From exotic phases to microscopic Hamiltonians via quantum dimer models

Kumar S. Raman

UIUC

PITP/Les Houches Summer School on Quantum Magnetism;
Les Houches, France, June 6-23, 2006

The search for exotic phases

The phase diagrams of many strongly correlated systems (i.e. high Tc materials) are not easily describable in the "usual" way:

Phases described by local order parameters.

Transitions described by symmetry breaking.

This led to proposal of phases with non-local, "exotic" ordering properties:

Spin liquids

- long-range RVB (Anderson)
- short-range RVB (Rokhsar, Kivelson, Sethna)
- algebraic spin liquid (Affleck, Marston, Hermele et. al.)
- many other varieties (Wen)

Stripe-like phases (Emery, Kivelson, Fradkin, and coworkers)

(high-order) spatially modulated structures in the absence of long-range interactions or symmetry breaking in the Hamiltonian.

Collaborators

Construction of spin liquid phase (cond-mat/0502146)

Roderich Moessner (ENS -- Paris)

Shivaji L. Sondhi (Princeton)

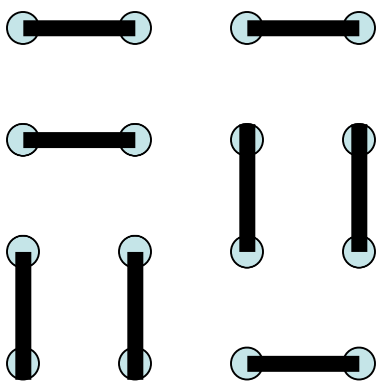
Construction of stripe-like phases (to appear 2006)

Stefanos Papanikolaou (UIUC)

Eduardo Fradkin (UIUC)

Part 1: What is an RVB liquid?

system: spins on a lattice



 means spins paired in a singlet

valence bond state $\left|c\right\rangle$

resonating valence bond (RVB) state: $$\left|RVB\right\rangle = \sum_c \left|c\right\rangle$$

Applications: Quantum magnetism (Fazekas, Anderson 1972)
High Tc (Anderson 1987), Quantum computing (Kitaev)

Comment: restriction to nn singlets = short range RVB

RVB liquids: kinematics

- Energetically competitive to Neel ordering.
- Translational and rotational invariance.
- No local order parameter (non-magnetic).
- Global order: “topological order”.
- Deconfined fractionalized excitations (spinons).

RVB liquids: dynamics

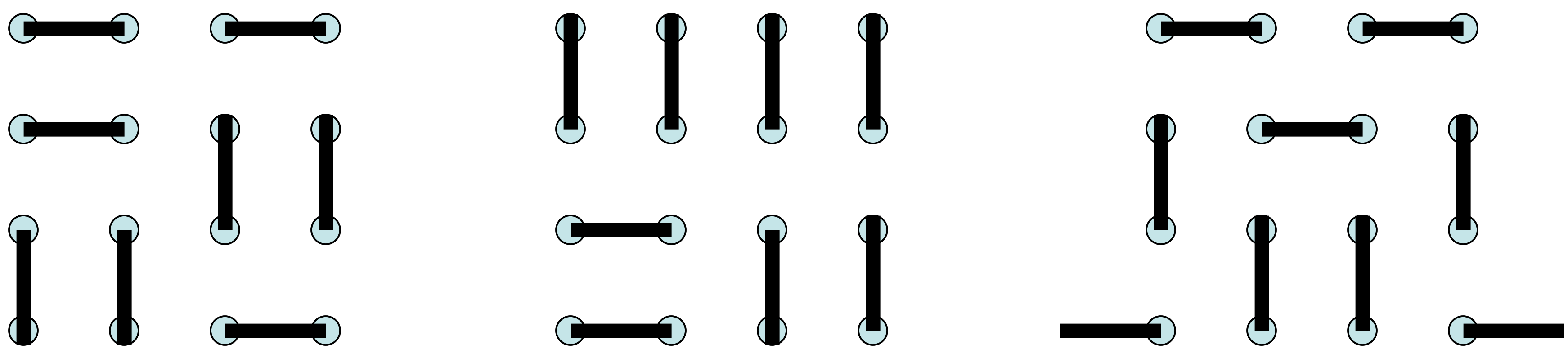
What kind of Hamiltonian has an RVB ground state?

We will construct a rotationally invariant spin Hamiltonian that shows an RVB liquid state in its ground state phase diagram.

Short range RVB and dimers

-- Rokhsar, Kivelson (1988): Quantum dimer model. Low energy theory for short-range RVB (Rokhsar, Kivelson, Sethna 1987).

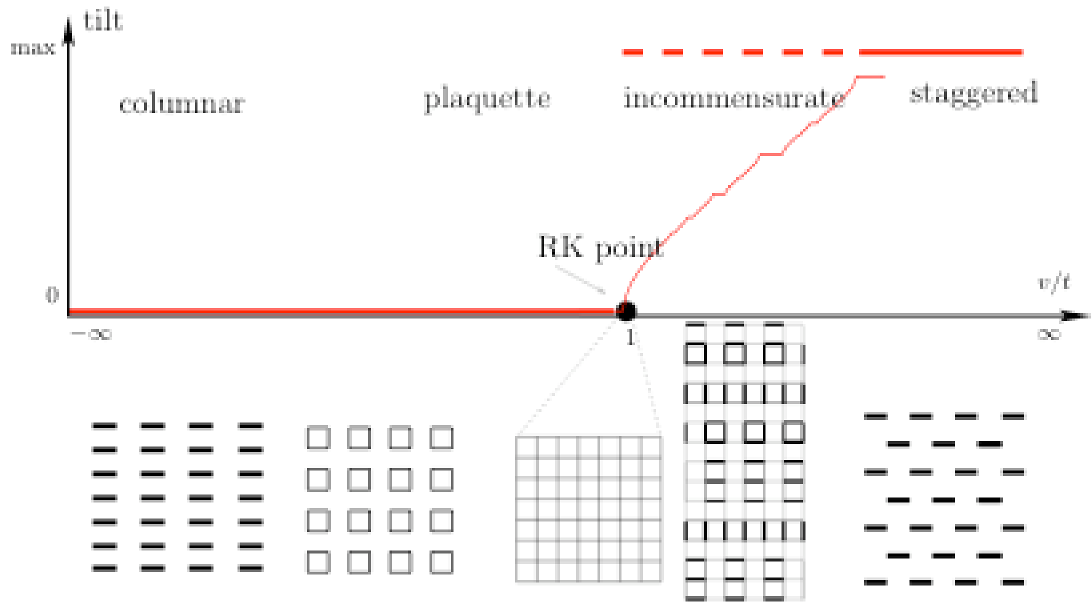
-- dimer coverings = orthonormal basis vectors of a dimer Hilbert space.



$$H_{QDM} = \sum -t\left(|\,\mathrm{I\,I}\,\rangle\langle\,=\,| + h.c.\right) + v\left(|\,\mathrm{I\,I}\,\rangle\langle\,\mathrm{I\,I}\,| + |\,=\,\rangle\langle\,=\,|\right)$$

