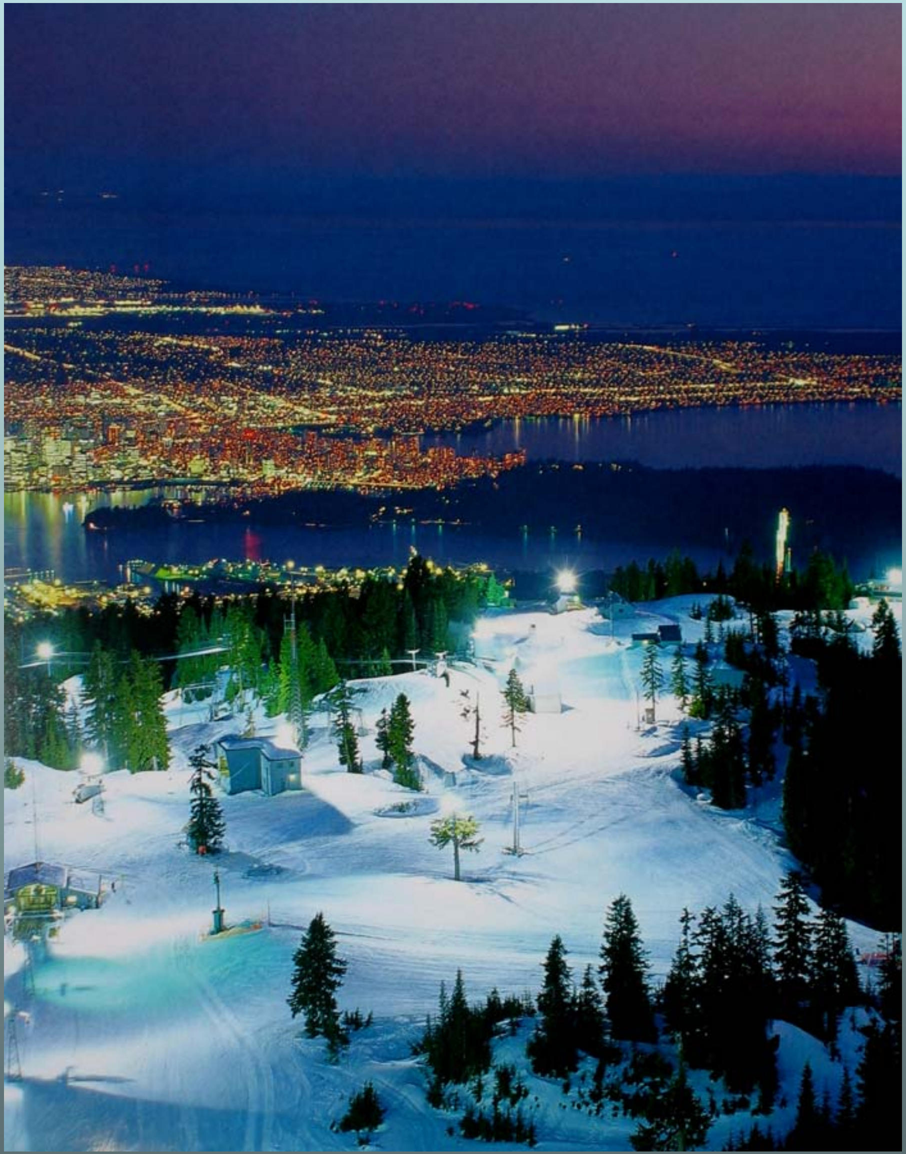
PCE STAMP

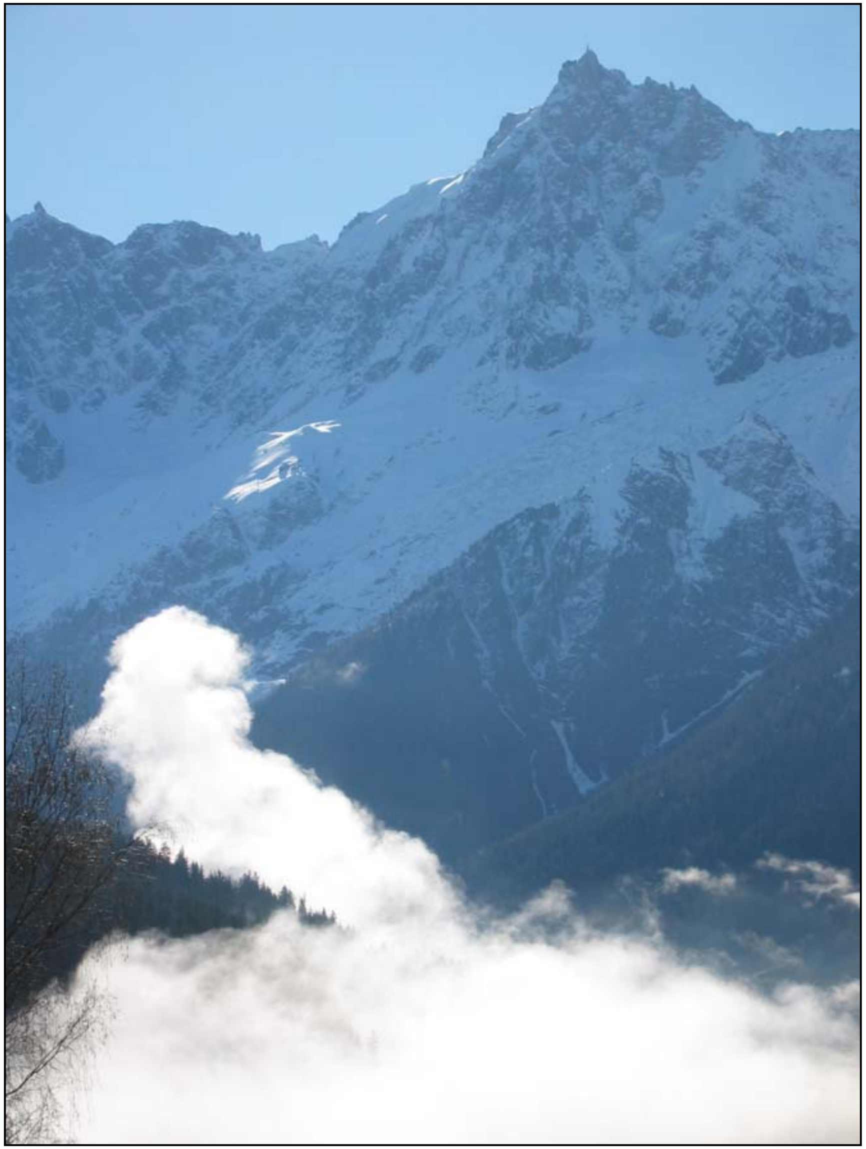
Physics & Astronomy
UBC
Vancouver



Pacific Institute
for
Theoretical Physics

http://pitp.physics.ubc.ca/index.html

PITP/Les Houches Summer School on QUANTUM MAGNETISM



http://pitp.physics.ubc.ca/upcoming/leshouches/
then go to "archives" for the lecture notes.

TOPIC: LARGE SCALE QUANTUM PHENOMENA in MAGNETS

LECTURER: PCE STAMP (UBC & PITP)

LECTURE 1: INTRODUCTION. The important questions, challenges, & mysteries. The important physical systems, important theoretical models, & their quantitative derivation. Key milestones

LECTURE 2: MODELS OF Q. ENVIRONMENT. Derivation of these models. Oscillator & spin baths. Qubit and spin net models. More abstract models. Mappings to other models.

LECTURE 3: DECOHERENCE. Dynamics of key models (Q Relaxation and Decoherence). Some useful techniques. Application to experiments in magnetic & other systems. Some key exptl. predictions

LECTURE 4: LATTICE & FIELD THEORIES. A general class of lattice models for spin nets and QUIP systems; mapping to field theories; results for phase diagram & dynamics. Connection to expts, & to topics in string theory.

LECTURE 1: INTRODUCTION and EFFECTIVE HAMILTONIANS

1.1: INTRODUCTORY REMARKS and MAIN THEMES

To set the scene for this course one needs to clarify some of the important distinguishing features of large-scale quantum phenomena in magnetic systems. One crucial feature differentiates magnetic systems from superfluids or superconductors – spin current, unlike mass or charge current, is not conserved. This makes it harder to see quantum effects in transport phenomena than in superfluids (see however the lectures of Haldane, MacDonald, and Zhang). However magnetic systems provide new quantum effects, absent in superfluids, connected with spin phase- these are central to many of the lectures in this school. In this course we are interested particularly in subtle dynamical phenomena involving the quantum phases of many spins, & one has to be very careful about applying traditional ideas like linear response to analyse these.

There is now a wealth of interesting experiments in this field, to be discussed in the 2nd and 3rd lectures (see also the lectures of Aeppli, Barbara, Christou, & Osheroff). One of the key theoretical questions is how one can force magnetic systems to display large-scale quantum phenomena in the face of strong phase decoherence. This necessitates a proper theoretical understanding of decoherence, which is far less trivial than one would guess from much of the literature.

SOME of the IMPORTANT QUESTIONS

1. LARGE-SCALE QUANTUM PHENOMENA in MAGNETIC SYSTEMS: interference, entanglement, coherence in the dynamics of spin, spin current, solitons, etc., in phenomena involving tunneling spins, spin networks, etc.

2. LOW -T EFFECTIVE HAMILTONIANS: Fundamental understanding of relevant interactions in complex quantum systems – particularly at very low E and/or T, & where multi-spin interactions are involved.

3. DECOHERENCE: Fundamental understanding of mechanisms of decoherence & how this translates into disentanglement dynamics. Touches upon basic issues in non-equilibrium physics, in stat mech, and in the interpretation of QM. Low-T spin dynamics in quantum regime, dynamics of nets, etc. A general field theoretical framework for decoherence?

4. SOLID-STATE Q. INFORMATION PROCESSING: A general understanding of "QUIP", in terms of models of spin nets and/or Quantum walks. Relevance of errors & error correction. Decoherence & QUIP. QUIP in magnetic systems?

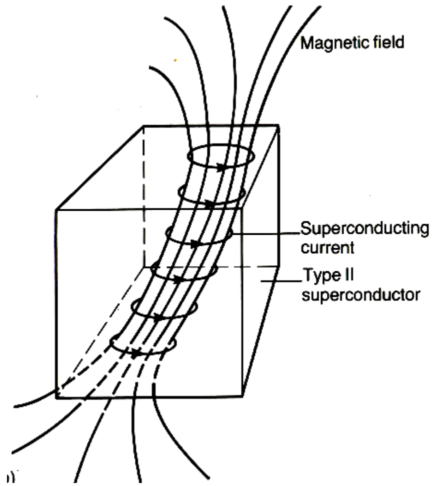
5. EXPERIMENTS on COHERENCE, ENTANGLEMENT, & DECOHERENCE: Experiments & predictions in magnetic systems (wires, molecules, quantum spin glasses, other spin networks, plus SQUIDs & ion traps. What do they/can they show?

