Infrared Quantum Gravity

1) Effective field theory – UV vs IR
   - non-analytic, non-local

2) Some low energy theorems of quantum gravity
   - soft theorems at one loop
   - non-geodesic motion/ EP violation

3) Beyond scattering amplitudes
   - non-linear non-local effective actions
   - hint of singularity avoidance

4) Some future issues
   - IR singularities
   - cumulative effects – the far IR

AMHERST CENTER FOR FUNDAMENTAL INTERACTIONS
Physics at the interface: Energy, Intensity, and Cosmic frontiers
University of Massachusetts Amherst

John Donoghue
Galiano Island
08/20/15

Quantum GR at low energies is an “effective field theory”

Effective field theory is a (now) standard technique:

- calculate quantum effects at a given energy scale
- shifts focus from U.V. to I.R.
- handles main obstacle
  - – quantum effects involve **all** scales

Much active focus on UV physics of gravity

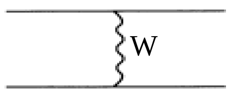
- unknown and interesting new physics at Planck scale
- but physics is expt. science – prospects for resolution at Planck scale?
- IR effects are small
- but IR is where quantum gravity is reliable
- goal is explore quantum effects in GR

Many good quantum calculations do not use phrase EFT, but EFT helps explain why they are good.

Local vs Nonlocal in Effective Field Theory

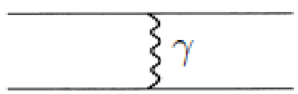
EFT separates known from unknown (or irrelevant) physics

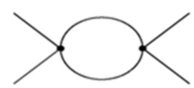
High energy effects are local



δm^2

Low energy = nonocal





Known aspects – massless (or light) degrees of freedom

- couplings near zero energy

Procedures:

1) General local Lagrangian, ordered by energy expansion

$$S = \int d^4x \sqrt{-g} \left[-\frac{2}{\kappa^2} R \; + \; c_1(\mu) R^2 \; + \; c_2(\mu) R_{\alpha\beta} R^{\alpha\beta} + \ldots \right]$$

2) Apply quantum field theory – perturbation theory

   Feynman-DeWitt

3) Renormalize Lagrangian

$$\Delta \mathcal{L}_0^{(1)} = \frac{1}{8\pi^2} \frac{1}{\epsilon} \left\{ \frac{1}{120} R^2 + \frac{7}{20} R_{\mu\nu} R^{\mu\nu} \right\}$$

'tHooft
Veltman

4) What are the quantum predictions?

   non-analytic in momentum space

$$Amp \sim q^2 \ln(-q^2) \quad , \quad \sqrt{-q^2}$$

   nonlocal in coordinate space

"Low energy theorems"

- independent of UV completion
- depend only on IR structure

Nature of the expansion:

Power counting theorem ($\Lambda = 0$) for quantum effects:

- one loop – extra $E^2 \sim \partial^2$
- two loop - $E^4 \sim \partial^4$

Vary from process to process

Amplitudes

$$Amp_i \sim Amp_i^{(0)} \left[1 + a_i G m \sqrt{-q^2} \ + \ b_i G q^2 \ln \frac{-q^2}{\mu^2} \ + \ c_i(\mu) G q^2 + ... \right]$$

Local terms

One loop divergences

Action

$$S = \int d^4x \sqrt{-g} \ \left[-\frac{2}{\kappa^2} R \ + \ c_1(\mu) R^2 \ + \ c_2(\mu) R_{\alpha\beta} R^{\alpha\beta} + ... \right]$$

Effective Field Theory in Action:

Chiral Perturbation Theory

-QCD at very low energies –pions and photons

Non-linear lagrangian required by symmetry:

$$\mathcal{L} = F_\pi^2 Tr(D_\mu U D^\mu U^\dagger) + L_1[Tr(D_\mu U D^\mu U^\dagger)]^2 + ...$$

$$\boxed{U = exp[i\frac{\tau \cdot \phi}{F_\pi}]}$$

Very well studied: Theory and phenomenology

- energy expansion, loops, symmetry breaking, experimental constraints, connection to QCD.

