PCE STAMP: LECTURE 3

QUANTUM SOLITONS in MAGNETIC SYSTEMS: DYNAMICS, DISSIPATION & DECOHERENCE

Some of the work herein was done with

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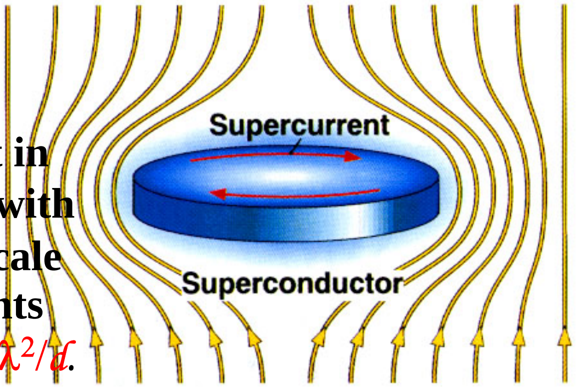
L Thompson

3.1: INTRODUCTION to QUANTUM VORTICES

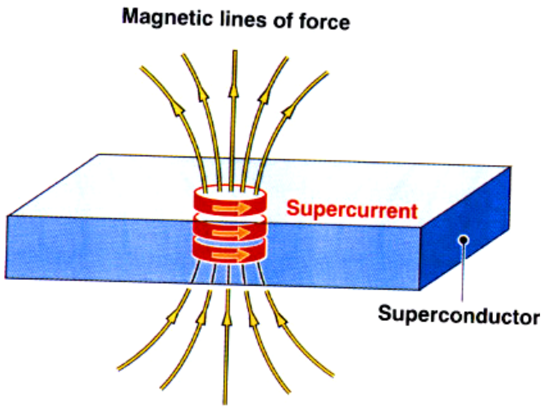
Here a quick survey of the many different kinds of quantum ‘string’ excitation exist, ie., 1-dimensional topological excitations of some quantum field. The low-T laboratory is often the best place to study these, and many quite interesting ideas about the role of stringy excitations in elementary particle physics and cosmology have come from such studies.

Finally it is noted how the fundamental question of how quantum vortices MOVE (ie., their equation of motion) is still unanswered. This is despite some 40 years of debate.

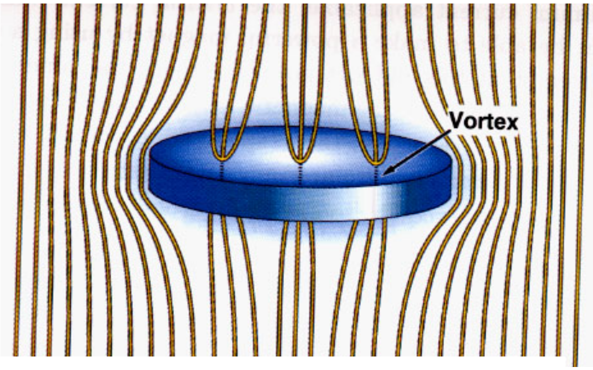
VORTICES in SUPERCONDUCTORS



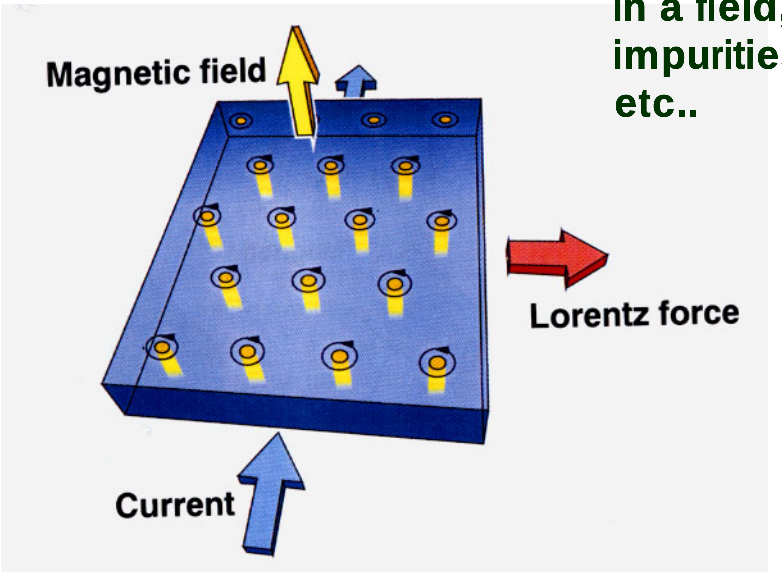
Superconductors show the Meissner effect (expulsion of flux), but in Type-II superconductors, flux penetrates through quantized vortices, with flux $\Phi_o = h/e$. The flux is confined by screening currents to a length scale λ (penetration depth). In thin films (thickness d), the screening currents spread to a much larger length $\Lambda = \lambda^2/d$.

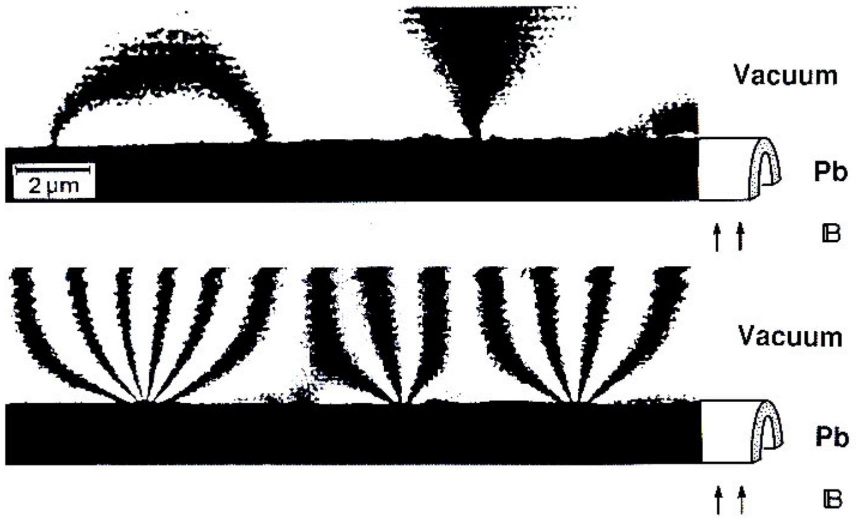


Outside the superconductor, the flux balloons out into space, and can be photographed using Aharonov-Bohm phase effects.



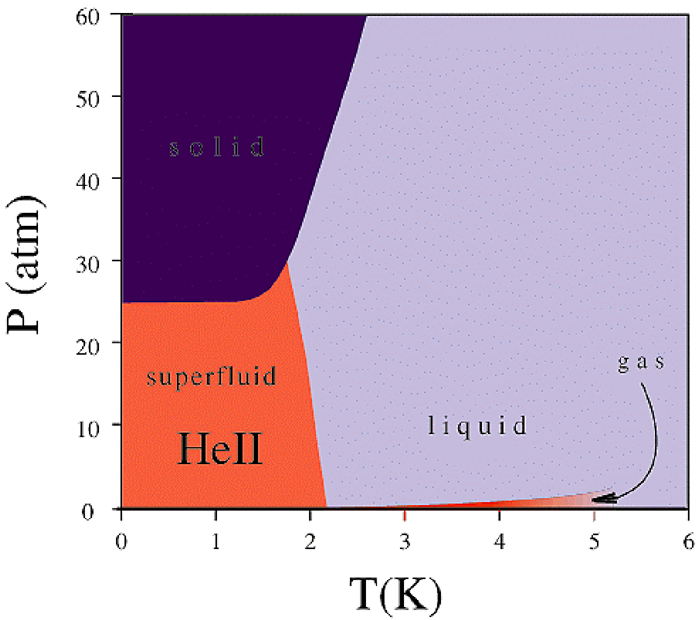
In an applied field the vortices in a type-II superconductor form a lattice, which feels a Lorentz force in a field, but which is pinned by impurities, defects, etc..

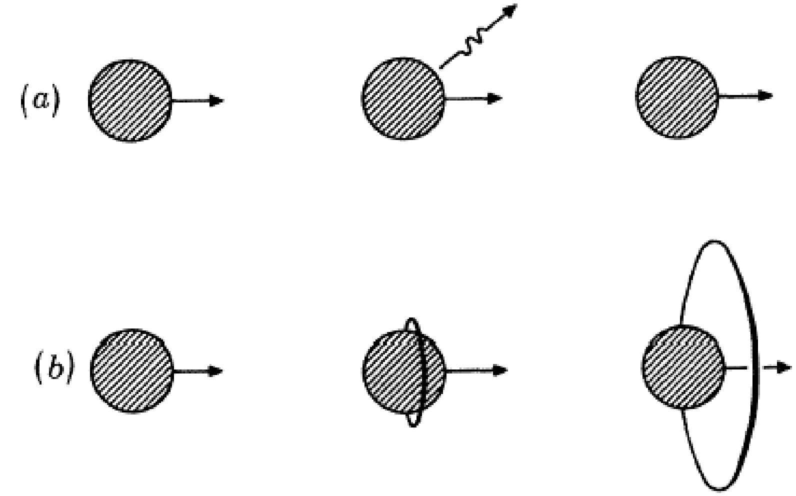




VORTICES in superfluid ^{4}He

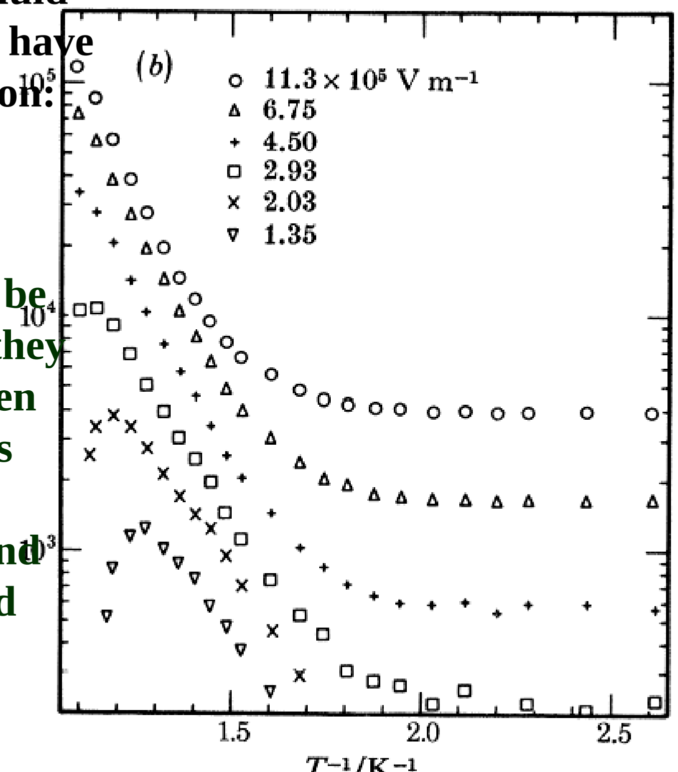
He-4 low temperature phases



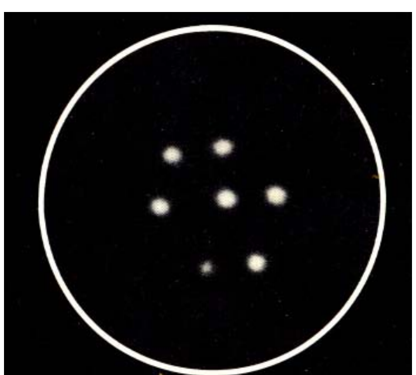


In a neutral superfluid like ^{4}He, the vortices have a quantized circulation:

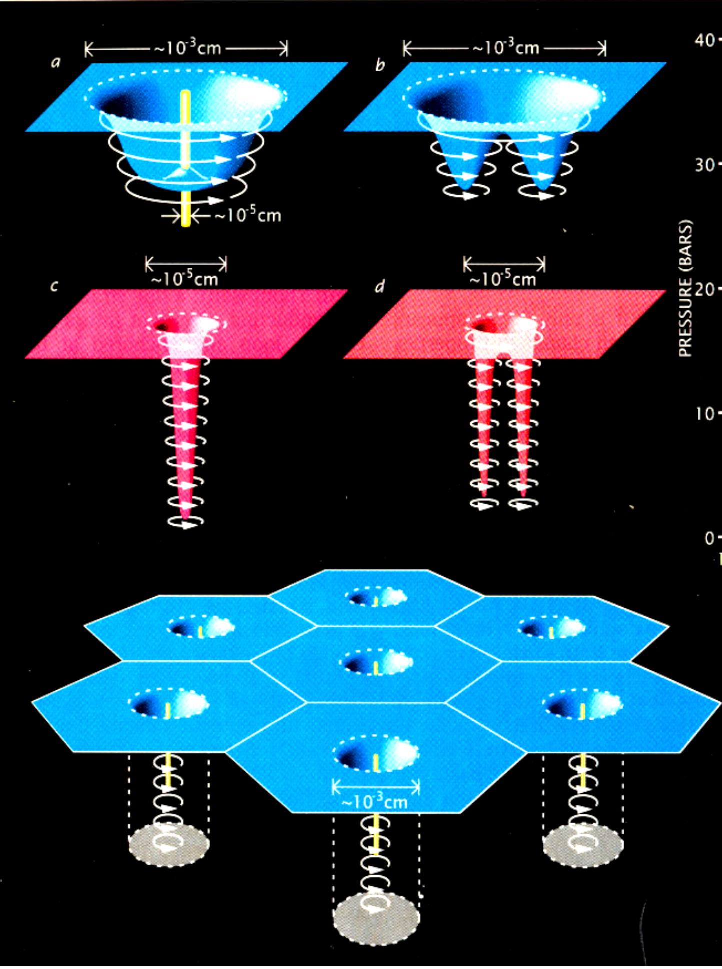
$$\kappa = h/m_4$$

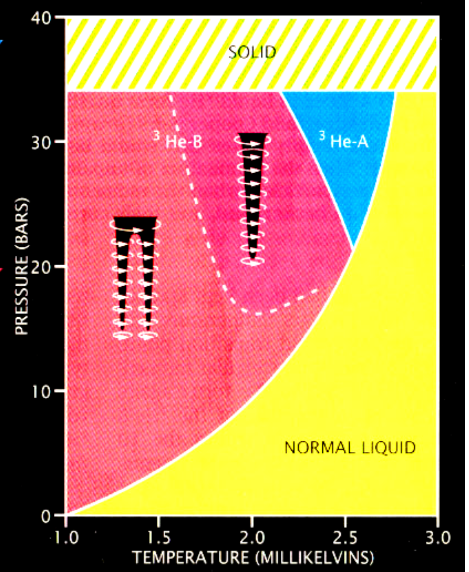


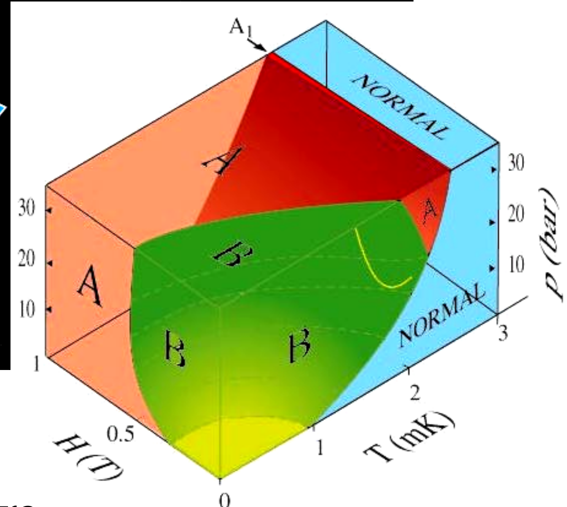
These vortices cannot be observed directly, but they can be decorated with electrons and then Photographed (by sucking the electrons out); and single vortex rings can be produced by quantum nucleation around ions which move through the superfluid at high velocities).

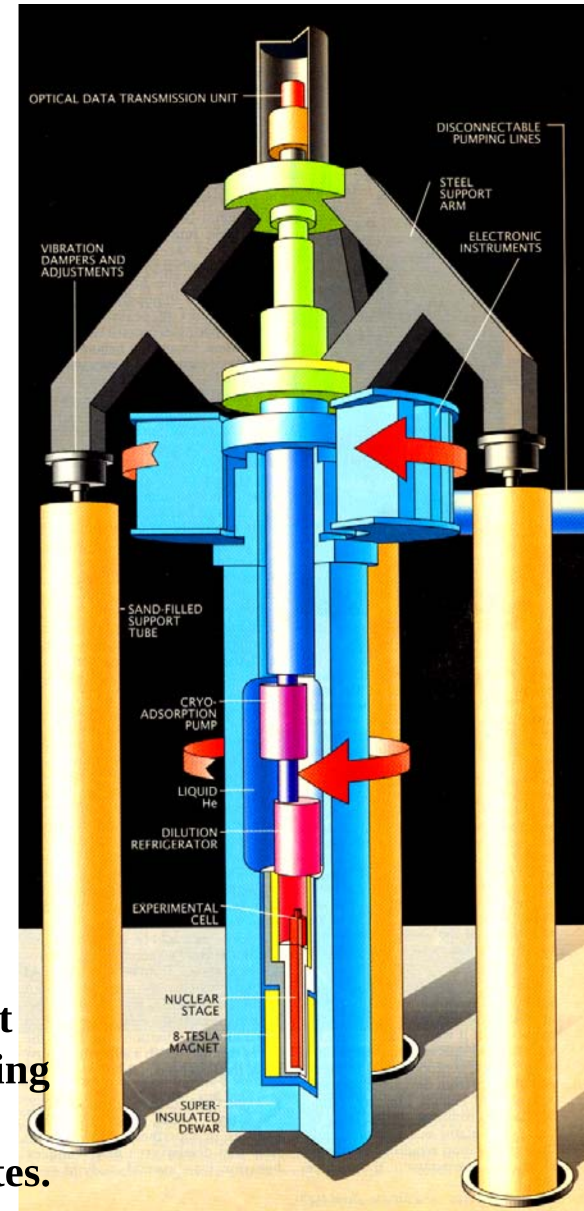


SUPERFLUID ^{3}He



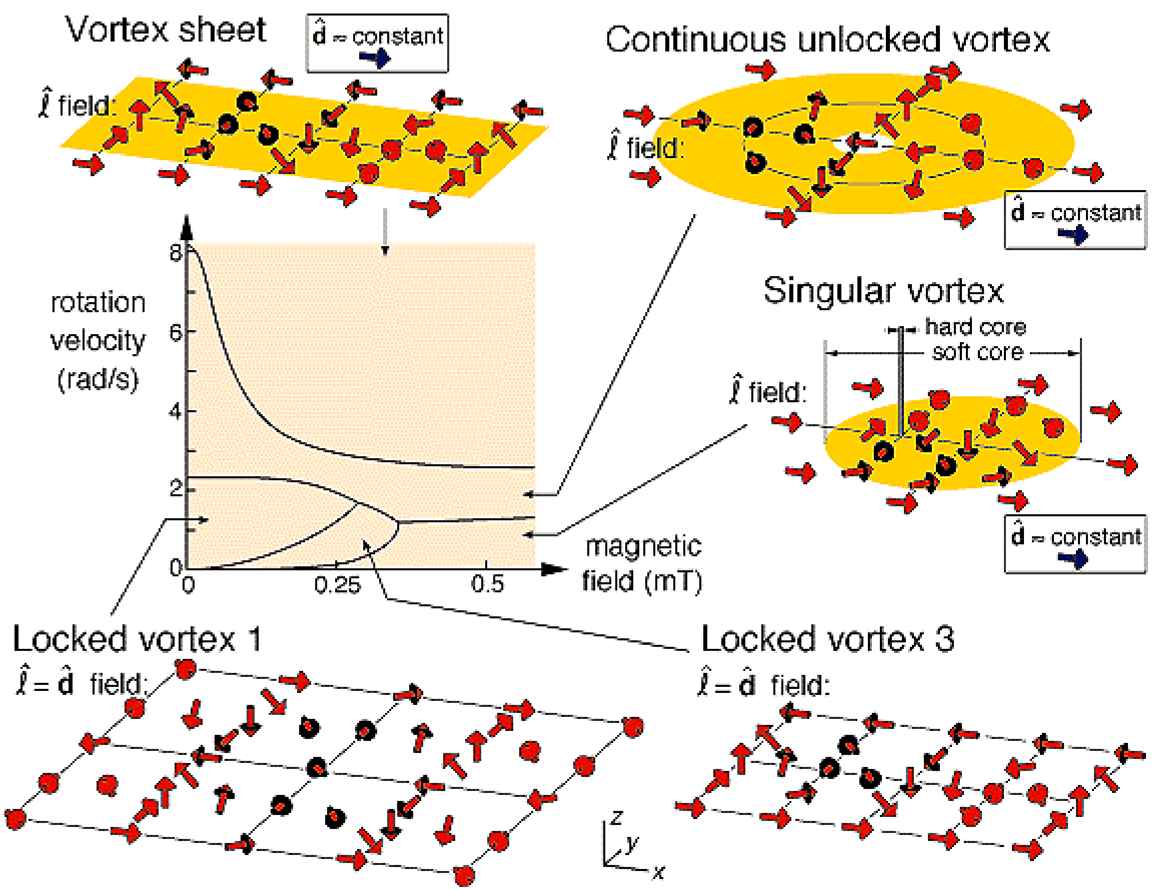




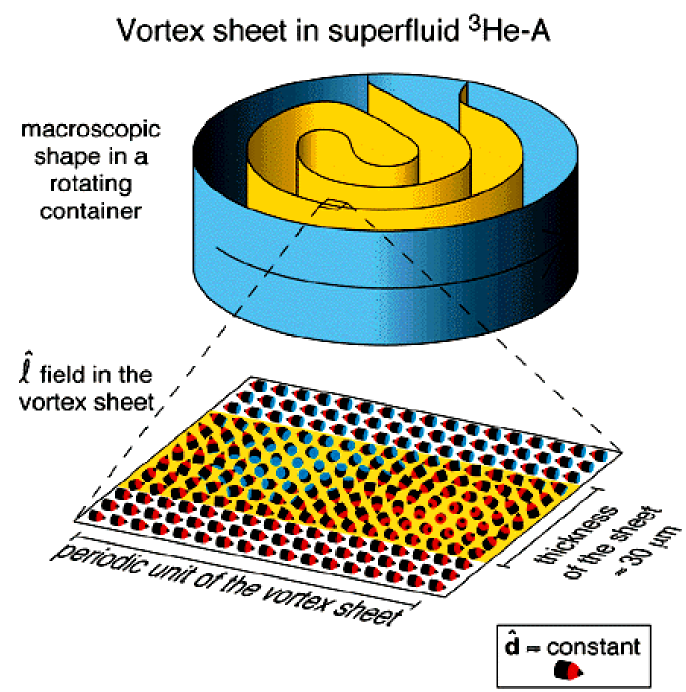


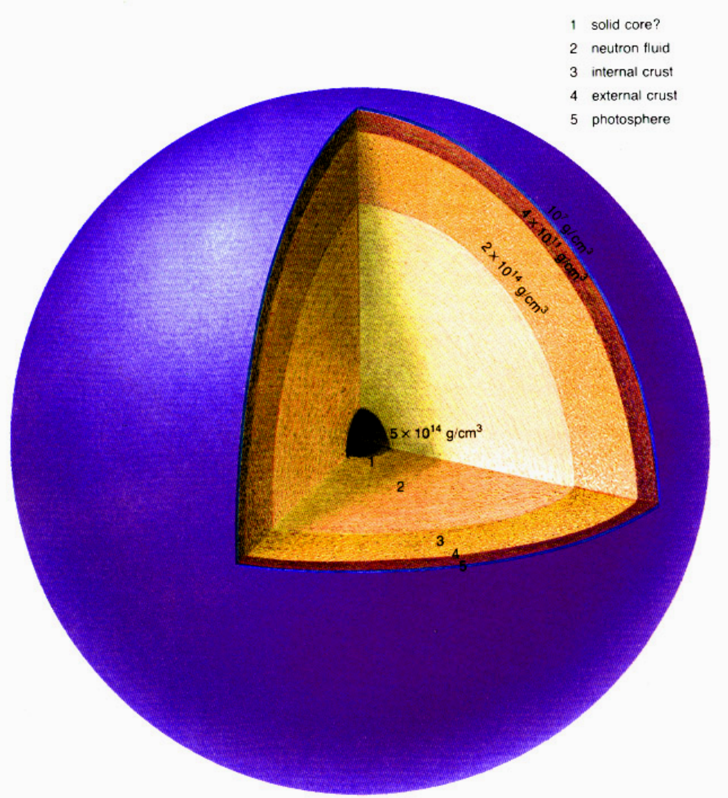
In superfluid ^{3}He a huge variety of vortices can be seen- their propeties are observed using sensitive NMR measurements, sometimes in rotating cryostats. The coherence length is long (15 nm at T=0) and many of the core textures are not even singular (and depending on which phase one is in there will be many different possible vortex phases). The cores are thus full of bound or resonant quasiparticle states.