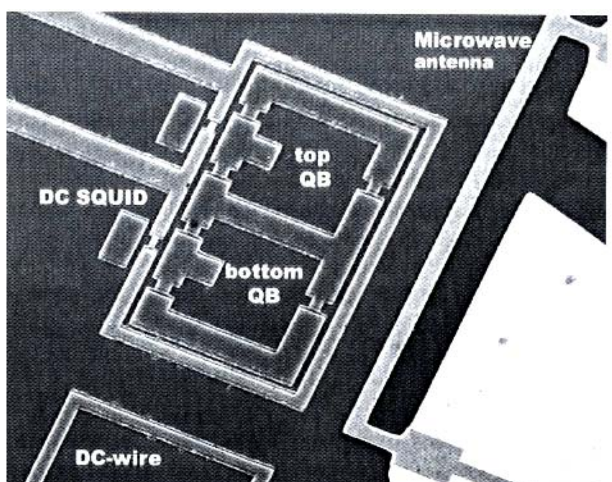
6. MATERIALS & APPLICATIONS: What kinds of material should we be looking at, and how to make 'designer quantum devices'? How can we suppress decoherence, what things can be built and how?

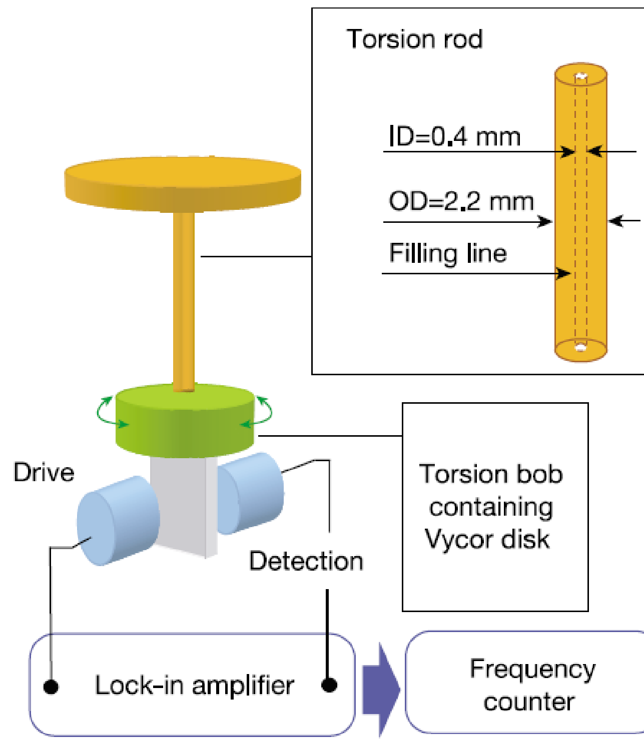
LARGE – SCALE QUANTUM PHENOMENA

We are all accustomed to the idea of macroscopic quantum phenomena in superfluids and superconductors- things like superflow, the Meissner and Hess-Fairbank effects, the Josephson effects, etc.- and most dramatically, macroscopic quantum tunneling and coherence in SQUIDs. But can one envisage large-scale quantum phenomena in any other system?

NB: the initial discovery of these phenomena severely challenged theory. We now think we understand them using effective Hamiltonians & order parameters.







A BASIC PROBLEM in the FIELD

The standard apparatus of STATISTICAL MECHANICS, RENORMALISATION GROUP, and EFFECTIVE HAMILTONIANS (along with things like linear response theory, 1- and 2-particle correlation functions, critical points, etc.) works very well for static phenomena, and for systems which are near equilibrium and only weakly perturbed.

It is NOT adequate when we have to deal with

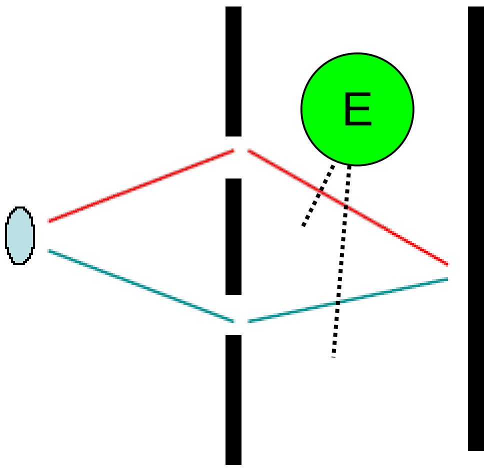
(1) The dynamics of quantum phases – particularly when we are dealing with multi-particle entanglement phenomena. In this case very small interactions can play a crucial role, as can initial conditions, boundary conditions, etc. The separation of energy scales inherent in RG may be neither useful nor even applicable. We shall see this explicitly in some of the models.

(2) Systems far from equilibrium (or where equilibrium is irrelevant to the dynamics). As one lowers the temperature of a system, and/or progressively decouples it from its surroundings, it becomes ever harder to keep it in equilibrium with any bath – often we DON'T WANT TO. Moreover the bath itself is subject to manipulation, and is often time-dependent in non-trivial ways.

So – we need to keep a sharp eye on

(a) very low energy excitations & weak couplings
(b) the detailed quantum dynamics of the bath
(c) multi-particle (or multi-spin) phases.

Question: WHAT is "DECOHERENCE" ?



When some quantum system with coordinate Q interacts with any other system (with coordinate x) , the result is typically that they form a combined state in which there is some entanglement between the two systems.

Example: **In a 2-slit expt., the particle coordinate Q couples to photon coordinates, so that we have the following possibility:**

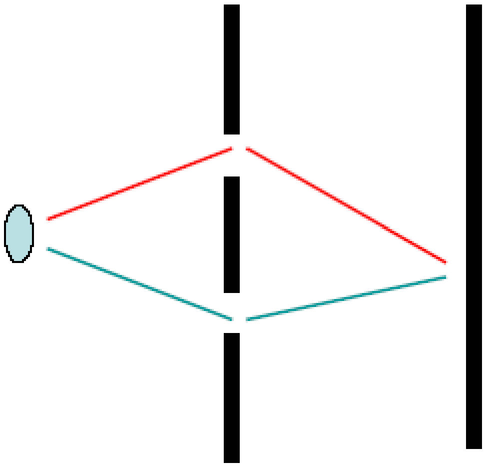
$$\Psi_o(Q)\ \Pi_q\, \phi_q^{in} \rightarrow [a_1\, \Psi_1(Q)\, \Pi_q\, \phi_q^{(1)} + a_2\, \Psi_2(Q)\, \Pi_q\, \phi_q^{(2)}$$

But now suppose we do not have any knowledge of, or control over, the photon states- we m average over these states, in a way consistent with the experimental constraints. In the extre case this means that we lose all information about the PHASES of the coefficients a_1 & a_2 (a particular the relative phase between them). This process is called DECOHERENCE

NB 1: **In this interaction between the system and its "Environment" E (which is in effect per measurement on the particle state),** *there is no requirement for energy to be exchanged between the system and the environment-* **only a communication of phase information.**

NB 2: **Nor is it the case that the destruction of the phase interference between the 2 paths n associated with a noise coming from the environment- what matters is that the state of the environment be CHANGED according to the what is the state of the system. In fact,** *noise is necessary nor sufficient for decoherence***, except under rather special circumstances.**

WARNING: 3rd PARTY DECOHERENCE

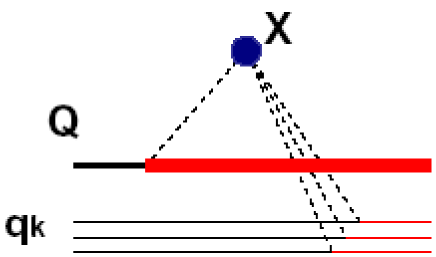


This is fairly simple- it is decoherence in the dynamics of system A (coordinate Q) caused by *indirect* entanglement with an environment E- the entanglement is achieved via 3rd party B (coordinate X).

Ex: Buckyball decoherence

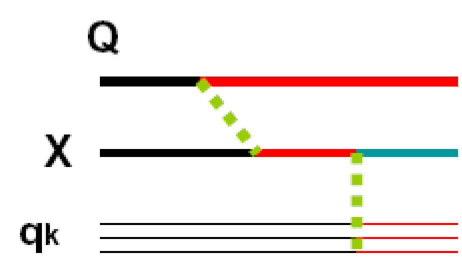
Consider the 2-slit expt with buckyballs. The COM coordinate Q of the buckyball does not couple directly to the vibrational modes {q_k} of the buckyball- by definition. However BOTH couple to the slits in the system, in a distinguishable way.

Note: the state of the 2 slits, described by a coordinate X, is irrelevant- it does not need to change at all. We can think of it as a scattering potential, caused by a system with infinite mass (although recall Bohr's response to Einstein, which includes the recoil of the 2 slit system). It is a PASSIVE 3rd party.



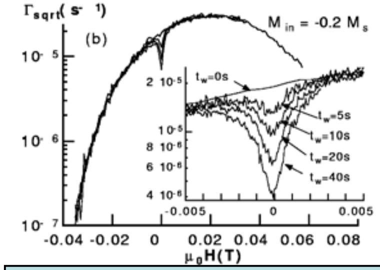
ACTIVE 3rd PARTY: **Here the system state correlates with the 3rd party, which then goes on to change the environment to correlate with Q. We can also think of the 3rd party X as PREPARING the states of both system and environment. Alternatively we can think of the system and the environment as independently measuring the state of X. In either case we see that system and environment end up being correlated/entangled.**

Note the final state of X is not necessarily relevant- it can be changed in an arbitrary way after the 2nd interaction of X. Thus X need not be part of the environment. Note we could also have more than one intermediary- ie., X, Y, etc.- with correlations/entanglement are transmitted along a chain (& they can wiped out before the process is finished).

