Polarized Electric Current in Semiclassical Transport with Spin-Orbit Interaction

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SFB TR 12

“The emerging field of spintronics ...”

Plan

1. **Introduction.**
2. **Semiclassical solution.**
3. **Out of plain polarization σ_z.**
4. **Polarized currents.**
    a. Shavrin conductance.
    b. QPC.
5. **Classical trajectories** *vs* **Quantum states.**
6. **Conclusions.**

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Spintronics – Spin Manipulation by means of the Electric Field

The Hamiltonian (Rashba Spin-Orbit interaction + usual electrostatic gates)

$$H = \frac{p^2}{2m} + \lambda(p_y\sigma_x - p_x\sigma_y) + V(x,y).$$

Relativistic correction $\sim \frac{\hbar e}{c^2}(\vec{E}\cdot[\vec{p}\times\vec{\sigma}])$.

Many proposal are based on spatial variation of $\vec{E}$ (or λ), which is hard to achieve experimentally.

What may be simpler?

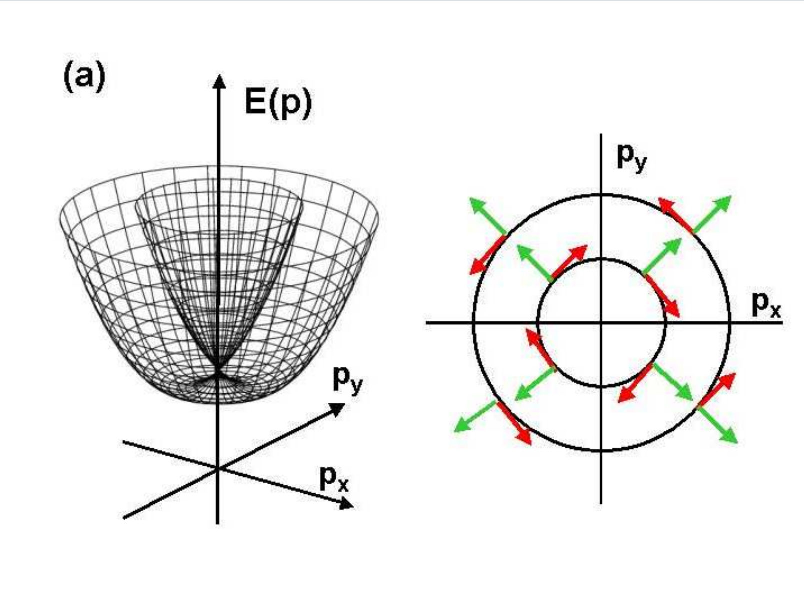
We are looking for the semiclassical solutions.
Which means a smooth external potential $V(x,y)$.

Littlejohn & Flynn
PRA 1991

How to do semiclassics in case of Spin-Orbit?

$$H = \frac{p^2}{2m} + \lambda(p_y\sigma_x - p_x\sigma_y) + V(x, y).$$

What is semiclassics?



W K B

$$H = \frac{p^2}{2m} + V(x, y)$$

a Semiclassical wave function

$$\psi(x, y) = \sqrt{\rho(x, y)}e^{iS(x,y)/\hbar}$$

Hamilton-Jacobi and continuity equations:

$$|\nabla S|^2 = 2m(E - U), \nabla(\rho\nabla S) = 0.$$

How to do semiclassics in case of Spin-Orbit?

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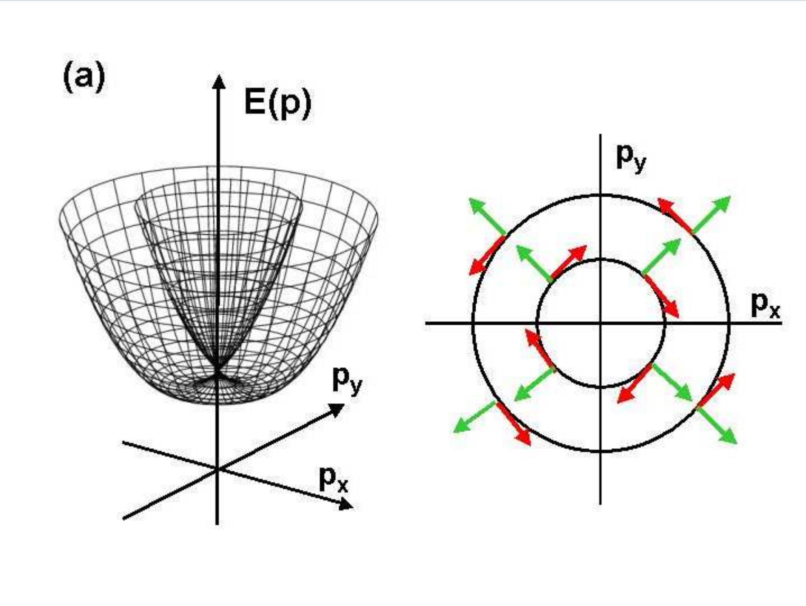