Vortices in Superfluid ^{3}He-A

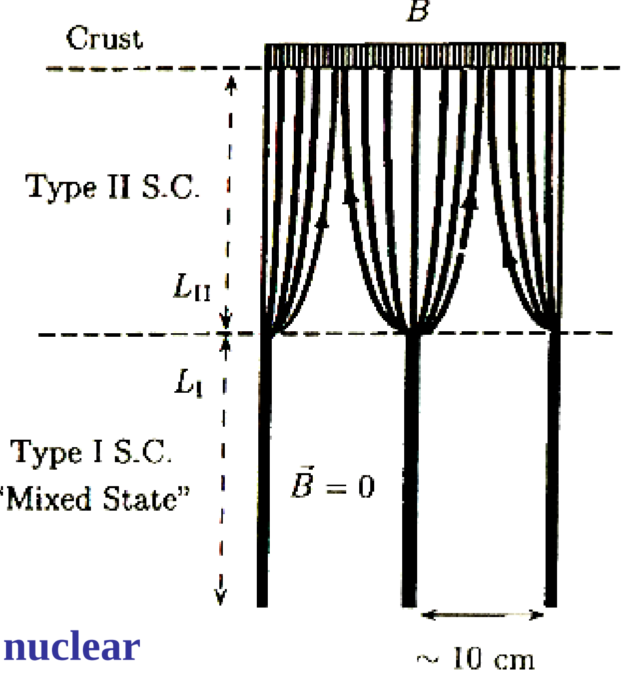


One can even have vortex sheets in ^{3}He superfluid, in the A phase. A large variety of phases is supported under different conditions of rotatio magnetic field, temperature, etc., either as disordered 'vortex tangles' or in lattice arrays.

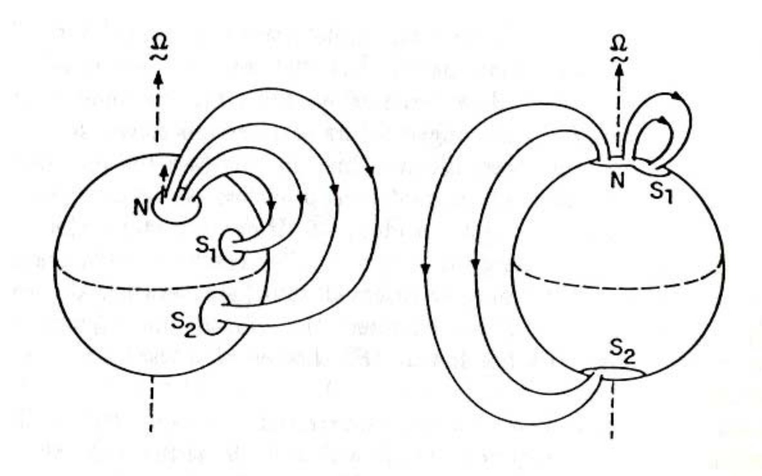


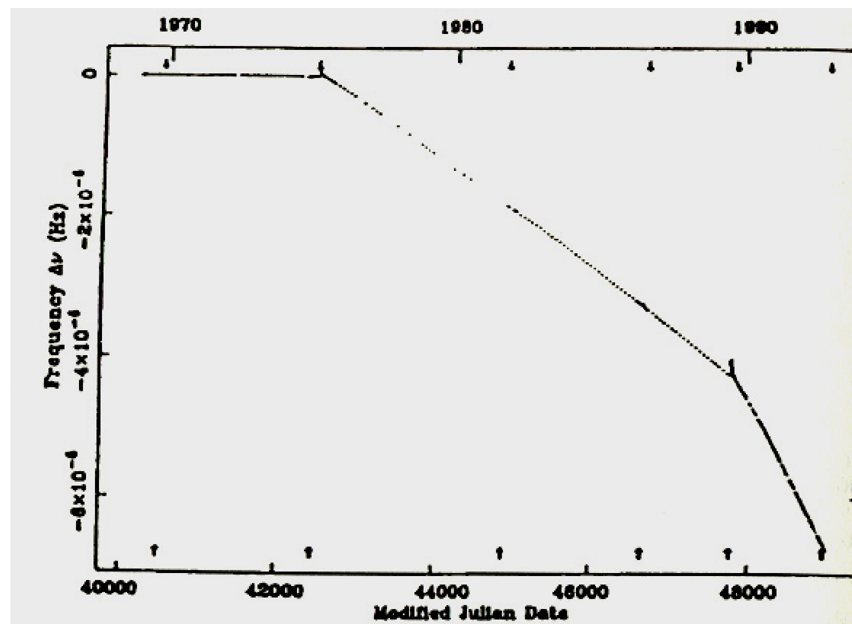


VORTICES in Neutron Stars

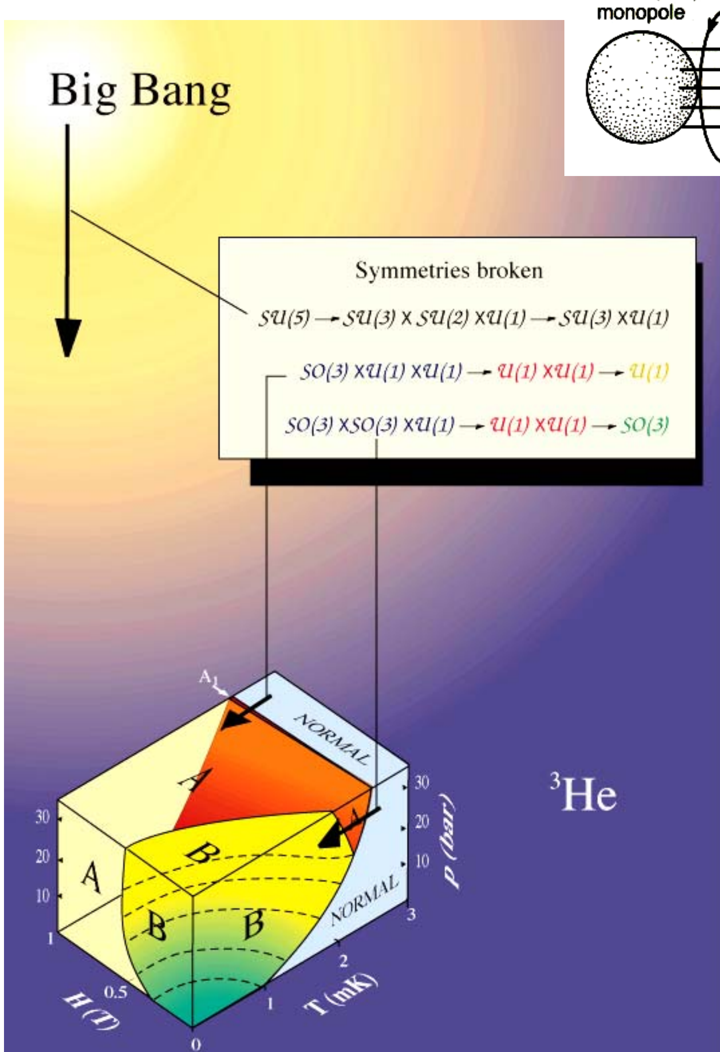


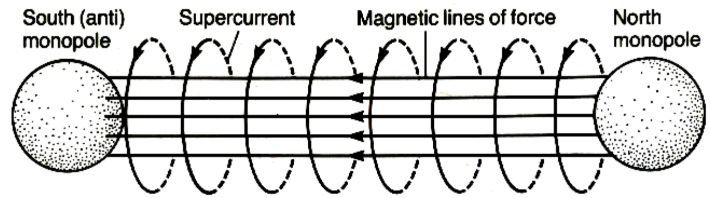
These can only be seen very indirectly, using the So-called 'glitch' phenomenon. There are many gaps in our understanding, because of incomplete knowledge of the nuclear equation of state at these densities

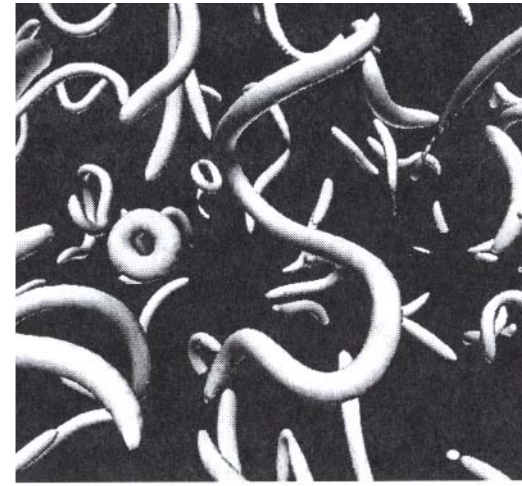




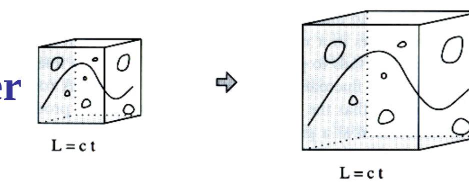
“STRINGS” & the UNIVERSE

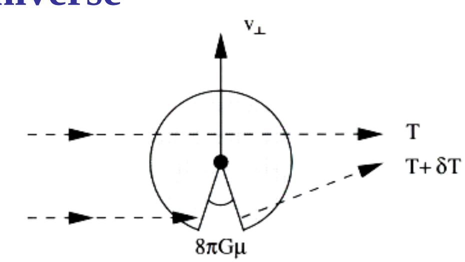


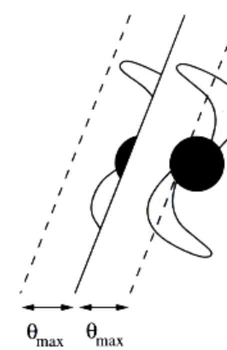




The relationship between cosmic strings (which have never been observed) and the strings of string theory has yet to be clarified. However theory does indicate a crucial role for cosmic strings in the history of the universe







WHAT is the EQUATION of MOTION of a QUANTUM VORTEX?

Vortex dynamics in neutral superfluids (both bosonic, like ^4He, or fermionic, like ^3He) has been discussed for over 50 years. In classical hydrodynamics, a Magnus force exists between the vortex and the superfluid, transverse to their relative motion. The quantum Magnus force $\mathbf{F}_M^\perp$ is proportional to the vector cross product $\rho_s \boldsymbol{\kappa} \times (\mathbf{v}_v - \mathbf{v}_s)$, where the vector $\boldsymbol{\kappa}$ gives the vortex circulation, $\rho_s(T)$ is the superfluid density, and $\mathbf{v}_v$, $\mathbf{v}_s$ are the vortex and superfluid velocities; this force comes from a Berry phase in the many-body wave-function [6]. There are also forces on the vortex coming from its interaction with normal fluid quasiparticles- in 1964 Iordanskii [7] derived a transverse force $\mathbf{F}_I^\perp \propto \boldsymbol{\kappa} \times (\mathbf{v}_v - \mathbf{v}_N)$, where $\mathbf{v}_N$ is the normal fluid velocity, and there is also a longitudinal dissipative force $\mathbf{F}^\parallel \propto (\mathbf{v}_v - \mathbf{v}_N)$. Similar forces have been discussed for superconducting vortices on earth and in neutron stars, and for cosmic strings.

Although there is little argument about the Magnus force, some of the other forces are highly controversial- arguments over their magnitude have continued for 40 years, and the very existence of the Iordanski force has been denied (again, on the basis of a Berry phase argument), causing a lively debate. The problem is further complicated by questions about the effective mass of a vortex, for which estimates vary enormously. Experiments have not resolved these questions- individual vortices are almost impossible to observe in neutral superfluids (unless one decorates them with electrons, which completely alters their structure), and in superconductors, vortices appear in lattices, and are often subject to pinning forces from defects and impurities. It is quite extraordinary that so fundamental a question as the basic dynamics of a quantum vortex is still unanswered.

One can ask similar (and even more difficult) questions about domain walls.

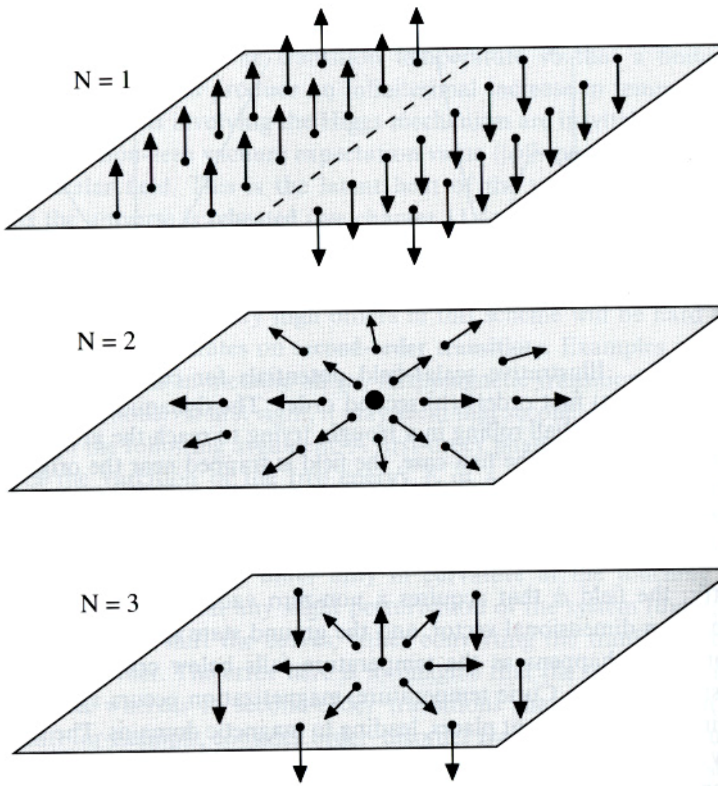
However, superfluids are not the only systems we can look at.....