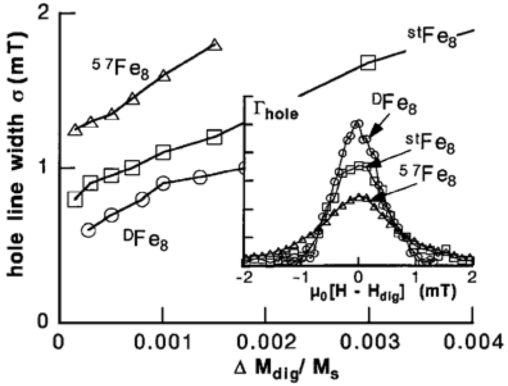


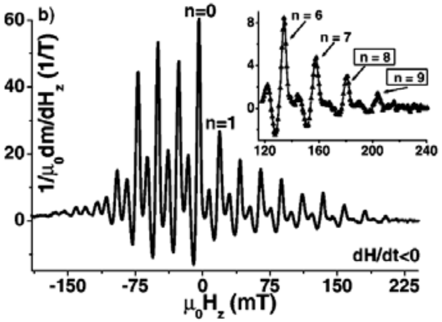
SOME EXPTS

RIGHT: Expts on Tunneling magn. molecules & Ho ions



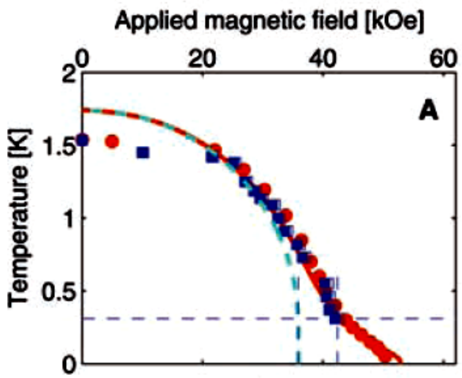
Wernsdorfer et al, PRL 82, 3903 (1999); and
PRL 84, 2965 (2000); and
Science 284, 133 (1999)

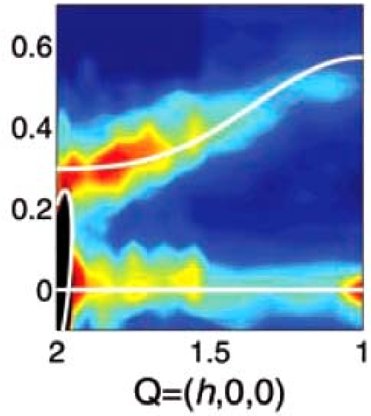


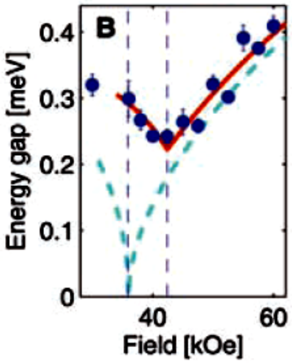


R. Giraud et al., PRL 87, 057203 (2001)

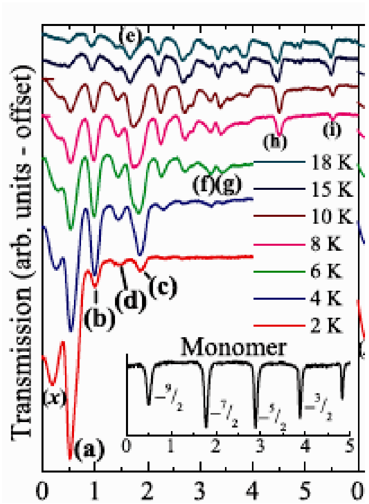
Expts on the quantum phase transition in $LiHoF_4$







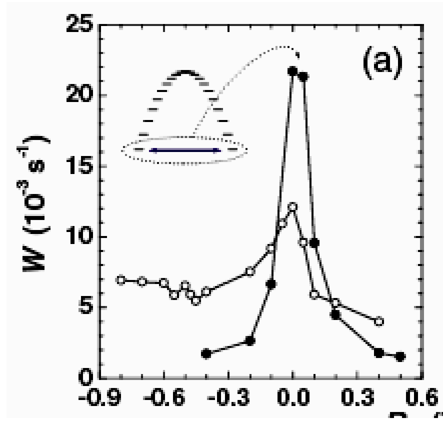
H.M. Ronnow et al., Science 308, 389 (2005)



LEFT: ESR on Mn dimer system

S Hill et al, Science 302, 1015 (2003)

RIGHT: NMR on Mn-12 tunneling molecules



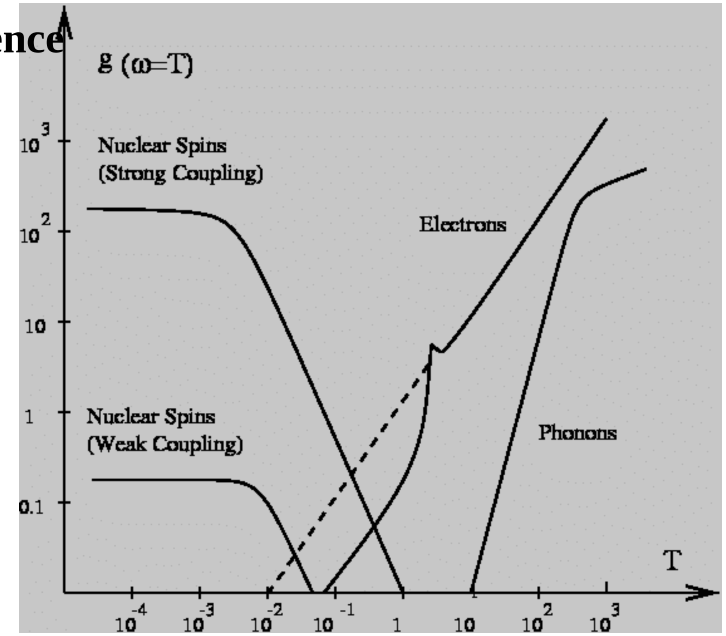
A. Morello et al., PRL 93, 197202 (2004)

EXPERIMENTS in Superconducting Qubits

(1) The oscillator bath (electrons, photons, phonons) decoherence rate:

$$\tau_\phi^{-1} \sim \Delta_o \; g(\Delta,T) \coth(\Delta/2kT)$$

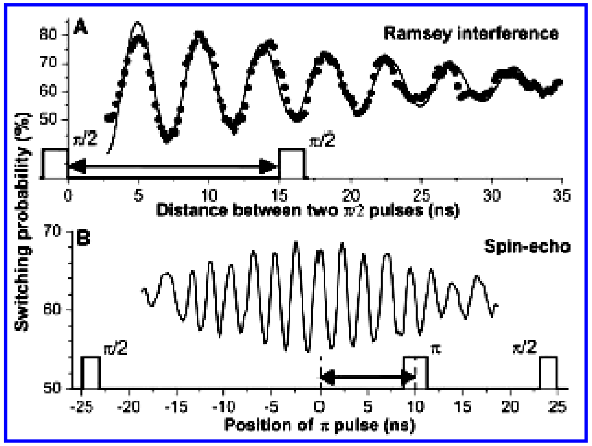
(Caldeira-Leggett). This is often many orders of magnitude smaller than the experimental decoherence rates.



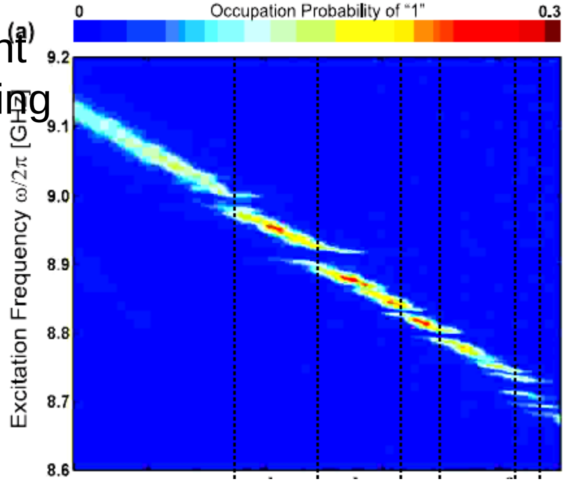
(2) The spin bath decoherence will be caused by a combination of charge & spin (nuclear & paramagnetic) defects- in junction, SQUID, and substrate.

$$1/\tau_\phi = \Delta_o \; (E_o/8\Delta_0)^2$$

The basic problem with any theory-experiment comparison here is that most of the 2-level systems are basically just junk (coming from impurities and defects), whose characteristics are hard to quantify. Currently 8 groups have seen coherent oscillations in superconducting qubits, and 2 have seen entanglement between qubit pairs.



I Chiorescu et al., Science 299, 1869 (2003)



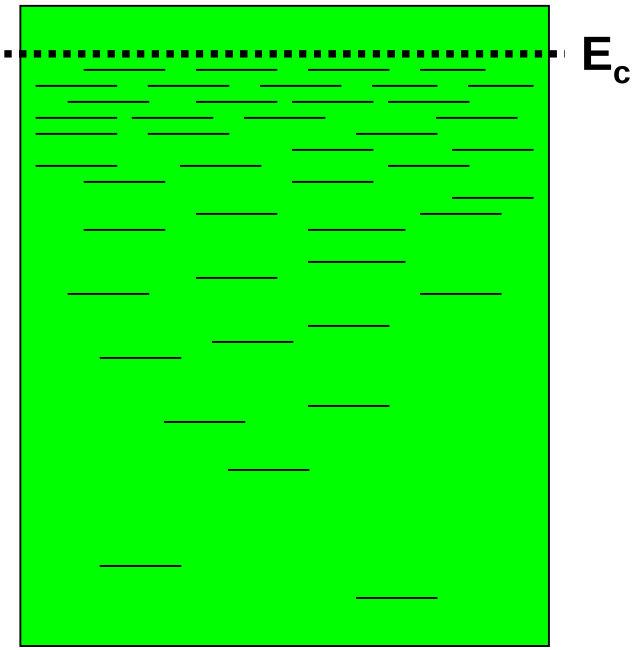
RW Simmonds et al., PRL 93, 077003 (2004)

1.2: GENERAL REMARKS on EFFECTIVE HAMILTONIANS

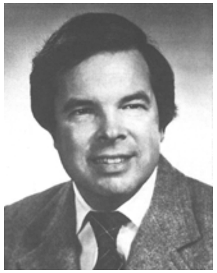
In the usual discussions of effective Hamiltonians, one discusses a hierarchy of energy scales, and proceeds to ‘integrate out’ the high-energy modes down to the energy scale of interest, in order to derive an effective Hamiltonian. This is done either perturbatively, using renormalisation group theory (or some equivalent), or by separating out ‘fast’ and ‘slow’ modes (using, eg., a Born-Oppenheimer procedure). The assumption is that such a ‘renormalisation’ procedure does not miss out any of the physics (a highly questionable assumption in general).

