for interacting fermions.

But phenomenon should be more general:

MAny many-body Hamiltonians with quenched randomness?

Even classical?

Back to the specific model:

$$H = \sum_i \Big[(c_i^\dagger c_{i+1} \cdots) + W_i c_i^\dagger c_i + V n_i n_{i+1} \Big]$$

$$= H_W + \sum_i V c_i^\dagger c_i c_{i+1}^\dagger c_{i+1}$$

noninteracting fermions.

many-body eigenstates (of H_W): each localized single-particle state either empty or occupied. $n_\alpha = 0$ or 1

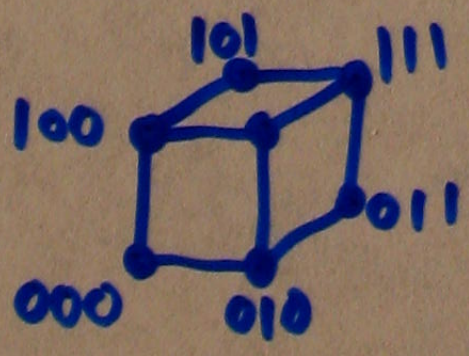
$$H = \sum_\alpha \epsilon_\alpha n_\alpha + \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta$$

single-particle eigenstates of H_W

interaction. (all states overlap the same sites)

Many-body Hilbert (Fock) space: all binary strings of $\{n_\alpha\}$ $\quad (n_\alpha = 0$ or $1)$

Corners of a N-dimensional hypercube $\alpha = 1, 2, \cdots N$



....

$$H = \underbrace{\sum_{\alpha} \epsilon_\alpha n_\alpha}_{\text{gives a (correlated) random potential on the hypercube (state space)}} + \underbrace{\sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} c^\dagger_\alpha c^\dagger_\beta c_\gamma c_\delta}_{\text{gives short-range (4 steps) hopping on the hypercube.}}$$

gives a (correlated) random potential on the hypercube (state space)

gives short-range (4 steps) hopping on the hypercube.
(also short-range in real space.)

Many-body system in 1D, length N

becomes

A single "particle" on a N-dimensional hypercube.

Should have a localized phase for small $V \neq 0$.

Appears to be perturbatively self-consistent.
(Basko, Aleiner, Altshuler)

How to test it numerically? (Vadim Oganesyan)

One clean, well-understood distinction between diffusive + localized phases:

- Spectral statistics of H -

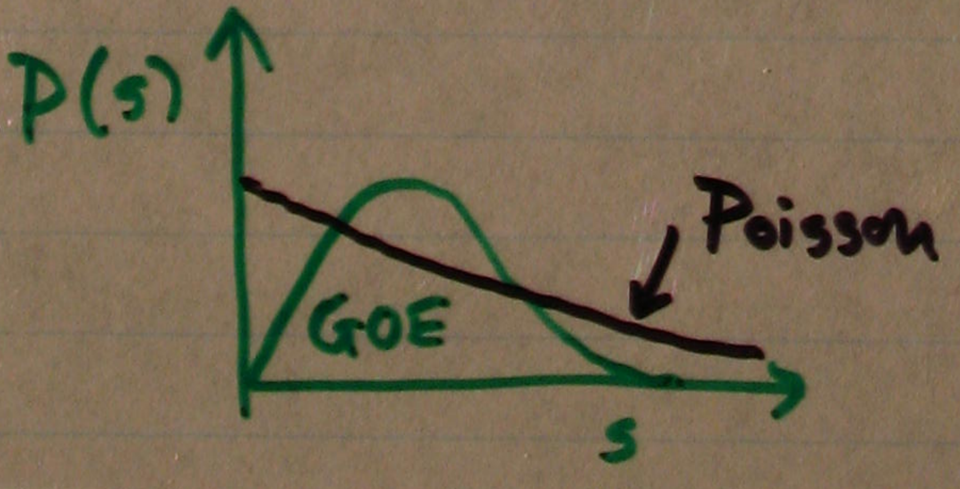
(level repulsion)

Level repulsion between nearly degenerate states occurs if and only if they occupy overlapping regions in Hilbert space.

Localized, Insulating phase: no level repulsion, eigenenergies Poisson-distributed (uncorrelated).

Diffusive phase: states are extended:

Wigner-Dyson-Mehta GOE (random matrix) level statistics.