My semiclassics does not assume a large spin. Usually one takes $\vec{S} = \hbar\vec{\sigma}/2$-fixed, $\hbar \to 0$. Here we are interested in the semiclassical description of two-component wave function.

Also λ is large enough to change the classical trajectories.

Chiral states

$$E_\pm = \frac{(p \pm m\lambda)^2}{2m},$$

$$\psi = \frac{1}{\sqrt{2|p|}} \begin{pmatrix} \sqrt{p_y - ip_x} \\ \pm\sqrt{p_y + ip_x} \end{pmatrix}.$$

What changes in case of smooth potential $V(x,y)$?

Simply:

$$\mathcal{H}_{eff} = \frac{(p \pm m\lambda)^2}{2m} + V(r) \; ?$$

The "Classical" dynamics is determined by the Effective Hamiltonian

$$\mathcal{H}_{eff} = \frac{(p \mp m\lambda)^2}{2m} + V(x, y)$$

This is possible if and only if the "Quantum" spin is automatically adjusted to the direction perpendicular to the momentum $(\sigma_x, \sigma_y) \propto \pm(-p_y, p_x)$.

The semiclassical wave function now takes a form

$$\psi = \sqrt{\frac{\rho}{2|p|}} \begin{pmatrix} \sqrt{p_y - ip_x} \\ \pm\sqrt{p_y + ip_x} \end{pmatrix} e^{iS/\hbar},$$

where the momentum $\vec{p} \equiv \nabla S$. Hamilton-Jacobi: $\mathcal{H}_{eff}(\vec{p}) \rightarrow \mathcal{H}_{eff}(\nabla S)$.

The continuity equation however

$$\nabla \cdot \rho\vec{v} = 0$$

now contains a velocity $\vec{v} = \frac{\vec{p}}{m} \mp \lambda\frac{\vec{p}}{p}$. $\left(\vec{v} = \frac{\partial E}{\partial \vec{p}}\right)$

Direct calculation of $H\psi$ shows that this is indeed the solution.

Out of plain polarization $\psi^\dagger\sigma_z\psi \neq 0$ appears as a quantum correction

(Spin-Hall effect). $\psi^\dagger\sigma_z\psi = \hbar\frac{\pm p - m\lambda}{m\lambda p^4}(p_y p_i \partial_i p_x - p_x p_i \partial_i p_y)\rho + \hbar\frac{p_y\partial_x\rho - p_x\partial_y\rho}{p^2}.$

A simple example: Potential depending only on x, $V = V(x)$.

y-momentum is conserved $p_y = const$.

y-velocity is not $v_y \neq const$.

$\psi \propto e^{ip_y y/\hbar}$.

The "semiclassical" density is

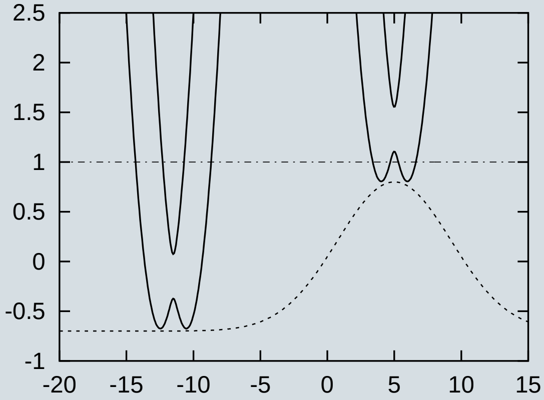
$$|\psi_1|^2 + |\psi_2|^2 \sim \frac{1}{|v_x|} \;, \; v_x = \frac{p_x}{m} \mp \lambda \frac{p_x}{p},$$

and the momentum $p_x = p_x(x)$ may be found from

$$E = \frac{(p \mp m\lambda)^2}{2m} + V(x) \quad , \quad p = \sqrt{p_x^2 + p_y^2}.$$