QDM in d=2: Bipartite lattices

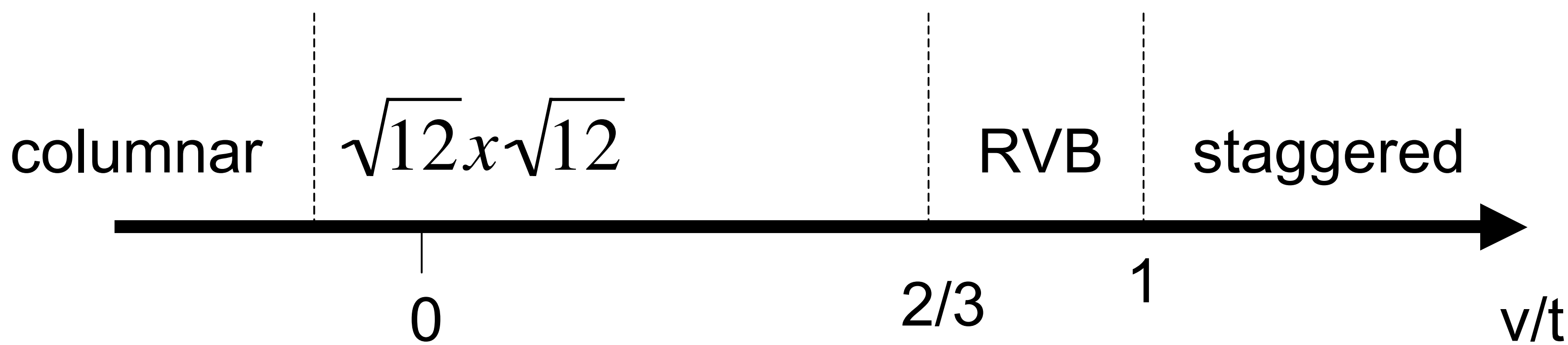
i.e. square, honeycomb



1. Moessner, Sondhi, Chandra (2001): Plaquette phase for v<t implying that the equal amplitude state occurs only at v=t.
2. Fradkin, Huse, Moessner, Oganesyan, and Sondhi. (2004): Small perturbations of the basic QDM drive the system into Cantor deconfined phases for v>t.

QDM in d=2: Non-bipartite lattices

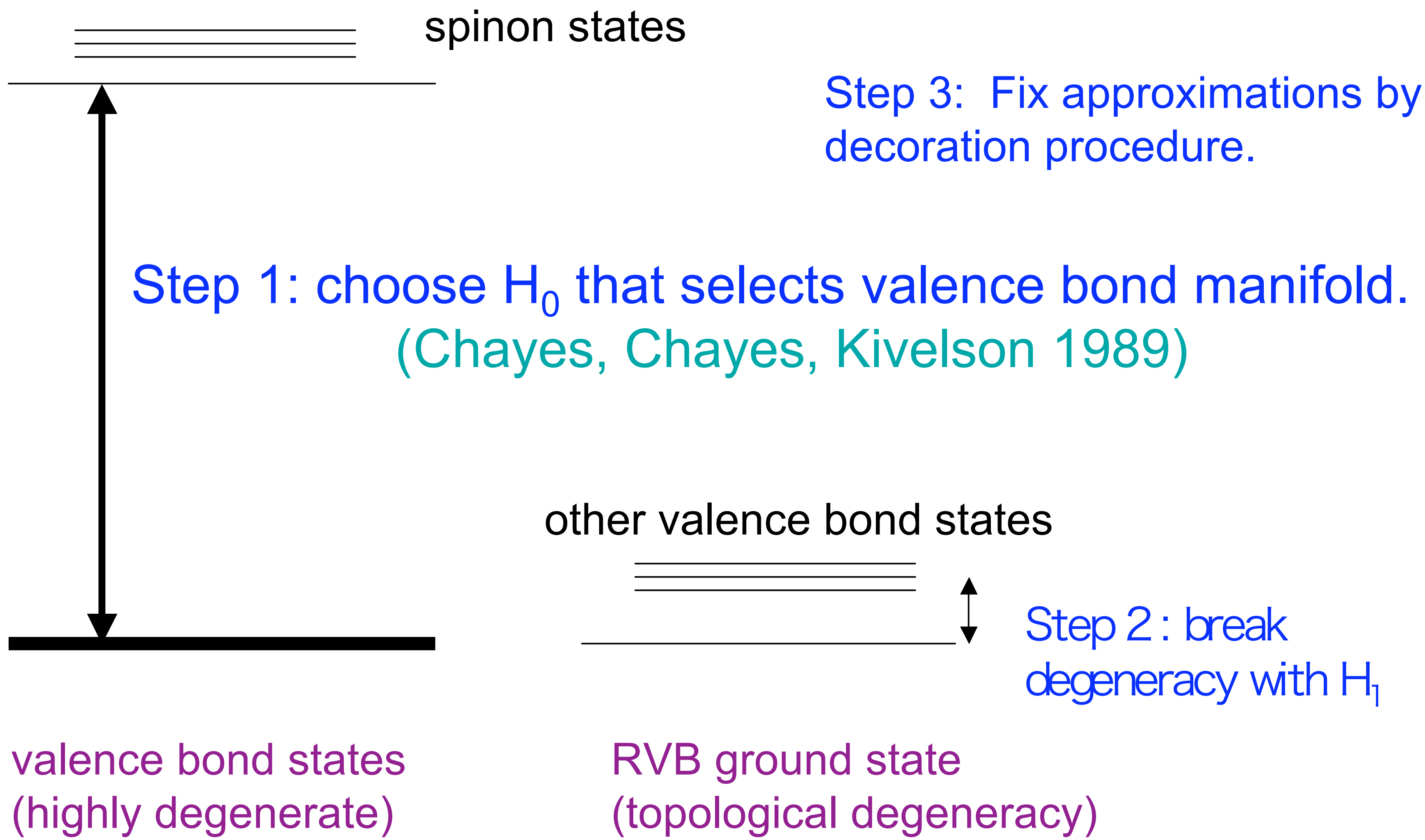
i.e. triangular, pentagonal



Moessner, Sondhi (2001)

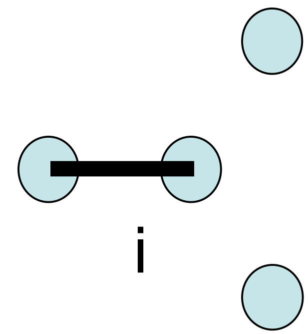
Relating QDM phases to quantum spin models

Outline of strategy



1. Choosing H_0: Klein models

$$H_o = \sum_{i \in \Lambda} \hat{P}_{N(i)}$$



$\hat{P}_{N(i)}$ projects the cluster of i and its (z-1) neighbors onto its highest total spin state. (e.g. S=2)

e.g. z=4

Note: If spin i forms a singlet with one of its neighbors, then the state will have zero projection. Valence bond coverings of the lattice are zero energy ground states of H_0

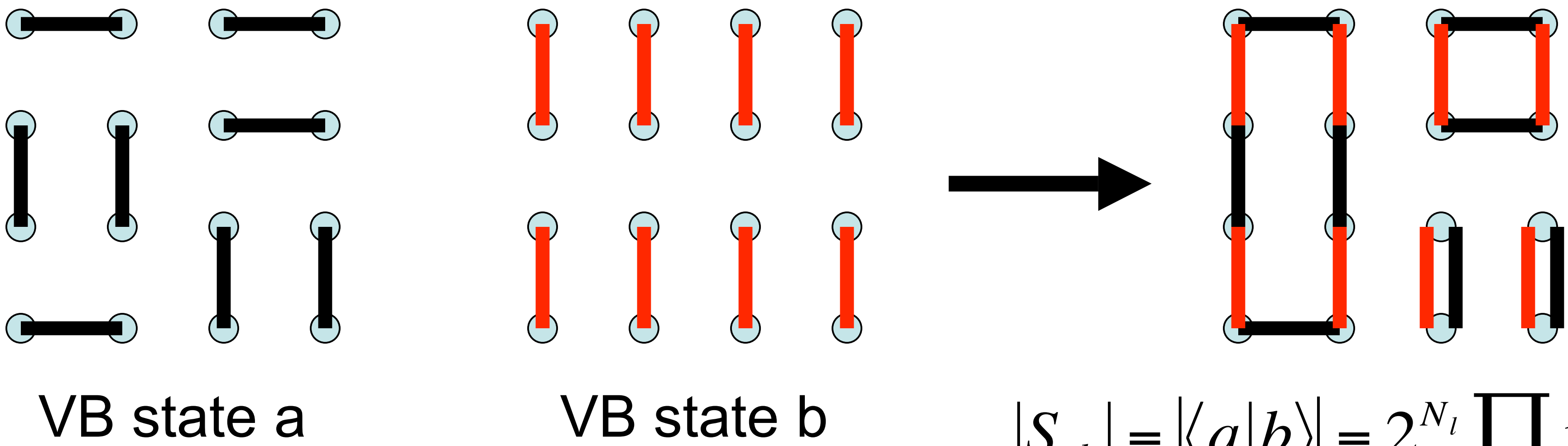
e.g. honeycomb

$$H_o = \sum_{\langle i,j \rangle} \vec{s}_i \cdot \vec{s}_j + \frac{1}{2} \sum_{\langle\langle i,j \rangle\rangle} \vec{s}_i \cdot \vec{s}_j + \frac{2}{5} \sum_{ijkr}^{\prime} (\vec{s}_i \cdot \vec{s}_j)(\vec{s}_k \cdot \vec{s}_r)$$