3.2: QUANTUM DYNAMICS of a MAGNETIC QUANTUM VORTEX

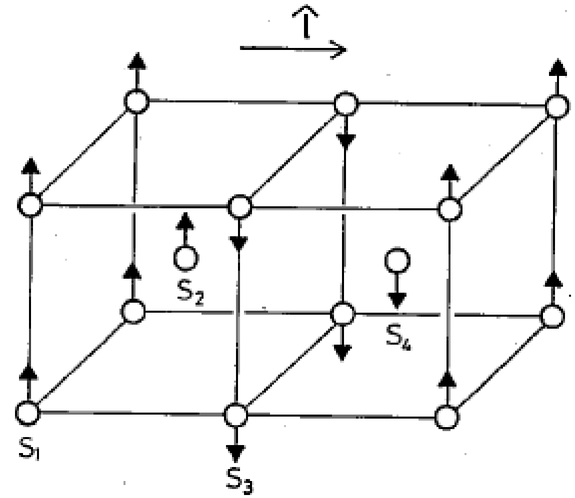
Here we look at the problem of a magnetic vortex, where (i) one can get a much better theoretical handle on the fundamental question posed in the last section, and (ii) where experiments should be a lot easier. The answer turns out to be surprising – the magnon environment gives a non-local dissipation which causes strong long-time memory effects. Thus the real dynamics of the vortex is much more complex than was previously thought.

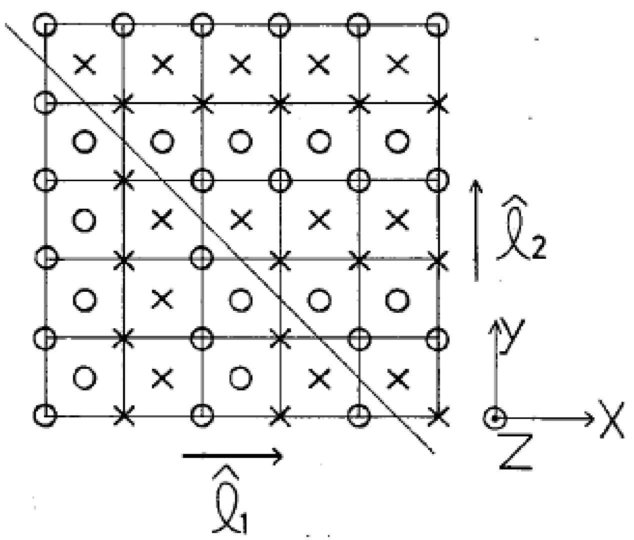
Topological Solitons in MAGNETS

SOLITONS in 2D MAGNETS



SOLID ^{3}He





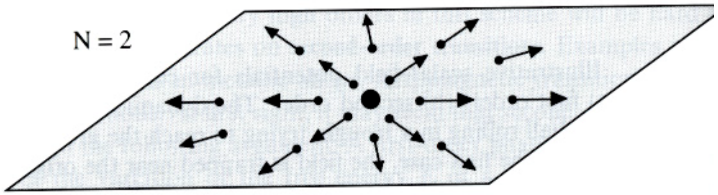
Quantum Vortex in 2D Easy-plane Ferromagnet

L. Thompson, PCE Stamp,   to be published
L Thompson, MSc thesis (UBC)

Lattice Hamiltonian $H = -\sum_{<i,j>} \tilde{J}\mathbf{S}_i \cdot \mathbf{S}_j + \sum_i \tilde{K} S_{iz}^2$

Continuum Limit $H = \int \frac{d^2 r}{a^2} \frac{J}{2} (\overrightarrow{\nabla} S)^2 + K S_z^2$

In Lagrangian: $\omega_B = -S \int \frac{d^2 r}{a^2} \cos\theta \dot{\phi}$ **(Berry phase)**



VORTEX PROFILE

$$
\begin{aligned}
\phi_v(\mathbf{r}) &= q\xi + \delta \\
\cos\theta_v(\mathbf{r}) &= p \begin{cases} 1 - c_1 \left(\frac{r}{r_v}\right)^2, & r \to 0; \\ c_2 \sqrt{\frac{r_v}{r}} \exp(-\frac{r}{r_v}), & r \to \infty. \end{cases}
\end{aligned}
$$

MAGNON SPECTRUM

$$
\begin{aligned}
\omega &= ckQ \\
Q^2 &= k^2 + k_o^2
\end{aligned}
$$

Core Radius

$$
\begin{aligned}
r_v^2 &= J/2K \\
k_o^2 &= 2K/J
\end{aligned}
$$

Spin Wave velocity $c = SJa^2$

VORTEX DYNAMICS

Density matrix propagator $\rho(Q_1, Q_2; t) = K(Q_1, Q_2; Q_1', Q_2'; t, t')\rho(Q_1', Q_2'; t, t')$

$$K(Q_2, Q_2'; Q_1, Q_1'; t, t') = \int_{Q_1}^{Q_2} \mathcal{D}q \int_{Q_1'}^{Q_2'} \mathcal{D}q' \; e^{-i/\hbar(S_o[q] \; - \; S_o[q'])} \mathcal{F}[q, q']$$

Influence Functional $F = \exp\left(\frac{i}{\hbar}\Phi - \frac{1}{\hbar^2}\Gamma\right)$

For the Magnetic vortex we find

$$F = \exp - \int \frac{a^2 d^2 k}{\hbar (2\pi)^2} \int_0^T \int_0^t dt ds \left(f_{\mathbf{k}}[\mathbf{X}(t)] - f_{\mathbf{k}}[\mathbf{Y}(t)] \right)$$
$$\left(\alpha_k(t-s) f_{-\mathbf{k}}[\mathbf{X}(s)] - \alpha_k^*(t-s) f_{-\mathbf{k}}[\mathbf{Y}(s)]\right)$$

Effective interaction $\alpha_k(t-s) = \frac{Sk}{2Q}\left(e^{-i\omega_k(t-s)} + \frac{2\cos\omega_k(t-s)}{e^{\hbar\omega_k\beta} - 1}\right)$

Effective coupling $f_{\mathbf{k}}[\mathbf{X}] = \frac{2i\pi q}{a^2} \frac{\dot{\mathbf{X}} \cdot \hat{\varphi}_{\mathbf{k}}}{kQr_v} e^{i\mathbf{k}\cdot\mathbf{X}}$

KINETIC/BERRY PHASE TERMS

Assembly of Vortices:

$$H = \sum_i E_{vi} + \frac{1}{2}\sum_{i\neq j} V_{ij} + O(\varphi^2, \vartheta^2)$$

$$\omega_B = \sum_{ij} S \int \frac{d^2 r}{a^2}(-\cos\theta_i + \sin\theta_i\ \vartheta)(\dot{\phi}_j + \dot{\varphi})$$

One immediately recovers gyrotropic "Magnus" force: $\mathbf{F}_i^{gyro} = -\frac{\pi S p_i q_i}{a^2}\hat{\mathbf{z}} \times \dot{\mathbf{X}}$

and for vortex assembly a NON-LOCAL MASS:

$$M_{ij} = \frac{\pi q_i q_j r_v^2}{J a^4}\begin{cases} \ln\frac{R_S}{r_{ij}} + \frac{1}{2}\Big((\hat{\mathbf{X}}_i \cdot \hat{\mathbf{e}}_{ij})(\hat{\mathbf{X}}_j \cdot \hat{\mathbf{e}}_{ij}) \\ -(\hat{\mathbf{X}}_i \times \hat{\mathbf{e}}_{ij}) \cdot (\hat{\mathbf{X}}_j \times \hat{\mathbf{e}}_{ij})\Big), & i \neq j; \\ \ln\frac{R_S}{\sqrt{a^2 + r_v^2}}, & i = j. \end{cases}$$

$$\hat{\mathbf{e}}_{ij} = \frac{\mathbf{X}_i - \mathbf{X}_j}{|\mathbf{X}_i - \mathbf{X}_j|}$$

Influence Functional: Phase terms

Total Phase

$$\Phi = \int_0^T dt \frac{1}{2} M_{ij}(\dot{X}_i(t)\dot{X}_j(t) - \dot{Y}_i(t)\dot{Y}_j(t)) + \Phi_{||} + \Phi_\Delta + \Phi_\perp$$

(1) Longitudinal Phase terms

$$\begin{aligned}\Phi_{||} = -\int_0^T dt \int_0^t ds \Big(& \gamma_{||}^{ij}(\Delta_{XX})\dot{\mathbf{X}}_i(s)\cdot\mathbf{X}_j(t) \\ & -\gamma_{||}^{ij}(\Delta_{YY})\dot{\mathbf{Y}}_i(s)\cdot\mathbf{Y}_j(t) \\ & +\gamma_{||}^{ij}(\Delta_{YX})\dot{\mathbf{Y}}_i(s)\cdot\mathbf{X}_j(t) \\ & -\gamma_{||}^{ij}(\Delta_{XY})\dot{\mathbf{X}}_i(s)\cdot\mathbf{Y}_j(t)\Big)\end{aligned}$$

$\Delta_{XX} = |\mathbf{X}_i(t) - \mathbf{X}_j(s)|$ **, etc.**

$$\begin{aligned}\gamma_{||}^{ij}(t-s,\Delta) &= \frac{S^2 J\pi q_i q_j}{2}\int kdk \frac{\cos\omega_0(t-s)J_0(k\Delta)}{Q^2 r_v^2} \\ \gamma_{\Delta}^{ij}(t-s,\Delta) &= \frac{S^2 J\pi q_i q_j}{2}\int kdk \frac{\cos\omega_0(t-s)J_2(k\Delta)}{Q^2 r_v^2} \\ \gamma_{\perp}^{ij}(t-s,\Delta) &= -\gamma_\Delta(t-s,\Delta)\end{aligned}$$

(2) 'Mixed' memory term:

$$\Phi_\Delta = -\int_0^T dt \int_0^t ds \gamma_\Delta^{ij}(\Delta_{XX}) \dot{X}_i(s) \big(\hat{\mathbf{X}}_i(s) \cdot \hat{\mathbf{e}}_{ij}\big) \times \big(\mathbf{X}_j(t) \cdot \hat{\mathbf{e}}_{ij}\big) + Y^2 \text{ and } XY \text{ terms}$$

(3) Transverse Damping term

$$\Phi_\perp = -\int_0^T dt \int_0^t ds \gamma_\perp^{ij}(\Delta_{XX}) \dot{X}_i(s) \big(\hat{\mathbf{X}}_i(s) \cdot \hat{\mathbf{e}}_{\perp ij}\big) \times \big(\mathbf{X}_j(t) \cdot \hat{\mathbf{e}}_{\perp ij}\big) + Y^2 \text{ and } XY \text{ terms}$$

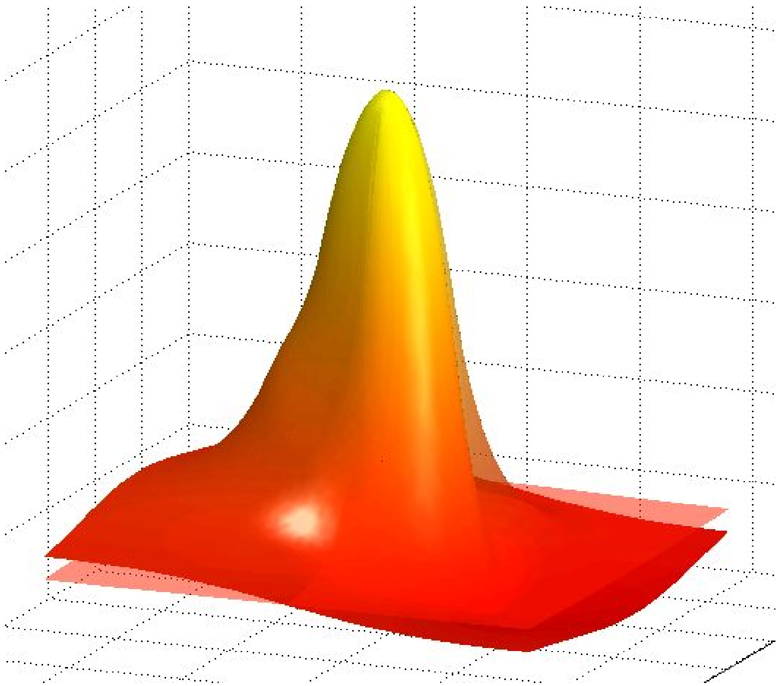
It immediately becomes clear that the real dynamics of a vortex, magnetic or otherwise, has both reactive and dissipative terms that are more complex than those that have been discussed so far.