The wave function now takes a form

$$\psi = \frac{1}{\sqrt{2p_x(p \mp m\lambda)}} \begin{pmatrix} \sqrt{p_y - ip_x} \\ \pm\sqrt{p_y + ip_x} \end{pmatrix} e^{ip_y y/\hbar + i\int p_x dx/\hbar}.$$

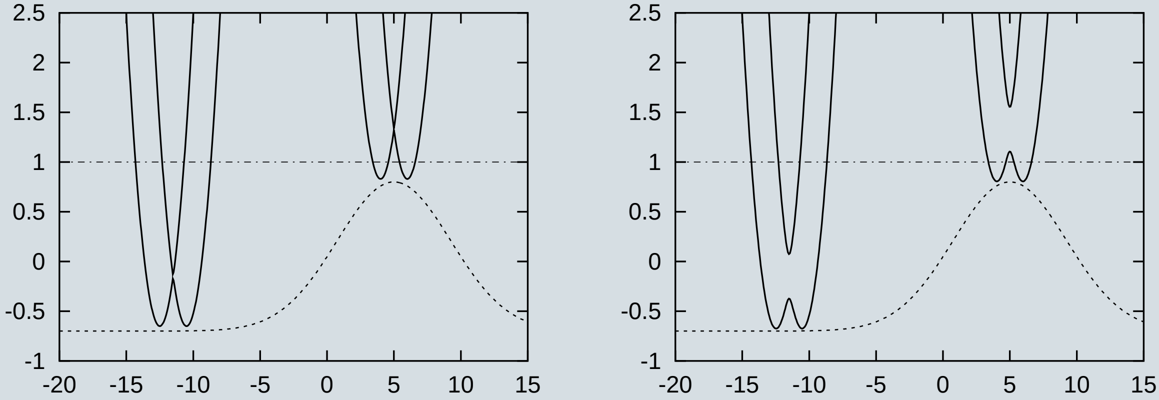


$$E = \frac{(\sqrt{p_x^2 + p_y^2} \mp m\lambda)^2}{2m} + V(x)$$

Immediate consequence:

Scattering by a smooth barrier $V = V(x)$. For $p_y \neq 0$ the barrier is open for transmission in the lower band and closed for transmission in the upper band. The spin of transmitted electrons is polarized in plane and perpendicular to current.

Number of solutions for p_x in a given subband may be $0, 2$ or 4.



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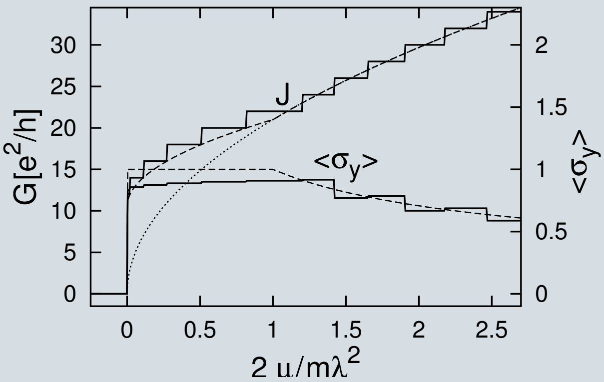
Nothing unusual happens in case of normal to the barrier trajectory $p_y \equiv 0$.

Both bands are equally transmitting. (Kramers doublets?)

Sharvin Conductance

$$V = V(x)$$



Without Spin-Orbit interaction conductance increases like $\sqrt{\mu}$. The massive degeneracy of the lowest energy electron state in case of Rashba spin orbit leads to step-like rise of the conductance at the pinch-off.

Conductance of a long barrier (length L)

$$G = \frac{e^2}{h}\frac{L}{\pi\hbar}\left(\sqrt{2\mu m} + m\lambda\right) \quad \text{for} \quad \mu < \frac{m\lambda^2}{2},$$

and

$$G = \frac{e}{h}\frac{L}{\pi\hbar}2\sqrt{2\mu m} \quad \text{for} \quad \mu > \frac{m\lambda^2}{2}.$$

Here μ is the chemical potential. The top of the barrier corresponds to $\mu = 0$.