Valence bonds vs. dimers

1. Linear independence: geometry dependent.

2. Non-orthogonality: overlap issues.



$$|S_{ab}| = |\langle a|b\rangle| = 2^{N_l} \prod_i x^{L_i}$$

$$x = 1/\sqrt{2}$$

3. Sign convention.

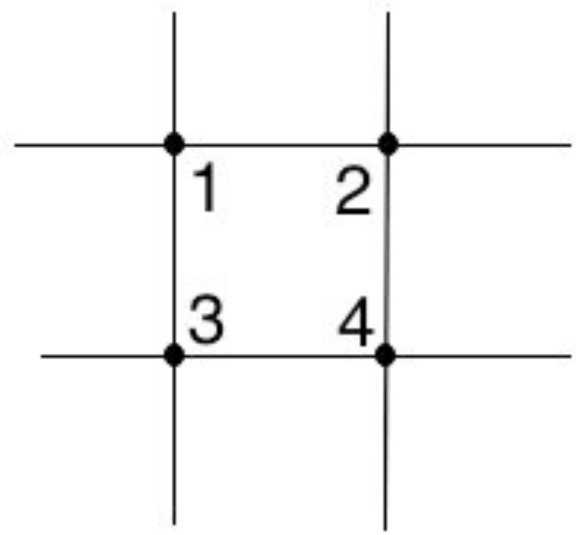
$$\pm \frac{1}{\sqrt{2}} (1_\uparrow 2_\downarrow - 1_\downarrow 2_\uparrow)$$

2. Perturb and get H_{eff} by expanding in x

$$\delta H = J\sum_{\langle ij\rangle}\vec{s}_i\cdot\vec{s}_j + v\sum_{P}\Big((\vec{s}_1\cdot\vec{s}_2)(\vec{s}_3\cdot\vec{s}_4) + (\vec{s}_1\cdot\vec{s}_3)(\vec{s}_2\cdot\vec{s}_4)\Big)$$

$$|\alpha\rangle = \sum_a \left(S^{-\frac{1}{2}}\right)_{\alpha a}|a\rangle$$

$$H_{eff} = S^{-1/2}(\delta H)S^{-1/2}$$

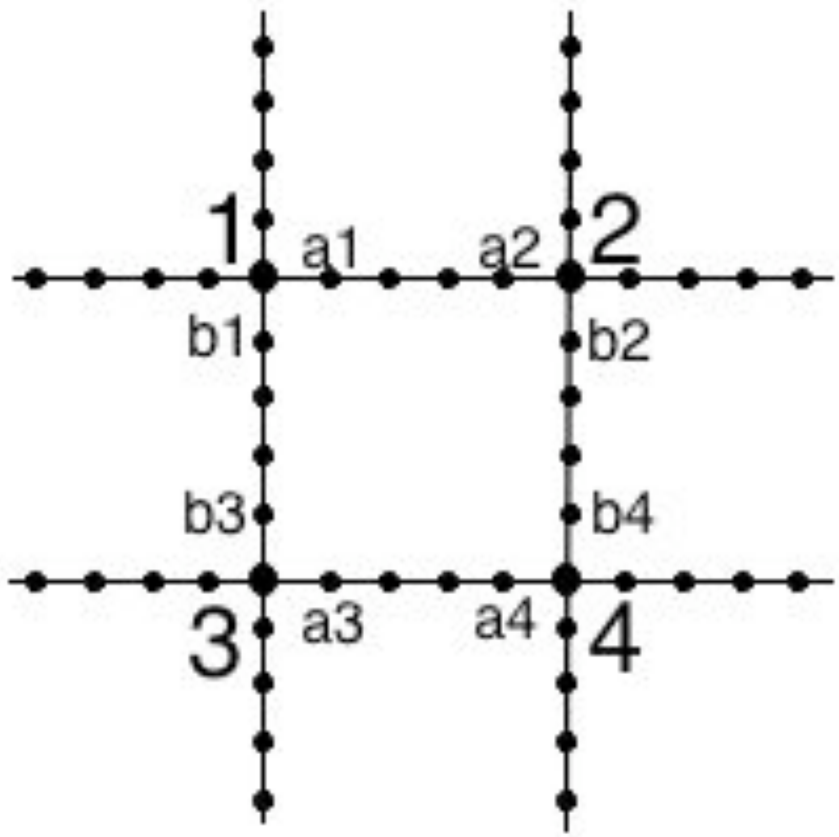


$$\boxed{\left(H_{eff}\right)_{\alpha\beta} = -tR_{\alpha\beta} + vn_{fl,\alpha}\delta_{\alpha\beta} + O(vx^4 + tx^2)}$$

$$t = Jx^4$$

3. Decoration procedure

Add even number N of sites in between. (N=4 in figure)



$$\delta H = J\sum_{\langle ij\rangle}\vec{s}_i\cdot\vec{s}_j + v\sum_P\Big((\vec{s}_1\cdot\vec{s}_{a1})(\vec{s}_3\cdot\vec{s}_{a3}) + (\vec{s}_1\cdot\vec{s}_{b1})(\vec{s}_2\cdot\vec{s}_{b2})\Big)$$

QDM physics in spin system

$$\boxed{\left(H_{eff}\right)_{\alpha\beta} = -tR_{\alpha\beta} + vn_{fl,\alpha}\delta_{\alpha\beta} + O(vx^{4N} + tx^{2N})}$$

Reproduce the physics of the QDM to arbitrary accuracy. Relatively small value of N will produce phases with the desired qualitative behavior.

Conclusion: We have constructed an SU(2) realization of bipartite QDM physics in d=2, which includes Cantor deconfinement, plaquette phases, etc. Can apply the same construction to triangular lattice QDM to realize its phases including the RVB liquid phase!

Excitations

- Moessner et. al. (2001): Collective dimer modes are gapped on the triangular lattice.
- Shastry and Sutherland (1981): Spinons are the natural excitations of the 1d chains. Spinons localized on the decorated edges should be gapped in the high decoration limit.
- Potential energy cost of violating Klein model defeats hopping energy from having a spinon localized at chain crossing. (present work).

The construction gives a stable (gapped) RVB liquid phase!

Higher dimensional examples

Our construction is a general way to transcribe the phases of a QDM in d dimensions into an SU(2) invariant quantum spin model in d dimensions.