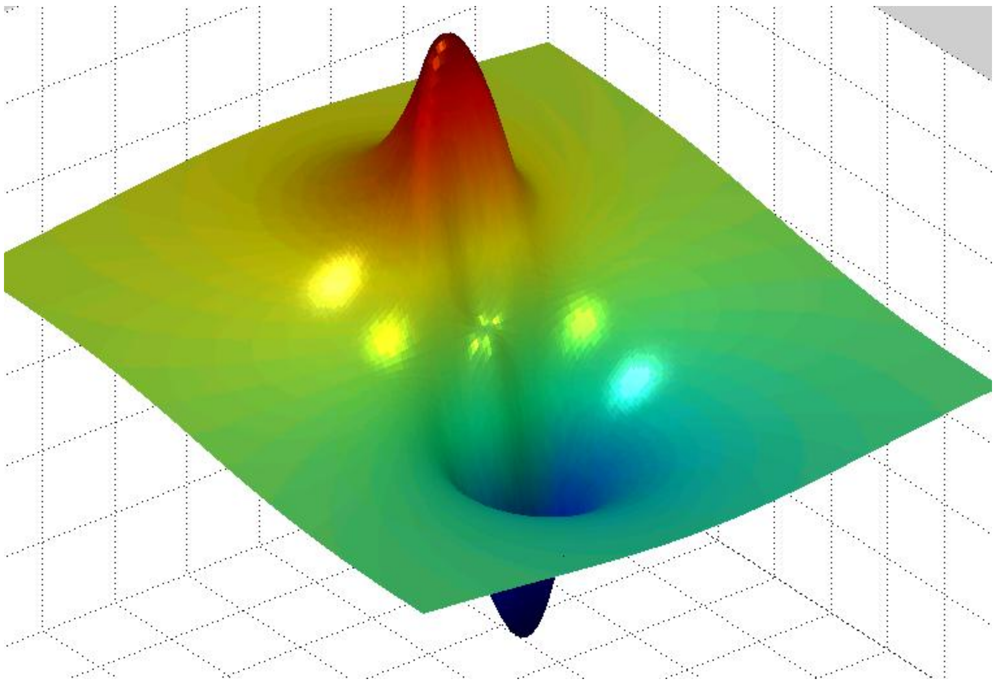
We note that there is definitely a transverse dissipative force having the symmetry of the Iordanskii term! If we have more than one vortex these have non-local contributions.

There are however other terms which come in, not discussed ever before for vortices in superfluids or superconductors- these should however exist in these systems as well….

SHAPE of a MOVING VORTEX

The profile of the vortex slowly distorts as it moves more quickly; the in-plane spins are forced slightly out of the plane, even some distance away from the vortex core. This distortion is very important – not only does it increase the energy of the vortex (leading to a kinetic energy term, and defining the effective mass of the vortex), but it also creates an extra scattering potential for the spin waves in the system, contributing to the forces acting on the vortex.





Influence Functional: Damping/Q Noise terms

Multi-vortex damping/noise term:

$$\Gamma = \int_0^T \int_0^t dtds \Big(A_{ij}(t-s) \mathbf{X}_i(t) \cdot \mathbf{X}_j(s) J_0(k\Delta_{X_i X_j})$$

$$+ A_{ij}(t-s) \Big((\mathbf{X}_i(t) \cdot \hat{\mathbf{e}}_{ij})(\mathbf{X}_j(s) \cdot \hat{\mathbf{e}}_{ij})$$

$$- (\mathbf{X}_i(t) \times \hat{\mathbf{e}}_{ij}) \cdot (\mathbf{X}_j(s) \times \hat{\mathbf{e}}_{ij}) \Big) J_2(k\Delta_{X_i X_j}) \Big) + \textbf{etc.}$$

with propagator:

$$A_{ij}(t-s) = \frac{\hbar \pi S q_i q_j}{2a^2} \int dk \frac{\coth \frac{\hbar \omega_k \beta}{2}}{Q^3 r_v^2} \omega_k^2 \cos \omega_k (t-s)$$

This gives a "quantum noise" term on the right hand-side of a Quantum Langevin equation. However the noise is not only non-Markovian (highly coloured in fact) but also non-local.

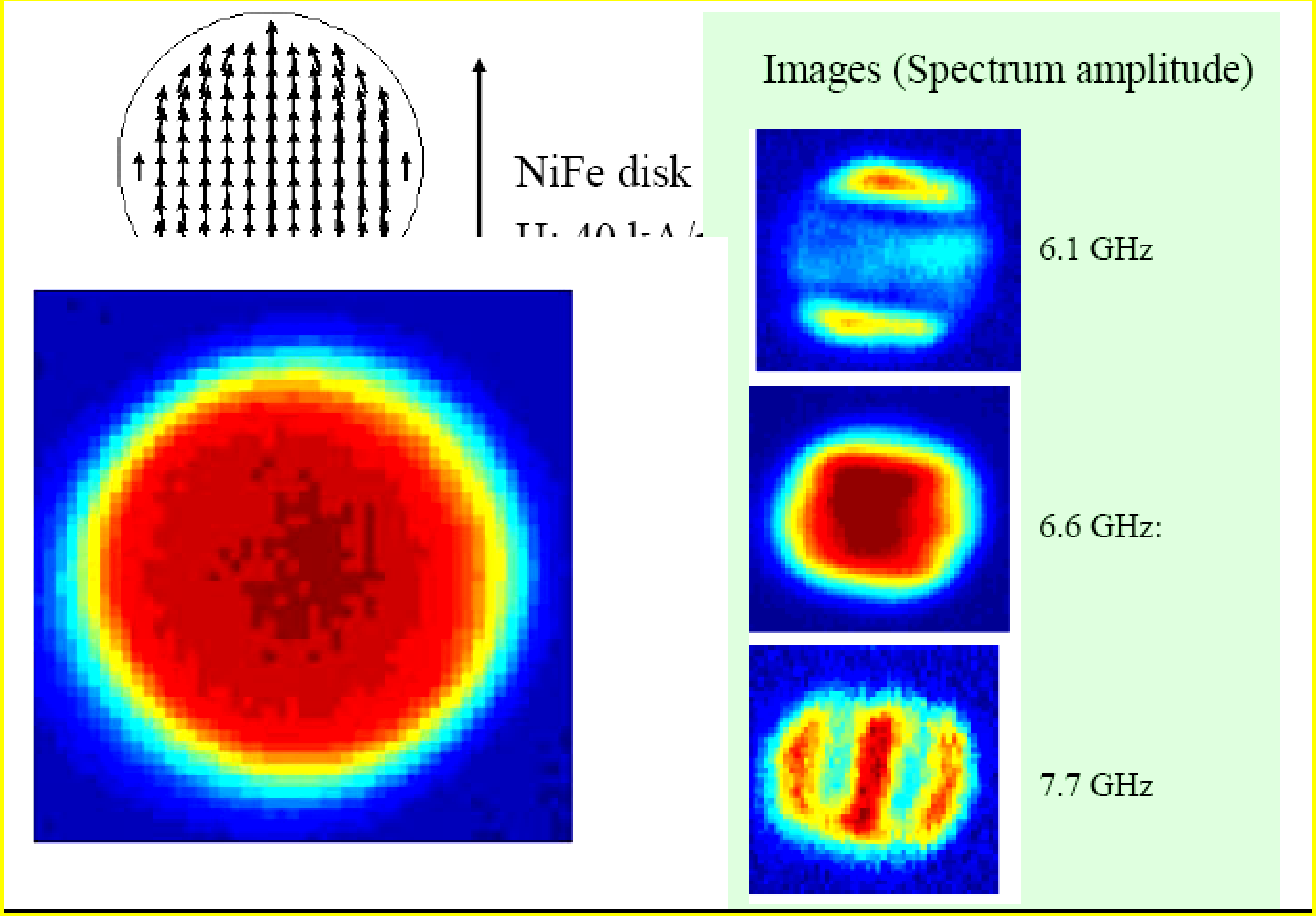
CONCLUSIONS

(1) There is no reason whatsoever to exclude transverse dissipative forces. In fact they are even more complex than previously understood

(2) The equations of motion for an assembly of vortices involve all sorts of forces (non-local in time and space) that have not previously been studied.

EXPERIMENTS on VORTICES in MAGNETS?

Example of Time-resolved Imaging in magnetic disc (courtesy M Freeman)



3.3: QUANTUM DYNAMICS of DOMAIN WALLS

The interesting thing about a domain wall is that the really thick & soft (ie., flexible) walls are the lightest ones, and therefore the ones most likely to show large-scale quantum behaviour. Naïve estimates then indicate that really very large walls, containing ~ 10^{12} spins, should have large quantum fluctuations and show, eg., tunneling behaviour. Walls themselves can have interesting quantum numbers, like chirality, and also have interesting internal ordering and modes and internal modes.

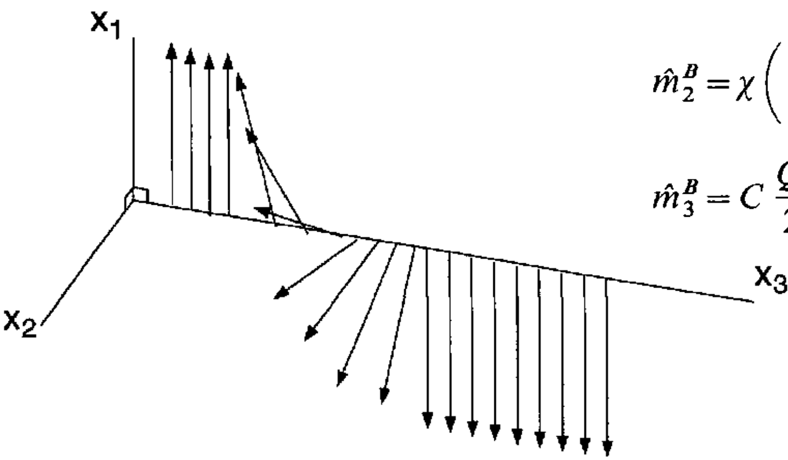
However one finds that the effect of decoherence can be rather drastic. For example, both phonons and transverse spin bath modes quickly kill off chirality fluctuations. In spite of this, detailed theoretical work does predict Tunneling of very large domain walls, and some evidence for this has appeared in experiments on magnetic wires (a great deal more needs to be done). This tunneling is on a large scale, similar to that found in SQUIDs. However spin bath effects on magnetic tunneling tend to be far more serious than that in SQUIDs (curiously, spin bath effects on COHERENCE in SQUIDs are very large, as already predicted a very long time ago).

QUANTUM DYNAMICS of a BLOCH WALL

Consider the Hamiltonian $\mathscr{H} = \frac{1}{2}\int d\mathbf{r}\left[J(\nabla\mathbf{M})^2 - K_{\parallel}(M_1)^2 + K_{\perp}(M_3)^2 - \frac{\mu_0}{2}(\mathbf{H}_{dm} + \mathbf{H}_e)\cdot\mathbf{M}\right]$

Where in the flat wall approximation $K_\perp = K_{\perp,i} + \mu_0 M_0^2/2$

We then have the wall profile $\hat{m}_1^B = C \tanh\left(\frac{x_3 - Q(t)}{\lambda_B}\right)$



$$\hat{m}_2^B = \chi\left(1 - \frac{\dot{Q}^2(t)}{8c_0^2}\right)\operatorname{sech}\left(\frac{x_3 - Q(t)}{\lambda_B}\right)$$

$$\hat{m}_3^B = C\,\frac{\dot{Q}(t)}{2c_0}\operatorname{sech}\left(\frac{x_3 - Q(t)}{\lambda_B}\right)$$

with $\chi = \pm 1$ (chirality)

and $C = \pm 1$ (“charge”)

One then has a wall thickness $\lambda_B = (J/K_{\parallel})^{1/2}$
(in lattice units)
and a wall surface energy $\sigma_0 = 4(JK_{\parallel})^{1/2}$

Now let us assume slow wall velocities $\dot{Q}(\tau) \ll c_0$, where $c_0 = \frac{2\gamma_g}{M_0}(JK_{\parallel})^{1/2}\left[\left[1 + \frac{K_\perp}{K_{\parallel}}\right]^{1/2} - 1\right]$ is the ‘Walker velocity’.

Then the wall behaves as a particle, with effective Hamiltonian $H_w = \frac{1}{2} M_w \dot{Q}^2$

The effective mass is $M_w = \frac{S_w M_0^2}{\gamma_g^2 (JK_{\parallel})^{1/2}}\left[\frac{1}{(1 + K_\perp/K_{\parallel})^{1/2} - 1}\right]^2$ per unit area.

