• Strong randomness renormalization group.

To determine the behavior of "quenched-random" materials, start ~~not~~ not from $\mathcal{H}$, a Hamiltonian,

but instead from $\boxed{P(\mathcal{H})}$,

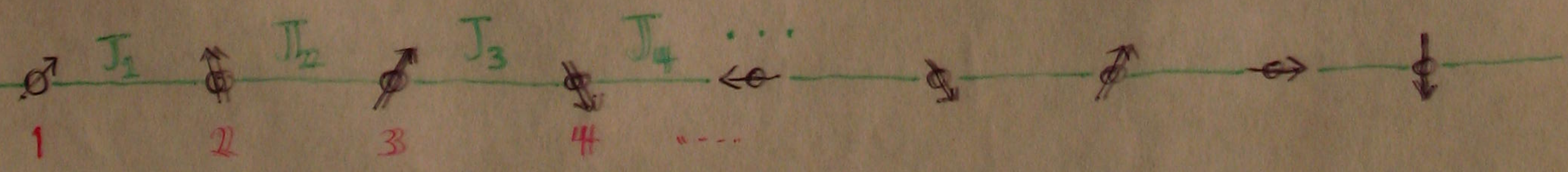
the probability distribution of the Hamiltonian.

(and don't approximate $P(\mathcal{H})$ as Gaussian ["Bell curve"]!)

A "simple" example, the random-exchange spin-½ chain:

$$\mathcal{H} = \sum_i J_i \left(\vec{S}_i \cdot \vec{S}_{i+1} \right)$$

Each J_i is random, but all antiferromagnetic $J_i > 0$



What is the ground state? What are the low-energy excitations?

First:

nonrandom $S=\frac{1}{2}$ AF chain:

$$H = J \sum_i \vec{S}_i \cdot \vec{S}_{i+1} \qquad J>0$$

ground state solved by Bethe ('30's)

gapless spectrum: continuum of "massless" low-lying excitations ("spinons")

Power-law ground state correlations

$$\langle \vec{S}_n \cdot \vec{S}_{n+m} \rangle \sim (-1)^m \frac{1}{m^\eta} \qquad (\eta=1)$$

A quantum critical point.

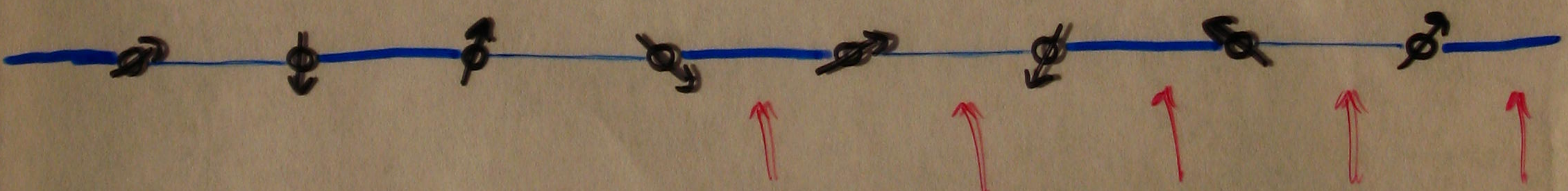
Separating what phases?

Now allow bond-alternation: (still "pure", not random)

$$H = J \sum_n \left[1 + \delta(-1)^n\right] \vec{S}_n \cdot \vec{S}_{n+1}$$

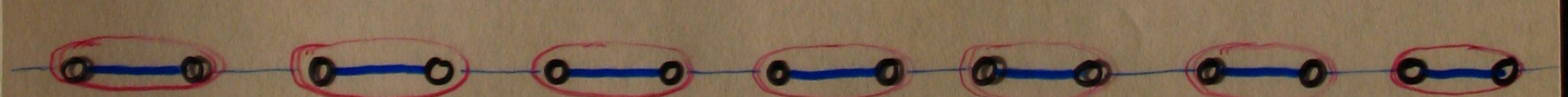
$(|\delta| \leq 1)$

For $\delta \neq 0$:



Bonds alternate: strong-weak-strong-weak-str

Ground state is:



singlet: $\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$ on each strong bond.

+ quantum fluctuations.

