Cosmological initial conditions: new type of hill-top inflation from the CFT driven cosmology

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New concept of initial conditions in the form of the microcanonical cosmological density matrix

Application to cosmological model driven by conformal field theory

Initial conditions problem

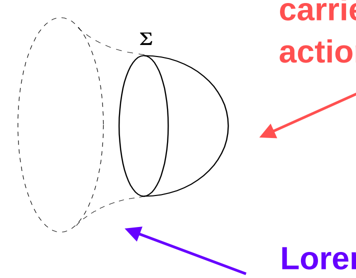
Ijjas, Steinhardt and Loeb, arXiv:1304.2785; Miao and Woodard, arXiv:1506.07306; … :

Unlikeliness of plateau-like inflation models, fine tuning is not enough, protection against quantum corrections, naturalness, …

Two known prescriptions for a *pure* initial state:

no-boundary (Hartle-Hawking) wavefunction

$$\boldsymbol{\Psi}(\varphi) \sim$$



Euclidean spacetime carries Euclidean action S_E<0

Lorentzian spacetime

$$\boldsymbol{\Psi}_{HH} \sim \exp(-S_E) = \exp\left(\frac{12\pi^2 M_P^4}{V(\varphi)}\right) \to \infty,$$

$$\frac{V(\varphi)}{M_P^2} = \Lambda_{eff} \to 0$$

Infrared catastrophe of $\Lambda_{eff} \to 0$, insufficient amount of inflation at minima of potential

"tunneling" wavefunction (Linde, Vilenkin, Rubakov, Zeldovich-Starobinsky, …)

$$\boldsymbol{\Psi}_T \sim \exp(+S_E)$$

Cosmology debate:
no-boundary vs tunneling

$$\boldsymbol{\Psi}_{HH,T} \sim \exp(\mp S_E)$$

Hyperbolic Wheeler-DeWitt equation

Both no-boundary (EQG path integral) and tunneling (WKB approximation) do not have a clear operator interpretation

Fokker-Planck equation (coarse-graining) and eternal inflation, Multiverse, Boltzmann brains, etc – measure problem on the space of universes .

Plan

Cosmological initial conditions – microcanonical density matrix of the Universe:

Projector on physical states (Sum over ``Everything”) and EQG path integral

A.B. & A.Kamenshchik, JCAP, 09, 014 (2006) Phys. Rev. D74, 121502 (2006);

A.B., Phys. Rev. Lett. 99, 071301 (2007)

CFT driven cosmology -- Universe dominated by quantum matter conformally coupled to gravity :

constraining the range of Λ (string landscape problem)

thermal cosmological instantons and elimination of vacuum instantons (no $\Lambda \to 0$ IR catastrophe)

initial conditions for inflation, $\Lambda \to V(\phi)$ – selection of inflaton potential $V(\phi)$ maxima (new type of hill-top inflation)

A.B, C.Deffayet and A.Kamenshchik, JCAP 05 (2008) 020; JCAP 05 (2010) 034

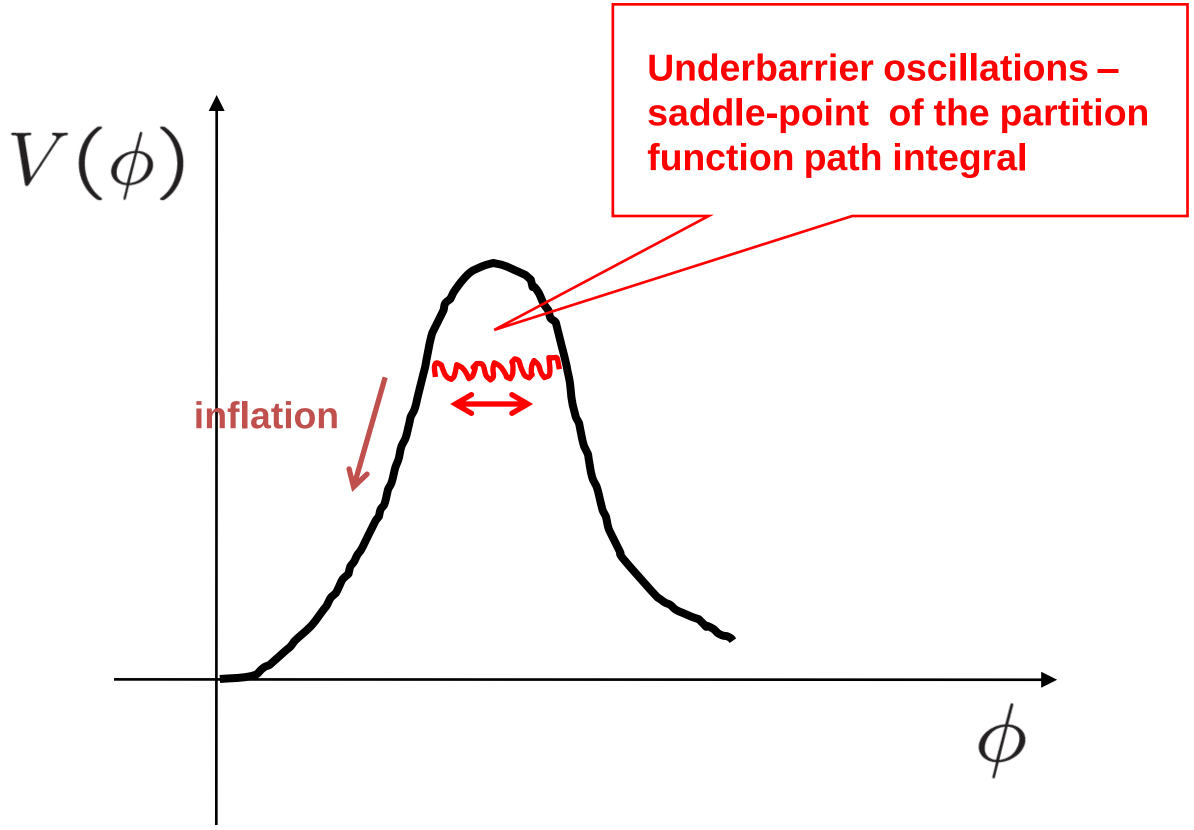
non-minimal coupling and quantum mechanism of hill-top potential