One often assumes $K_\perp \sim \mu_0 M_0^2/2 \gg K_{\parallel}$ so that $c_0 = \mu_0 \gamma_g \lambda M_0/2$

The effective mass is then $M_w = 2S_w/\mu_0\gamma_g^2\lambda$ per unit area

- PCE Stamp, PRL **66**, 2802 (1991)
- G Tatara H Fukuyama, PRL **72**, 772 (1994)
- M Dube PCE Stamp, J Low Temp Phys **110**, 779 (1998)

RELEVANT TERMS in the HAMILTONIAN

Assume the lattice form $$L = s\sum_j \frac{d\phi_j}{dt}\cos\theta_j(t) + L_{s\phi} - H_M - H_{EM} - H_{\text{imp}} - H_{\text{def}}$$

(1) Electronic Spin terms $$H_M = \tfrac{1}{2}\sum_{\langle ij\rangle} J_{ij}\mathbf{s}_i\cdot\mathbf{s}_j + \tfrac{1}{2}K\sum_j s_j^x s_j^x + H_{\text{dip}}$$

Often go to the continuum form $$\mathscr{H} = \frac{1}{2}\int d\mathbf{r}\left[J(\nabla\mathbf{M})^2 - K_{||}(M_1)^2 + K_\perp(M_3)^2 - \frac{\mu_0}{2}(\mathbf{H}_{dm}+\mathbf{H}_e)\cdot\mathbf{M}\right]$$

where in flat wall approximation: $K_\perp = K_{\perp,\prime} + \mu_0 M_0^2/2$

(2) Photon-spin interactions $$H_{sA} = \int d^3r\sum_{\mathbf{k}\lambda}\left(\frac{\hbar\omega_{\mathbf{k}\lambda}}{2\epsilon(\omega)}\right)^{1/2}[\hat{\mathbf{e}}_{\mathbf{k}\lambda}\cdot\mathbf{M}(\mathbf{r},t)] \times[\alpha_{\mathbf{k}\lambda} - \alpha^\dagger_{-\mathbf{k}\lambda}]\,.$$

(3) Spin-phonon interactions $$L_{\text{int}} = -\int d^3\mathbf{r}\, A_{ijkl}U_{ij}(\mathbf{r})(\hat{m}_k(\mathbf{r})\,\hat{m}_l(\mathbf{r}) - \hat{m}_k^0\hat{m}_l^0) + R_{ijklmn}U_{ij}(\mathbf{r})\,U_{kl}(\mathbf{r})(\hat{m}_m(\mathbf{r})\,\hat{m}_n(\mathbf{r}) - \hat{m}_m^0\hat{m}_n^0)$$

$$U_{kl} = (\partial_k u_l + \partial_l u_k)/2$$

(4) Static impurity/defect pinning potential $V(Q) = -V_0\,\text{sech}^2(Q/\lambda_B)$

(5) Dynamic impurities (Nuclear spins, paramagnetic impurities)

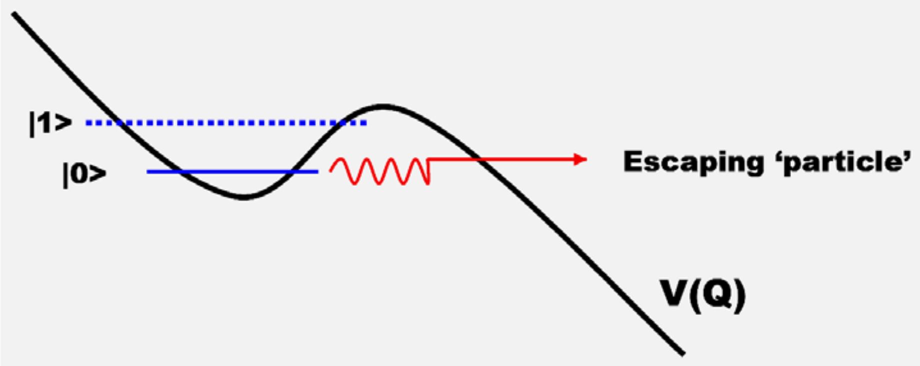
$$H = H_m + \sum_{k=1}^{N}\omega_k\mathbf{s}_k\cdot\mathbf{I}_k + \tfrac{1}{2}\sum_k\sum_{k'}V_{kk'}^{\alpha\beta}I_k^\alpha I_{k'}^\beta$$

WALL TUNNELING PROBLEM

The naïve Hamiltonian is

$$H_w = \tfrac{1}{2} M_w \dot{Q}^2 + V(Q) - 2S_w \mu_0 M_0 H_e Q$$

$$V(Q) = -V_0 \operatorname{sech}^2(Q/\lambda_B)$$



One easily finds a 'coercive field' $\mu_0 H_c = \frac{2}{3\sqrt{3}} \frac{V_0}{\lambda S_w M_0}$

Now assume we are close to the coercive field. Define $\varepsilon = 1 - H_e/H_c$

Then the potential is $\tilde{V}(\varepsilon) \sim (h\gamma_g \mu_0 H_c)\, N_0 \varepsilon^{3/2}$ with escape point $Q_0 = \frac{\sqrt{3}}{2} \lambda \varepsilon^{1/2}$

This wall contains a total number of spins $N_0 = \lambda S_w / a^3$

Then the naïve tunneling rate is given by $\Gamma_0 = \left[\frac{30}{\pi} \frac{B(\varepsilon)}{h} \right]^{1/2} \Omega_0 e^{-B_0(\varepsilon)/\hbar}$

with exponent: $\frac{1}{h} B_0(\varepsilon) = \frac{54}{5} \frac{S_w \lambda}{\gamma_g h} (M_0 H_c)^{1/2} \varepsilon^{5/4} \sim N_0 \left(\frac{H_c}{M_0} \right)^{1/2} \varepsilon^{5/4}$

The exponent can also be written as $\frac{1}{h} B_0(\varepsilon) = \frac{8}{15} M_w \Omega_0 Q_0^2$

with frequency $\Omega_0^2 = \frac{3\sqrt{3}}{4} (\mu_0 \gamma_g)^2 (M_0 H_c)\, \varepsilon^{1/2}$

The crossover to tunneling occurs at a temperature $k_B T_0 \sim \Omega_0 / 2\pi$

PUTTING IN SOME NUMBERS

Iron Garnet (YIG) and nickel. YIG is an insulator with a bcc cubic structure, a saturation magnetisation $\mu_0 M_0 = 0.24$ T and with exchange and anisotropy energies $J = 1 \times 10^{-11}$ J/m and $K_{||} = 580$ J/m^3. The width of the domain wall is $\lambda = 860$ Å, with a mass per unit area 2×10^{-9} kg/m^2. Nickel is a conductor, again with a cubic structure. The saturation magnetisation $\mu_0 M_0 = 0.6$ T, with exchange and anisotropy $J = 3 \times 10^{-11}$ J/m and $K_{||} = 4500$ J/m^3, giving a domain width $\lambda = 500$ Å and a mass 6×10^{-10} kg/m^2.

Now let $N_0 = 10^6$

Assuming $\varepsilon = 10^{-3}$ and $H_c/M_0 = 0.01$, we get $B_0/h \sim 20$, $\Omega_0 \sim 6 \times 10^9\ \mathrm{sec}^{-1}$ so that $\Gamma \sim 200\ \mathrm{sec}^{-1}$.

$$T_0 \sim 0.02\ \mathrm{K}$$

BASIC FORM of COUPLINGS

(1) OSCILLATOR BATH:

$$L_{CL} = \frac{M}{2}\dot{\mathbf{Q}}^2 - V(\mathbf{Q}) + \frac{1}{2}\sum_{\mathbf{k}} m_{\mathbf{k}}(\dot{\mathbf{x}}_{\mathbf{k}}^2 + \omega_{\mathbf{k}}^2 \mathbf{x}_{\mathbf{k}}^2)$$

$$-\sum_{\mathbf{k}} [F_{\mathbf{k}}(\mathbf{Q},\dot{\mathbf{Q}})\,\mathbf{x}_{\mathbf{k}} + G_{\mathbf{k}}(\mathbf{Q},\dot{\mathbf{Q}})\,\dot{\mathbf{x}}_{\mathbf{k}}] + \Phi(\mathbf{Q},\dot{\mathbf{Q}})$$

This gives a term:

$$\Delta S_{\text{eff}} = \frac{1}{2}\int_0^{1/T} d\tau \int_0^{1/T} d\tau'\, \alpha(\tau-\tau')(\mathbf{Q}(\tau) - \mathbf{Q}(\tau'))^2$$

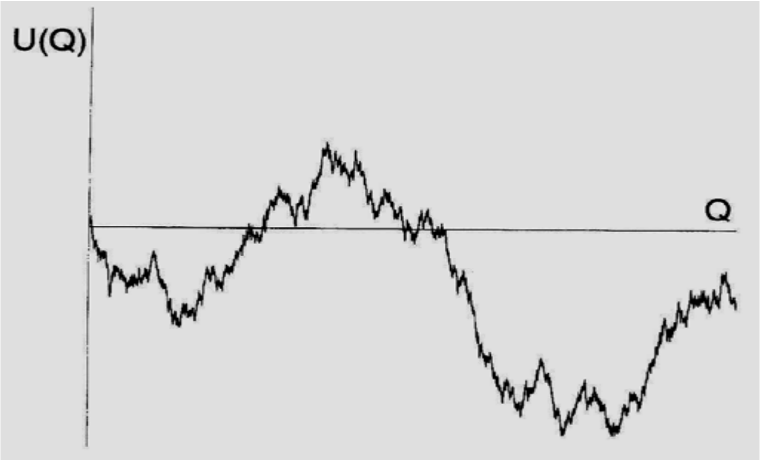
with $\alpha(\tau-\tau') = \frac{1}{4\pi} T \sum_n \int_0^\infty d\omega\, \omega J(\omega) \frac{e^{i\omega_n(\tau-\tau')}}{\omega_n^2 + \omega^2}$ and coupling $J(\omega) = \frac{\pi}{2}\sum_{\mathbf{k}} \frac{C_{\mathbf{k}}^2}{m_{\mathbf{k}}\omega_{\mathbf{k}}}\delta(\omega - \omega_{\mathbf{k}})$

(2) SPIN BATH:

$$H_{SB} = H_0(\mathbf{P},\mathbf{Q}) + H_B(\{\boldsymbol{\sigma}_k\}) + H_{\text{int}}(\mathbf{P},\mathbf{Q},\boldsymbol{\sigma}_k\})$$

We have the standard bath Hamiltonian

$$H_B(\{\boldsymbol{\sigma}_k\}) = \sum_k \mathbf{h}_k \cdot \boldsymbol{\sigma}_k + \tfrac{1}{2}\sum_k \sum_{k'} V_{kk'}^{\alpha\beta}\sigma_k^\alpha \sigma_{k'}^\beta$$



The interaction takes the form:

$$H_{\text{int}}(\mathbf{P},\mathbf{Q},\boldsymbol{\sigma}_k\}) = \sum_k (F_k^z(\mathbf{P},\mathbf{Q})\,\sigma_k^z + \tfrac{1}{2}(F_k^+(\mathbf{P},\mathbf{Q})\,\sigma_k^- + F_k^-(\mathbf{P},\mathbf{Q})\,\sigma_k^+))$$

Typically we introduce an effective potential $U(\mathbf{Q}) = \sum_k F_k^z(\mathbf{Q})\,\sigma_k^z$

which fluctuates over a distribution: $W(U) \sim (2\pi E_0^2)^{-1/2}\exp[-U^2/2E_0^2]$

WALL-MAGNON INTERACTIONS

We assume a 'collective coordinate' decoupling of the wall profile from the spin waves, ie., a separation $\mathbf{M}(\mathbf{r}) = \mathbf{M}_W(\mathbf{r}, Q) + \delta\mathbf{M}(\mathbf{r})$

The effective Lagrangian takes the form, in terms of Holstein-Primakoff bosons:

$$L_{mag} = \int d^3r \left\{ \lambda^2 \Delta_0 (\nabla b)^2 - \Delta_0 \left[1 - \mathrm{sech}^2 \left(\frac{\tilde{z}}{\lambda} \right) \right] b^\dagger b - i\hbar\gamma K \left(\frac{32s}{l_0^3} \right)^{1/2} \mathrm{sech}\left(\frac{\tilde{z}}{\lambda} \right) \left[\tanh\left(\frac{\tilde{z}}{\lambda} \right) - \lambda\nabla \right] (b^\dagger - b) b^\dagger b \right\}$$