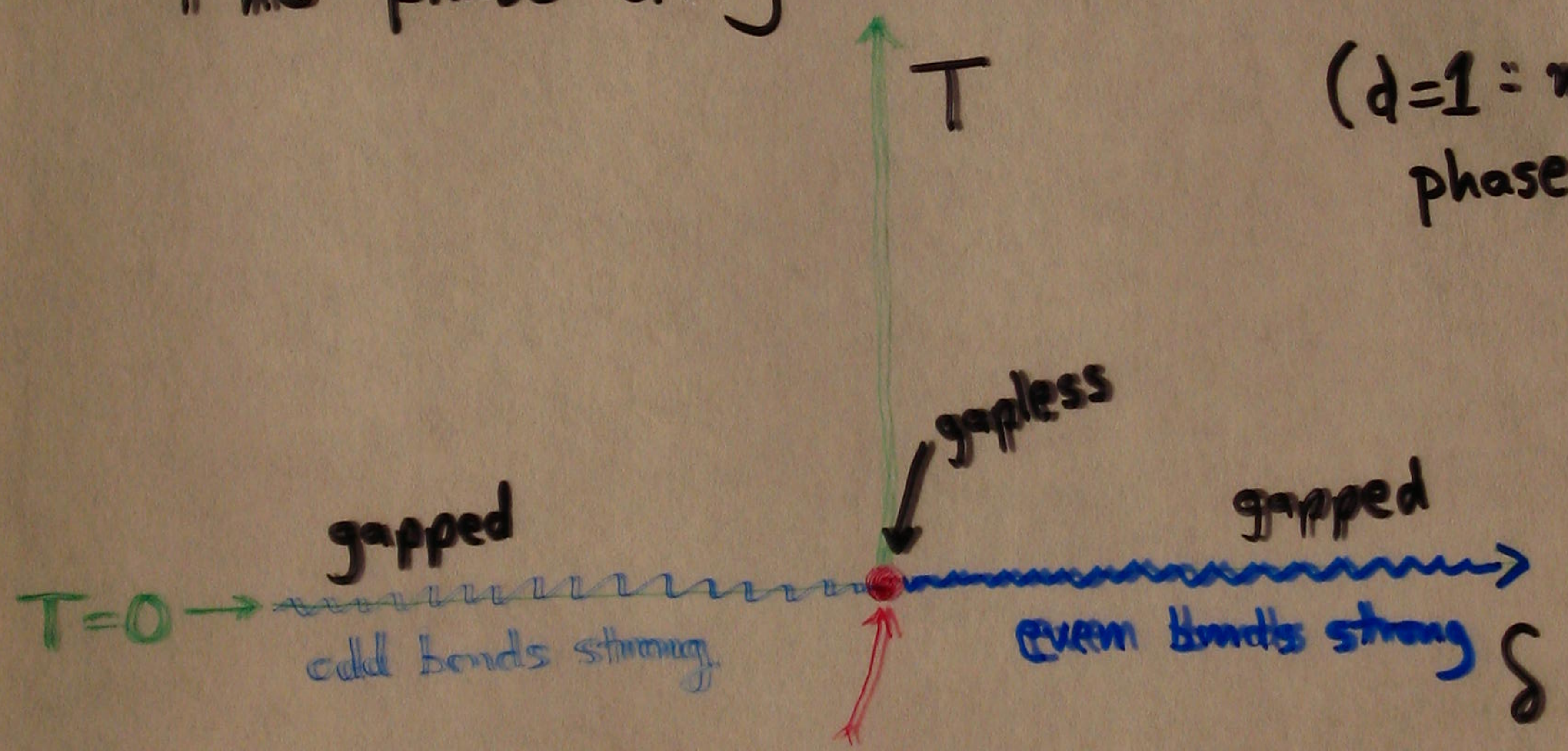
Energy gap to ("massive") excitations. for $\delta \neq 0$

$$\mathcal{H} = J \sum_n \left[1 + \delta(-1)^n\right] \vec{S}_n \cdot \vec{S}_{n+1}$$

(still no randomness)

The phase diagram:

(d=1: no T>0 phase transitions)



Bethe's quantum critical point: a quantum (T=0) phase transition
(δ=0, T=0)

"odd"/"even" phases characterized by their edge-state properties:

cut weak bond: no zero-energy edge "excitations"

semi-∞ chains

cut strong bond: spin-½ left (unpaired) at end

ground-state now 2-fold degenerate.

Back to random-exchange AF chain:

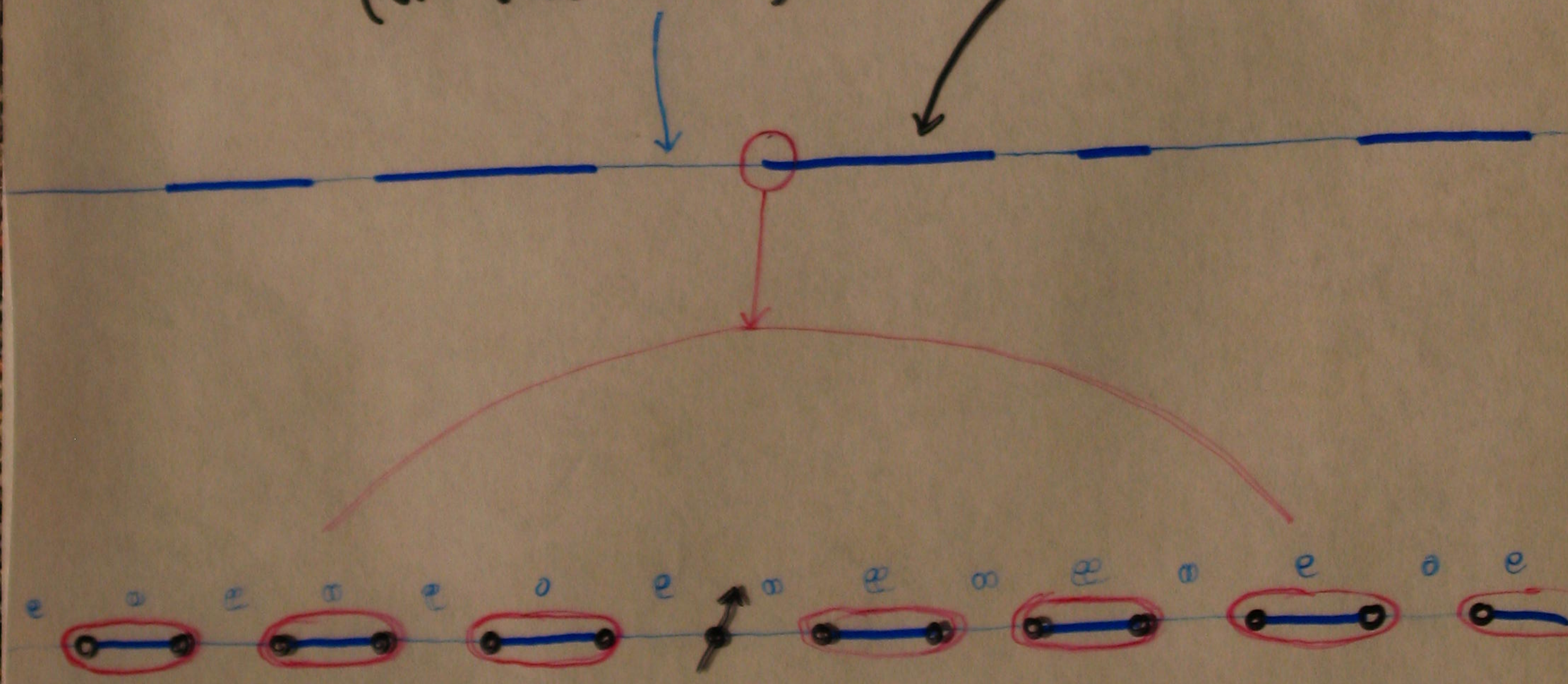
$$\mathcal{H} = \sum_n J_n \vec{S}_n \cdot \vec{S}_{n+1}$$