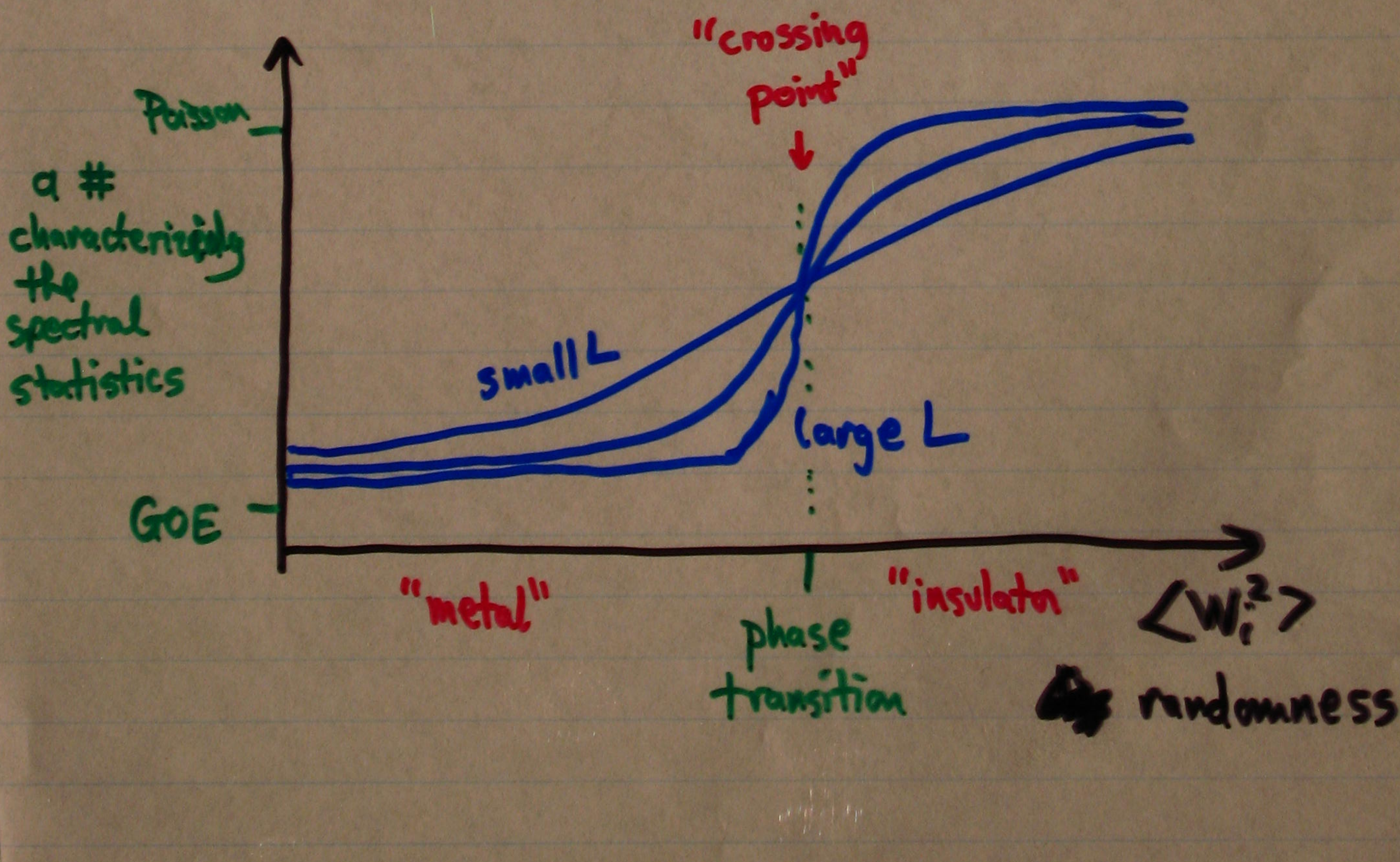
$$s = \frac{\text{gap}}{\langle \text{gap} \rangle}$$

Finite-size effects, finite-size scaling.

