INTRO. (Some general discussion about quenched randomness)

Consider: Solids with quenched*-in, random spatial inhomogeneities due to

- varying local chemical composition (e.g. randomly-placed dopants)
- varying local structural arrangements (e.g. in a glass)

* Temperature T low enough so atoms do not diffuse

Remaining active (not quenched) degrees of freedom:

spins
electrons
phonons
and/or

phenomena:

magnetism

(super) conduction/localization

(thermodynamic, dielectric, other transport properties)

Local properties

e.g. Susceptibility $\chi_{loc}(\vec{r})$ (magnetic)

Conductivity $\sigma_{loc}(\vec{r})$

specific heat $C_{loc}(\vec{r})$

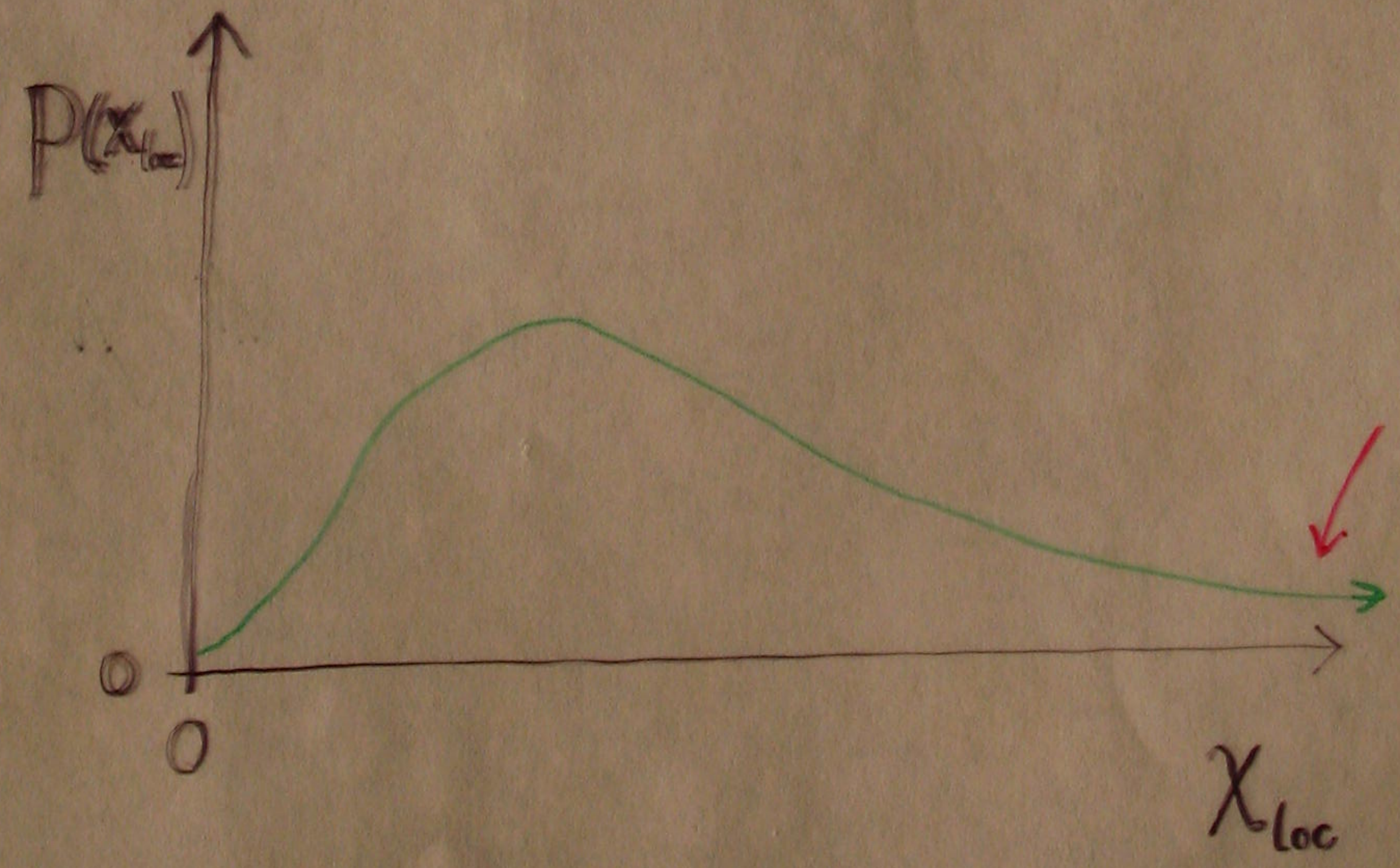
may vary strongly from place to place.

$$\chi = \frac{\partial m}{\partial H}$$

Have probability distributions $P(\chi_{loc})$, etc.