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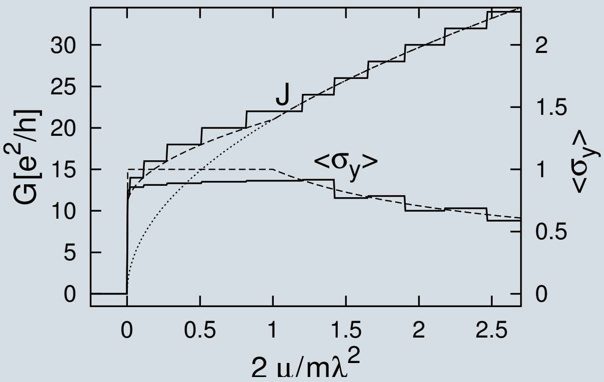

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$$\langle\sigma_y\rangle = \langle\psi^\dagger\sigma_y v_x\psi\rangle / \langle\psi^\dagger v_x\psi\rangle = \min(1, \sqrt{m\lambda^2/2\mu})$$

QPC

Classical motion at the saddle point. The effective Hamiltonian

$$H_{eff} = \frac{(|p| - m\lambda)^2}{2m} - \frac{m\Omega^2 x^2}{2} + \frac{m\omega^2 y^2}{2}.$$

Classical equations of motion now have a form

$$\dot{x} = \frac{p_x}{m} - \lambda \frac{p_x}{p} \ , \ \dot{y} = \frac{p_y}{m} - \lambda \frac{p_y}{p} \ , \ \dot{p_x} = m\Omega^2 x \ , \ \dot{p_y} = -m\omega^2 y.$$

The kinetic energy has a degenerate minimum at the circle $|p| = m\lambda$. It is convenient therefore to shift the momentum

$$p_x = \cos\alpha m\lambda + P_x \quad , \quad p_y = \sin\alpha m\lambda + P_y ,$$

and write the linearized equations of motion

$$\frac{\dot{x}}{\cos\alpha} = \frac{\dot{y}}{\sin\alpha} = \left[\cos\alpha \frac{P_x}{m} + \sin\alpha \frac{P_y}{m}\right] \ , \ \frac{\dot{P_x}}{m} = \Omega^2 x \ , \ \frac{\dot{P_y}}{m} = -\omega^2 y \ .$$

The equation for the momentum along "dangerous" direction P has a simple form

$$P \equiv \cos\alpha P_x + \sin\alpha P_y \quad , \quad \ddot{P} + (-\Omega^2 \cos\alpha^2 + \omega^2 \sin\alpha^2) P = 0 \ .$$

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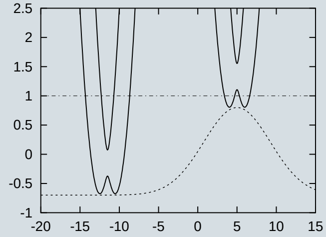

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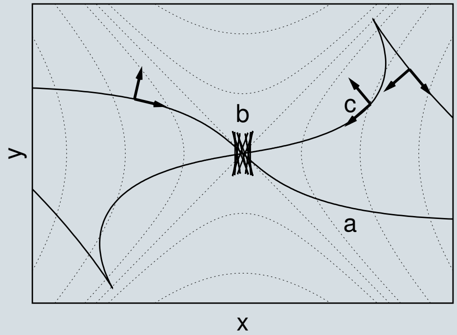


Because of the massive degeneracy of the ground state, trajectories within the angle

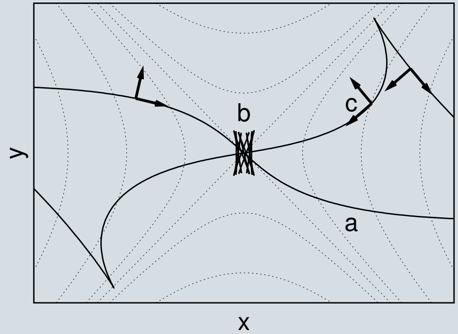
$$|\tan\alpha| < \frac{\Omega}{\omega}$$