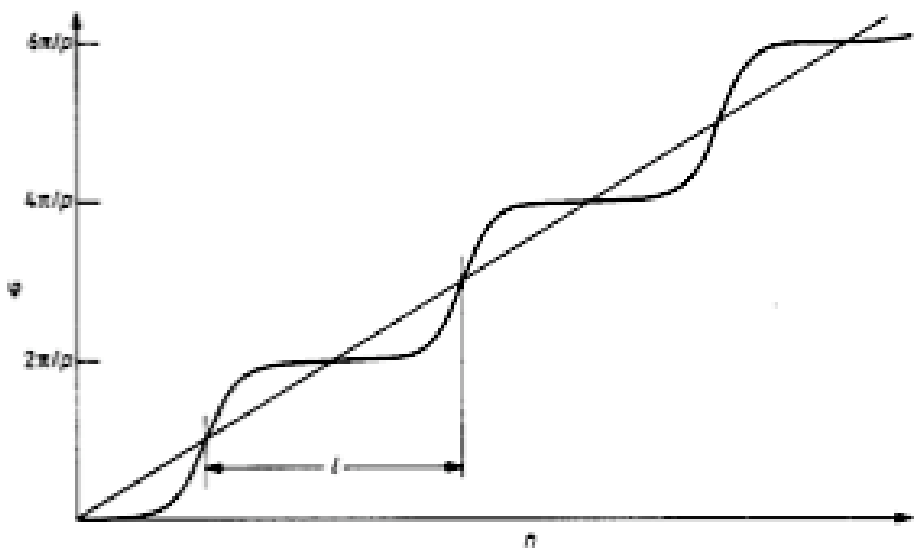
- Bipartite lattices in d=3 (cubic, diamond): Coulomb phases, i.e. U(1) RVB liquid.
- Non-bipartite lattice in d=3 (FCC): Z_2 RVB liquid phase.
- An interesting example: pyrochlore lattice. Can construct two different models which show the two different types of liquid behavior.

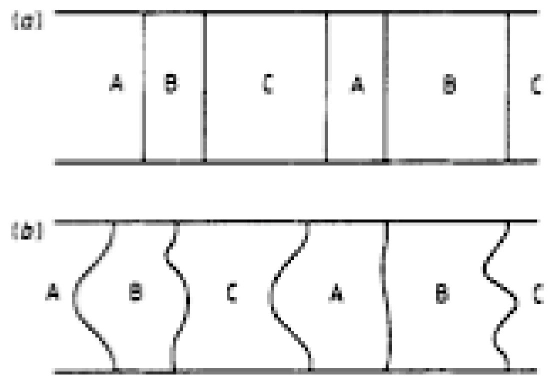
Part 2: What do I mean by "stripe"?

Classical example: fluctuating domain walls (Pokrovsky/Talapov)

$$H = \int dn \Big[\frac{1}{2} \Big(\frac{d\phi}{dn} - \delta \Big)^2 + V(1 - \cos p\phi) \Big]$$

At critical value of parameters:



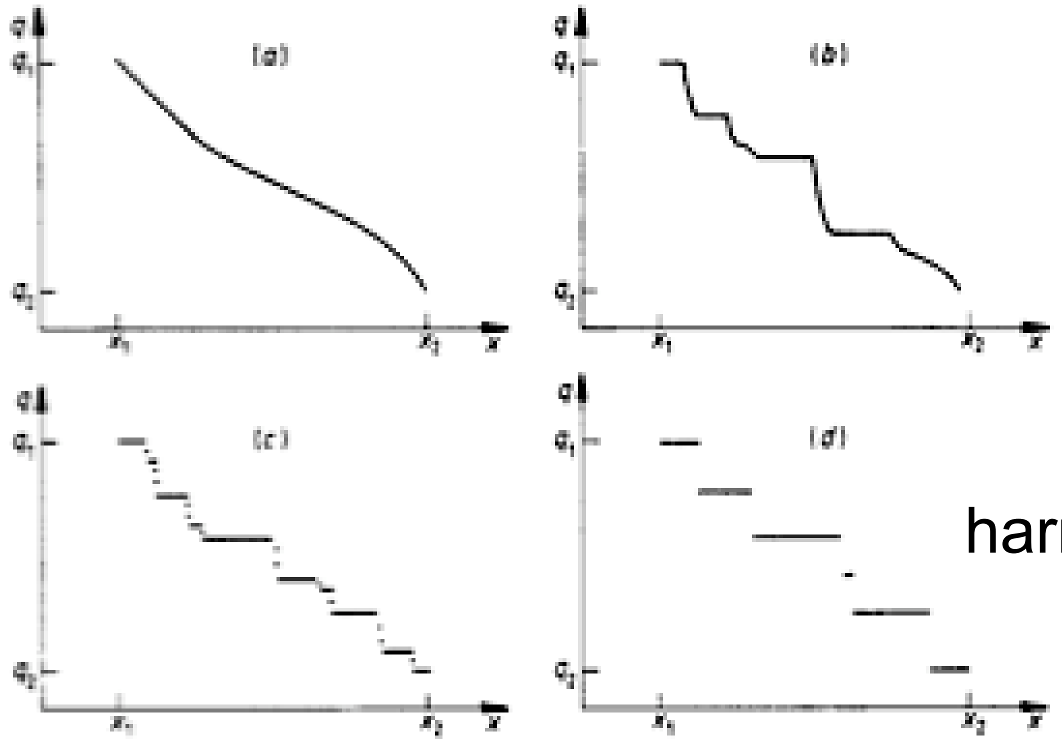


In principle, scale of modulation can be large compared to scale of interaction.