thermally corrected CMB spectrum – temperature of the CMB temperature

A.B, arXiv:1308.4451 JCAP 1310 (2013) 059

hierarchy problem and conformal higher spin theory (CHS)

Picture of a new type hill-top inflation



Microcanonical ensemble in cosmology and EQG path integral

$H_\mu = 0$ constraints on initial value data – corner stone of any diffeomorphism invariant theory.

Physical states:

$$\hat{H}_\mu |\Psi\rangle = 0 \qquad \hat{H}_\mu \equiv \underbrace{\hat{H}_\perp(\mathbf{x}),\ \hat{H}_i(\mathbf{x})}$$

operators of the Wheeler-DeWitt equations

$\mu = (\perp\mathbf{x},\ i\mathbf{x}),\ i = 1,2,3$
$\mathbf{x}$ – spatial coordinates

Microcanonical density matrix – projector onto subspace of quantum gravitational constraints

$$|\Psi\rangle \to \hat{\rho}, \qquad \hat{H}_\mu\,\hat{\rho} = 0$$

$$\hat{\rho} = e^{\Gamma} \prod_\mu \delta(\hat{H}_\mu)$$

A.B., Phys. Rev. Lett. 99, 071301 (2007)

Statistical sum

$$e^{-\Gamma} = \mathrm{Tr} \prod_\mu \delta(\hat{H}_\mu)$$

Motivation: aesthetical (minimum of assumptions – Occam razor)

A simple analogy — an unconstrained system with a conserved Hamiltonian $\hat{H}$ in the microcanonical state with a fixed energy E

$$\hat{\rho} \sim \delta(\hat{H} - E)$$

Spatially closed cosmology does not have *freely specifiable* constants of motion. The only conserved quantities are the Hamiltonian and momentum constraints H_μ, all having a particular value --- zero

$$\hat{\rho} \sim \prod_\mu \delta(\hat{H}_\mu)$$

A.B., Phys.Rev.Lett. 99, 071301 (2007)

is a natural candidate for the quantum state of the closed Universe – ultimate equipartition in the physical phase space of the theory --- *Sum over Everything*.

Creation of the Universe from *everything* is conceptually more appealing than creation from *nothing*, because the democracy of the microcanonical equipartition better fits the principle of the Occam razor than the selection of a concrete state.

EQG path integral representation of the statistical sum

BFV/BRST method
A.B. JHEP 1310 (2013) 051,
arXiv:1308.3270

↓

$$e^{-\Gamma} \equiv \mathrm{Tr} \prod_\mu \delta(\widehat{H}_\mu) = \int_{\text{periodic}} D[\, g_{\mu\nu}, \phi \,]\, e^{-S_E[\, g_{\mu\nu}, \phi \,]}$$

↑

Euclidean metric

↓

EQG density matrix
D.Page (1986)

$$-i\infty < N < i\infty, \quad g^{44} = +N^2$$

important for convexity of the Euclidean action at saddle points – provides ``conformal" rotation

Lorentzian signature path integral
=
EQG path integral with integration over the *imaginary* lapse

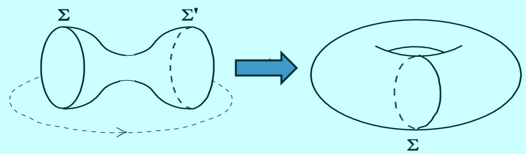
$$ds^2_{\text{Euclidean}} = N^2_{\text{Euclidean}} dt^2 + g_{ab}(dx^a + N^a dt)\,(dx^b + N^b dt),$$

$$N_{\text{Euclidean}} = i N_{\text{Lorentzian}}$$

Spacetime topology in the statistical sum:

S^3 topology of spatially closed cosmology

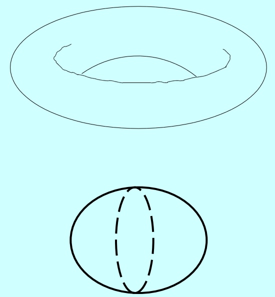
$$\mathrm{tr}\,\hat{\rho} = 1$$



$$e^{-\Gamma} = \int_{\text{periodic}} D[\,g_{\mu\nu},\phi\,]\; e^{-S_E[\,g_{\mu\nu},\phi\,]}$$