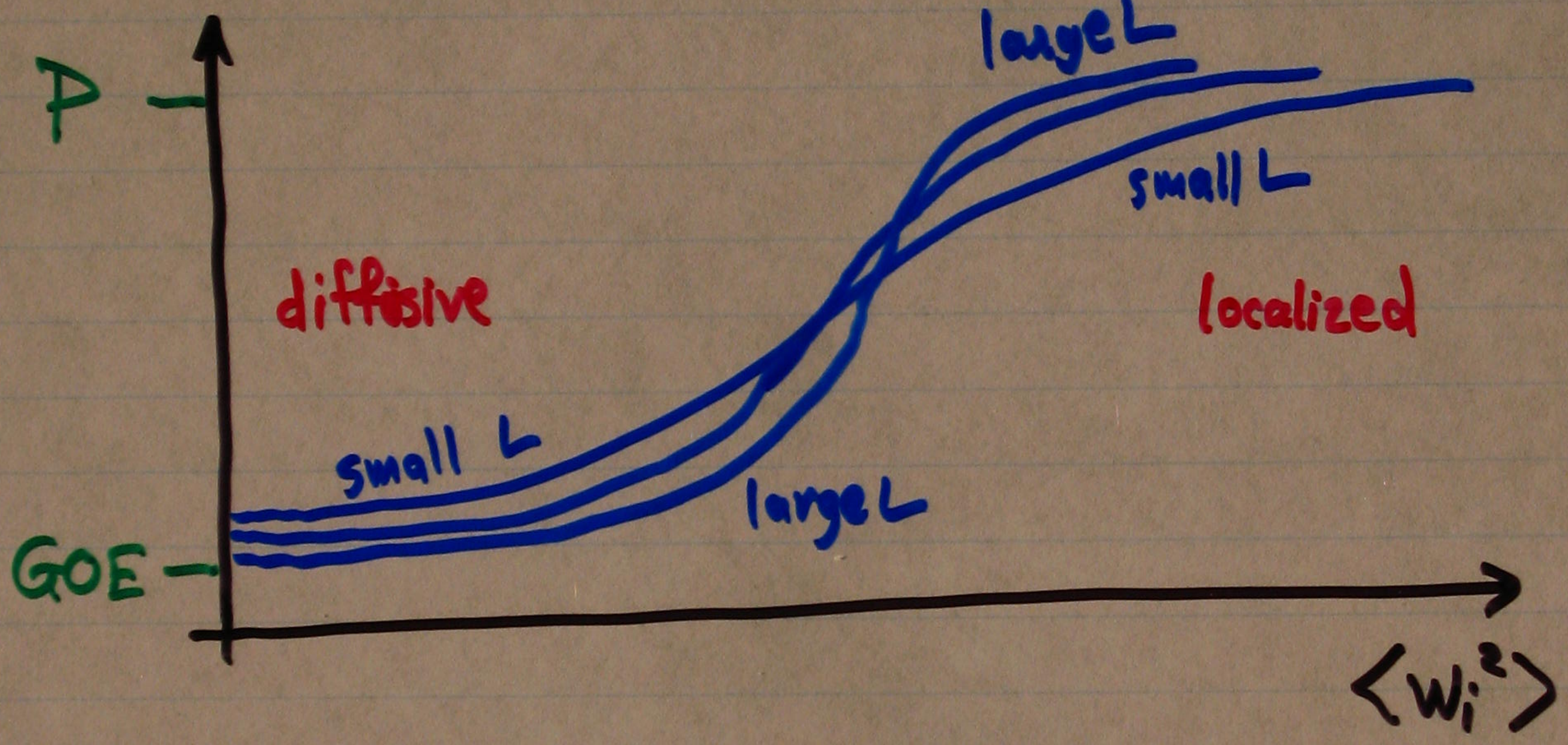
$L < \infty$: Spectral statistics are "intermediate" between Poisson + GOE

In $\begin{matrix}\text{Localized}\\ \text{Diffusive}\end{matrix}$ phase, they converge for $L \to \infty$ to $\begin{matrix}\text{Poisson}\\ \text{GOE}\end{matrix}$.

Ideally:



Less ideal:



If crossing point "drifts" towards localized phase as L increases (from 6 to 18), it raises worry that localized phase might not survive in strict $L \to \infty$ limit. (?)

Conclusions

- It has been argued that quantum many-body systems with quenched randomness can have a localization transition at $T>0$.
  (Gornyi et al., Basko et al.)

- This issue can be explored within models of quantum magnetism.

- Can we produce strong numerical results addressing this question? (Looks promising)

- What model would be best for this purpose?

Many-body localization = Loss of a heat bath.

" " ergodicity.

" " decoherence..