Types of spatial modulation

floating

incomplete devil's staircase

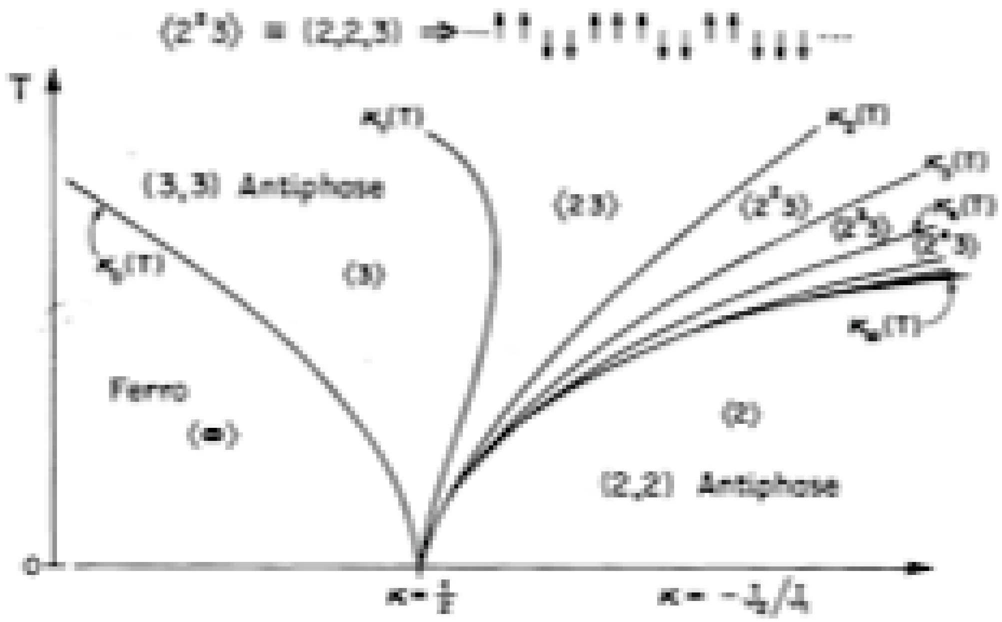


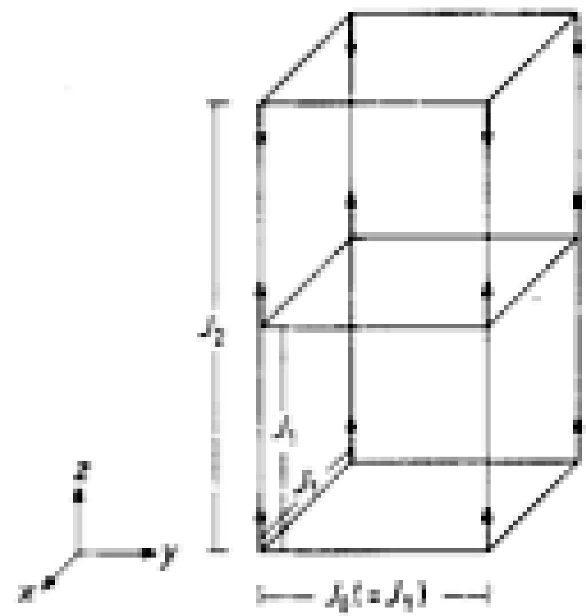
complete devil's staircase

harmless staircase

P. Bak (1982)

Classical example: ANNNI MODEL



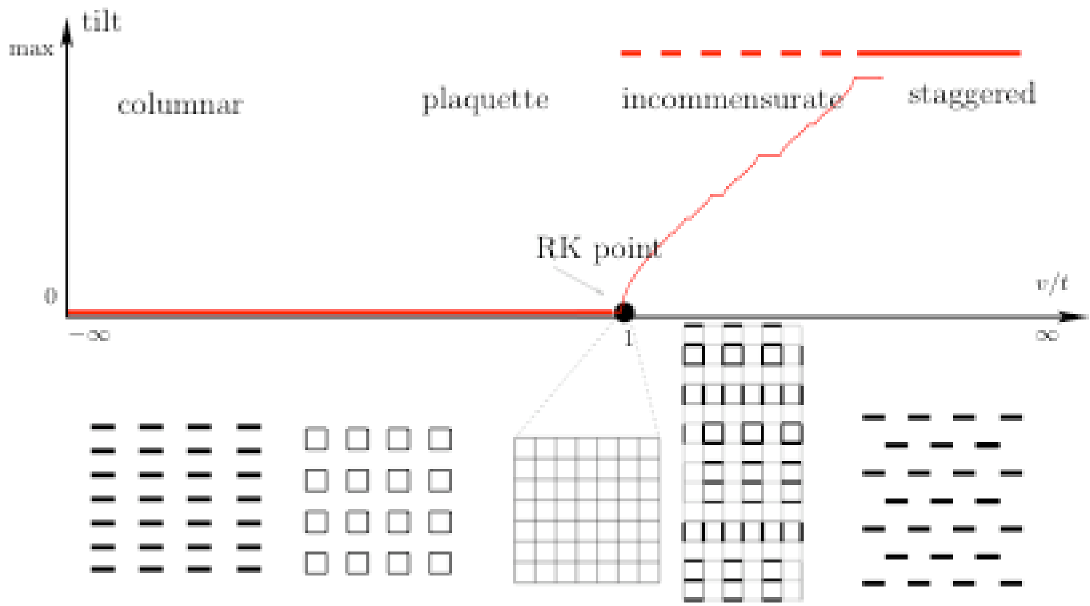


Fisher and Selke (1980), P. Bak (1982)

Question: Can we realize very high order spatially modulated structures in a quantum model without breaking lattice or spin symmetries and in the absence of long range interactions?

Basic strategy 1: Reduce problem to dimers

FLAT



STEEP

bipartite lattices in 2d: square, honeycomb

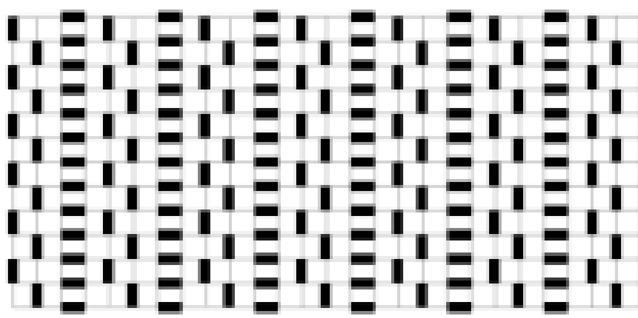
Field theoretic arguments suggest that stripe-like staircase structures can be realized in quantum dimer models

Fradkin, Huse, Moessner, Oganesyan, and Sondhi. (2004)

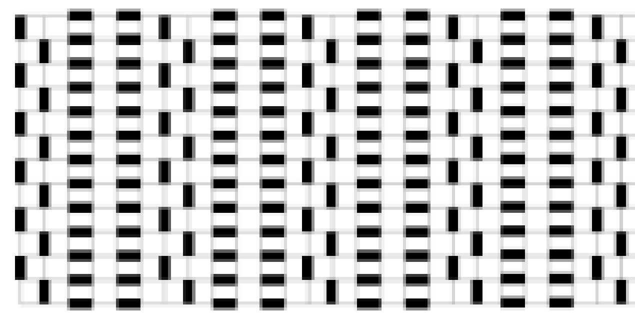
Obtain an SU(2) invariant spin model by similar procedure as I just described.

The [1n] states (simplest tilted states)

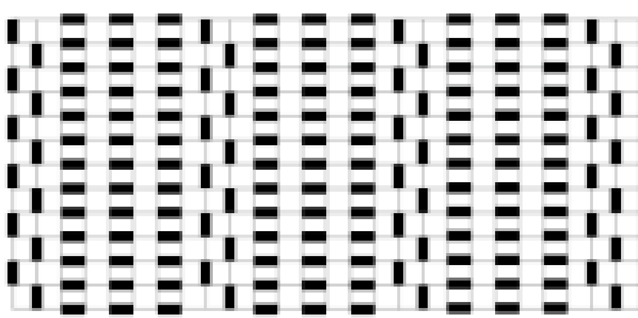
[11]



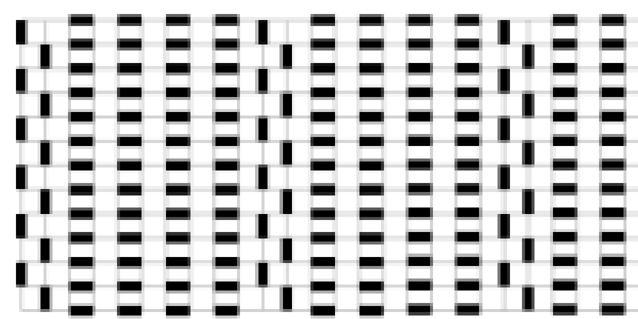
[12]



[13]



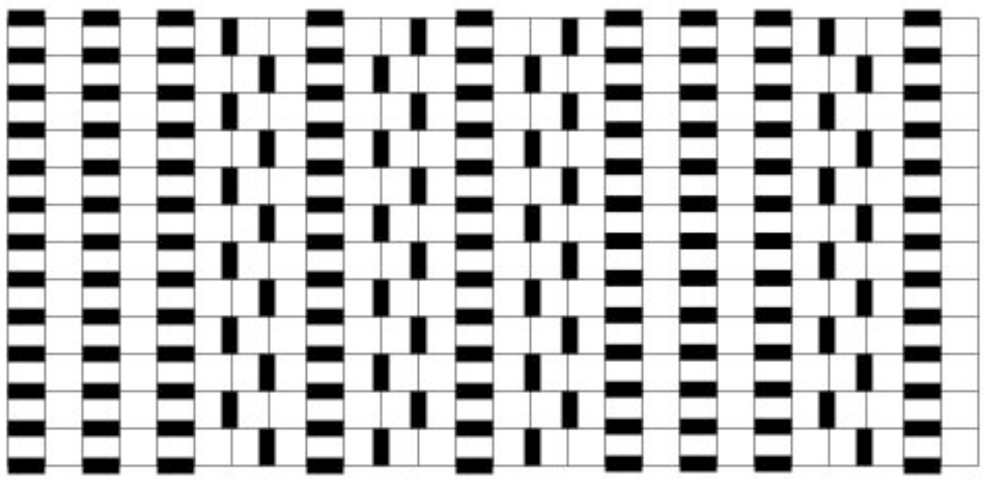
[14]



Staggered states are like domain walls separating columnar regions.

