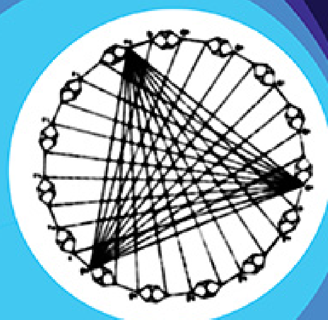
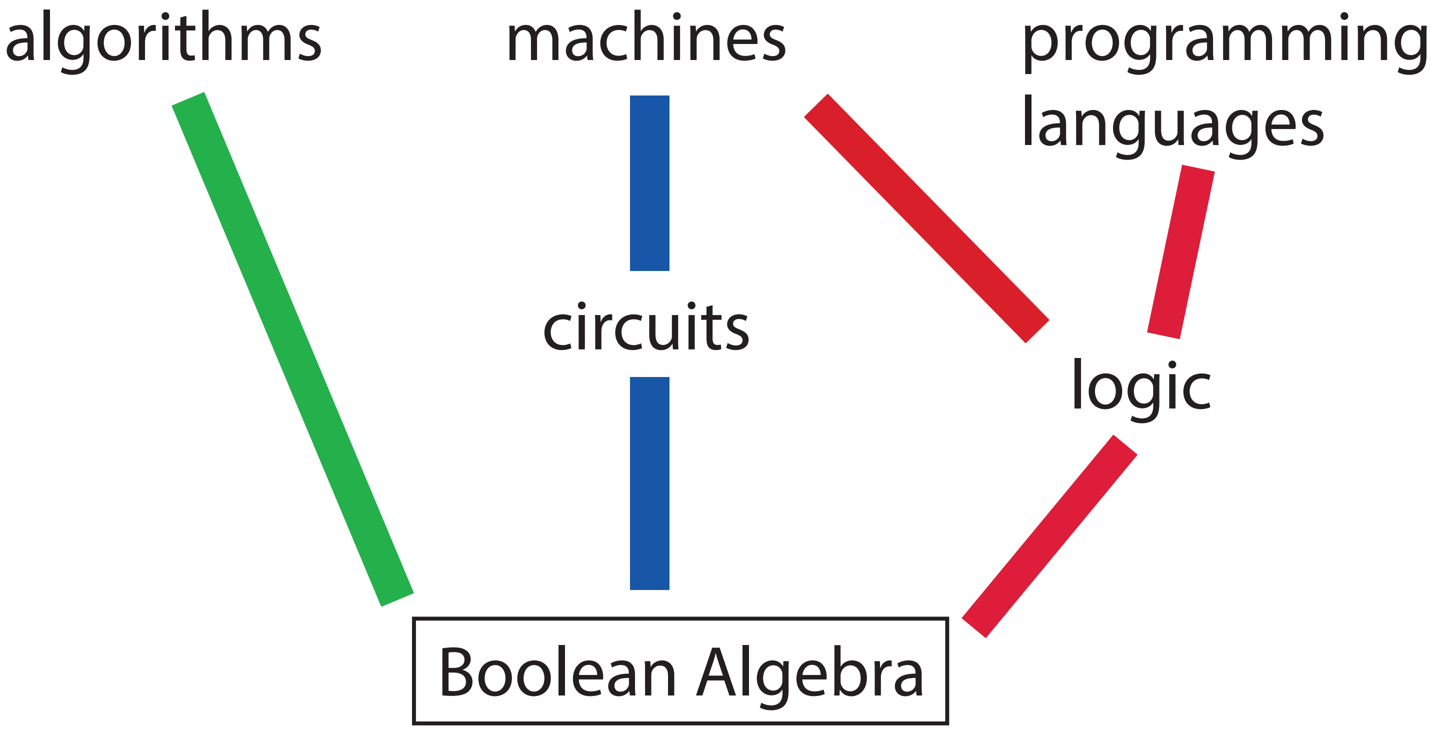