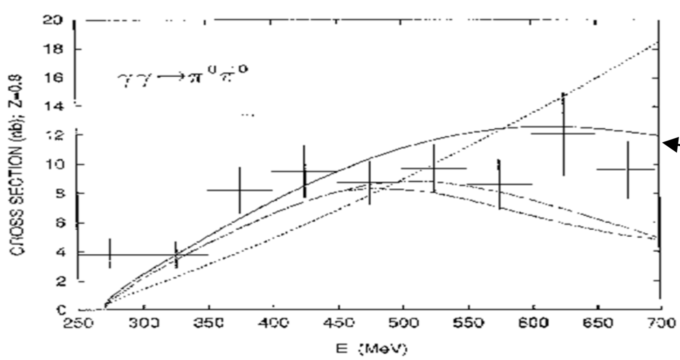
Sample calculation:

-no direct couplings at low energy

– pure loops

-essentially parameter free at low energy

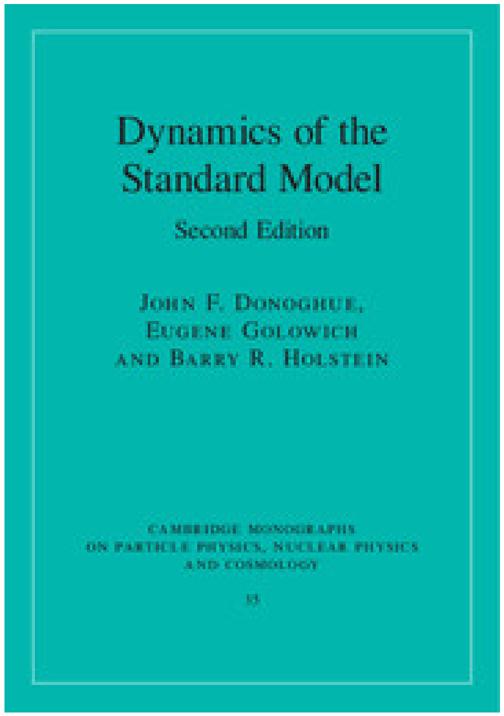
$$\gamma\gamma \longrightarrow \pi^0\pi^0$$



one loop

two loops

Advertisment: Read all about it!



Second edition

Some low energy theorems of quantum gravity

Quantum loop effects independent of any UV theory
- assumption is GR at low energy
- leading couplings and gravitons

1) Gravitational scattering of masses
   - universal/ soft theorem at one loop

2) Graviton-gravition scattering
   - IR singularities

3) Light bending at one loop
   - non-universal behavior / EP violation
   - motion not on null geodesics

Corrections to Newtonian Potential

JFD 1994
JFD, Holstein, Bjerrum-Bohr 2002
Khriplovich and Kirilin
Other references later

Here discuss scattering potential of two heavy masses – S matrix element.

$$\begin{aligned} \langle f|T|i\rangle &\equiv (2\pi)^4\delta^{(4)}(p-p')(\mathcal{M}(q)) \\ &= -(2\pi)\delta(E-E')\langle f|\tilde{V}(\mathbf{q})|i\rangle \end{aligned}$$

Potential found using from

$$V(\mathbf{x}) = \frac{1}{2m_1}\frac{1}{2m_2}\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{x}}\mathcal{M}(\vec{q})$$

Classical potential has been well studied

Iwasaki
Gupta-Radford
Hiida-Okamura

What to expect:

General expansion:

$$V(r) = -\frac{GMm}{r}\left[1 + a\frac{G(M+m)}{rc^2} + b\frac{G\hbar}{r^2c^3}\right] + cG^2Mm\delta^3(r)$$

Classical expansion parameter

Quantum expansion parameter

Short range

Relation to momentum space:

$$\int \frac{d^3q}{(2\pi)^3}\; e^{i\mathbf{q}\cdot\mathbf{r}}\frac{1}{|\mathbf{q}|^2} = \frac{1}{4\pi r}$$

$$\int \frac{d^3q}{(2\pi)^3}\; e^{i\mathbf{q}\cdot\mathbf{r}}\frac{1}{|\mathbf{q}|} = \frac{1}{2\pi^2 r^2}$$

$$\int \frac{d^3q}{(2\pi)^3}\; e^{i\mathbf{q}\cdot\mathbf{r}}\ln(\mathbf{q}^2) = \frac{-1}{2\pi r^3}$$

Momentum space amplitudes:

$$V(q^2) = \frac{GMm}{q^2}\left[1 + a'G(M+m)\sqrt{-q^2} + b'G\hbar q^2\ln(-q^2) + c'Gq^2\right]$$

Classical

quantum

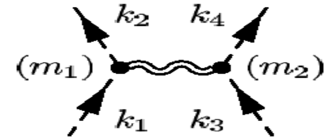
short range

Non-analytic

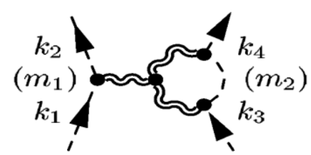
analytic

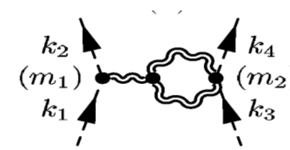
The calculation:

Lowest order:

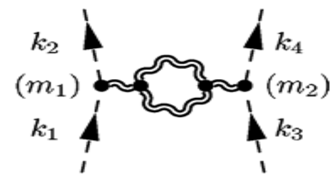


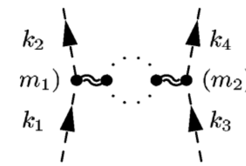
Vertex corrections:



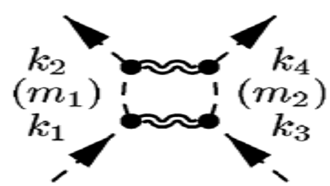


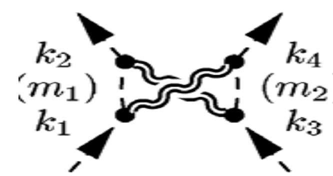
Vacuum polarization:
(Duff 1974)



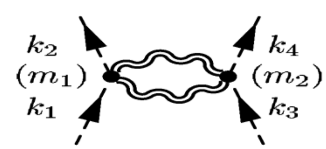


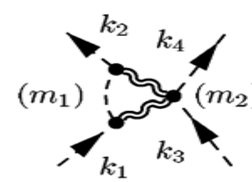
Box and crossed box





Others:





Results:

Pull out non-analytic terms:
-for example the vertex corrections:

$$
\begin{aligned}
M_{5(a)+5(b)}(\vec{q}) &= 2G^2 m_1 m_2 \left(\frac{\pi^2(m_1+m_2)}{|\vec{q}|} + \frac{5}{3}\log \vec{q}^2 \right) \\
M_{5(c)+5(d)}(\vec{q}) &= -\frac{52}{3} G^2 m_1 m_2 \log \vec{q}^2
\end{aligned}
$$

Sum diagrams:

$$
\boxed{V(r) = -\frac{Gm_1m_2}{r}\left[1 + 3\frac{G(m_1+m_2)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]}
$$

Gives precession
of Mercury, etc
(Iwasaki ;
Gupta + Radford)

Quantum
correction

Comments

1) Both classical and quantum emerge from a one loop calculation!
   - classical first done by Gupta and Radford (1980)

1) Unmeasurably small correction:
   - best perturbation theory known(!)

3) Quantum loop well behaved - no conflict of GR and QM

4) Other calculations
   (Radikowski, Duff, JFD; Muzinich and Vokos; Hamber and Liu; Akhundov, Bellucci, and Sheikh ; Khriplovich and Kirilin)
   -other potentials or mistakes

Aside: Classical Physics from Quantum Loops:

JFD, Holstein
2004 PRL

Field theory folk lore:

Loop expansion is an expansion in $\hbar$

"Proofs" in field theory books

This is not really true.

- numerous counter examples – such as the gravitational potential

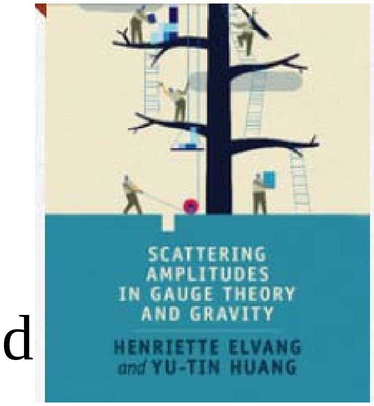
- can remove a power of $\hbar$ via kinematic dependence

$$\sqrt{\frac{m^2}{-q^2}} = \frac{m}{\hbar\sqrt{-k^2}}$$

- classical behavior seen when massless particles are involved

On-shell techniques and loops from unitarity

JFD, Bjerrum-Bohr
Vanhove

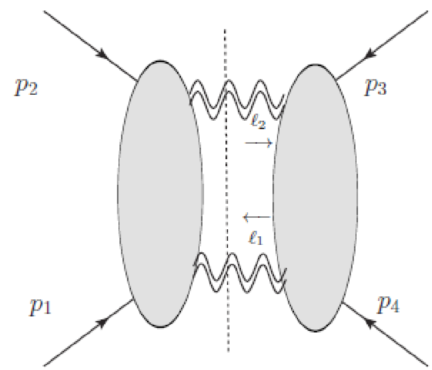


- On-shell amplitudes only
- No ghosts needed – axial gauge
- Both unitarity cuts and dispersion relation method
- Gravity is square of gauge theory