Basic strategy 2: Construct dimer model

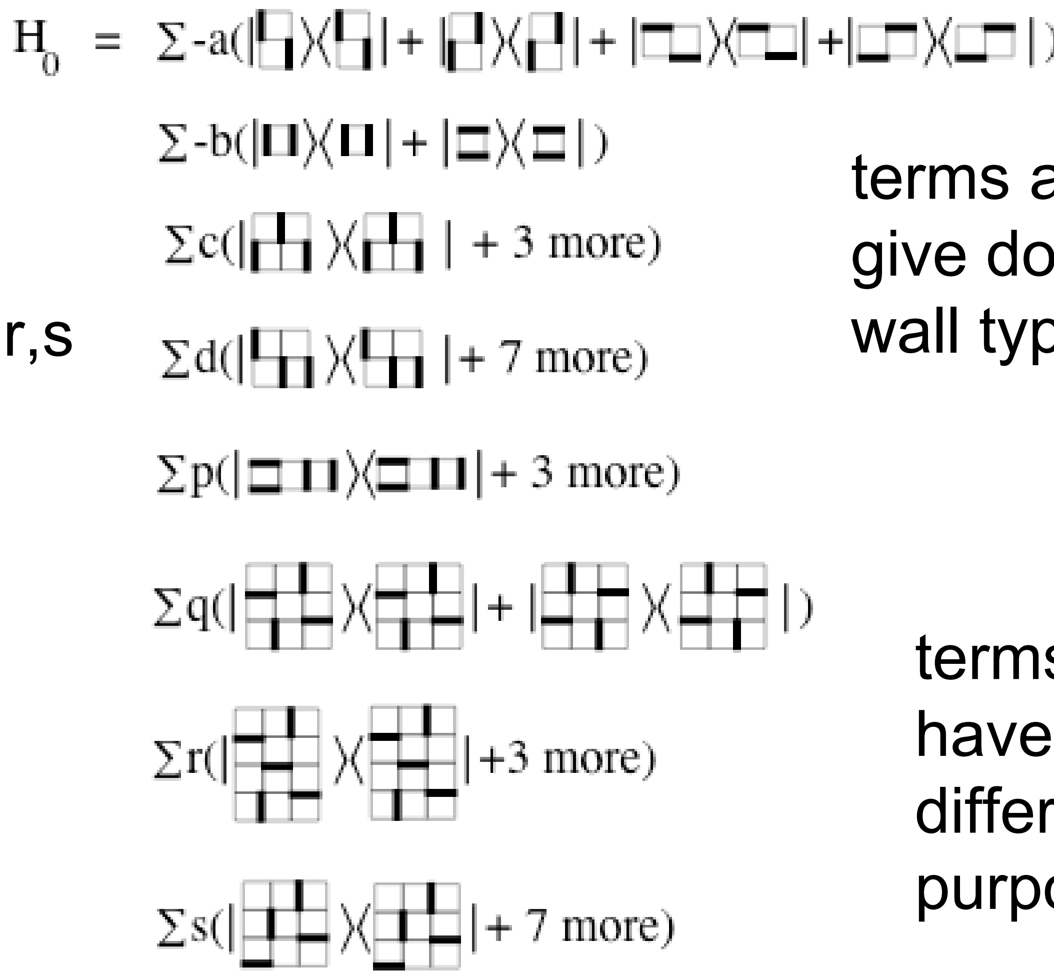
1. Construct a diagonal parent Hamiltonian with a large ground state degeneracy at a special point. Design it to favor quasi-1D domain wall-type states:



Note:
(a) staggering comes in two orientations
(b) direction of staggered vs. columnar is opposite
(c) staggered regions one column wide

2. Make the domain walls fluctuate via an off-diagonal resonance term (i.e. Pokrovsky-Talapov)

Parent Hamiltonian

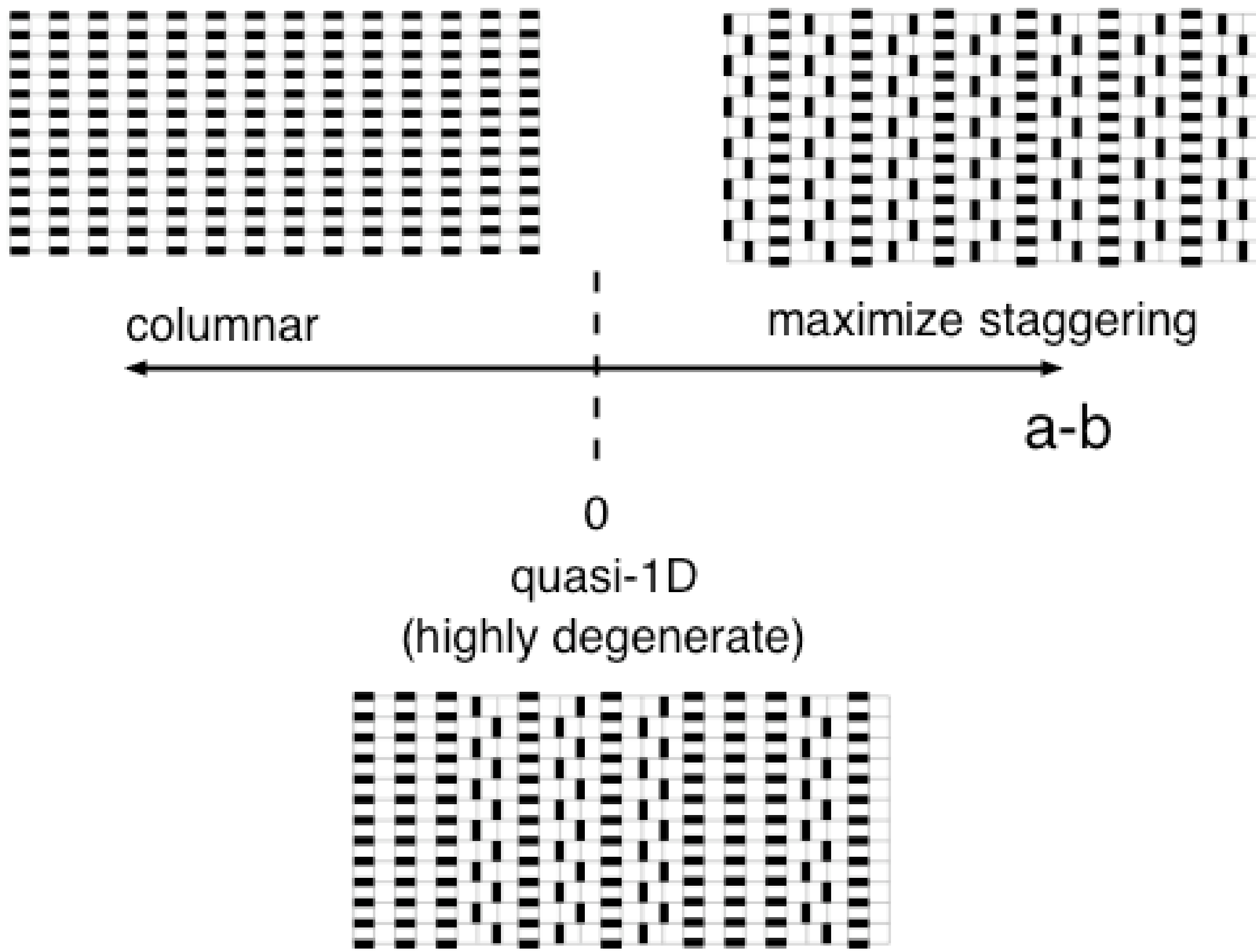


Require:
a,b < c,d << p,q,r,s

terms a to d give domain wall type states.

terms p to s have a different purpose.