These bosons are defined along the local axes defined by the wall profile:

$$\delta M_w(\mathbf{r}) = -4\gamma_g b^+(\mathbf{r})\, b(\mathbf{r})$$

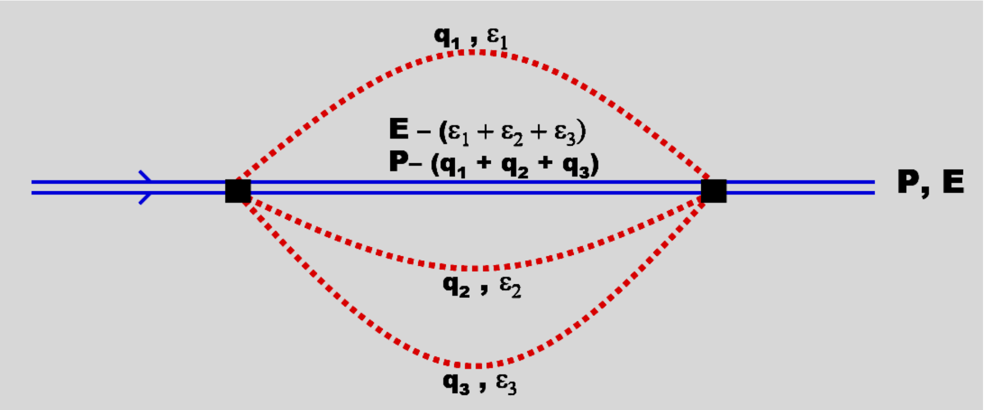
$$\delta M_+(\mathbf{r}) = (4\gamma_g M_0)^{1/2} \left(1 - \frac{2\gamma_g}{M_0} b^+(\mathbf{r})\, b(\mathbf{r}) \right)^{1/2} b(\mathbf{r})$$

$$\delta M_-(\mathbf{r}) = (4\gamma_g M_0)^{1/2}\, b^+(\mathbf{r}) \left(1 - \frac{2\gamma_g}{M_0} b^+(\mathbf{r})\, b(\mathbf{r}) \right)^{1/2}$$

with $\delta M_\pm(\mathbf{r}) = \delta M_u(\mathbf{r}) \pm \delta M_v(\mathbf{r})$

The key point is that quadratic interactions with magnons cause no dissipation or decoherence; one needs TRIPLETS of magnons to do this. The triplets involve combinations of wall & bulk magnons, & give a dissipation rate at low velocity of

$$\eta(T) = \frac{A_W}{16\pi^2\gamma^2\lambda^3} \left(\frac{kT}{\Delta_0^2} \right) e^{-\Delta_0/kT}$$



WALL-PHONON INTERACTIONS

M Dube, PCE Stamp, J Low Temp Phys 110, 779 (1998)

Free phonons $\mathscr{L}_p(\mathbf{x}, \tau) = \frac{1}{2} \rho_v \dot{u}_i^2 + \frac{1}{2} C_{ijkl} U_{ij} U_{kl}$ with $U_{kl} = (\partial_k u_l + \partial_l u_k)/2$

Interaction:

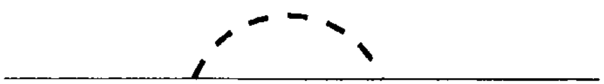
$$L_{\text{int}} = -\int d^3\mathbf{r}\, A_{ijkl} U_{ij}(\mathbf{r})(\hat{m}_k(\mathbf{r})\,\hat{m}_l(\mathbf{r}) - \hat{m}_k^0 \hat{m}_l^0) + R_{ijklmn} U_{ij}(\mathbf{r})\, U_{kl}(\mathbf{r})(\hat{m}_m(\mathbf{r})\,\hat{m}_n(\mathbf{r}) - \hat{m}_m^0 \hat{m}_n^0)$$

Here $C \sim 10^{11}\mathrm{J/m^3}$

These contribute to an effective action $e^{-S_{\text{eff}}[\hat{\mathbf{m}}]} = e^{-S_0[\hat{\mathbf{m}}]} \dfrac{\int D[\mathbf{u}]\, e^{-S_0[\mathbf{u}] - S_I[\hat{\mathbf{m}}, \mathbf{u}]}}{\int D[\mathbf{u}]\, e^{-S_0[\mathbf{u}]}}$

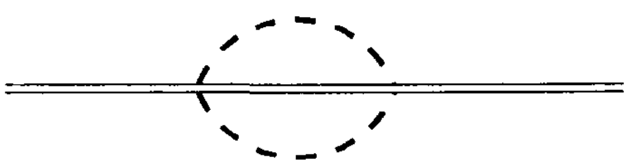
One-phonon couplings

$$S_I^{(i)}[\mathbf{u}, \hat{\mathbf{m}}] = \frac{i}{2} A_{ab} T \sum_n \sum_{\mathbf{q}} \int_0^{1/T} d\tau\, e^{i\mathbf{q}\cdot\mathbf{Q}(\tau)} e^{-i\omega_n \tau} \mathscr{M}_b(-\mathbf{q}) \times [q_{a_1} u_{a_2}(\mathbf{q}, i\omega_n) + q_{a_2} u_{a_1}(\mathbf{q}, i\omega_n)]$$

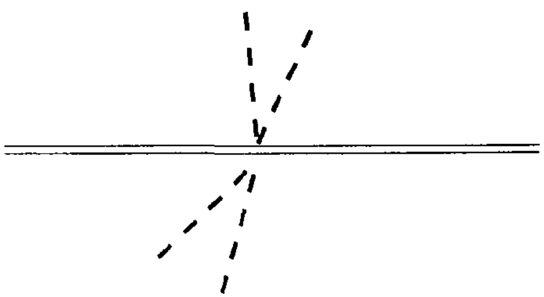


Two-phonon couplings

$$S_I^{(ii)}[\mathbf{u}, \hat{\mathbf{m}}] = -\tfrac{1}{4} R_{abc} T^2 \sum_{nn'} \sum_{\mathbf{kk}'} \int_0^{1/T} d\tau\, e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{Q}(\tau)} e^{-i(\omega_n + \omega_{n'})\tau} \times \mathscr{M}_c(-\mathbf{k} - \mathbf{k}')[k_{a_1} u_{a_2}(\mathbf{k}, i\omega_n) + k_{a_2} u_{a_1}(\mathbf{k}, i\omega_n)] \times [k'_{b_1} u_{b_2}(\mathbf{k}', i\omega'_n) + k'_{a_2} u_{b_1}(\mathbf{k}', i\omega'_n)]$$



Matrix elements $\mathscr{M}_a(\mathbf{q}) = \int d^3\mathbf{r}\, e^{-i\mathbf{q}\cdot\mathbf{r}}(\hat{m}_{a_1}(\mathbf{x})\,\hat{m}_{a_2}(\mathbf{r}) - \delta_{a_1, 1}\delta_{a_2, 1})$



We see that this problem is complex because we again have to take account of non-linear terms in the coupling – in fact the calculations show that interactions with pairs of phonons (above) and 4-phonon scattering processes (left) are crucial.

The phonons also completely suppress CHIRALITY Fluctuations (thus these are suppressed even if we have no spin bath). For all details see reference above.

EFFECT OF NUCLEAR SPINS & PARAMAGNETIC IMPURITIES

One deals with both of these in the same way. The key here is that this is the spin bath, & so it can cause a great deal of decoherence.

Let's take the interaction with nuclear spins: $$H = H_m + \sum_{k=1}^{N} \omega_k \mathbf{s}_k \cdot \mathbf{I}_k + \tfrac{1}{2} \sum_{k} \sum_{k'} V_{kk'}^{\alpha\beta} I_k^{\alpha} I_{k'}^{\beta}$$

As we saw before, we should split this into longitudinal & transverse parts. With some algebra we transform this to an interaction between nuclear spins & the domain wall coordinate $H_{\text{int}}(Q, \{\mathbf{I}_k\}) = {}^{\parallel}H_{\text{int}} + {}^{\perp}H_{\text{int}}$

where
$$^{\parallel}H_{\text{int}} = \sum_{k=1}^{N} \omega_k \int \frac{d^3r}{\gamma_g} \delta(\mathbf{r} - \mathbf{r}_k)((M_W(\mathbf{r}, Q)\, I_k^w - \mathbf{M}_0 \cdot \mathbf{I}_k) + I_k^w \delta M_w(\mathbf{r}))$$

$$^{\perp}H_{\text{int}} = \frac{1}{2} \sum_{k=1}^{N} \omega_k \int \frac{d^3r}{\gamma_g} \delta(\mathbf{r} - \mathbf{r}_k)(\delta M_+(\mathbf{r})\, I_k^- + \delta M_-(\mathbf{r})\, I_k^+)$$

We then find that one depends on the chirality, & the other on the topological charge

Longitudinal:
$$^{\parallel}H_{\text{int}}^{\text{Bloch}}(Q, \{\mathbf{I}_k\}) = \frac{\omega_0 M_0}{\gamma_g} \sum_{k=1}^{N} \int d^3r\, \delta(\mathbf{r} - \mathbf{r}_k) \times \left(\left(1 - C \tanh\left(\frac{x_3 - Q}{\lambda_B}\right)\right) + \delta m_w(\mathbf{r})\right) I_k^w$$

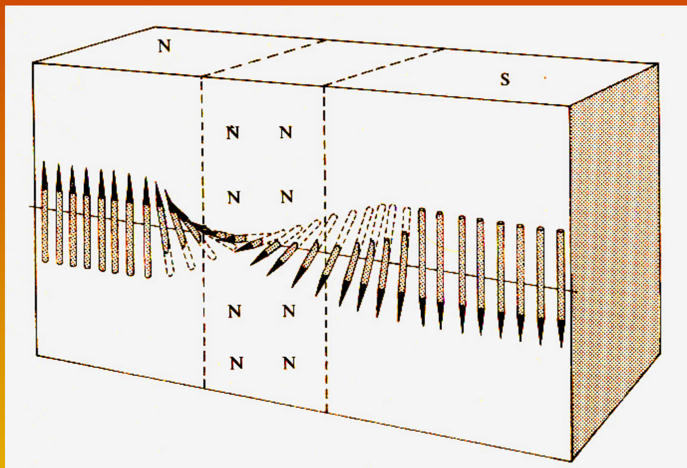
Transverse:
$$^{\perp}H_{\text{int}}^{\text{Bloch}}(Q, \{\mathbf{I}_k\}) = \frac{\omega_0 M_0}{2\gamma_g} \sum_{k=1}^{N} \int d^3r\, \delta(\mathbf{r} - \mathbf{r}_k) \left(\chi \operatorname{sech}\left(\frac{x_3 - Q}{\lambda_B}\right)(I_k^+ + I_k^-) + (\delta m_-(\mathbf{r})\, I_k^+ + \delta m_+(\mathbf{r})\, I_k^-)\right)$$

The transverse spin bath fluctuations couple to the chirality – we already saw how crucial this is in studying the spin net.

The longitudinal term gives our effective potential: $$U(Q) = \frac{\omega_0 M_0}{\gamma_g} \sum_{k=1}^{N} \int d^3r\, \delta(\mathbf{r} - \mathbf{r}_k) \left(1 - C \tanh\left(\frac{x_3 - Q}{\lambda_B}\right)\right) I_k^w$$

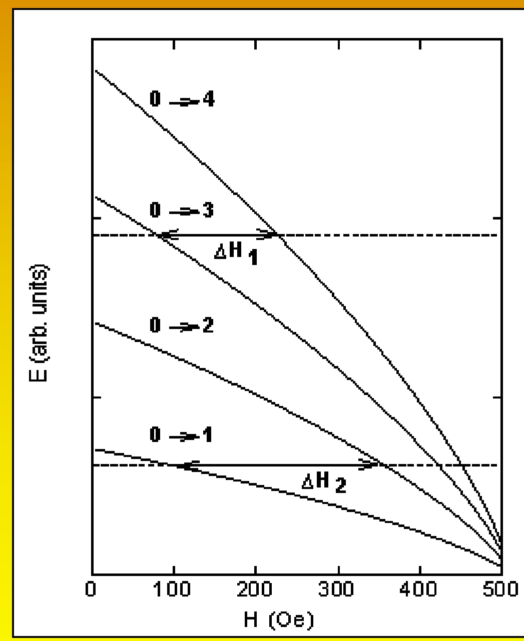
MQT in Magnets: Theory vs. Experiment.

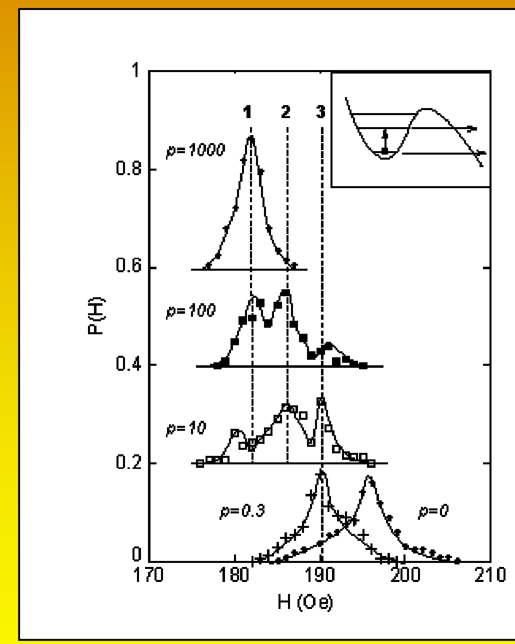
So, can we get Large Scale Quantum Phenomena with these solitons?



Initial theory on domain walls showed that any QUANTUM SOLITON in a magnet should show large-scale quantum Properties – provided the decoherence could be suppressed. Quantitative predictions could be made here- indicating that in some situations magnetic domain walls containing up to 10^{10} spins should be able to tunnel.