random J_n's ($J_n > 0$ AF)

Some stretches of chain:

"domains" where even J's > odd J's (just by chance)

(or vice versa)



unpaired spin at the "domain" boundary

(low-energy description of this system)

free spins at each domain boundary.

How do they interact? ... (a little later)

General summary (so far):

quantum phase transition

+ quenched randomness

(quantum McCoy-Griffiths singularities) '68

$\longrightarrow$ domains + domain boundaries

+ associated low-lying excitations. (usually spins)

Various other examples::

e.g.:: metal (all orbitals either full or empty at $T=0$: no spins, Pauli paramagnet)

-to-

insulator (localized half-full orbitals :: spins)

transition::

rare insulating domains make "local moments" in disordered metal. (e.g. P-doped Si)

Bhatt
D Fisher
Holcomb
Paalanen

~~Summary: (so far)~~

~~randomness near quantum phase transitions produces frozen-in domains + domain boundaries + associated low-lying excitations.~~ (quite generally)

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How to do this more systematically?

Renormalization Group

For nonrandom case: take $\mathcal{H}$, ← Hamiltonian

"integrate out" high-energy modes, get new

$\mathcal{H}'$ ("renormalized Hamiltonian")

that governs lower energy behavior. Repeat.

What $\mathcal{H}' \to$ in low-energy limit determines nature of ground state.

With randomness:

at energy E, $P(H)$ (prob. dist. of Hamiltonian)

renormalize to energy $E' < E$

each $H \to H'$ get

at energy E', $P'(H')$

renormalized probability distribution of Hamiltonians

What happens in low energy limit?

In other terms:

What is the renormalization-group fixed point governing the system's ground state + low-lying excitations?

3 possibilities for degree of randomness in low-E limit:

(i) asymptotically non-random (self-averaging)

Examples: Acoustic phonons in a disordered solid.
Fermi liquid behavior in a 3-d metal.
⋮

randomness can be treated as a perturbation on a pure system.

(ii) finite, non-zero randomness
ratios of energies, matrix elements of H's are $\mathcal{O}(1)$

Examples: Two-level-system model of glasses
spin-glass ordered phase
quantum Griffiths phases

(noninteracting) electrons at Anderson metal/insulator transition
various other quantum ~~phase~~ critical points!
(and classical)
⋮
random matrix theories

OR (this case is my focus)

→ (iii) asymptotically ∞ randomness at low energy

ratios of energies $\to 0$ or ∞ (very broad probability distributions)

Examples:

- hopping matrix elements in variable-range-hopping insulator

→ · various quantum critical states:

in random spin chains (1d) (many examples)

in quantum Ising model ($d=1,2,3,\ldots$?)

of particle-hole symmetric localization ($d=1,2.$)

(e.g. quasiparticles in superconductors)

Things "simplify" in the ∞-randomness limit

due to disparate energy scales.

Decoupling in energy "space"

(An essential principle that allows us to do physics: focus on one energy scale and safely ignore much higher or lower energy physics)

→ Allows controlled systematic solutions of these models.

A new class of renormalization-group fixed points

Back to our $S=\frac{1}{2}$ A.F. random-exchange chain:

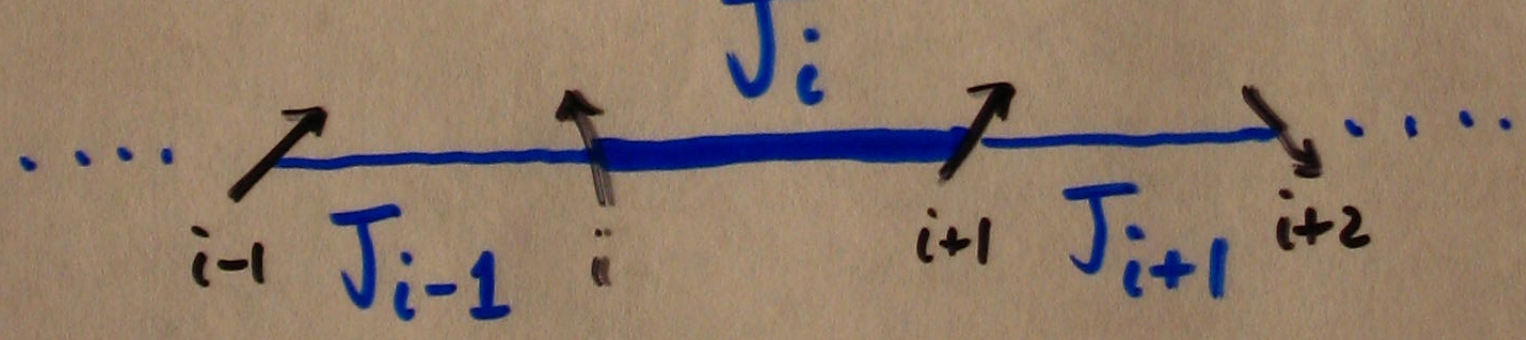
$$H = \sum_i J_i \vec{S}_i \cdot \vec{S}_{i+1} \; ; \quad P(J) \text{ (broad)}$$

RG: (Ma-Dasgupta-Hu, D. Fisher)

Find highest energy + deal with it first.

(R.G. to find ground state)

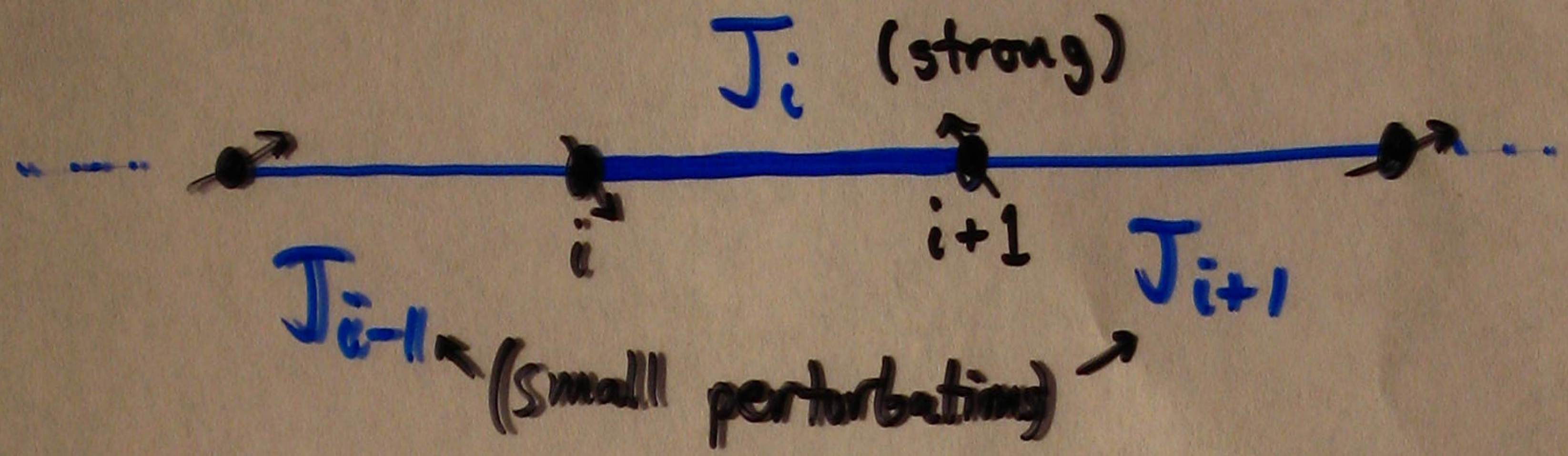
Largest J_i:



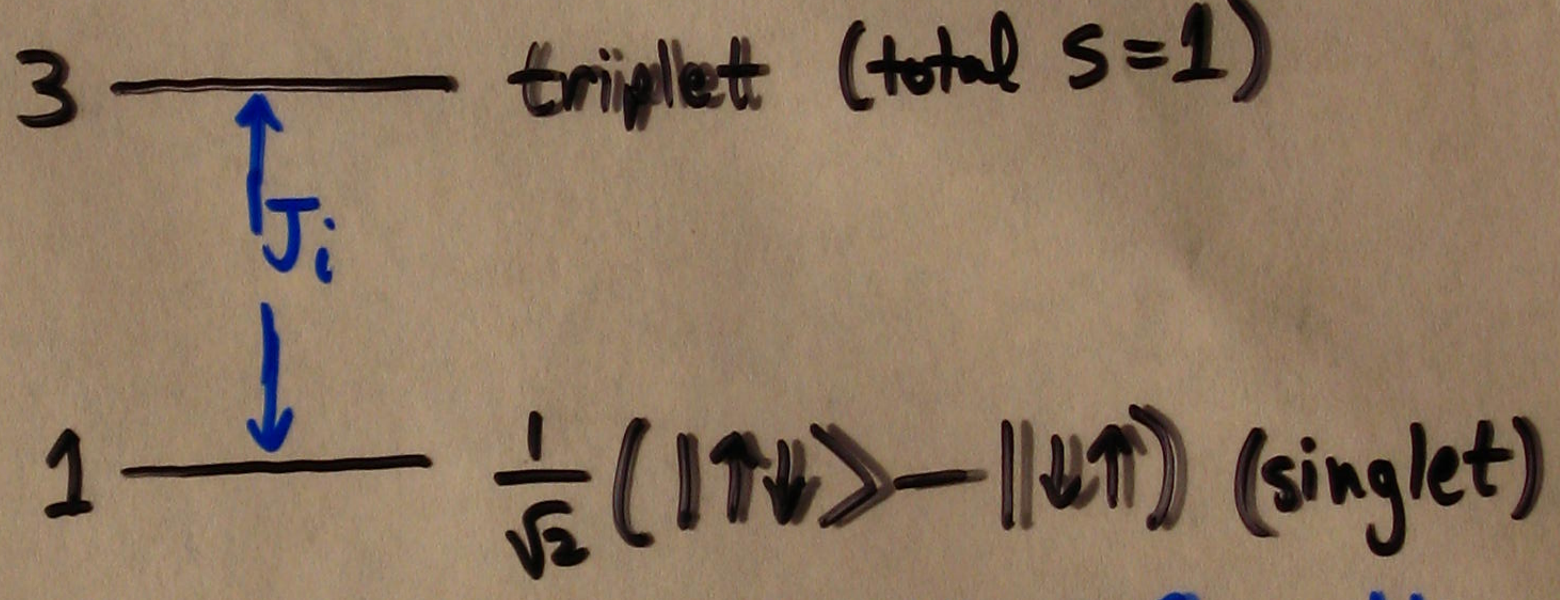
Strong randomness: $J_i >> J_{i+1}$ (with high probability)