One easily enumerates the energy scales in magnetic systems, & derive effective Hamiltonians using standard methods (see, eg., lectures of Aharony & Sawatzky) if one is trying to derive phase diagrams and other macroscopic equilibrium properties. However for the dynamics it has serious shortcomings, because very low-energy excitations play a crucial role, & because at low T the systems are usually very far from equilibrium. The key low-energy excitations in magnetic (& other solid state) systems are localised – defects, and localised spin excitations (paramagnetic, nuclear, etc), and these typically behave as a bath of two-level systems (the ‘spin bath’). The usual ideas of linear response and fluctuation – dissipation theorems are often very badly wrong.

Of great current interest is the study of effective Hamiltonians describing ‘spin nets’ of ‘qubits’ interacting with spin and oscillator baths. More exotic models are also of interest, particularly those describing magnetic solitons, or those relevant to quantum computation, or to topological quantum fluids.



Orthodox view of $\mathcal{H}_{\text{eff}}$



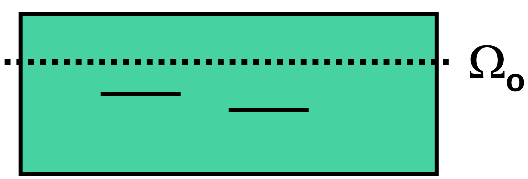
The RG mantra:

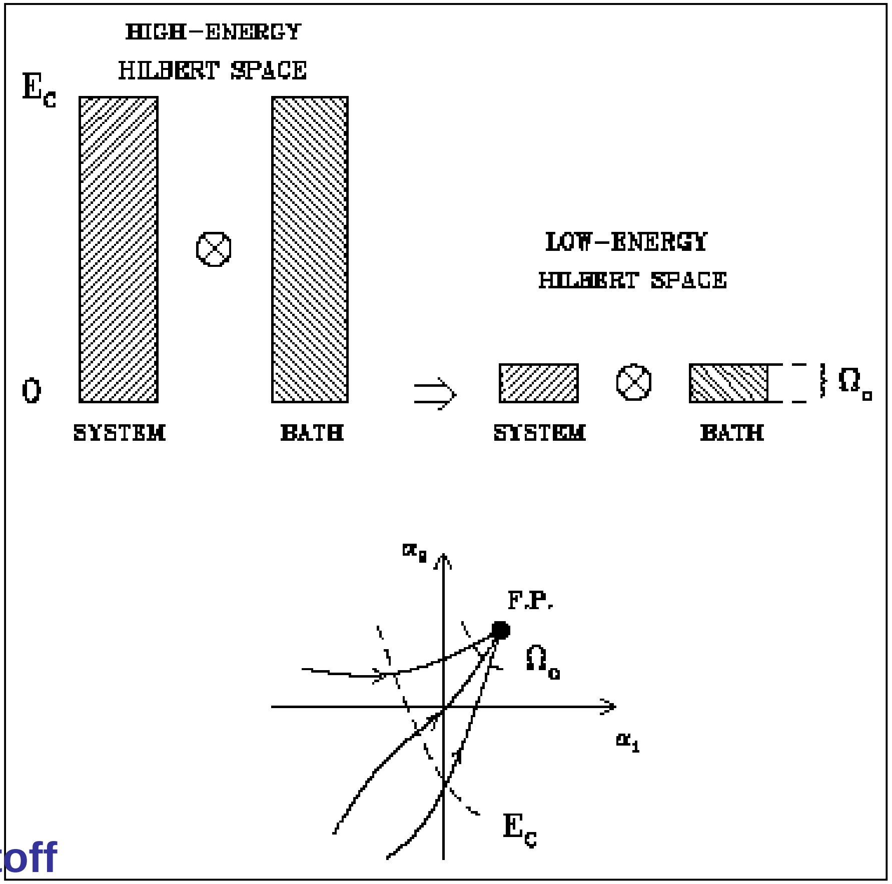
RG flow & fixed points

low-energy $\mathcal{H}_{\text{eff}}$

universality classes

Scale out High-E modes



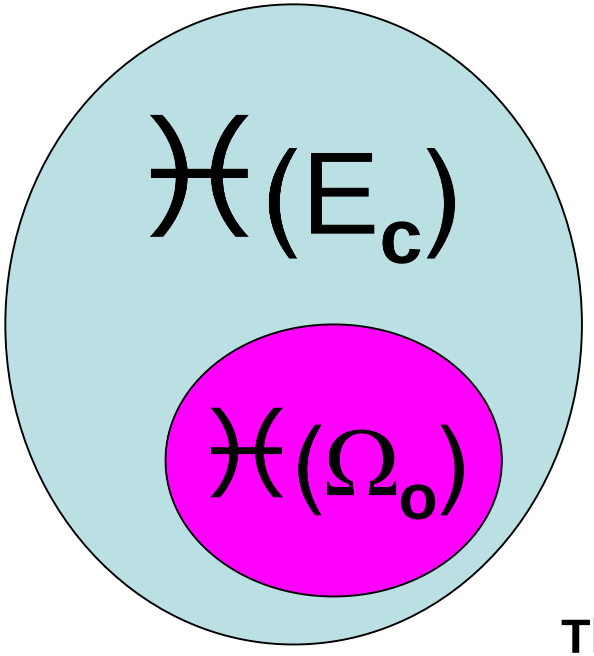


$$\mathcal{H}_{\text{eff}}(E_c) \rightarrow \mathcal{H}_{\text{eff}}(\Omega_o)$$

$$|\psi_i\rangle\ \mathbf{H_{ij}(E_c)}\ \langle\psi_j| \quad \rightarrow \quad |\phi_\alpha\rangle\ \mathcal{H}_{\alpha\beta}(\Omega_o)\ \langle\phi_\beta|$$

Flow of Hamiltonian & Hilbert space with UV cutoff

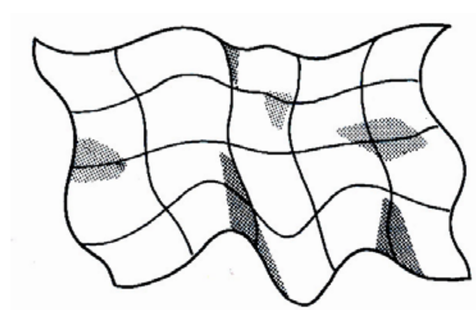
MORE ORTHODOXY

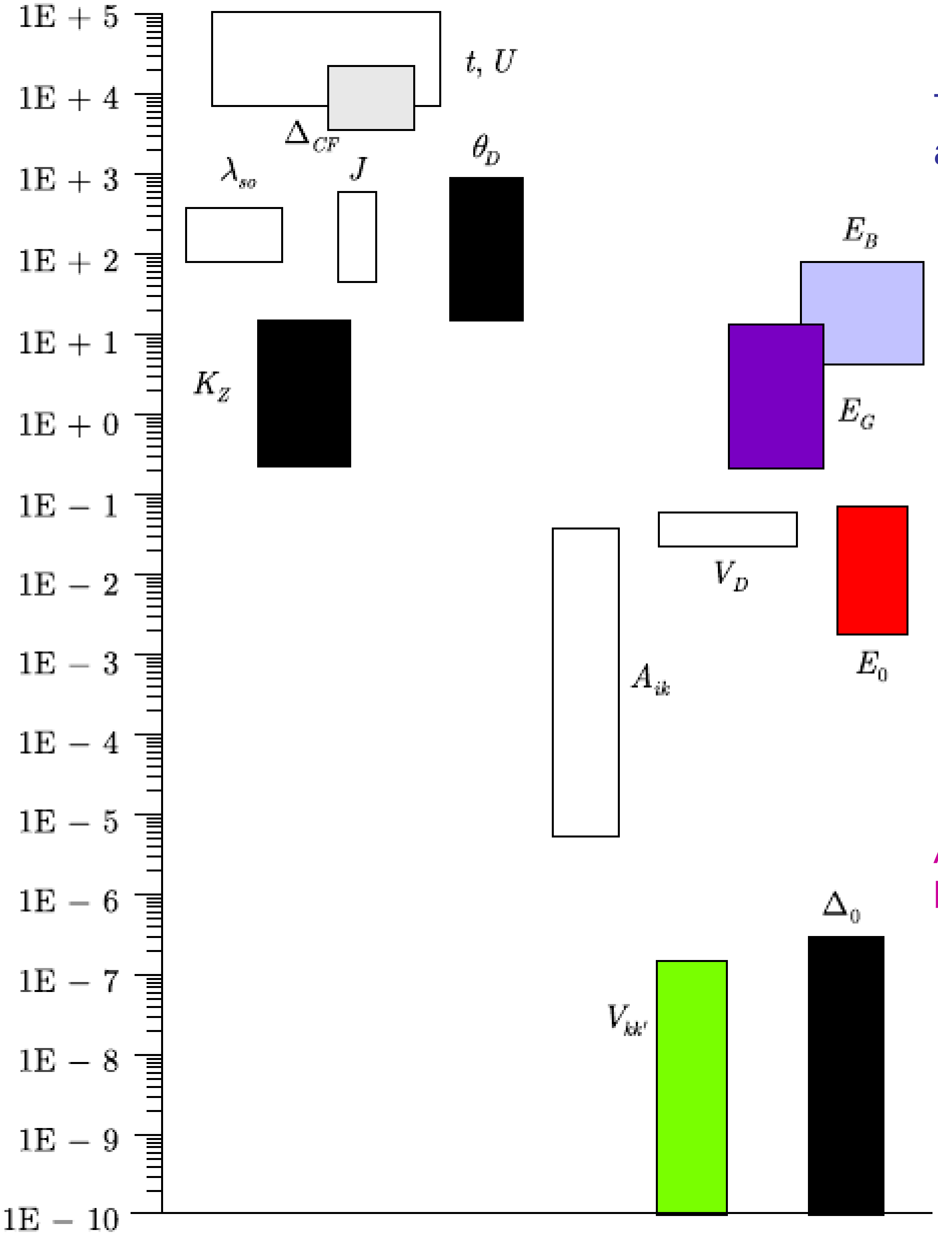


Continuing in the orthodox vein, one supposes that for a given system, there will be a sequence of Hilbert spaces, over which the effective Hamiltonian and all the other relevant physical operators (NB: these are effective operators) are defined.

Then, we suppose, as one goes to low energies we approach the '*real vacuum';* the approach to the fixed point tells us about the excitations about this vacuum. This is of course a little simplistic- not only do the effective vacuum and the excitations change with the energy scale (often discontinuously, at phase transitions), but the effective Hamiltonian is in any case almost never one which completely describes the full N-particle states.