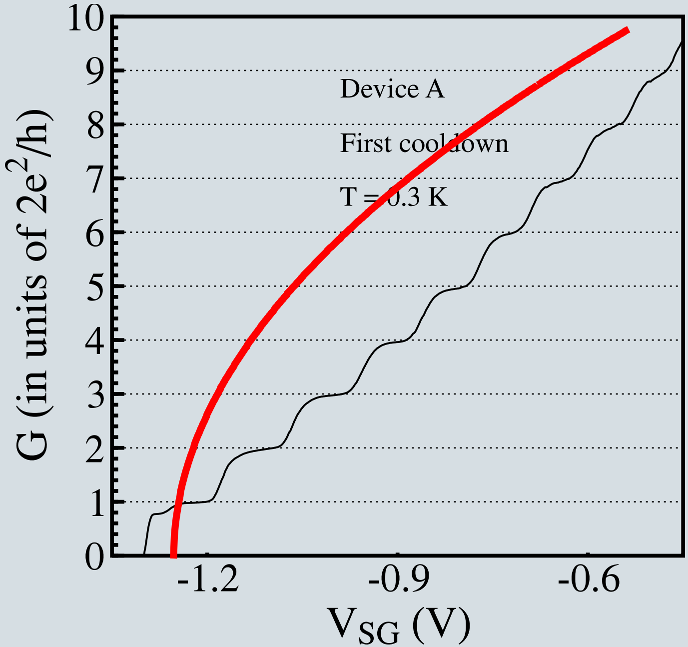
are transmitted even at the pinch off.

Conductance of the QPC

The main result

$$G = \sin\alpha \, \frac{e^2}{h} \, \frac{8\lambda\sqrt{2m\mu}}{h\omega} \, .$$





Valid for $\mu \ll m\lambda^2$.
Crosses over to

$$G = 2\frac{e^2}{h}\frac{\mu}{\hbar\omega} \, , \qquad \text{for } \mu > m\lambda^2.$$

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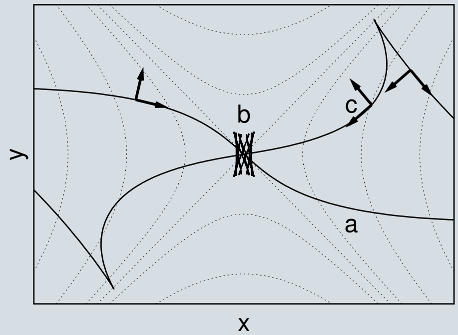

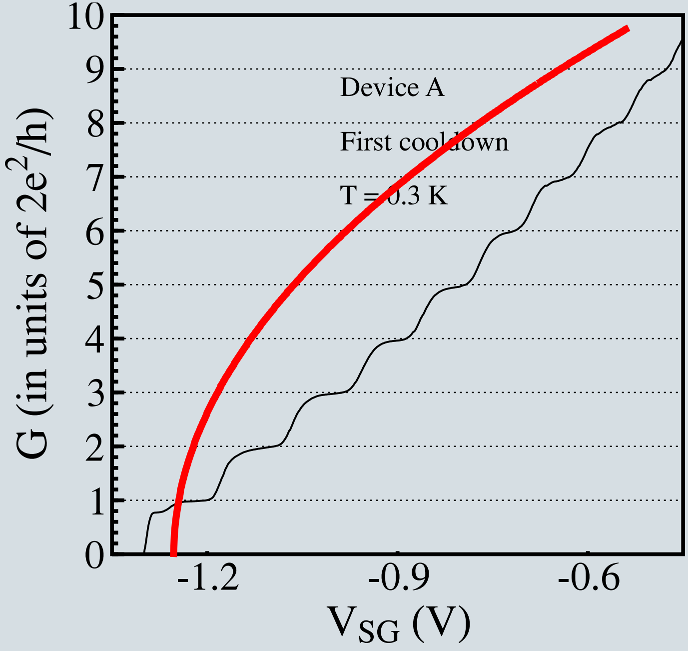

Conclusions

100% polarized current (=nonequilibrium spin-density).

No need in:

- Direct measurement of spin.
- Spatial modulation of Spin-Orbit interaction.
- Electron beam Collimation.

Conductance of the QPC

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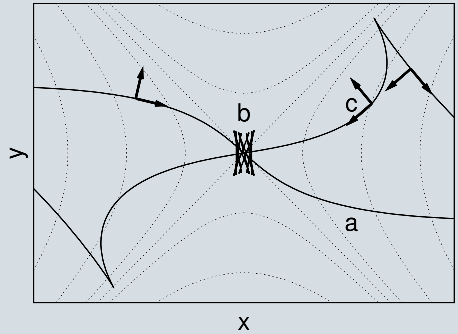