$J_i >> J_{i-1}$

gives small parameters: $\frac{J_{i\pm1}}{J_i}$, controlled calculation.



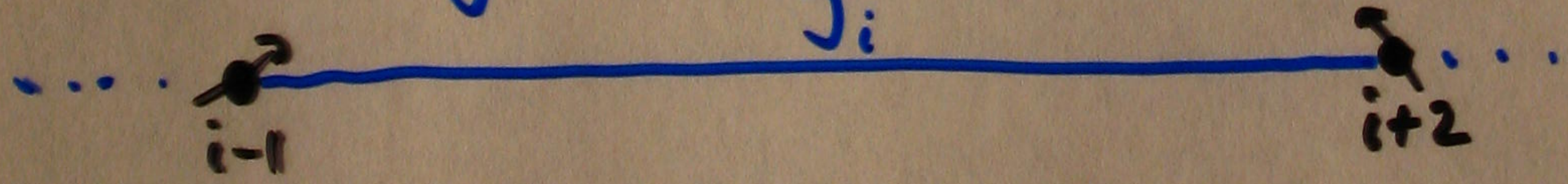
spectrum of $J_i \vec{S}_i \cdot \vec{S}_{i+1}$



ground state before adding perturbations.

Then treat J_{i+1}, J_{i-1} in simple pert. theory:

$$J' = \frac{J_{i+1} J_{i-1}}{J_i}$$



R.G. step: two spins "integrated out" (singlet ground state)

three bonds deleted, (including strongest)

one new renormalized J added

note: $J' \ll J_{i\pm1}$ is <u>weak</u>

Result:

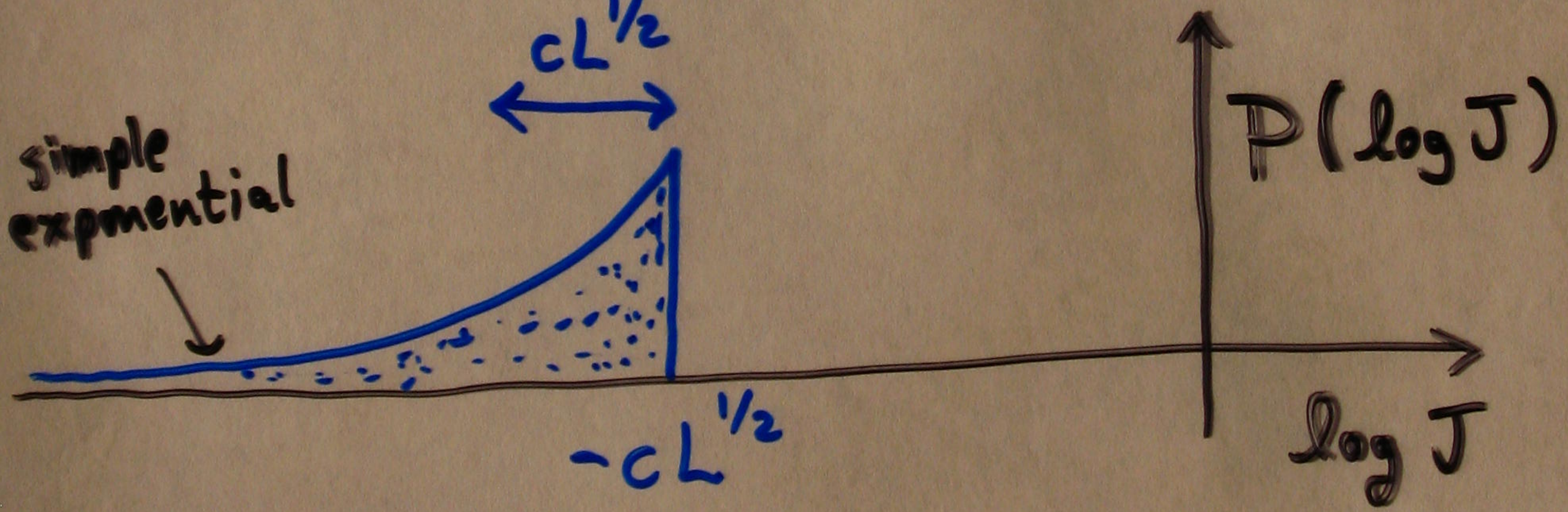
probabillity distribution $P(J)$

is renormalized.