Our interest today: cases where distribution is broad, has long "tails":

e.g.:



Some "rare" locations have very large χ_{loc} far above (below) median

This can Happen at low T, + particularly near phase transitions.
(easily "flipped" magnetic moments) (later)

Some measurements:

$$\chi\,,\quad C=\frac{dE}{dT}\,,\quad S(\vec{q},\omega)$$ (Scattering intensity)

are simple (arithmetic) averages and can be dominated by rare regions.

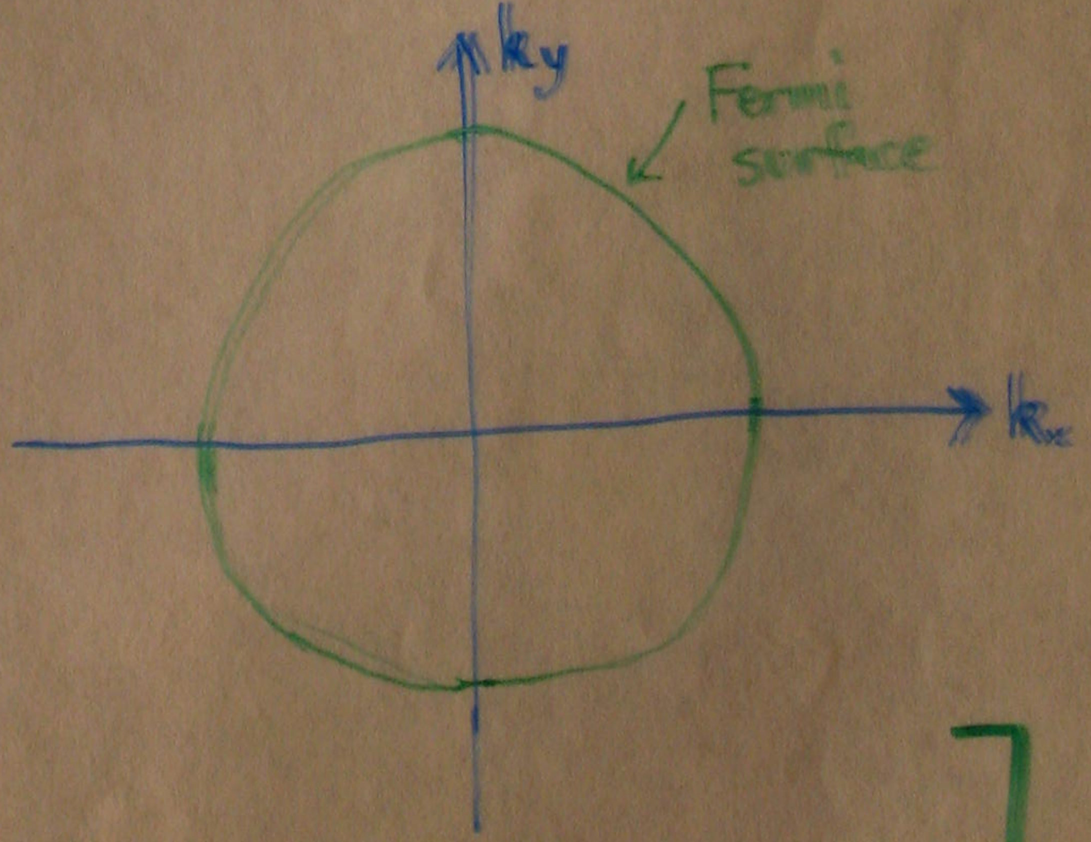
(usually: places with low-energy excitations)

[ASIDE:

Nonrandom (pure) systems are uniform in real-space, but always non-uniform in momentum-space. E.g.: metal:

Many Low-T ($T \ll E_F$) properties are dominated by "rare" electrons near Fermi surface.