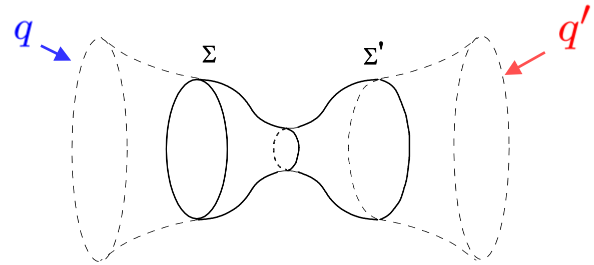
on $S^3 \times S^1$ (thermal)

including as a limiting (vacuum) case S^4



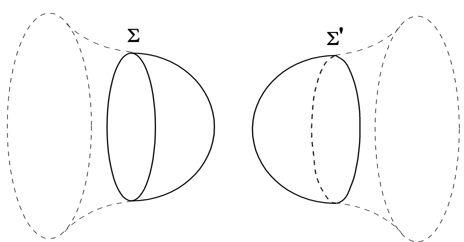
Hartle-Hawking state as a vacuum member of the microcanonical ensemble:

$$\hat{\rho}_{\rm mixed} = \rho_{\rm mixed}(q, q')$$

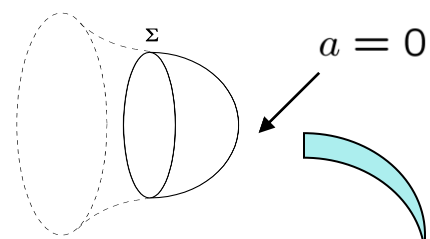


pinching a tubular spacetime

$$|\Psi_{HH}\rangle\langle\Psi_{HH}| = \rho_{HH}(q, q')$$



$$|\Psi_{HH}\rangle$$



density matrix representation of a pure Hartle-Hawking state – vacuum state of zero temperature, inverse of the conformal time $T \sim 1/\eta = 0$

$$\eta = \int_{a=0} d\tau \, \frac{N}{a} \to \infty$$

(see below)

Application to the CFT driven cosmology

$$S[g_{\mu\nu},\phi] = -\frac{M_P^2}{2}\int d^4x\, g^{1/2}\left(R-2\Lambda\right) + S_{CFT}[g_{\mu\nu},\phi]$$

$\Lambda = 3H^2$ -- primordial cosmological constant

$N_s \gg 1$ conformal fields of spin s=0,1,1/2

Conformal invariance → "exact" calculation of S_{eff}:

Assumption of $N_{cdf} \gg 1$ conformally invariant quantum fields

Recovery of the action from the conformal anomaly and the action on a static Einstein Universe

Path integral calculation: disentangling the minisuperspace sector

$$[\, g_{\mu\nu}, \phi \,] = [\, a(\tau), N(\tau); \Phi(x) \,]$$

minisuperspace sector

cosmological perturbations:

$$\Phi(x) = (\varphi(x), \psi(x), A_\mu(x), h_{\mu\nu}(x), \ldots)$$

$$ds^2 = N^2\, d\tau^2 + a^2\, d^2\Omega^{(3)}$$

lapse

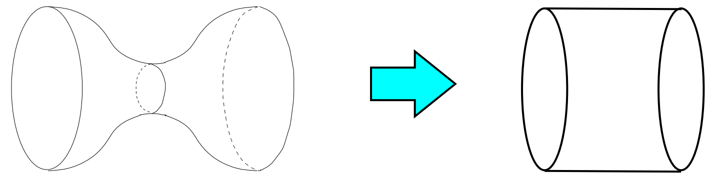
scale factor

Decomposition of the statistical sum path integral:

$$e^{-\Gamma} = \int_{\text{periodic}} D[\, a, N \,]\, e^{-S_{\text{eff}}[\, a, N \,]}$$

quantum effective action of Φ on minisuperspace background

$$e^{-S_{\text{eff}}[\, a, N]} = \int_{\text{periodic}} D\Phi(x)\, e^{-S_E[\, a, N; \Phi(x)\,]}$$



Starobinsky (1980);
Fischetty,Hartle,Hu;
Riegert; Tseytlin;
Antoniadis, Mazur & Mottola;
……

$$ds^2 = a^2(\eta)(d\eta^2 + d^2\Omega^{(3)}) \Rightarrow d\bar{s}^2 = d\eta^2 + d^2\Omega^{(3)}$$

conformal time

$$S_{\text{eff}} = \text{classical part} + \Gamma_A + \Gamma_{EU}$$

anomaly contribution

Einstein universe contribution

$$g_{\mu\nu}\frac{\delta\Gamma_A}{\delta g_{\mu\nu}} = \frac{1}{4(4\pi)^2} g^{1/2}\left(\alpha\Box R + \beta E + \gamma C^2_{\mu\nu\alpha\beta}\right)$$

spin-dependent coefficients

Gauss-Bonnet term

Weyl term

$$\beta = \tfrac{1}{360}(2N_0 + 11N_{1/2} + 124N_1)$$

N_s # of fields of lspin s

Effective Friedmann equation for saddle points of the path integral:

$$\frac{\delta S_{\rm eff}[\,a,N\,]}{\delta N(\tau)} = 0$$

amount of radiation and Casimir energy constant

$$\frac{1}{a^2} - \frac{a'^2}{a^2} - \frac{B}{2}\left(\frac{1}{a^2} - \frac{a'^2}{a^2}\right)^2 = \frac{\Lambda}{3} + \frac{C - B/2}{a^4},$$

$$a' \equiv \frac{1}{N}\frac{da}{d\tau}$$ time parameterization invariance

$$C = \frac{B}{2} + \frac{1}{6\pi^2 M_P^2}\sum_\omega \frac{\omega}{e^{\omega\eta} \pm 1}$$