It broadens as we go to low E,

making perturbative approx. improve.

R.G. "flow" to ∞-randomness:

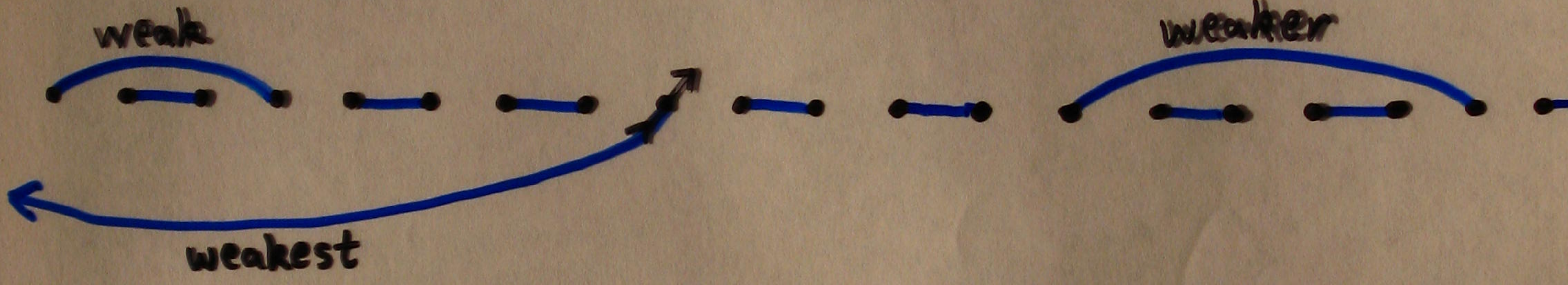


when density of spins left is $1/L$.

remaining renormalized J's $\sim e^{-cL^{1/2}}$.

(L = avg. distance between remaining spins: domain size)

Random singlet ground state:



Spins pair into singlets •—• on all length and energy scales.

At energy $E = e^{-cL^{1/2}}$

density of ~~unpaired~~ spins is

$$\frac{1}{L} \approx \frac{c^2}{(\log E)^2}$$

Tippie + Clark
Q-TCNQ
(1981)

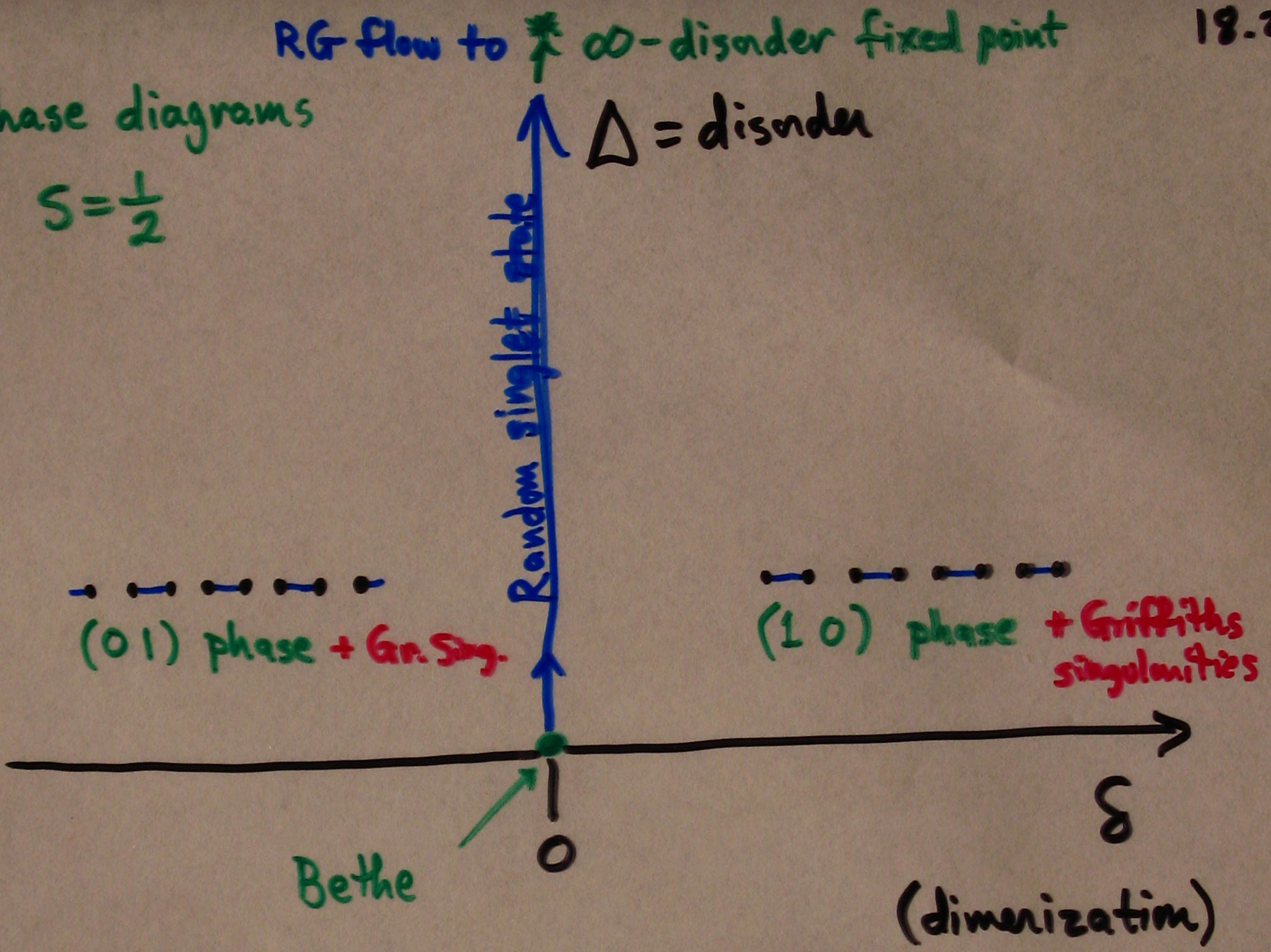
At temperature T: magnetic susceptibility