Ground state phase diagram of H_0



Perturbation

$$-tV \;=\; \sum -t(|\;\rangle\langle\;| + |\;\rangle\langle\;| + \text{h.c.})$$

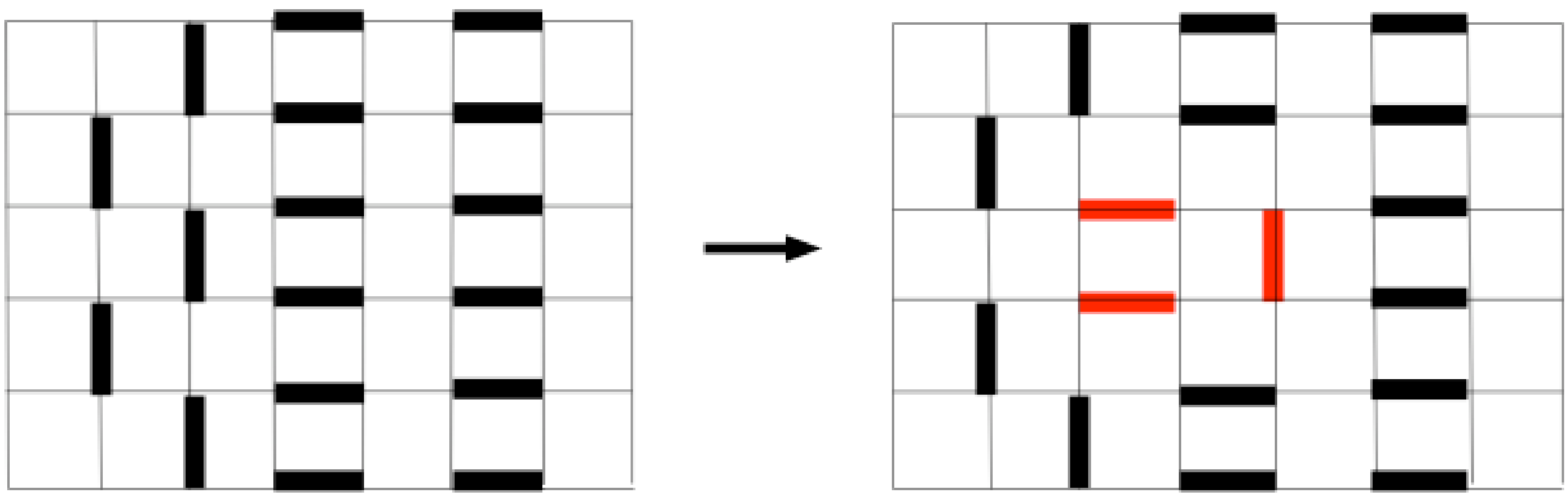
Non-diagonal resonance term causes staggered strips to fluctuate which stabilizes staggering relative to columnar strips.

Second order in perturbation theory:

$$E_N = \epsilon_n - t^2 {\sum_m}' \frac{V_{nm}V_{mn}}{\epsilon_m - \epsilon_n} + O(t^4)$$

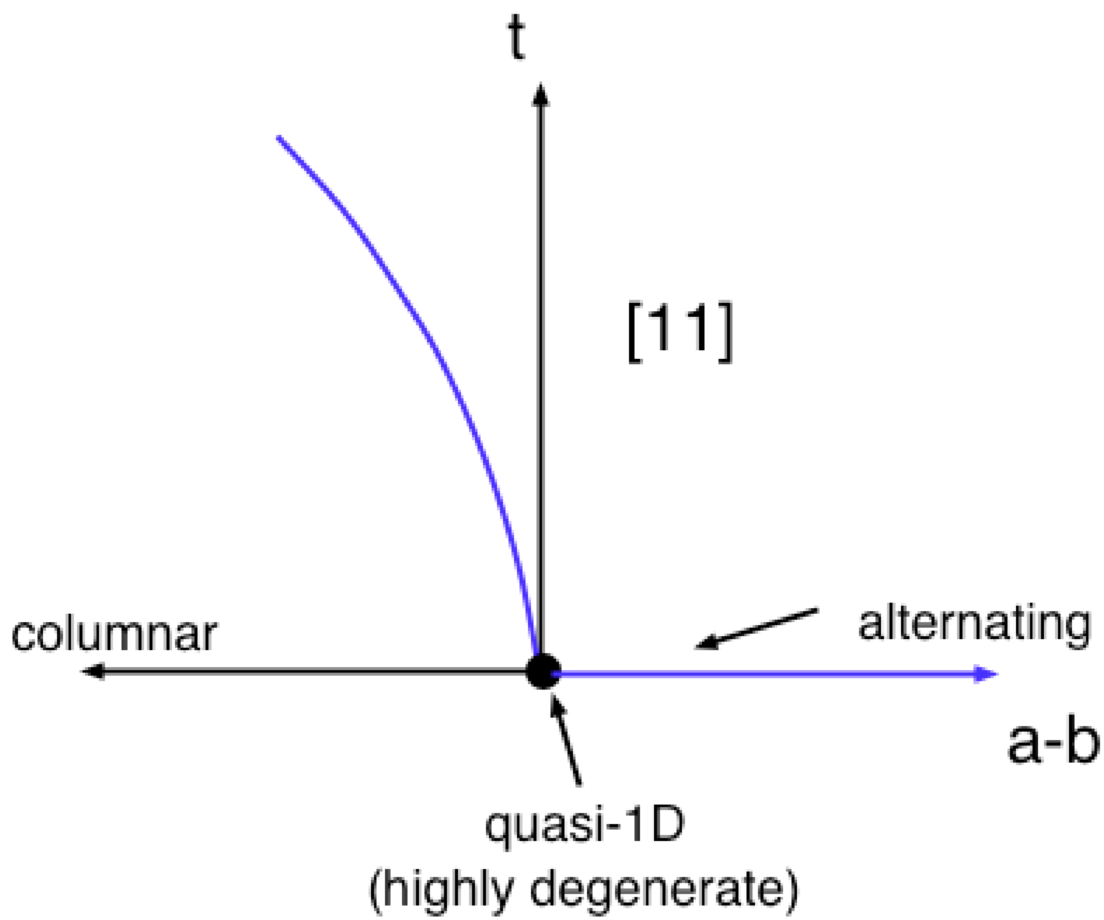
Calculate the correction to each of the quasi-1D states.