Cohomological Framework for Contextual Quantum Computation

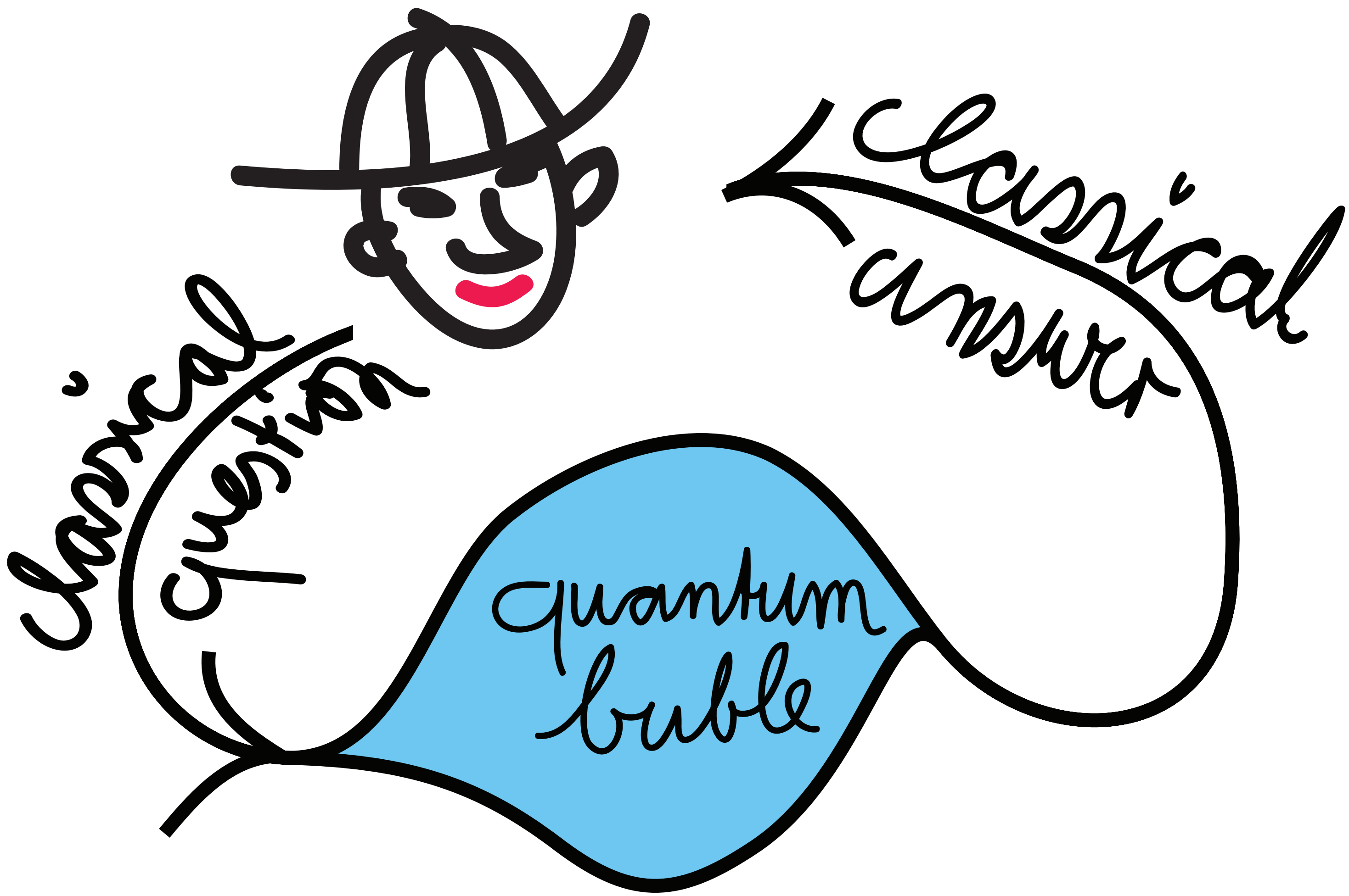
Robert Raussendorf, UBC Vancouver * UBC, March 2016