$$\chi \sim \frac{1}{T}\frac{1}{L} \sim \frac{1}{T(\log T)^2}$$

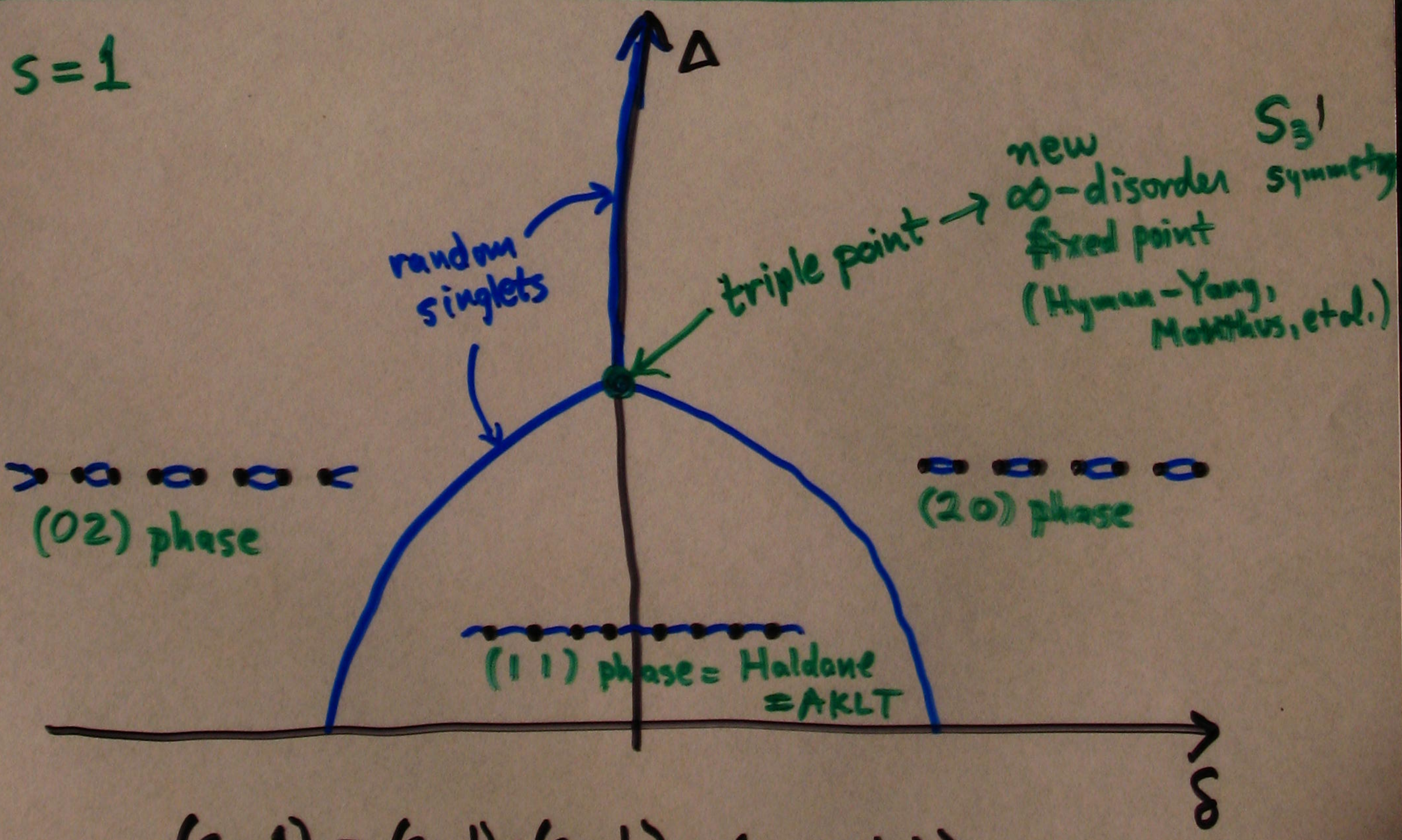
diverges as $T \to 0$.