Nevertheless, most believe that the basic structure is correct - that the effective Hamiltonian (*& note that ALL Hamiltonians or Actions are effective)* ***captures all the basic physics***





MICROSCOPIC ENERGY SCALES in MAGNETS

The standard electronic coupling energies are (shown here for Transition metals):

Band kinetic & interactions:	t, U
Crystal field:	D_{CF}
Exchange, superexchange	J
Spin-orbit:	l_{so}
Magnetic anisotropy	K_Z
inter-spin dipole coupling	V_D
p/m impurities (not shown)	J, T_K

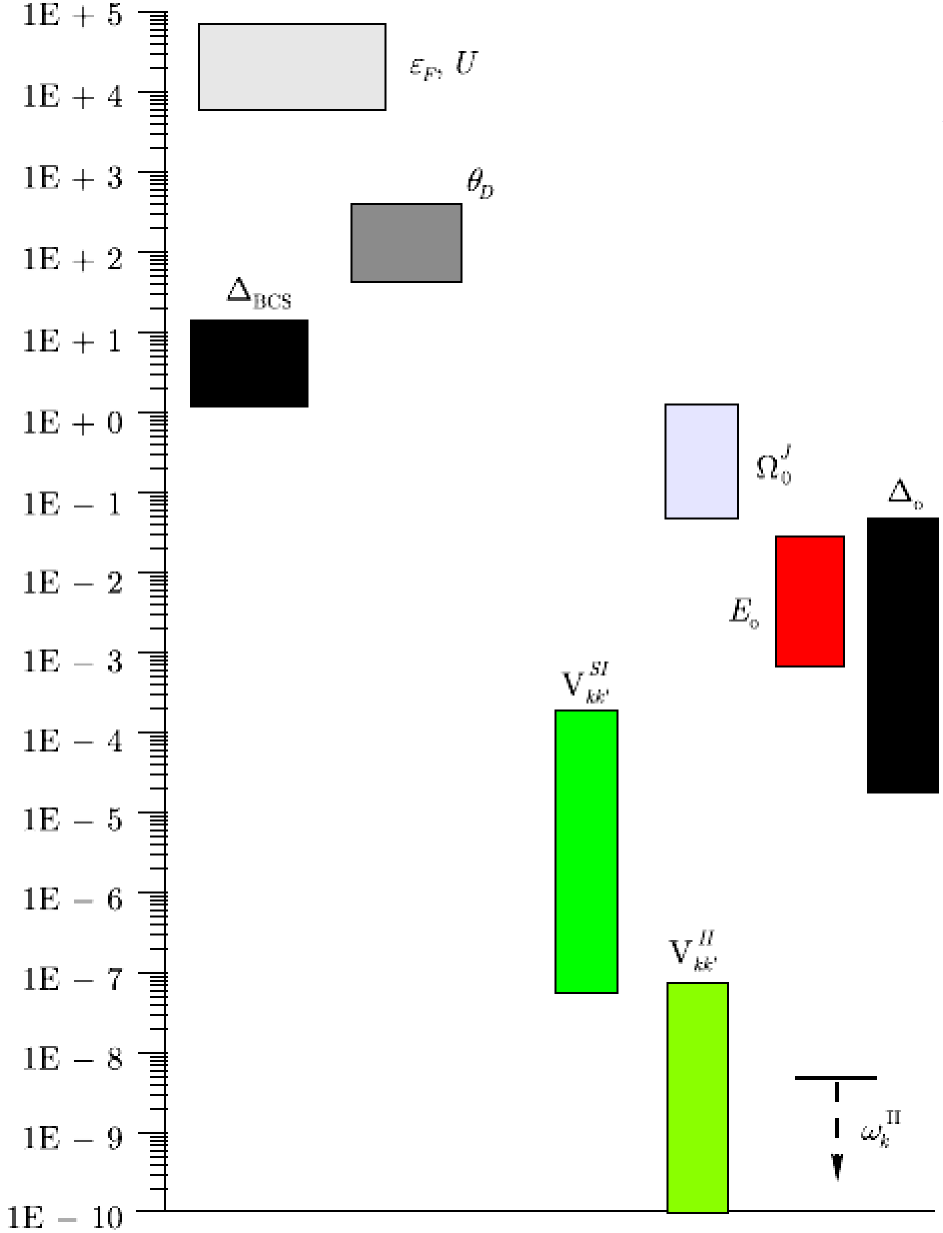
which for large spin systems lead to

Anisotropy barriers:	$E_B \sim EK_Z$
small oscillation energies	$E_G \sim K_Z$
Spin tunneling amplitude	Δ_0

Also have couplings to various "thermal baths", with energy scales:

Debye frequency:	θ_D
Hyperfine couplings	A_{ik}
Total spin bath energy	$E_0 \sim N^{1/2}\omega_k$
Inter-nuclear couplings	$V_{kk'}$

NOTE: all of these are parameters in effect Hamiltonians for magnets at low T.



ENERGY SCALES in SUPERCONDUCTORS

Again one has a broad hierarchy of energy scales (here shown for conventional s/c):

electronic energy scales:	U, ε_F, (or t)
phonon energies:	θ_D
Gap/condensation energy	Δ_{BCS}
p/m impurities (not shown)	J, T_K
Coupling of $\psi(r)$ to spins:	ω_k
Total coupling to spin bath:	$E_0 \sim N^{1/2}\omega_k$

A superconducting device has other energy scales – eg., in a SQUID:

Josephson plasma energy	$\Omega_0{}^J$
Tunneling splitting	Δ_0

These are of course not all the energies that can be relevant in a superconductor. However we note that in general magnetic systems have a more complex hierarchy of interactions than a superconductor.

REDUCTION to a LOW-ENERGY OSCILLATOR BATH FORM

Quantum Dynamics

Classical Dynamics

$\mathcal{H}_{\mathrm{eff}}$

Suppose we want to describe the dynamics of some quantum system in the presence of decoherence. As pointed out by Feynman and Vernon, if the coupling to all the enevironmental modes is WEAK, we can map the environment to an 'oscillator bath, giving an effective Hamiltonian like:

$$H_{\mathrm{eff}}(\Omega_0) = H_0(P,Q) + \sum_{q=1}^{N} [F_q(P,Q)x_q + G_q(P,Q)p_q] + \frac{1}{2}\sum_{q=1}^{N}\left(\frac{p_q^2}{m_q} + m_q\omega_q^2 x_q^2\right)$$

A much more radical argument was given by Caldeira and Leggett- that for the purposes of the predictions of QM, one can pass between the classical and quantum dynamics of a quan system in contact with the environment via H_{eff}. Then, it is argued, one can connect the class dissipative dynamics directly to the low-energy quantum dynamics, even in the regime where quantum system is showing phenomena like tunneling, interference, coherence, or entanglement; and even where it is MACROSCOPIC.

This is a remarkable claim because it is very well-known that the QM wave-function is far richer than the classical state- and contains far more information.

Feynman & Vernon, Ann. Phys. 24, 118 (1963)

Caldeira & Leggett, Ann. Phys. 149, 374 (1983)

AJ Leggett et al, Rev Mod Phys 59, 1 (1987

CONDITIONS for DERIVATION of OSCILLATOR BATH MODELS

Starting from some system interacting with an environment, we want an effective low-energy Hamiltonian of form

$$H_{\text{eff}}(\Omega_0) = H_0(P,Q) + \sum_{q=1}^{N}[F_q(P,Q)x_q + G_q(P,Q)p_q] + \frac{1}{2}\sum_{q=1}^{N}\left(\frac{p_q^2}{m_q} + m_q\omega_q^2 x_q^2\right)$$

(1) PERTURBATION THEORY

Assume environmental states $\phi_\alpha(\mathbf{X})$ and energies ϵ_α

The system-environment coupling is $V(Q,\mathbf{X})$

Weak coupling: $|V_{\alpha\beta}| \ll |(\epsilon_\alpha - \epsilon_\beta)|$ where $V_{\alpha\beta} = \int d\mathbf{X}\, \phi_\alpha^*(\mathbf{X})\, V(Q,\mathbf{X})\, \phi_\beta(\mathbf{X})$

In this weak coupling limit we get oscillator bath with $\omega_q \equiv (\epsilon_\alpha - \epsilon_\beta)$
and couplings $F_q(Q) = V_q(Q)$

(2) BORN-OPPENHEIMER (Adiabatic) APPROXIMATION

Suppose now the couplings are not weak, but the system dynamics is SLOW, ie., Q changes with a characteristic low frequency scale E_o . We define slowly-varying environmental functions as follows:

Quasi-adiabatic eigenstates: $\tilde{\phi}_\alpha(\mathbf{X},Q)$ Quasi-adiabatic energies: $\tilde{\epsilon}_\alpha(Q)$

'Slow' means $E_o \ll \tilde{\epsilon}_\alpha$

Then define a gauge potential $iA_{\alpha\beta} = \int d\mathbf{X}\, \tilde{\phi}_\alpha^*(\mathbf{X})\, \partial/\partial Q\, \tilde{\phi}_\beta(\mathbf{X})$

We can now map to an oscillator bath if $|A_{\alpha\beta}| \ll |(\tilde{\epsilon}_\alpha - \tilde{\epsilon}_\beta)|$

Here the bath oscillators have energies $\omega_q \equiv (\tilde{\epsilon}_\alpha - \tilde{\epsilon}_\beta)$

and couplings $F_q(P,Q) = \omega_q^2 \int_0^Q dQ'\, Re A_q(P,Q')$

The oscillator bath models are good for describing delocalised modes; then

$F_q(Q) \sim O(1/N^{1/2})$

(normalisation factor)