Action of perturbation



initial excited

Ground state phase diagram to $O(t^2)$

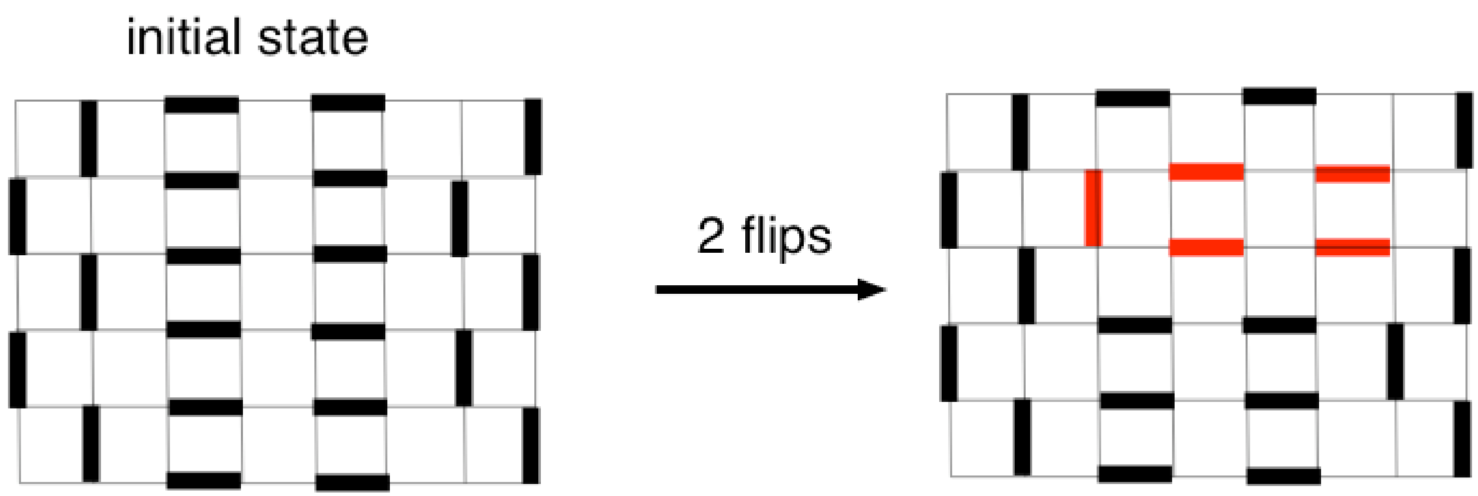


Fourth order perturbation theory

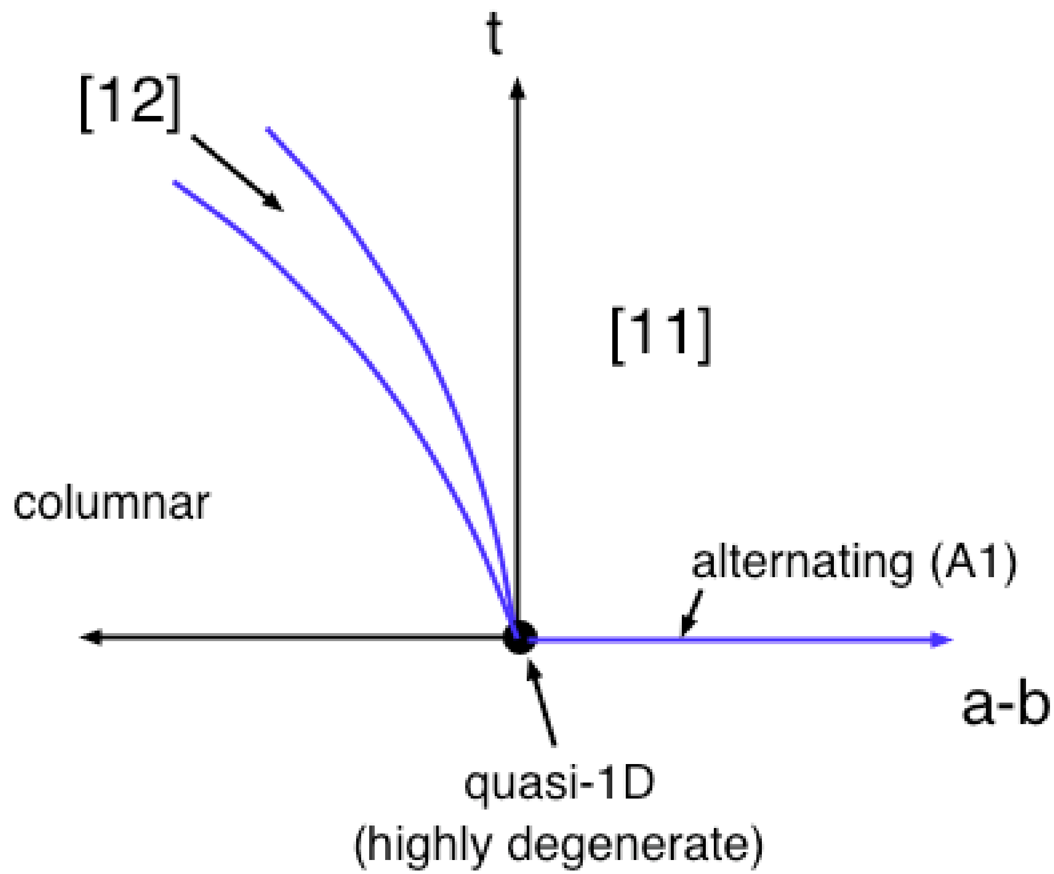
$$E_N = \epsilon_n - t^2 \sum_m{}' \frac{V_{nm}V_{mn}}{\epsilon_m - \epsilon_n}$$

$$-t^4 \Big[\sum_{ml}{}' \frac{V_{nm}V_{ml}V_{lk}V_{kn}}{(\epsilon_m - \epsilon_n)(\epsilon_l - \epsilon_n)(\epsilon_k - \epsilon_n)} - \sum_{ml}{}' \frac{V_{nm}V_{mn}V_{nl}V_{ln}}{(\epsilon_m - \epsilon_n)^2(\epsilon_l - \epsilon_n)} \Big] + O(t^6)$$

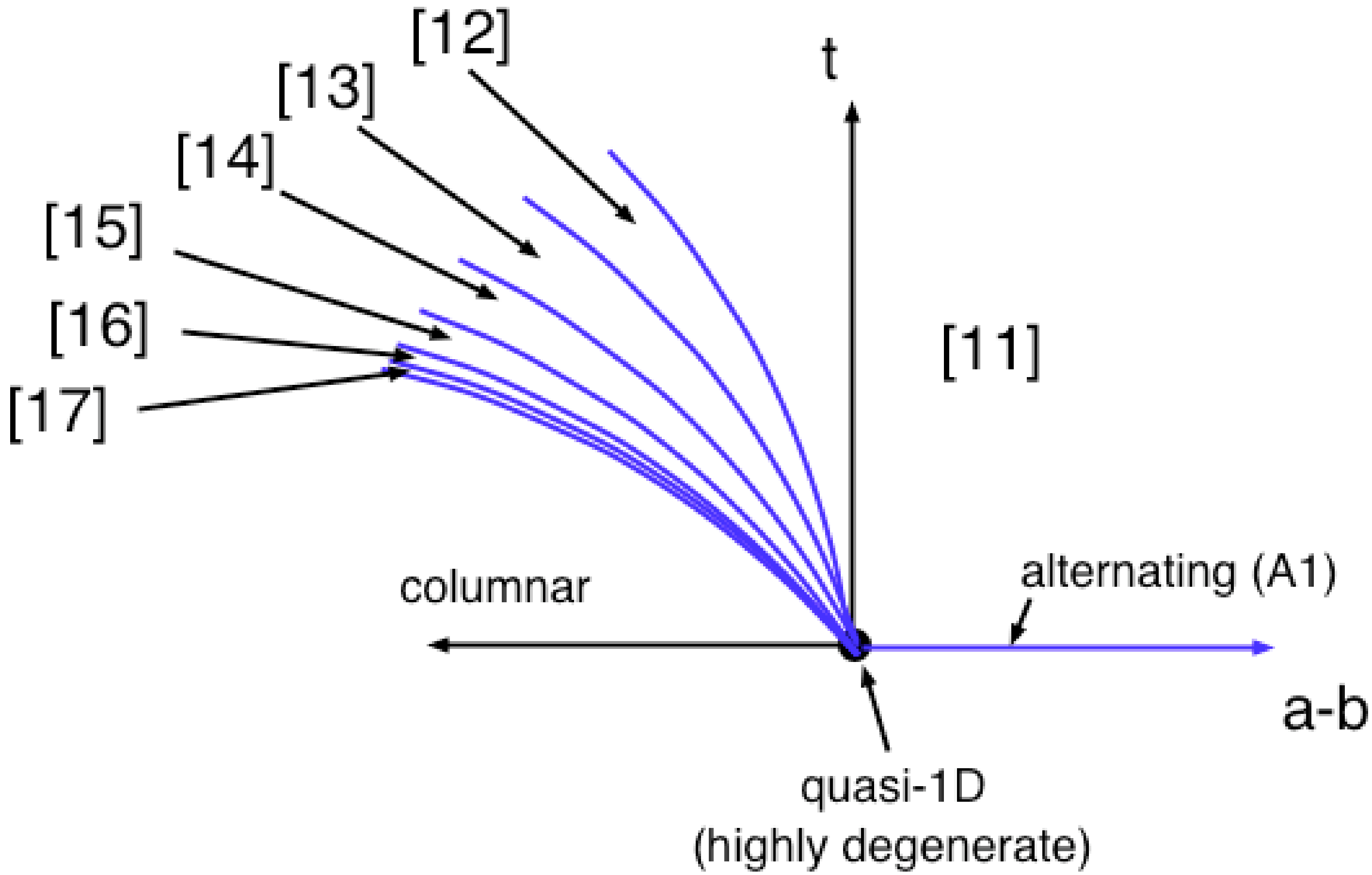
Terms p,q,r,s ensure the only important fourth order process is:



Ground state phase diagram to $O(t^4)$



Ground state phase diagram to O(t^{2N})



Conclusions:

1. Explicit demonstrations that spin liquids and high-order striped states can, in principle, exist in systems with only short-range interactions and spin rotation and lattice symmetries.
2. The value of dimer models as a means of answering these sorts of questions was (hopefully) conveyed.