phase diagrams

$S=\frac{1}{2}$



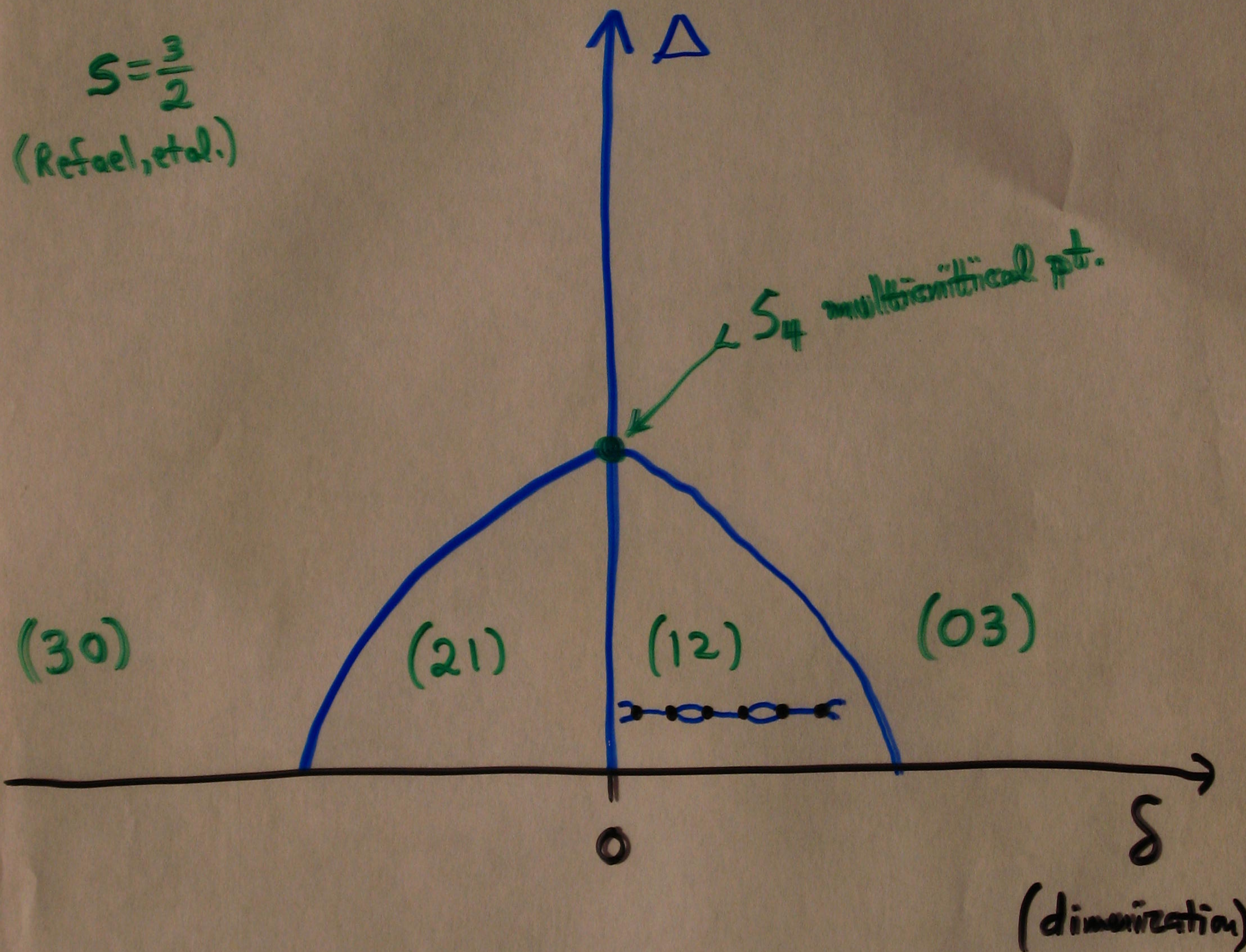
$s=1$



$(s=1) = (s=\frac{1}{2}) + (s=\frac{1}{2})$ (symmetric)

$S = \frac{3}{2}$

(Refael, et al.)



$$H = \sum_i J_i \vec{S}_i \cdot \vec{S}_{i+1}$$

$\Delta = \mathrm{var}(\log J_i)$

$\delta = \overline{\log J_{odd}} - \overline{\log J_{even}}$

blue lines all random-singlet states.

Other stable ∞-randomness fixed points:

– spin-$\frac{1}{2}$ chain with exchange anisotropy (D Fisher)

– Haldane/random-singlet transition in Spin-1 chain • show p. diags.
(Hyman + Yang, Monthus et al)

– Higher-order multicritical points for spin chains (Damle + Huse '02) •

– Quantum critical point of ~~transverse~~-field Ising model

For $d>1$ (Motrunich et al. PRB 11/1/00) QUANTUM PERCOLATION

– Hopping models with fermions (e.g. superconducting quasiparticles)

hopping on random chains + ladders + in $d=2$

(Motrunich, Damle, Huse, PRB's + ~~in preparation~~) cond-mat/0107582

⋮

more?

Currently exploring quantum entanglement in these critical ground states $S_E \approx \sigma \log L$ (for $d=1$)

pure: $\sigma = \frac{c}{6}$ (Wilczek + co.)

random: ?

For strong-randomness RG to work need:

Hamiltonian to remain "simple" after decimations:

- Hilbert space of local degrees of freedom remains small
- Only a finite set of types of interactions are generated (typically only one- and two-site terms)

So far, this has succeeded with - various spin models,

and for - spinless lattice fermions* with particle-hole symmetry and repulsive interactions.

* e.g., ~~...~~ quasiparticles in a superconductor with spin-orbit int'ns.

- insulating states with variable-range-hopping can be treated within this framework, also. (e.g., "Bose glass phase")

¿ Will it be useful for metal / insulator / superconductor or quantum Hall transitions??

¿ Can more complicated Hilbert spaces / Hamiltonians be dealt with on the computer??