Casimir energy

thermal energy

"bootstrap" equation: inverse temperature -- functional of geometry

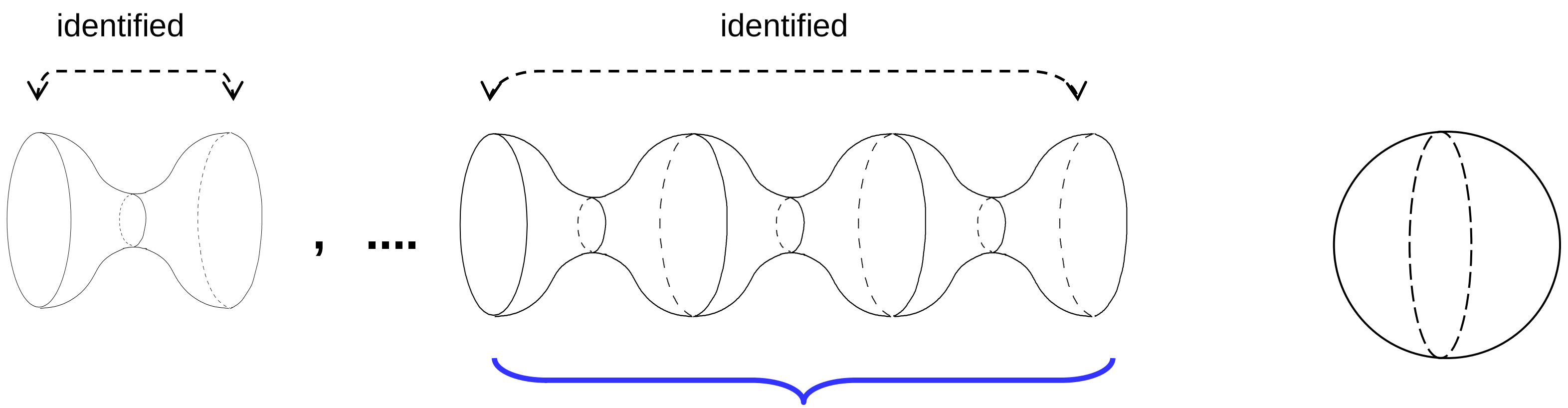
$$\eta = \oint d\tau\,\frac{N}{a}$$

Inverse (comoving) temperature

$$B = \frac{\beta}{8\pi^2 M_P^2}$$

-- coefficient of the Gauss-Bonnet term in the conformal anomaly

Saddle point solutions --- set of periodic (thermal) garland-type instantons with oscillating scale factor ($S^1 \times S^3$) and the vacuum Hartle-Hawking instantons (S^4)