(A small region in k-space.)



]

On the other hand,

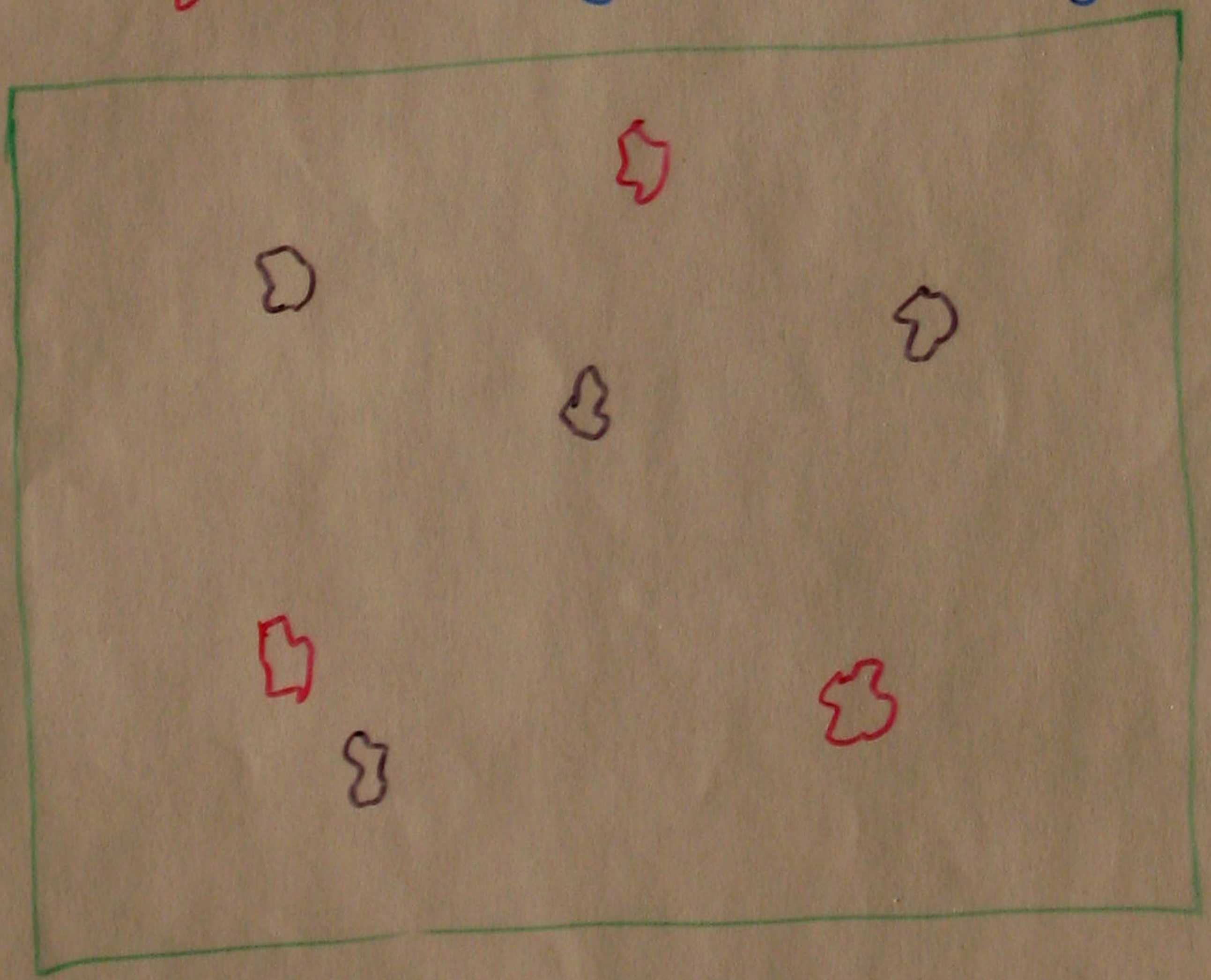
For $d>1$, transport coefficients + stiffnesses (including surface tensions + elastic constants) are given by something like the <u>median</u>, not the average.