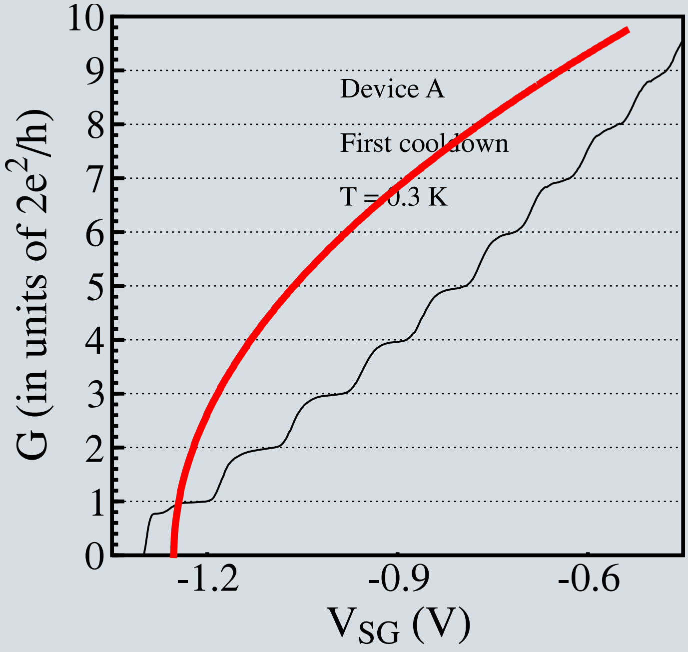

Conclusions?

Some Numbers

Taking for $InAs$ $k_F = 2.5 \times 10^8 \mathrm{m}^{-1}$, $\lambda\hbar = 2 \times 10^{-11} eVm$ and $m^* = 0.04 m_0$ we get $\hbar\lambda k_F = 5\mathrm{meV}$, $E_F = 60\mathrm{meV}$ and $m^*\lambda^2/2 = 0.1\mathrm{meV} = 1.2\mathrm{K}$. We may introduce a length associated with spin-orbit $l_R = \hbar/m^*\lambda = 100\mathrm{nm}$. A barrier with $L \gg l_R$, or QPC with $\hbar\omega \ll m^*\lambda^2/2$ may be used to achieve the spin-polarized transmission.

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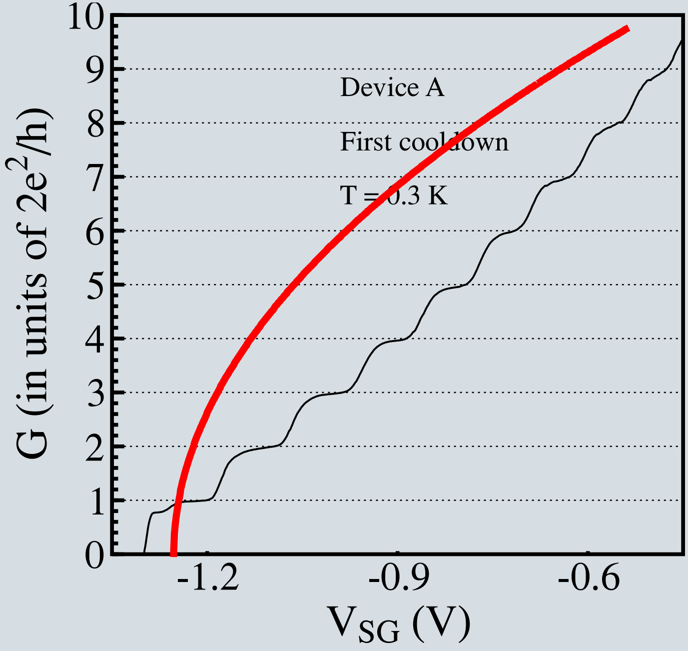

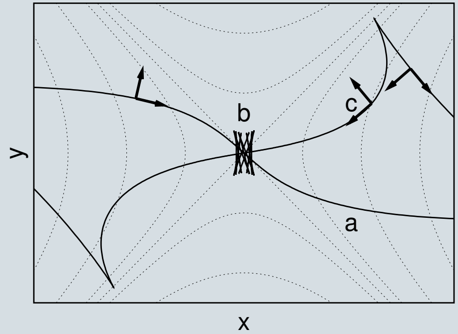

Conclusions?

Work in progress

Edge Spin-Hall effect $\langle\sigma_z\rangle$

$$[H, \sigma_y] = -2i\lambda p_y \sigma_z .$$

Work in progress in progress

Resonances, Rashba-Antidot?