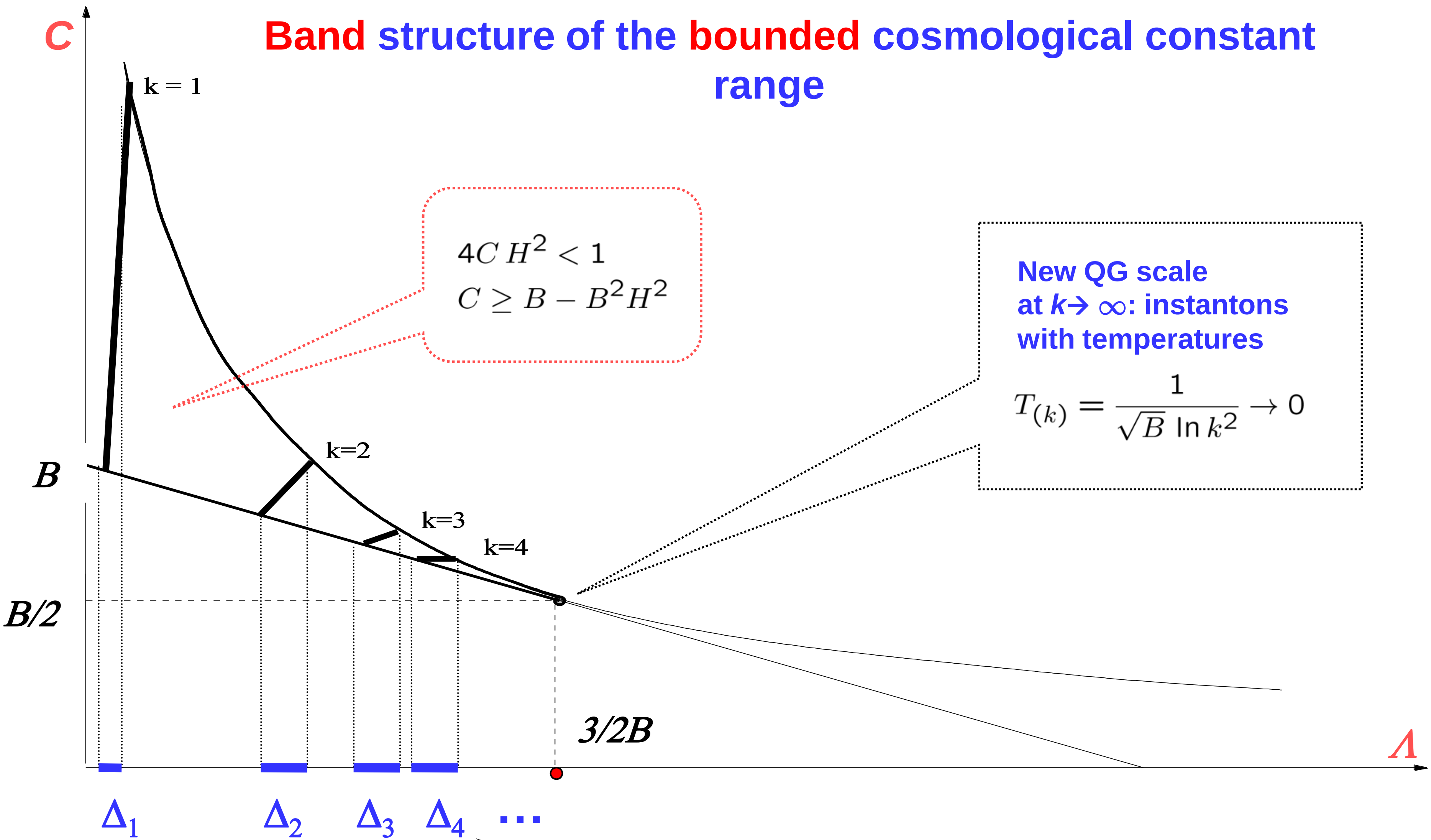
1- fold, *k*=1

***k*- folded garland, *k*=1,2,3,…**

***S*^{4}**

does not contribute: ruled out by *infinite positive* Euclidean action (effect of conformal anomaly)

Band structure of the bounded cosmological constant range



$$\Lambda_{\rm min} < \Lambda < \Lambda_{\rm max} = \frac{12\pi^2 M_P^2}{\beta}$$

Δ_k – cosmological constant band for k-folded garland

Constraining string landscape?

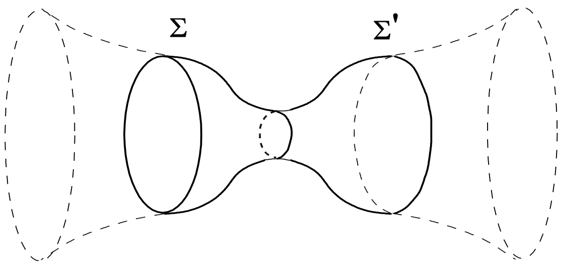
"SOME LIKE IT HOT" scenario

"SOME LIKE IT HOT" (SLIH) scenario recovers a new incarnation of Hot Big Bang -- it incorporates effectively thermal state at the onset of the cosmological evolution. Known inflation paradigm retracted the BB concept by replacing it with the initial vacuum state.

So how does SLIH scenario matches with inflation?

Hill-top inflation

Lorentzian Universe with initial conditions set by the saddle-point instanton. Analytic continuation of the instanton solutions:



$$\tau = it, \ a(t) = a_{Euclid}(it)$$

Expansion and quick dilution of primordial radiation

Generalization to Λ as a composite operator – inflaton potential in "slow roll" regime

$$\Lambda \to \frac{V(\phi)}{M_P^2}$$

decay of a composite Λ, exit from inflation and particle creation of conformally non-invariant matter:

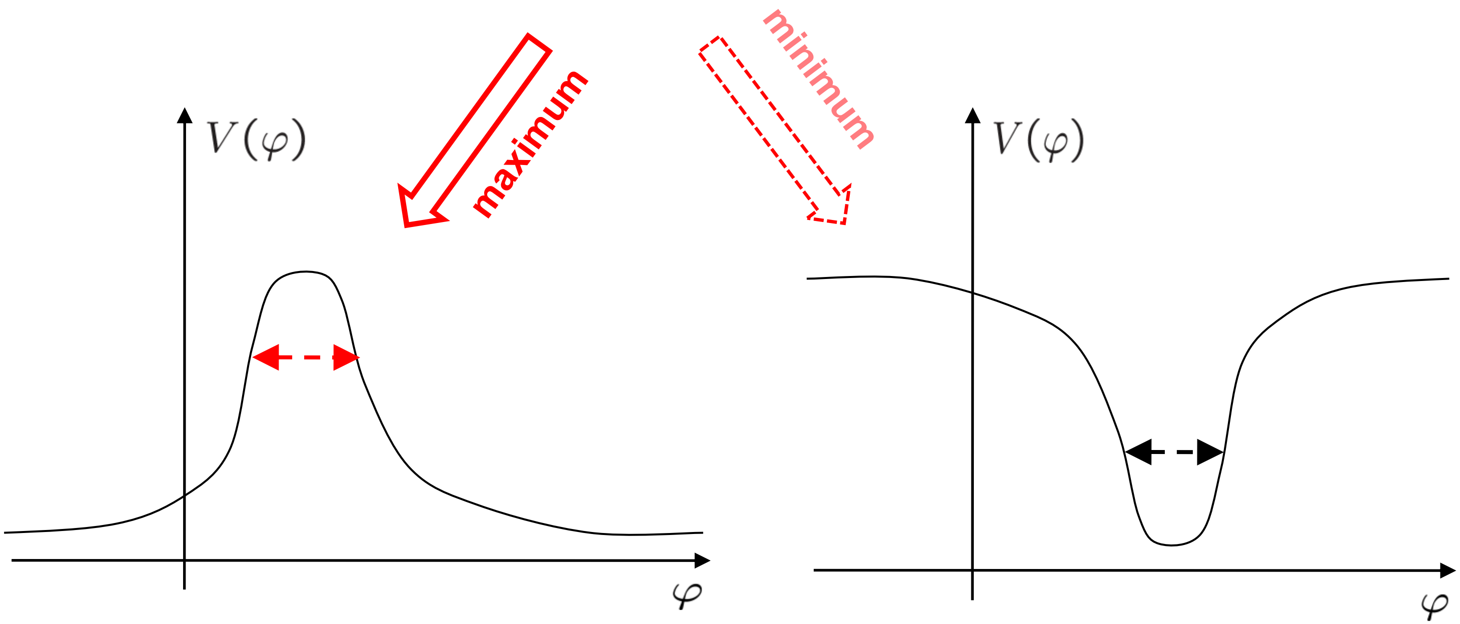
$$\frac{\Lambda}{3} + \frac{\mathcal{C}}{a^4} \Rightarrow \frac{8\pi G}{3}\varepsilon$$

energy density of non-conformal matter

Selection of inflaton potential *maxima* as initial conditions for inflation

Critical point: $\frac{d}{d\tau}a^3\dot{a} = a^3\frac{\partial V}{\partial\varphi} \;\Rightarrow\; \oint d\tau\, a^3\frac{\partial V}{\partial\varphi} = 0 \;\Rightarrow$

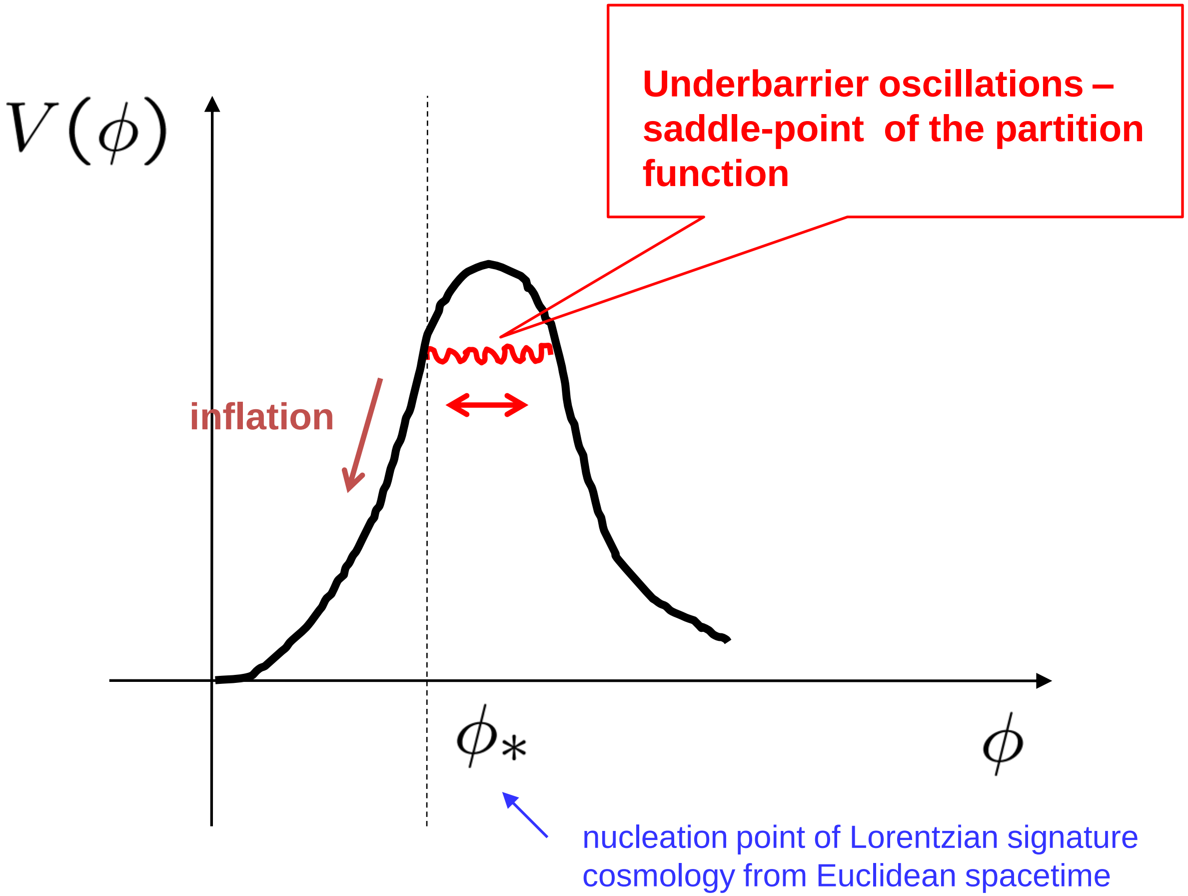
$\frac{\partial V}{\partial\varphi} \gtrless 0$ Potential extremum ``inside" instanton