Example: random conductor with <u>rare</u>

superconducting ($\rho=0$) and

insulating ($\sigma=0$) regions:

(σ_{loc}, ρ_{loc} **have ∞ average, st. deviation**)



rare regions do <u>not</u> dominate conductivity:

nor do averages.

No path to carry supercurrent across sample

No surface of insulator to block current ($d>1$)

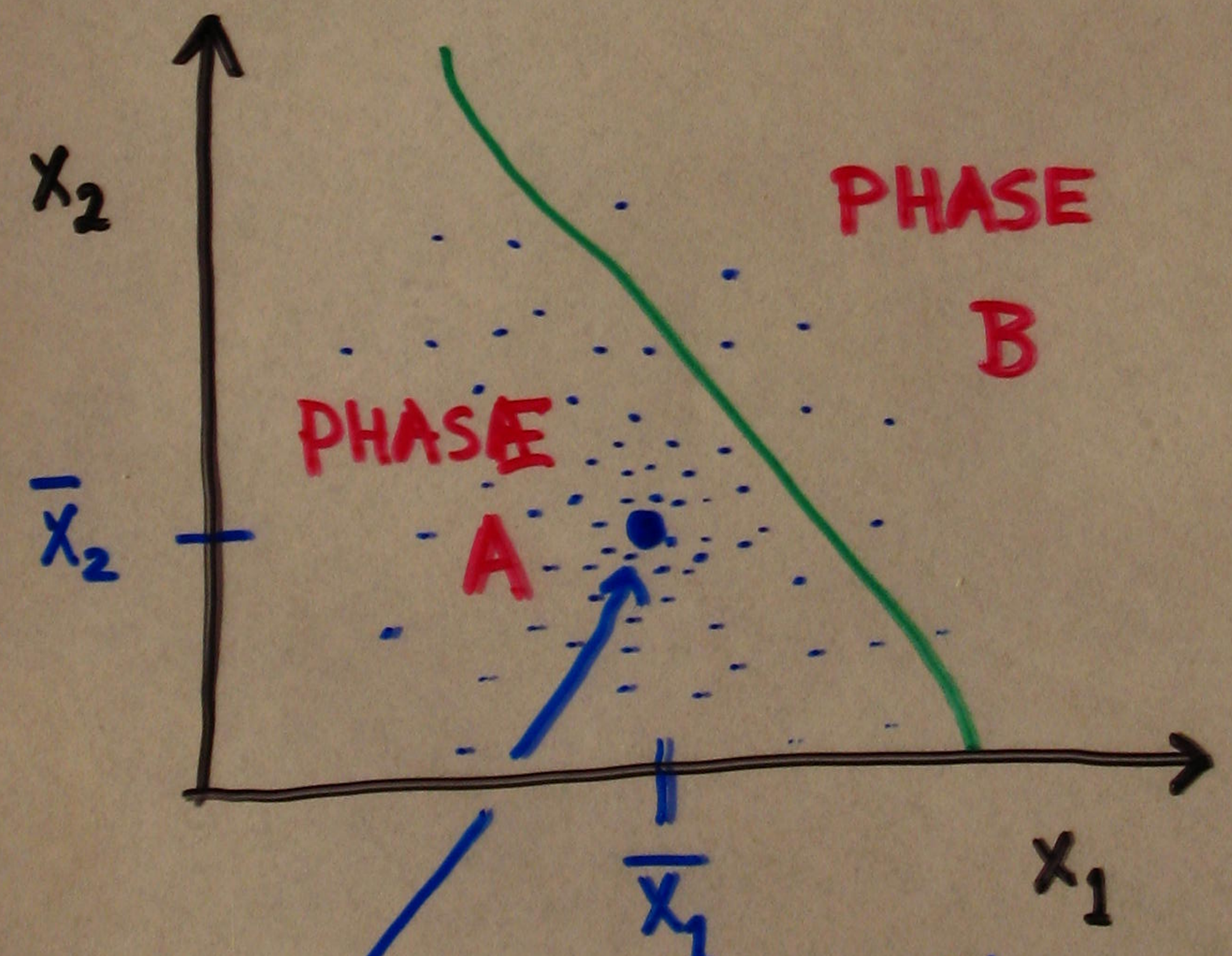
<u>Macroscopic</u> σ conductivity $\sim$ <u>median</u> of (meso) microscopic σ

$\rightarrow$ Ambegaokar, Halperin + Langer .....