Experiments in Ni wires and in large particles bore out the theory





Example:

THEORY: **P.C.E. Stamp, PRL 66, 2802 (1991)**
M.Dube, P.C.E.Stamp, JLTP 110, 779 (1998)

EXPT:
K. Hong,N. Giordano, Europhys. Lett.36, 147 (1996)
W Wernsdorfer et al. PRL 78, 1791 (1997)

Notice the problem – there is a very wide dispersion, which is caused by the interaction with the spin bath (which turns out to be a combination of paramgnetic O impurties and nuclear spins).

Macroscopic Quantum Tunneling (MQT) in SQUIDs: Theory vs Expt.

It is interesting to compare with this previously studied phenomenon. It was shown by Leggett that essentially all previous arguments against large-scale Quantum phenomena were flawed, because

(i) matrix elements between macroscopic states can be controlled by microscopic energies

(ii) Because what really matters is the behaviour in time of

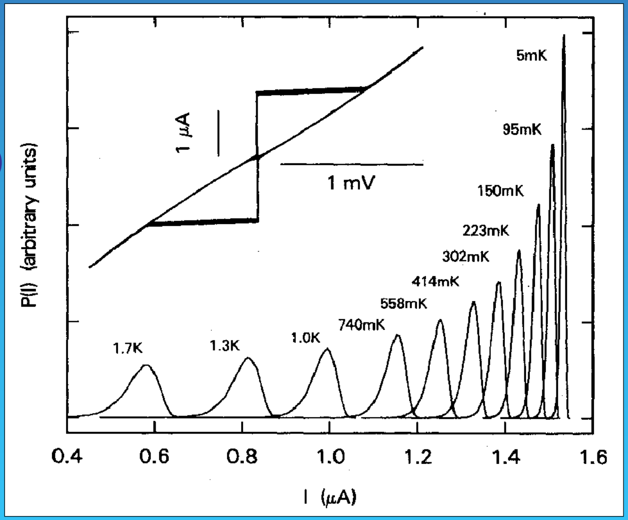
$$O_{AB}(t) = \langle \Psi(R_A;r_1,r_2,..)| e^{iH\, t} |\Psi(R_B;r_1,r_2,..)\rangle \qquad \text{(transition matrix)}$$

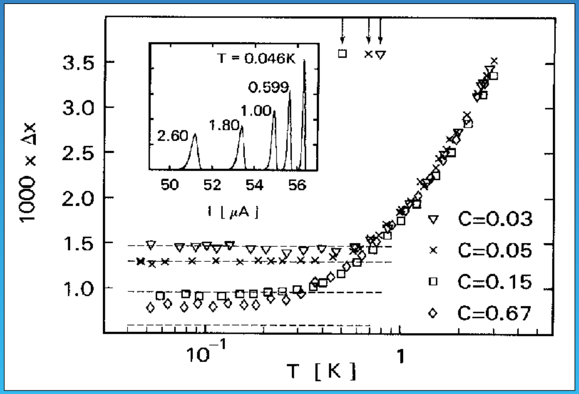
and that in particular, "Macroscopic Quantum Tunneling" between 2 different Flux states of a SQUID should be possible- a QUANTATIVE THEORETICAL PREDICTION was given.

Experimental confirmation came very quickly

Caldeira & Leggett: Ann. Phys. 149, 374 (1983)

Voss & Webb: PRL 47, 265 (1981)
Clarke et al: Science 239, 992 (88)

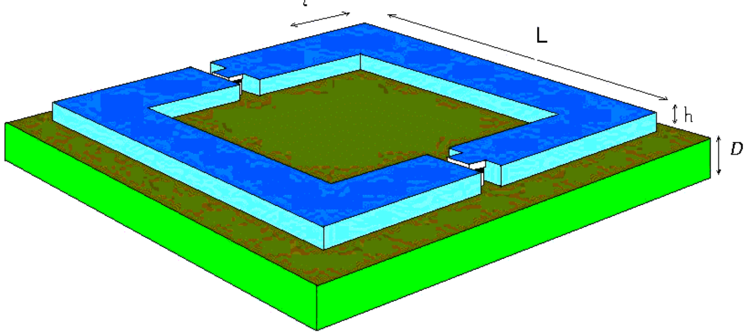




The comparison between theory and experiment for these MQT experiments was very good. The experiment ONLY took account of the oscillator bath environment, parametrised by the electronic circuitry – such modes cause all the dissipation.

However this theory fails completely to deal with DECOHERENCE in SQUIDS.....

SQUID DECOHERENCE RATES



We can always derive an effective Hamiltonian of spin bath form for a SQUID coupled to a spin bath. In the weak decoherence regime we get

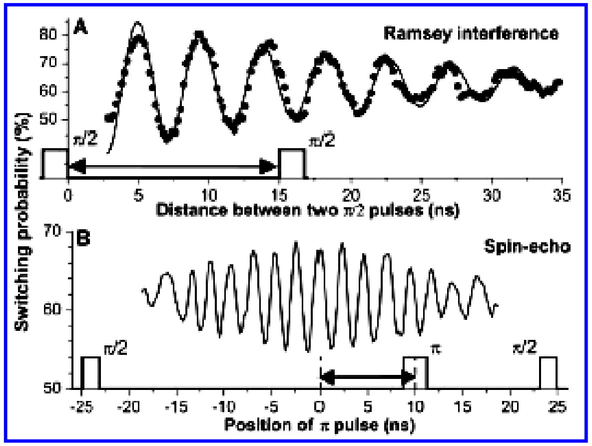
$$H_{\text{eff}} \sim \left[\Delta + \sum_{kk'} \frac{\omega_k^{\parallel}\omega_{k'}^{\parallel}}{2\Delta} \hat{\sigma}_k^x \hat{\sigma}_{k'}^x \right] \hat{\tau}_x + \sum_k \omega_k^{\perp} \hat{\sigma}_k^z$$

plus noise terms, giving a decoherence rate $\tau_\phi^{-1} \sim \sum_{kk'} \omega_k^{\parallel}\omega_{k'}^{\parallel}/8\Delta \;\approx\; E_o^2/8\hbar\Delta$

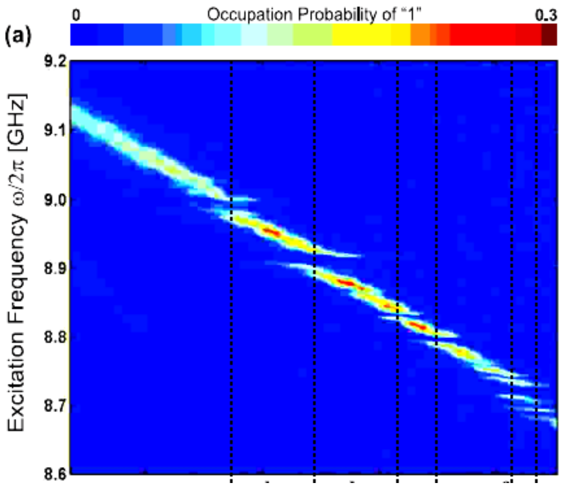
This rate is FAR higher than the dissipation rate. Experiments now confirm this is the main decoherence source

- PCE Stamp, PRL **61**, 2905 (1988)
- NV Prokof'ev, PCE Stamp, Rep Prog Phys 63, 669 (2000)

The basic problem with any theory-experiment comparison here is that most of the 2-level system are basically just junk (coming from impurities and defects), whose characteristics are hard to quantify. Currently ~10 grou have seen coherent oscillati in superconducting qubits, a several have seen entangler between qubit pairs. It has become clear in the last yea that most of the low-E decoherence is coming from TLS in the junction.



I Chiorescu et al., Science 299, 1869 (2003)



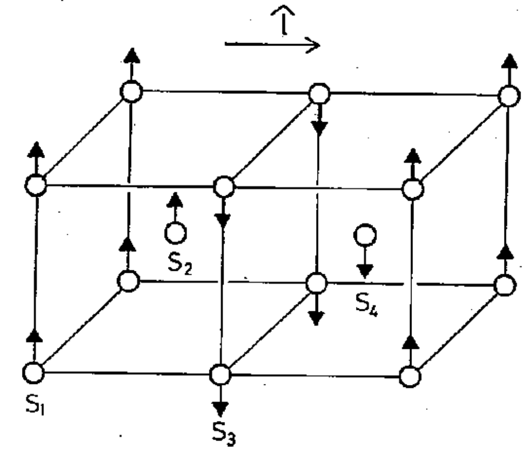
RW Simmonds et al., PRL 93, 077003 (2004)

Thus what is required is a parameter-free way of relating the theory to experiment, in which the distribution of couplings is extracted from expt. & used to predict other experimental properties. We do not yet have this.

3.4: DOMAIN WALLS in SOLID ^{3}He

There is one domain wall that has rather exceptional properties, at least on paper. This is the domain wall in solid 3He, which has never been explored experimentally. In fact solid 3He has a number of extraordinary properties, which we describe briefly below.

^{3}He SOLID: HAMILTONIAN and EQUATIONS of MOTION



The basic structure of the underlying U_2D_2 state has pairs of FM- coupled planes which are antiferromagnetically coupled to each other. This interesting combination of FM and AFM order arises because of the competition between different order FM and AFM exchange processes.

$$H=-\frac{1}{2}\sum_{n=1}^{3}J_n\sum_{(i,j)}^{(n)}\boldsymbol{S}_i\cdot\boldsymbol{S}_j$$

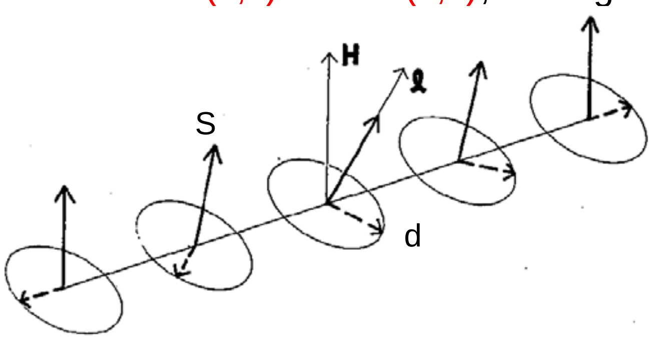
$$-\frac{1}{4}\sum_{\alpha=\mathrm{F,P}}K_\alpha\sum_{(i,j,k,l)}^{(\alpha)}[(\boldsymbol{S}_i\cdot\boldsymbol{S}_j)(\boldsymbol{S}_k\cdot\boldsymbol{S}_l)+(\boldsymbol{S}_i\cdot\boldsymbol{S}_l)(\boldsymbol{S}_k\cdot\boldsymbol{S}_j)-(\boldsymbol{S}_i\cdot\boldsymbol{S}_k)(\boldsymbol{S}_j\cdot\boldsymbol{S}_l)]$$

We will be interested in the equations of motion in the inhomogeneous case, so one generalises the OCF equations to get coupled equations for *d*(r,t) and *S*(r,t):

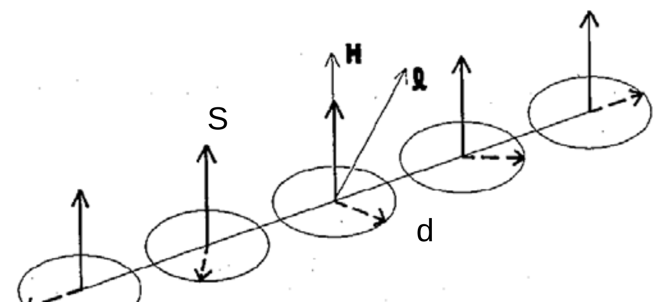
$$\dot{\hat{\boldsymbol{d}}}=\hat{\boldsymbol{d}}\times(\gamma_0\boldsymbol{H}-\gamma_0{}^2\chi_0{}^{-1}\boldsymbol{S}),$$

$$\dot{\boldsymbol{S}}=\gamma_0\boldsymbol{S}\times\boldsymbol{H}-\lambda(\hat{\boldsymbol{d}}\cdot\hat{\boldsymbol{l}})(\hat{\boldsymbol{d}}\times\hat{\boldsymbol{l}})+\chi_0\gamma_0{}^{-2}[c_{/\!/}{}^2(\hat{\boldsymbol{d}}\times\nabla_{/\!/}{}^2\hat{\boldsymbol{d}})+c_\perp{}^2(\hat{\boldsymbol{d}}\times\nabla_\perp{}^2\hat{\boldsymbol{d}})]$$

The first equation here has d(r,t) precessing in a combination of external field and the 'field' of *S*(r,t). The 2nd equation involves precession of *S* itself, and also a dipolar coupling between *l*(r,t) and *d*(r,t); and gradient terms.



A texture in d(r,t) for small H



A texture in d(r,t) for large H

The remaining material on He-3 was discussed on the blackboard

(For more on He-3 solid, see lectures of Osheroff)