classically forbidden (underbarrier) oscillation

classically allowed (overbarrier) oscillation --- ruled out because of *underbarrier* oscillations of scale factor

Hill-top inflation by nucleation from cosmological instanton



Approximation of two coupled oscillators → slow roll parameters

$$\epsilon \sim \eta^2 \ll |\eta|, \quad \eta < 0$$

Mechanism of hill-top potential

Not in Einstein frame, no shift symmetry, IR instability and breakdown of grad. expansion! Tsamis, Woodard, …

Gradient expansion in the Jordan frame:

non-minimal coupling

$$\Gamma[g_{\mu\nu},\varphi] = \int d^4x\, g^{1/2} \Big(V(\varphi) - U(\varphi)\, R(g_{\mu\nu}) + \frac{1}{2} G(\varphi)\, (\nabla\varphi)^2 + ... \Big)$$

$$V(\varphi) = \frac{\lambda}{4}\varphi^4 + \frac{\lambda\varphi^4}{64\pi^2}\boldsymbol{A}\ \ln\frac{\varphi}{\mu} + ..., \quad U(\varphi) = \frac{M_P^2}{2} + \frac{\xi\varphi^2}{2}\Big(1 + \frac{3\lambda}{8\pi^2}\ln\frac{\varphi}{\mu} + ...\Big),$$

$$G(\varphi) = 1 + \frac{F}{16\pi^2}\ln\frac{\varphi}{\mu} + ...$$

Transition to the Einstein frame:

$$V(\varphi) \to V_{EF}(\phi) = \frac{M_P^4}{4}\frac{V(\varphi)}{U^2(\varphi)} \sim \frac{M_P^4}{\lambda\xi^2}\frac{\boldsymbol{A}\ \ln\frac{\varphi}{\mu}}{\ln^2\frac{\varphi}{\mu}} \sim \frac{1}{\ln\frac{\varphi}{\mu}} \to 0, \quad \varphi\to\infty$$

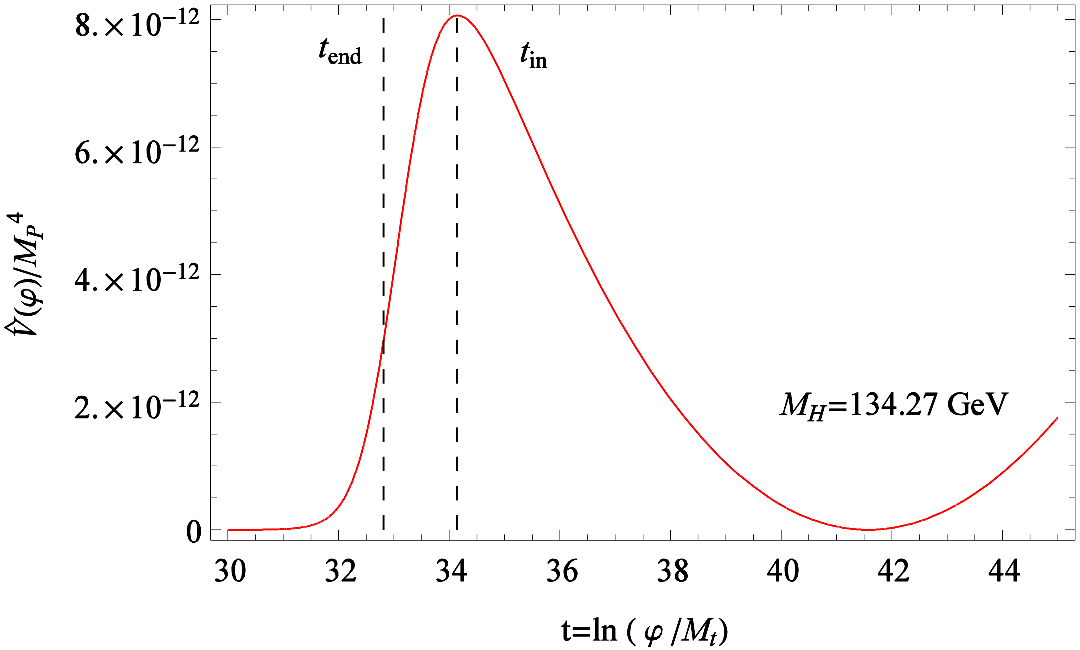
Any *l*-th loop order:

$$\frac{\ln^l\frac{\varphi}{\mu}}{\ln^{2l}\frac{\varphi}{\mu}} \sim \frac{1}{\ln^l\frac{\varphi}{\mu}} \to 0, \quad \varphi\to\infty$$

Resummation by RG confirms this.