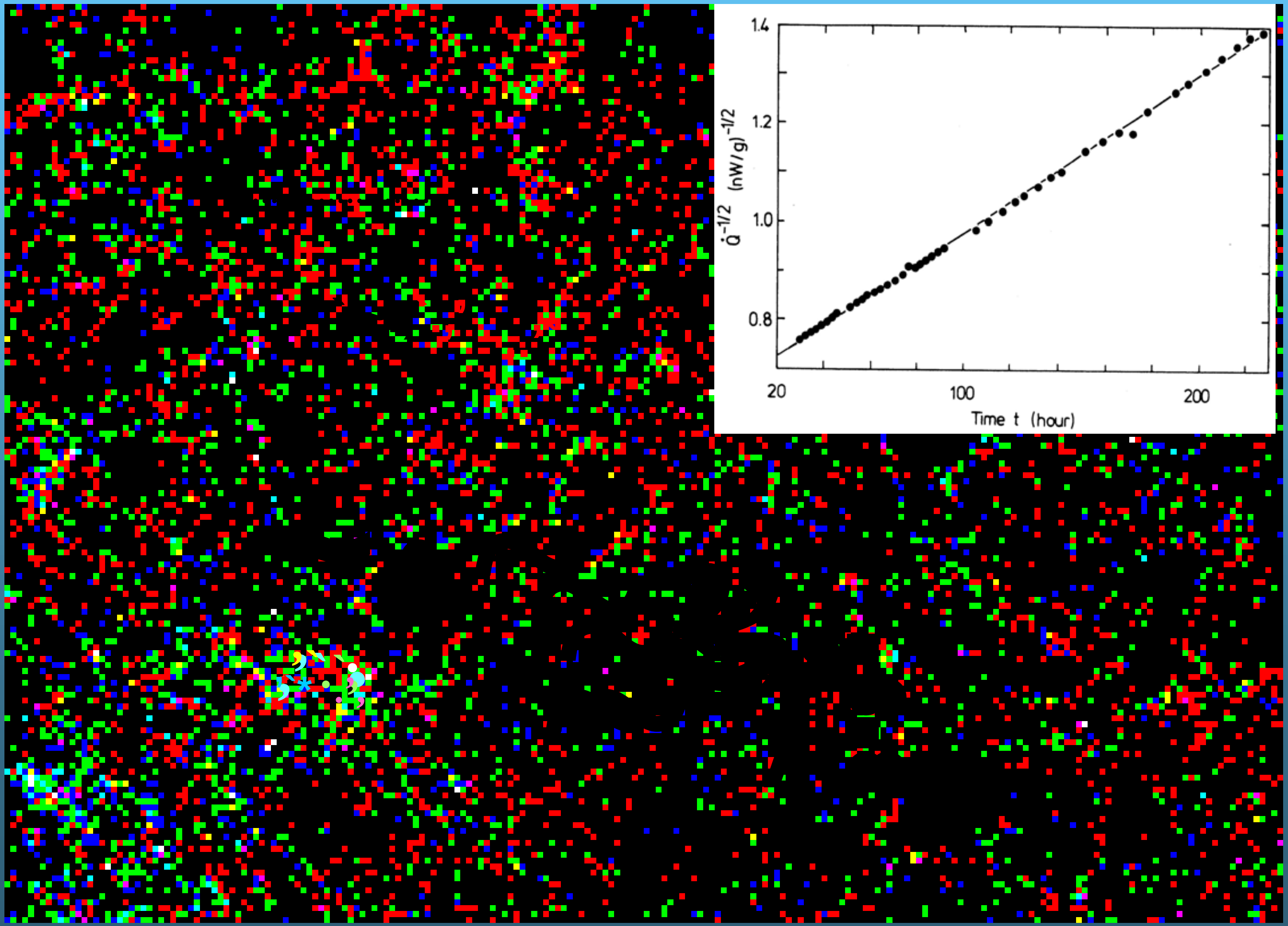
WHAT ARE THE LOW-ENERGY EXCITATIONS IN A SOLID ?

DELOCALISED

Phonons, photons, magnons, electrons, ………These always dominate at high energy/high T

LOCALISED

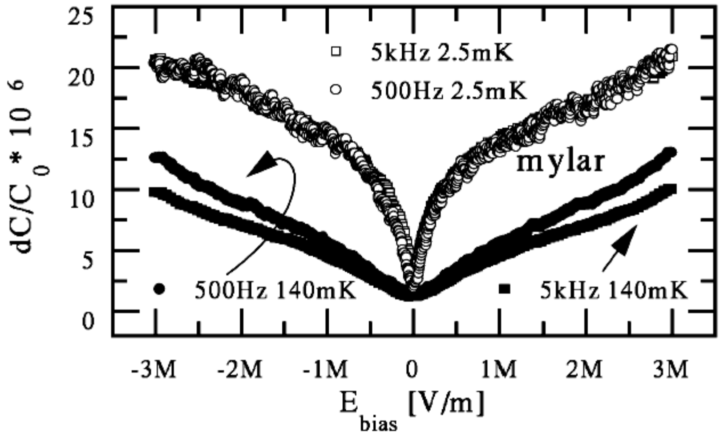
Defects,
Dislocations,
Paramagnetic impurities,
Nuclear Spins,
……. These always dominate at low T



At right- artist's view of energy distribution at low T in a solid- at low T most energy is in localised states.

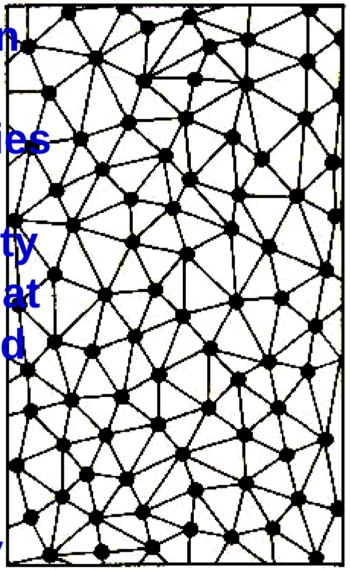
INSET: heat relaxation in bulk Cu at low T

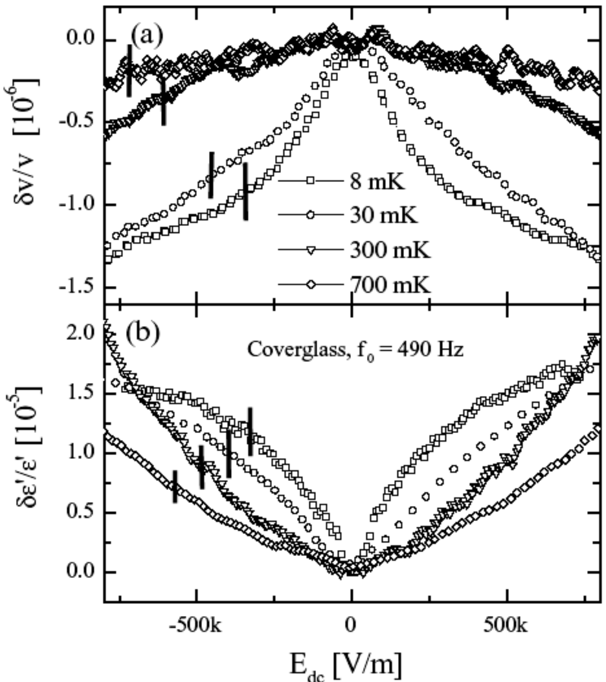
How do REAL Solids (%99.9999) behave at low Energy?

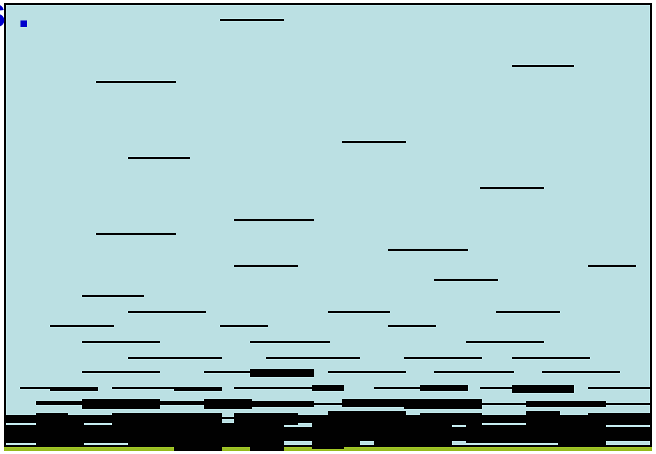


Results for Capacitance (*Above*) & Sound velocity and dielectric absorption (*Below*) for pure SiO_2 , at very low T

In almost all real solids, a combination of frustrating interactions, residual long-range interactions, and boundaries leads to a very complex hierarchy of states. These often have great difficulty communicating with each other, so that the long-time relaxation properties and memory/aging effects are quite interesting- for the system to relax, a large number of objects (atoms, spins, etc.) must simultaneously reorganise themselves . This happens even in pure systems







A model commonly used to describe the low-energy excitations (which is certainly appropriate for many of them) is the 'interacting TLS model', with effective Hamiltonian:

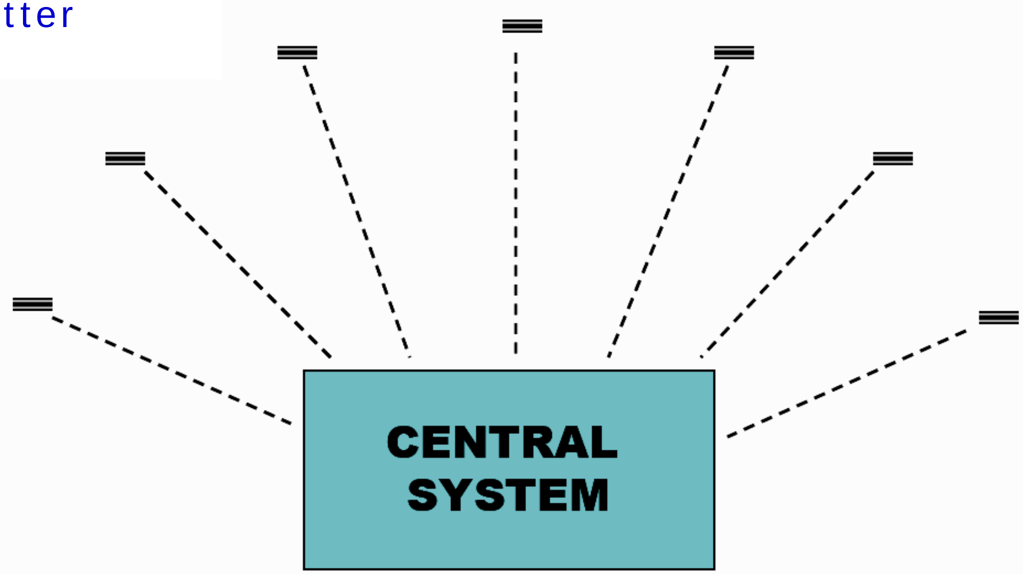
$$H^{sp}_{env} = \sum_{k}^{N_s} \mathbf{h}_k \cdot \boldsymbol{\sigma}_k + \sum_{k,k'}^{N_s} V^{\alpha\beta}_{kk'} \sigma^{\alpha}_k \sigma^{\beta}_{k'}$$

ABOVE: structure of low-energy eigenstates for interacting TLS model, before relaxation

QUANTUM ENVIRONMENTS of LOCALISED MODES

Consider now the set of localised modes that exist in all solids (and all condensed matter systems except the He liquids). As we saw before, a simple description of these on their own is given by the 'bare spin bath Hamiltonian'

$$H^{sp}_{env} = \sum_{k}^{N_s} \mathbf{h}_k \cdot \boldsymbol{\sigma}_k + \sum_{k,k'}^{N_s} V^{\alpha\beta}_{kk'} \sigma^{\alpha}_k \sigma^{\beta}_{k'}$$



where the 'spins' represent a set of discrete modes (ie., having a restricted Hilbert space). These must couple to the central system with a coupling of general form:

$$H^{sp}_{int} = \sum_{k}^{N_s} \boldsymbol{F}_k(P,Q) \cdot \boldsymbol{\sigma}_k$$