A question

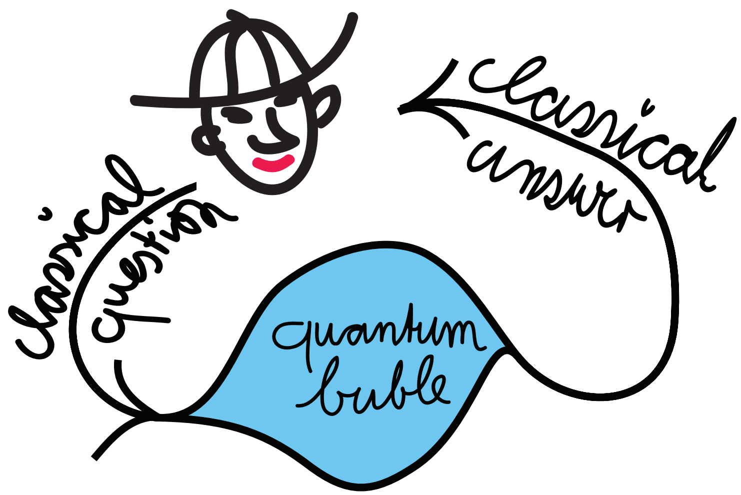


Which fundamental computational structures exist in Hilbert space?

Computational structures in Hilbert space



Computational structures in Hilbert space

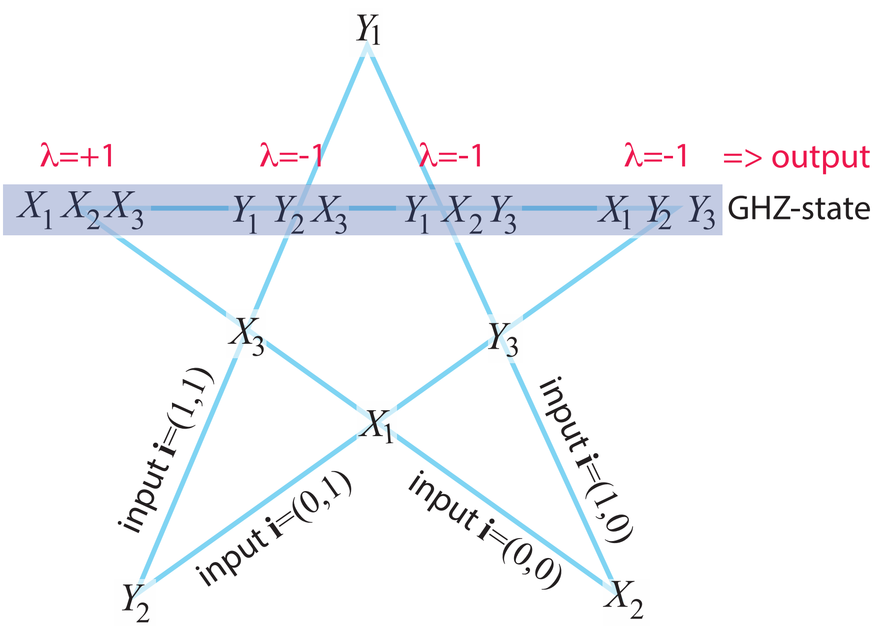


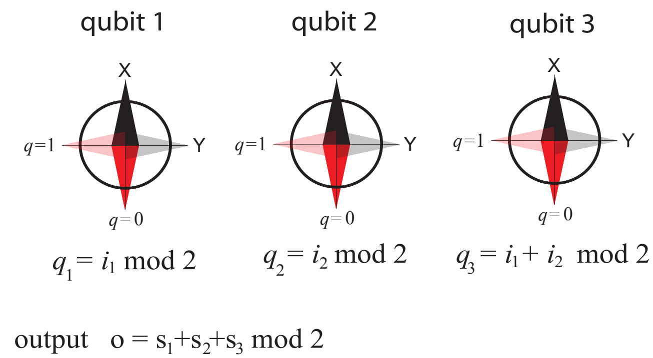
Two criteria:

- Must specify an input structure, an output structure, and a function computed.

- Must be genuinely quantum.

Mermin's KS proof computes!