Higgs inflation with non-minimally coupled inflaton

B.Spokoiny 1986,
A.Kamenshchik & A.B 1991,
Bezrukov,Shaposhnikov 2008
(Einstein frame calculations),
A.Kamenshchik, A.Starobinsky & A.B 2008



A.Kamenshchik, C.Kiefer, A.Starobinsky, C.Steinwachs, & A.B., JCAP 12 (2009) 003, arXiv:0904.1698

One-loop RG improved effective potential. Inflationary domain for $N = 60$ CMB perturbation is marked by dashed lines.

2-loop RG contribution leads to $M_{Higgs} \to M_{LHC}$= 126 GeV
(Bezrukov & Shaposhnikov 2009)

Thermal corrections to primordial power spectrum

Conventional T_{CMB} is subject to vacuum fluctuations

"SLIH" T_{CMB} is subject to thermal distribution with the temperature $T=1/\eta$
-- *temperature of the CMB temperature*

thermal contribution

$$\delta^2_\phi(k) = \langle\, \hat{\phi}_k(t)\hat{\phi}_k(t)\,\rangle_{\rm thermal} = |u_k(t)|^2\Big(1 + 2N_k(\eta)\Big)$$

vacuum part (under investigation)

$$N_k(\eta) = \frac{1}{e^{k\eta} - 1}$$

$$n_s(k) = n_s^{\rm vac}(k) + \Delta n_s^{\rm thermal}(k)$$

additional reddening of the CMB spectrum

$$\Delta n_s^{\rm thermal}(k_l) \simeq -\frac{20\,l}{(3\,\tilde{\beta})^{1/6}} e^{-10\,l/(3\,\tilde{\beta})^{1/6}} \ll 1, \quad \tilde{\beta} \sim \frac{\sum_s \beta_s N_s}{\sum_s N_s}$$

thermal part for low spins is negligible

$$\tilde{\beta}_s = \frac{\beta_s}{N_s} \sim s^4 \to \infty$$

contribution of higher conformal spins might have large thermal imprint on CMB

?

Hierarchy problem and higher spin conformal fields

$$H^2 \lesssim \frac{1}{2B} = \frac{4\pi^2}{\beta} M_P^4 \quad \Rightarrow \quad V_{\text{inflation}} = 3M_P^2 H^2 \sim \frac{12\pi^2}{\beta} M_P^4 \sim 10^{-11} M_P^4$$

E-coefficient of total conformal anomaly

Higgs inflation

$$\beta \simeq 10^{13}$$

Recent progress in HS field theory (Vasiliev) and CHS theory (Klebanov, Giombi, Tseytlin, etc) arXiv:1309.0785

$$\beta_s \sim s^6, \quad N_s \sim s^2$$

$$\beta_{\text{boson}} = \sum_{s=1}^{S} \beta_s \simeq \frac{S^7}{180} \sim 10^{13} \quad \Rightarrow \quad S \sim 100$$

$$\mathbb{N}_{\text{boson}} = \sum_{s=1}^{S} N_s \sim 10^6$$

We need a hidden sector of CHS with the tower of spins to $S\sim 100$ and # of polarizations $\sim 10^6$

This number of hidden sector fields gives a red thermal correction to CMB spectral index in the third (potentially observable) decimal order:

$$\tilde{\beta} \equiv \frac{\beta_{\text{boson}}}{\mathbb{N}_{\text{boson}}} \sim 10^6 \quad \Rightarrow \quad \Delta n_s^{\text{thermal}} \sim -0.001$$

Interesting case: anomaly free CHS theory – candidate for quantum consistent TOE (Giombi, Klebanov, Tseytlin)

$$S \to \infty, \quad \sum_{s=1}^{\infty} \beta_s = 0, \quad B = 0, \quad C < 0$$

negative radiation energy density ! (Beccaria, Tseytlin, 2014)

$$C < 0, \quad H^2 < 0, \quad S^1 \times \mathbb{S}^3 \to S^1 \times \mathbb{H}^3$$

negative curvature 3-hyperboloid

Periodic Lorentzian signature solutions of negative curvature:

$$ds^2 = -dt^2 + a^2(t)\, d\mathbb{H}^2_{(3)}$$

$$a^2(t) = \frac{1}{2\,|\,H^2|} - \frac{1}{2\,|\,H^2|}\sqrt{1 - \frac{72\pi^2 M_P^2}{11|H^2|}\cos\left(2|H|t\right)}$$

and

$$a(t) = \frac{1}{|\,H\,|}\sin\left(|H|t\right)$$

Exact Lorentzian AdS foliated by 3-hyperboloids

Playground for string theory and holography.

What about cosmology???

Conclusions

Microcanonical density matrix of the Universe – Sum over Everything

Application to the CFT driven cosmology with a large # of quantum species – a limited range of Λ -- cosmological (string?) landscape , elimination of IR dangerous no-boundary states

SLIH scenario, hill-top inflation, mechanism of hill-top potential, thermally corrected CMB spectrum, but rather cold

Attempt of solving hierarchy problem by CHS fields, but problems of naturalness, unitarity, etc.

SOME LIKE IT HOT

SOME LIKE IT COOL