We are thus led to a general description of a quantum system coupled to a 'spin bath', of the form shown at right. This is not the most general possible Hamiltonian, because the bath modes may have more than 2 relevant levels.

$$H = H_0(P,Q) + H_{\rm int}(P,Q;\{\hat{\vec{\sigma}}\}) + H_{\rm env}(\{\hat{\vec{\sigma}}\})$$

$$H_{\rm int}(P,Q;\{\hat{\vec{\sigma}}\}) = \sum_{k=1}^{N} [F_k^{\parallel}(P,Q)\hat{\sigma}_k^z + [F_k^{\perp}(P,Q)\hat{\sigma}_k^- + \text{h.c.}]]$$

$$H_{\rm env}(\{\hat{\vec{\sigma}}\}) = \sum_{k=1}^{N} \vec{h}_k \hat{\vec{\sigma}}_k + \sum_{k=1}^{N}\sum_{k'=1}^{N} V_{kk'}^{\alpha\beta} \hat{\sigma}_k^{\alpha} \hat{\sigma}_{k'}^{\beta}$$

A qubit coupled to a bath of delocalised excitations: the SPIN-BOSON Model

Feynman & Vernon, Ann. Phys. 24, 118 (1963)
PW Anderson et al, PR B1, 1522, 4464 (1970)
Caldeira & Leggett, Ann. Phys. 149, 374 (1983)
AJ Leggett et al, Rev Mod Phys 59, 1 (1987)

U. Weiss, "Quantum Dissipative Systems" (World Scientific, 1999)

Suppose we have a system whose low-energy dynamics truncates to that of a 2-level system τ. In general it will also couple to DELOCALISED modes around (or even in) it. A central feature of many-body theory (and indeed quantum field theory in general) is that

(i) under normal circumstances the coupling to each mode is WEAK (in fact $\sim O(1/N^{1/2})$), where N is the number of relevant modes, just BECAUSE the modes are delocalised; and

(ii) that then we map these low energy "environmental modes" to a set of non-interacting Oscillators, with canonical coordinates $\{x_q, p_q\}$ and frequencies $\{\omega_q\}$.

It then follows that we can write the effective Hamiltonian for this coupled system in the 'SPIN-BOSON' form:

$$
\begin{aligned}
\mathbf{H}(\Omega_o) = \{ & [\Delta_o \tau_x + \varepsilon_o \tau_z] && \text{qubit} \\
& + {}^1/_2 \Sigma_q (p_q^2/m_q + m_q \omega_q^2 x_q^2) && \text{oscillator} \\
& + \Sigma_q [c_q \tau_z + (\lambda_q \tau_+ + \text{H.c.})] x_q \} && \text{interaction}
\end{aligned}
$$

Where Ω_o is a UV cutoff, and the $\{c_q, \lambda_q\} \sim N^{-1/2}$.

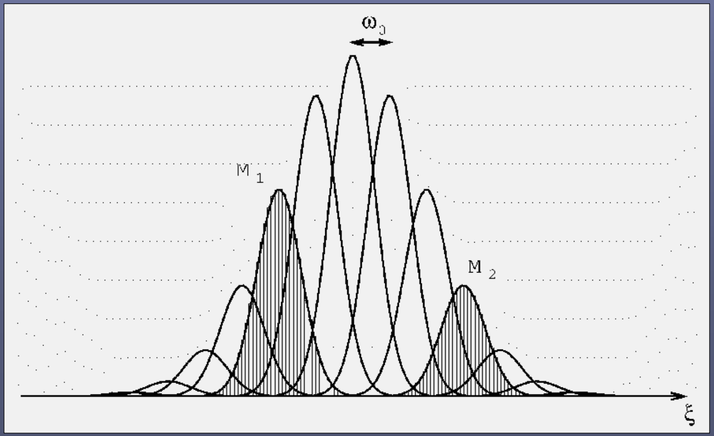
A qubit coupled to a bath of localised excitations: the CENTRAL SPIN Model

P.C.E. Stamp, PRL 61, 2905 (1988)
AO Caldeira et al., PR B48, 13974 (1993)
NV Prokof'ev, PCE Stamp, J Phys CM5, L663 (1993)
NV Prokof'ev, PCE Stamp, Rep Prog Phys 63, 669 (2000)

Now consider the coupling of our 2-level system to LOCALIZED modes.These have a Hilbert space of finite dimension, in the energy range of interest- in fact, often each localised excitation has a Hilbert space dimension 2. Our central Qubit is thus coupling to a set of effective spins; ie., to a "SPIN BATH". Unlike the case of the oscillators, we cannot assume these couplings are weak.

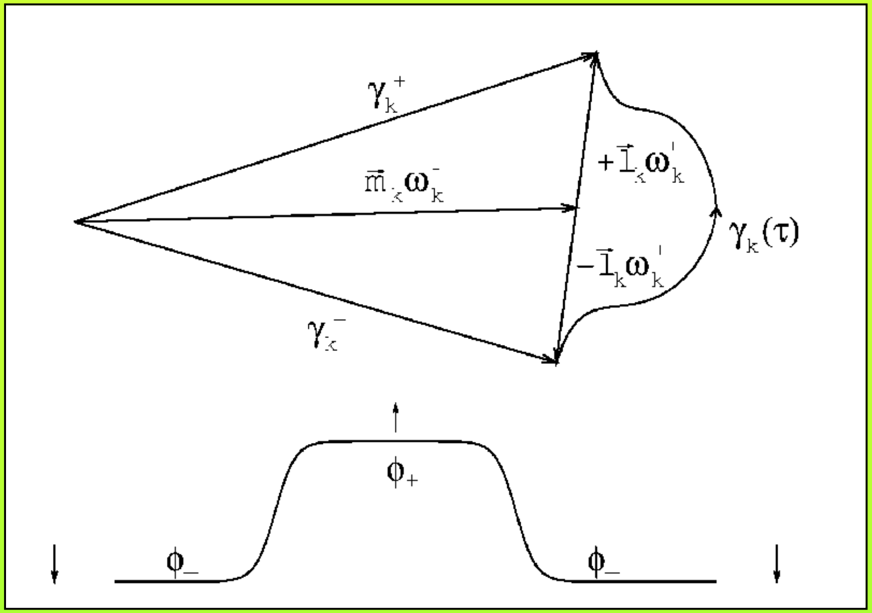
For simplicity assume here the bath spins are a set $\{\sigma_k\}$ of 2-level systems, which interact with each other only very weakly (because they are localised). We then get a low-energy effective Hamiltonian in which the central qubit couples to all of these. Since the spectral weight of the oscillator bath excitations vanishes at low energy, the spin bath always dominates at low energies! The Hamiltonian is:

$$H(\Omega_o) = \{ [\Delta\tau_+ \exp(-i \Sigma_k \alpha_k.\sigma_k) + H.c.] + \varepsilon_o\tau_z \qquad \text{(qubit)}$$
$$+ \tau_z \omega_k.\sigma_k + h_k.\sigma_k \qquad \text{(bath spins)}$$
$$+ \text{ inter-spin interactions}$$



Now the couplings ω_k , h_k to the bath spins (the 1st between bath spin & qubit, the 2nd to external fields) are often very STRONG (much larger than the inter-bath spin interactions or even than Δ).

Heuristic Derivation of "CENTRAL SPIN" Effective hamiltonian



This is done using instanton methods-but can be explained in pictures. To be definite do this for a SQUID and a "spin bath".

(i) Start with the k-th bath spin, and define the vector field

$$\gamma_k(\tau) = h_k \mathbf{m}_k + \omega_k \mathbf{l}_k(\tau)$$

which varies as shown, during the transitions of the SQUID qubit. The 'stationary states' of this define our qubit basis states, and the fields acting on them (including any longitudinal bias ε_o).

(ii) Define the "transfer matrix"

$$T_k^{\pm} = \exp\left\{-\frac{1}{\hbar}\int_-^+ d\tau \omega_k^{\parallel} \vec{l}_k(\tau)\cdot\vec{\sigma}_k\right\} \equiv e^{-i\varphi_k + \vec{\alpha}_k\cdot\vec{\sigma}_k}$$

where the scalar ϕ_k & the vector α_k are both complex. If we incorporate the scalar phase into a renormalised amplitude Δ then we get the non-diagonal 'qubit flip' term.

(iii) Add the bath inter-spin interactions

$$\sum_{k=1}^{N}\sum_{k'=1}^{N} V_{kk'}^{\alpha\beta}\hat{\sigma}_k^{\alpha}\hat{\sigma}_{k'}^{\beta}$$

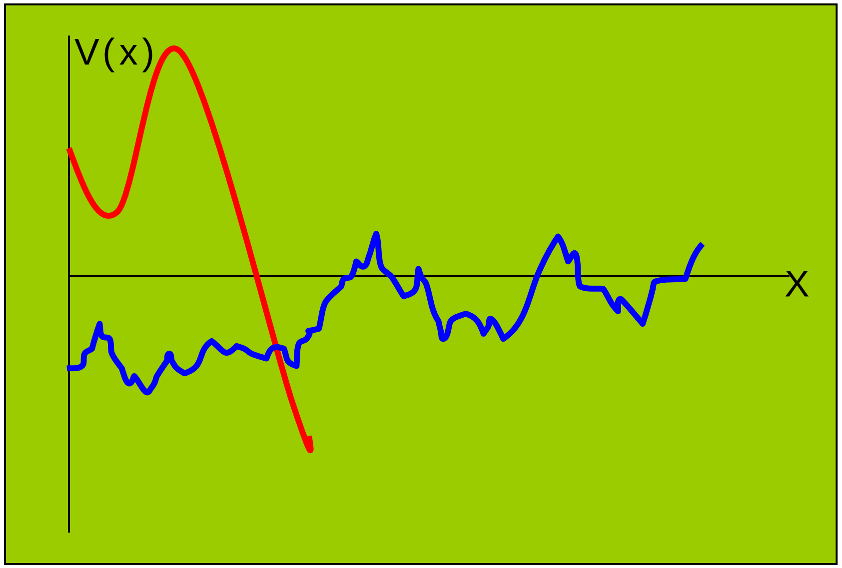
$$\begin{aligned} H(\Omega_o) = \{ &[\Delta\tau_+ \exp(-i\Sigma_k \alpha_k.\sigma_k) + \mathrm{H.c.}] + \varepsilon_o\tau_z \\ &+ \tau_z\omega_k.\sigma_k + h_k.\sigma_k \\ &+ \text{inter-spin interactions} \end{aligned}$$

PARTICLE coupled to a BATH

In many problems the central system reduces to a 'particle' (ie., a single degree of freedom) moving in some potential. The general Hamiltonian is still

$$H = H_0(P, Q) + H_{\rm int}(P, Q; \{\hat{\vec{\sigma}}\}) + H_{\rm env}(\{\hat{\vec{\sigma}}\})$$

And all the interest is in varying the kind of bath we have, and the different sorts of potential. Examples are legion.