Consider a • Griffiths-McCoy singularities •

SOLID with local fluctuations in structure/composition $\{x_i\}$ frozen in during its fabrication.

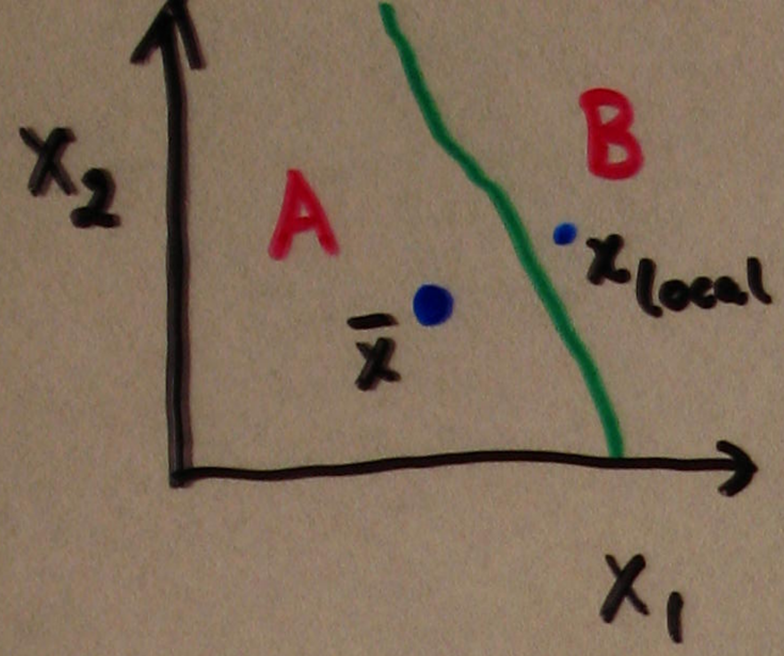
Phase diagram:



spatial average

Our sample has $\bar{x}_1, \bar{x}_2, \ldots$ and is in phase A.

But locally x_1, x_2 can "fluctuate" to phase B at some locations.



Griffiths-McCoy domains are:

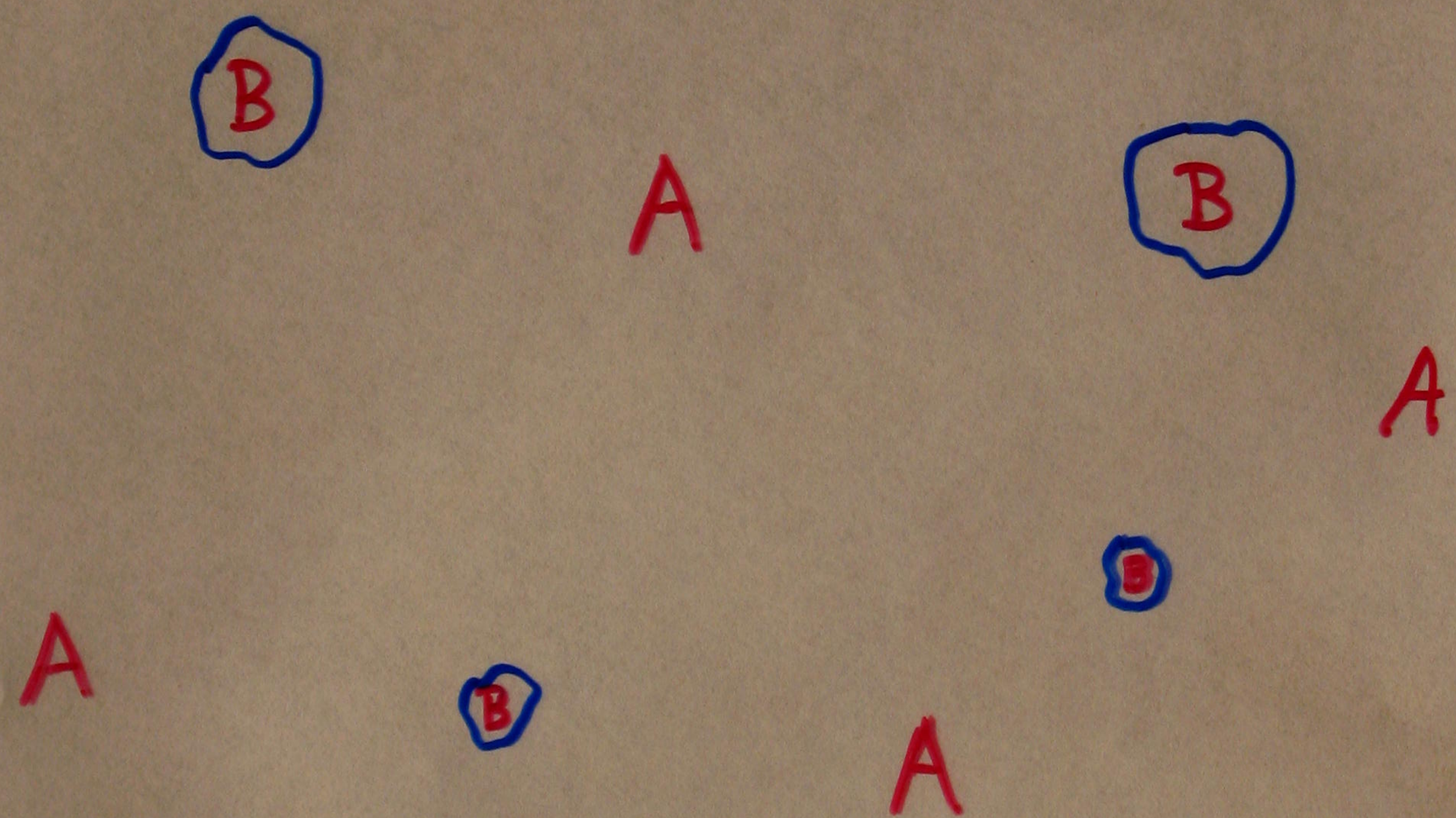
- region in sample where $\{x_1, x_2 \cdots\}$ are in phase $\underline{\underline{B}}$, although $\{\bar{x}_1, \vec{\bar{x}}_2, \cdots\}$ (averages) for sample are in phase $\underline{\underline{A}}$

Such domains of volume V occur with probability, density $\propto \exp(-aV)$
(rare thing happens V times)

(a depends on $\{\bar{x}_i\}$ and vanishes at phase boundary)

Griffiths-McCoy singularities: large V

Real-space "picture" of sample: (coarse-grained)



If B domains, or A/B domain walls

have much higher: - susceptibility

- relaxation time

- low-energy d.o.s. .....

Then Griffiths-McCoy domains may dominate the

low-T, low-ω, or long-time behavior.

Model with well-understood Griffiths-$\underbrace{\text{McCoy}}_{\text{quantum}}$ singularities:

Quantum Ising model: spin-$\frac{1}{2}$

$$H = -\sum_{\langle ij \rangle} \overset{\text{random}}{J_{ij}}\, S_i^z S_j^z \;-\; \sum_i \overset{\text{random}}{h_i}\, S_i^x$$

← But they do not <u>both</u> have to be random.