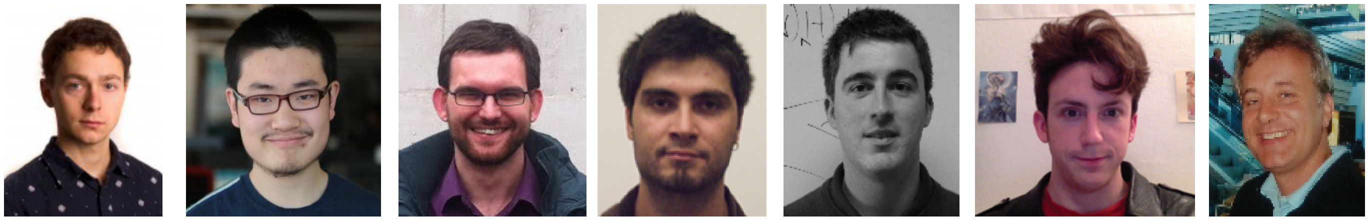
* Use GHZ state as computational resource
* Compute OR-gate

- Classical processing all *linear*, computed OR-gate *non-linear*.

$\Rightarrow$ Classical control computer promoted to classical universality.

J. Anders and D. Browne, PRL 102, 050502 (2009).

Recent work



Philippe Guerin, Vienna

Jake Bian, UBC & Silicon valey

Dan Browne, Imperial College

Cihan Okay, UWO

Nicolas Delfosse, Caltech

Juan Bermejo Vega, Berlin

- Nicolas Delfosse, Philippe Allard Guerin, Jacob Bian, Robert Raussendorf, *Wigner function negativity and contextuality in quantum computation on rebits*, Phys. Rev. X 5, 021003 (2015)

- Robert Raussendorf, Dan E. Browne, Nicolas Delfosse, Cihan Okay, Juan Bermejo-Vega, *Contextuality as a resource for qubit quantum computation*, arXiv:1511.08506

- R. Raussendorf, *Cohomological framework for contextual quantum computations*, arXiv:1602.04155

One result

Proposition. Consider a measurement-based quantum computation $\mathcal{M}$ and a classical computation $\mathcal{C}$, evaluating the same Boolean function $o : \mathbb{Z}_2^m \to \mathbb{Z}_2$. The classical computational cost C_{class} of $\mathcal{C}$ is bounded by the maximum violation Δ of the logical non-contextuality inequality for $\mathcal{M}$,

$$C_{\text{class}} \leq \Delta.$$

Substantial amount of contextuality required for speedup

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Substantial amount of contextuality required for speedup

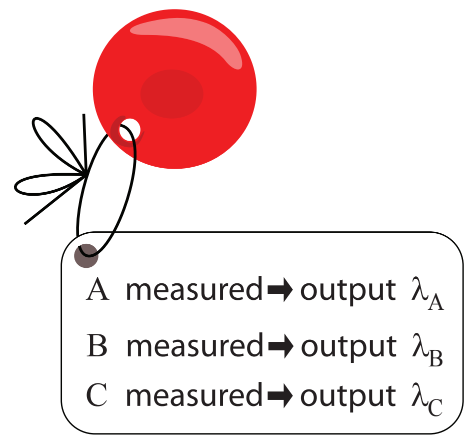
Contextuality of QM

What is a non-contextual hidden-variable model?