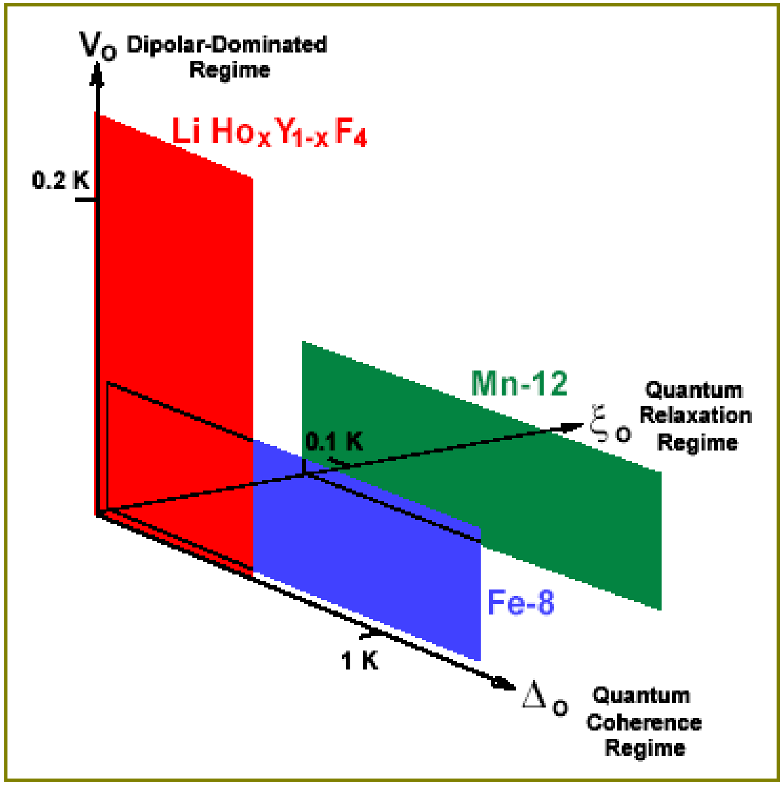


The INTERACTING 'SPIN NET'

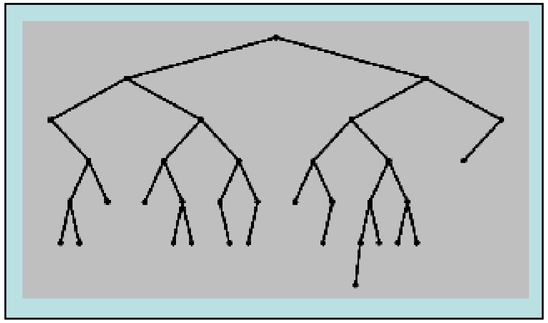
By this we mean a set of spins/qubits interacting Via a set of CONTROLLABLE couplings; they also interact with spin & oscillator baths. A typical example is the 'DIPOLAR SPIN NET':

$$\mathbf{H} \;=\; \Sigma_j\,(\Delta_j\,\tau_j^x + \varepsilon_j\,\tau_j^z) \;+\; \Sigma_{ij}\,V_{ij}^{zz}\,\tau_i^z\,\tau_j^z$$
$$+\; \mathbf{H}_{SB}(I_k) \;+\; \mathbf{H}_{osc}(x_q)$$
$$+\; \text{interactions}$$

For spin systems interacting with each other via dipolar forces, and individually with a nuclear bath, this leads to the picture at right.



Remarks on NETWORKS- the QUANTUM WALK



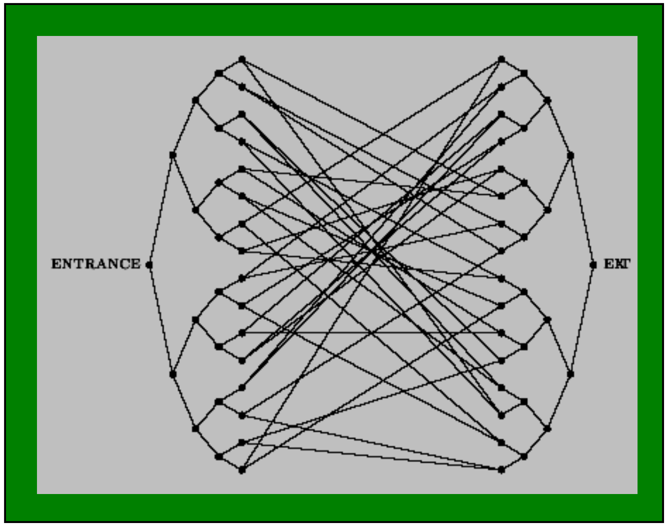
Computer scientists have been interested in RANDOM WALKS on various mathematical GRAPHS, for many years. These allow a general analysis of decision trees, search algorithms, and indeed general computer programmes (a Turing machine can be viewed as a walk). One of the most important applications of this has been to error correction- which is central to modern software.

Starting with papers by Aharonov et al (1994), & Farhi & Gutmann (1998), the same kind of analysis has been applied to QUANTUM COMPUTATION. It is easy to show that many quantum computations can be modeled as QUANTUM WALKs on some graph. The problem then becomes one of QUANTUM DIFFUSION on this graph, and one easily finds either power-law or exponential speed-up, depending on the graph. Great hopes have been pinned on this new development- it allows very general analyses, and offers hope of new kinds of algorithm, and new kinds of quantum error correction- and new 'circuit designs'.

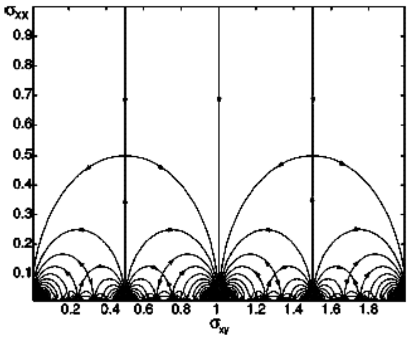
Thus we are interested in simple walks described by Hamiltonians like

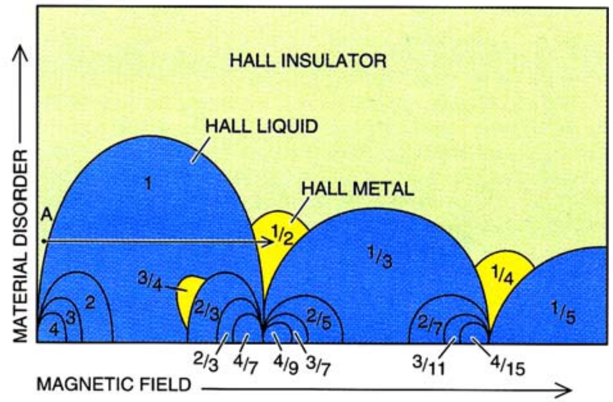
$$\hat{H} = -\sum_{\langle ij \rangle} \Delta_{jk} \left(\hat{c}_i^\dagger \hat{c}_j + \hat{c}_i \hat{c}_j^\dagger \right) + \sum_{k=0}^{2^N} V_k \hat{c}_k^\dagger \hat{c}_k$$

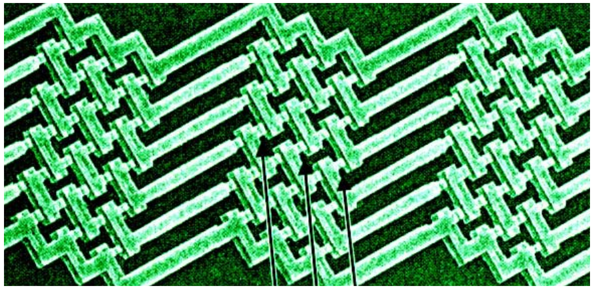
which can be mapped to a variety of gate Hamiltonians, spin Hamiltonians, and interacting qubit networks. Most of all we want to understand how decoherence affects the quantum walk dynamics; ie., we couple oscillator and spin baths to the walker.

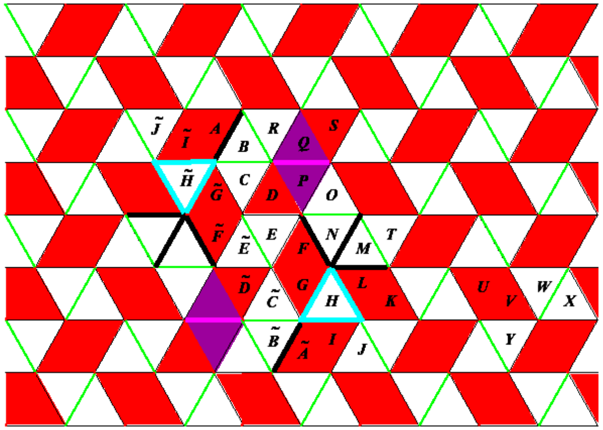


Decoherence in Topological Quantum Fluids









Certain kind of spin net have very interesting topological properties, and unconventional excitations exist in them. They are of central interest for strongly-correlated electronic systems (FQHE, 1-d junctions, 2-d spin liquids, etc.) & in topological quantum computation.

We learn a great deal about these by studying a 2-d lattice 'dissipative WAH' model, with

$$H = \frac{1}{2m}\left[-i\hbar\nabla - \frac{e}{c}\mathbf{A}\right]^2 + V(\mathbf{x}) + \sum_\alpha P_\alpha^2/2m_\alpha$$

$$+ \frac{1}{2}\sum_\alpha m_\alpha \omega_\alpha^2 (x_\alpha + q\lambda_\alpha/m_\alpha\omega_\alpha^2)^2$$

Here V(x) is periodic, and A(x) gives a uniform flux per plaquette. This model, & the simpler Schmid model (where flux = 0), have crucial connections with string theory, & have recently been re-evaluated with surprising results.

The underlying symmetry in the 2d parameter space is SL(2,Z), the same as that of an interacting set of vortices and charges. This is the same symmetry as that possessed by a large class of string theories.

THAT WAS JUST THE BEGINNING.......