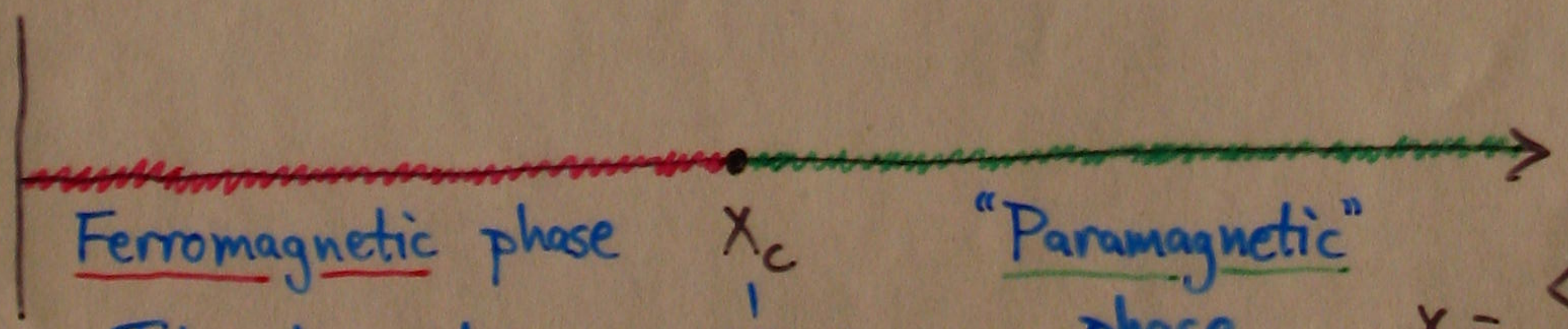
(Ferromagnetic) interactions: want ↑↑↑↑ or ↓↓↓↓ for ground state

transverse fields want → → → → for ground state

z

x

T=0 phase diagram:



Ferromagnetic phase

X_c

q. critical point

"Paramagnetic" phase

$$X = \frac{\langle h \rangle}{\langle J \rangle}$$

J's dominate:

↗ ↗ ↗ ↗ ↗

or

↘ ↘ ↘ ↘ ↘

<u>two</u> ground states, differing in M_z: $\chi = \frac{dM_z}{dH_z} \to \infty$

h's dominate:

→ → → →

<u>unique</u> ground state

"Griffiths" domain in paramagnetic phase:

Para.

Para.

Para.

Probability $\sim e^{-cV}$

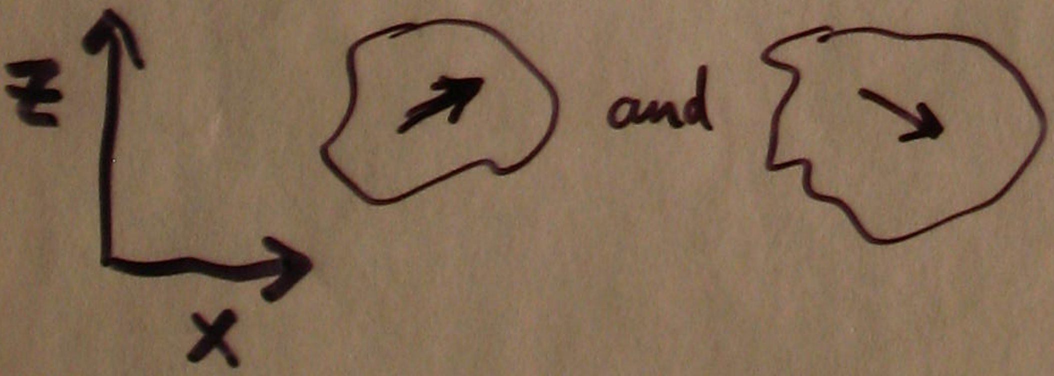
(rare for large V)

What is magnetic susceptibility of this domain of volume V? <u>at</u> $T=0$

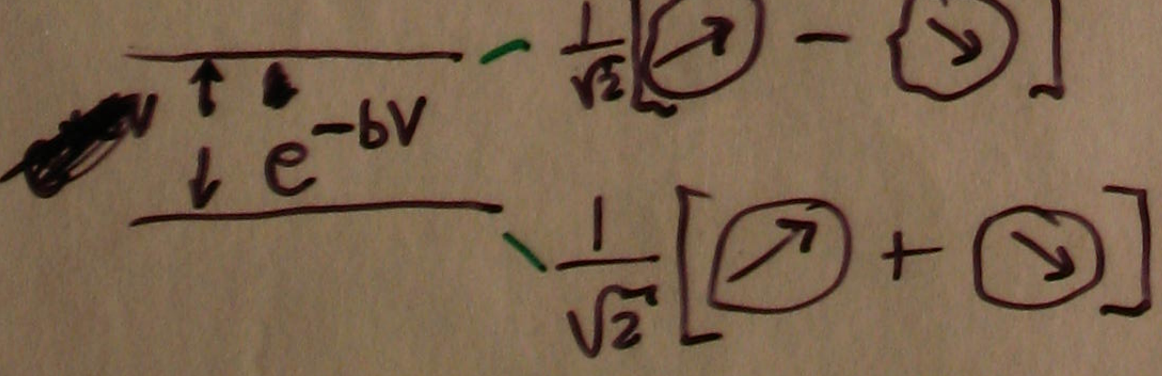
Key: <u>Discrete</u> broken symmetry in Ferro. phase.