quantum mechanics

$$|\Psi\rangle$$

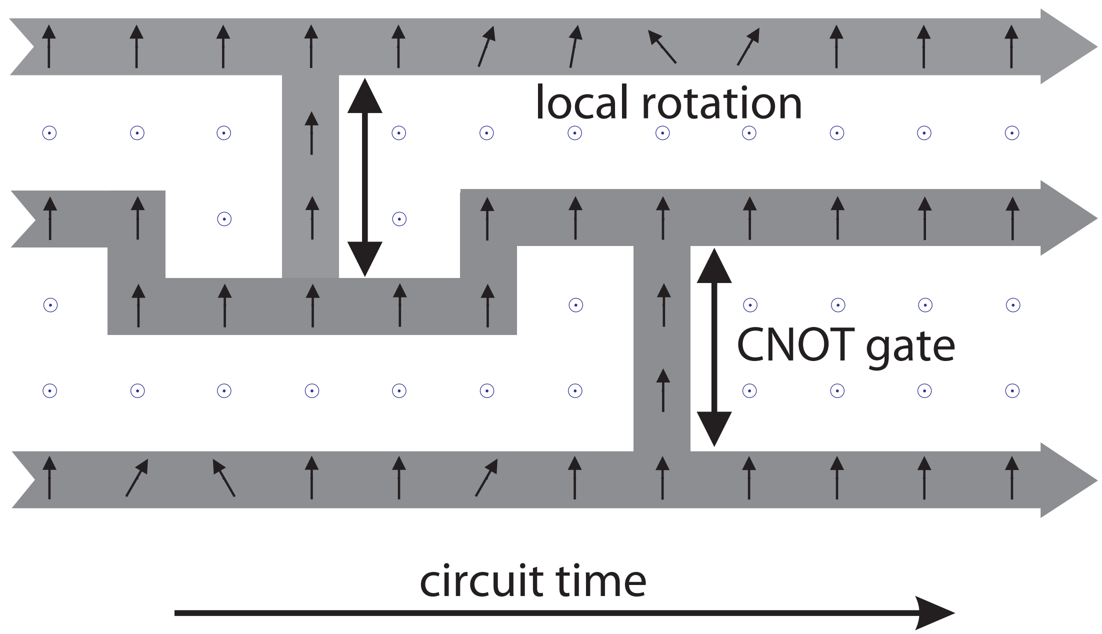
hidden-variable model



Noncontextuality: Given observables A,B,C: $[A, B] = [A, C] = 0$:
λ_A is *independent* of whether A is measured jointly with B or C.

Theorem [Kochen, Specker]: For $\dim(\mathcal{H}) \geq 3$, quantum-mechanics cannont be reproduced by a non-contextual hidden-variable model.

Quantum computation by measurement



- Information written onto a cluster state, processed and read out by one-qubit measurements only.
- The resulting computational scheme is *universal*.

Motivation

Theorem 1: Every MBQC which deterministically evaluates a non-linear Boolean function is contextual.

Remark: This can be extended to probabilistic computations.

But what does that mean?
A Boolean function is a classical thing.

Does the contextuality in the MBQC have any bearing on other means of evaluating a Boolean function?

=> Violation of a non-contextuality inequality bounds classical computational cost.

R. Raussendorf, PRA (2013).

What is a non-contextuality inequality?

- Consider an observable $A = \sum_i^N P_i$.

- P_i are projectors. Classically, they correspond to statements that cannot be simultaneously true.

- Expectation for A in a non-contextual HVM:

$$\langle A \rangle_{HVM} \leq N - \Delta$$

- Quantum mechanical expectation

$$\langle \psi | A | \psi \rangle \leq N.$$

  Maximum can be reached.

Our result

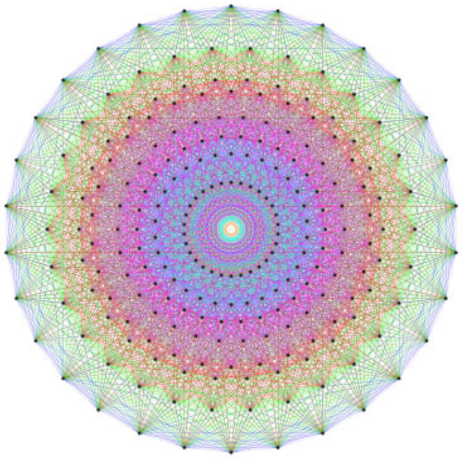
- Expectation for A in a non-contextual HVM/ in QM:

$$\langle A\rangle_{HVM} \leq N-\triangle, \quad \langle\psi|A|\psi\rangle \leq N.$$

- We show: For every MBQC computing a function o we can find an operator A such that $\triangle$ bounds the computational cost for classically computing o.

Outlook

- *Which computational structures exist in Hilbert space?*



Cohomology is part of the answer.