For quantum Ising:

Domain is ferromagnetic, has two low-lying states:



For finite V there is quantum tunnelling between these states:

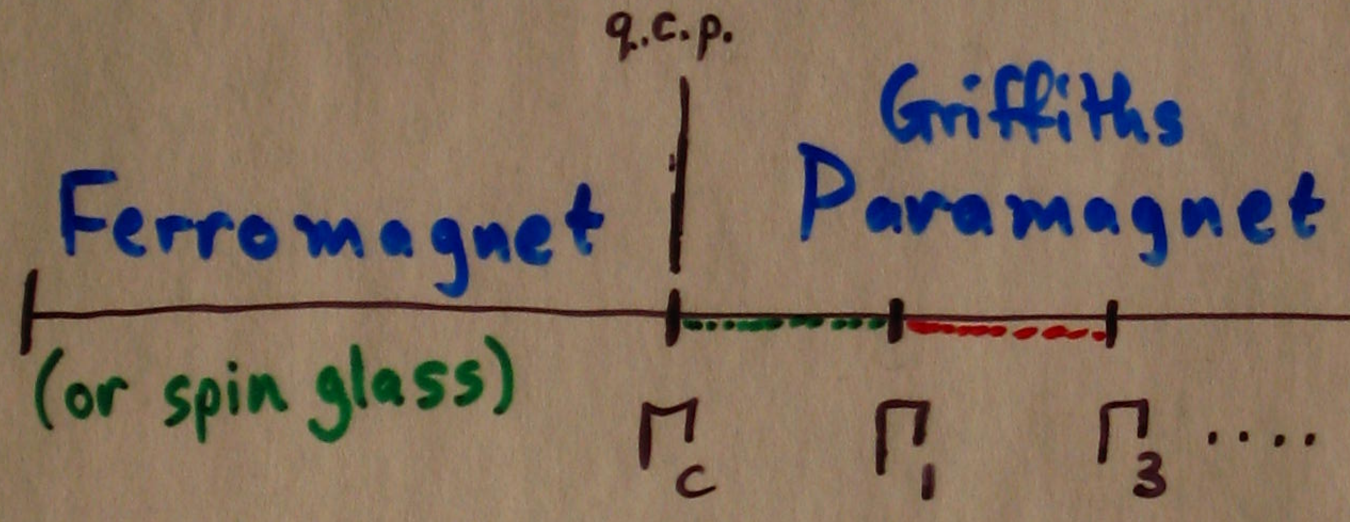


small splitting:

$$\chi \sim e^{+bV} \sim (\text{prob.})^{-b/c}$$

Power-law low E density of excited states

$T=0$ behavior of *random* quantum Ising model:



with power-law low E density of excited states.

$D(E) \sim E^{-\gamma}$

$\Gamma = \frac{\langle h \rangle}{\langle J \rangle}$

$$\chi = \frac{dM_z}{dh} = \infty$$

nonlinear susceptibility

$$\chi_3 = \frac{d^3 M_z}{dh^3} = \infty$$

divergences in para. phase:
Quantum
Griffiths – McCoy
singularities

(due to rare domains of Ferro. phase)

For $T>0$: $\chi(T) \sim T^{-g(\Gamma)}$, $C(T) \sim T^{1-g(\Gamma)}$

exponent depends on Γ

$g(\Gamma) \to 0$ at $\Gamma = \Gamma_1$
" $\to 1$ at Γ_c
" $\to -2$ at Γ_3

We know this from:

- Exact solution in $d=1$ McCoy + Wu; D. Fisher
- Quantum Monte Carlo in $d=2,3$ Reiger + Young; Guo, Bhatt + Huse + Pich + Kawashima
- Strong-randomness R. G.