$$iM^{1-\text{loop}}\Big|_{disc} = \int \frac{d^D\ell}{(2\pi)^D} \frac{\sum_{\lambda_1,\lambda_2} M^{\text{tree}}_{\lambda_1\lambda_2}(p_1,p_2,-\ell_2^{\lambda_2},\ell_1^{\lambda_1})(M^{\text{tree}}_{\lambda_1\lambda_2}(p_3,p_4,\ell_2^{\lambda_2},-\ell_1^{\lambda_1}))^*}{\ell_1^2\ell_2^2}\Bigg|_{cut},$$

$$iM_0^{\text{tree}}(p_1,p_2,k_1^+,k_2^+) = \frac{\kappa_{(4)}^2}{16}\frac{1}{(k_1\cdot k_2)}\frac{m^4\,[k_1\,k_2]^4}{(k_1\cdot p_1)(k_1\cdot p_2)},$$

$$iM_0^{\text{tree}}(p_1,p_2,k_1^-,k_2^+) = \frac{\kappa_{(4)}^2}{16}\frac{1}{(k_1\cdot k_2)}\frac{\langle k_1|p_1|k_2]^2\,\langle k_1|p_2|k_2]^2}{(k_1\cdot p_1)(k_1\cdot p_2)},$$



Confirm results (and gauge invariance)

One loop universality/soft theorem

Low
Gell-Mann
Goldberger
Weinberg
Gross
Jackiw

Tree level soft theorems

- Compton amplitudes and gravitational Compton amplitudes are universal at leading order
- Conservation of charge/energy and ang. mom.

One loop soft theorem

- E&M and gravitational potentials
- formed by square of Compton amplitudes
- quantum term down from classical by $\sqrt{-q^2}$
- first found in direct calculations by Holstein and Ross

Graviton –graviton scattering

Fundamental quantum gravity process

Lowest order amplitude:

$$\mathcal{A}^{tree}(++;++) = \frac{i}{4}\frac{\kappa^2 s^3}{tu}$$

Cooke;
Behrends Gastmans
Grisaru et al

One loop:

Incredibly difficult using field theory

Dunbar and Norridge –string based methods! (just tool, not full string theory)

$$\begin{aligned}
\mathcal{A}^{1-loop}(++;--) &= -i\,\frac{\kappa^4}{30720\pi^2}\left(s^2+t^2+u^2\right) \\
\mathcal{A}^{1-loop}(++;+-) &= -\frac{1}{3}\mathcal{A}^{1-loop}(++;--) \\
\mathcal{A}^{1-loop}(++;++) &= \frac{\kappa^2}{4(4\pi)^{2-\epsilon}}\frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}\mathcal{A}^{tree}(++;++)\times(s\,t\,u) \qquad (3) \\
&\quad\times\left[\frac{2}{\epsilon}\left(\frac{\ln(-u)}{st}+\frac{\ln(-t)}{su}+\frac{\ln(-s)}{tu}\right)+\frac{1}{s^2}f\left(\frac{-t}{s},\frac{-u}{s}\right)\right. \\
&\quad\left.+2\left(\frac{\ln(-u)\ln(-s)}{su}+\frac{\ln(-t)\ln(-s)}{tu}+\frac{\ln(-t)\ln(-s)}{ts}\right)\right]
\end{aligned}$$

where

$$\begin{aligned}
f\left(\frac{-t}{s},\frac{-u}{s}\right) &= \frac{(t+2u)(2t+u)\left(2t^4+2t^3u-t^2u^2+2tu^3+2u^4\right)}{s^6}\left(\ln^2\frac{t}{u}+\pi^2\right) \\
&\quad+\frac{(t-u)\left(341t^4+1609t^3u+2566t^2u^2+1609tu^3+341u^4\right)}{30s^5}\ln\frac{t}{u} \\
&\quad+\frac{1922t^4+9143t^3u+14622t^2u^2+9143tu^3+1922u^4}{180s^4}, \qquad (4)
\end{aligned}$$

Infrared safe:

JFD +
Torma

The $1/\epsilon$ is from infrared
-soft graviton radiation
-made finite in usual way
$1/\epsilon$ -> ln(1/resolution) (gives scale to loops)
-cross section finite

$$\left(\frac{d\sigma}{d\Omega}\right)_{tree} + \left(\frac{d\sigma}{d\Omega}\right)_{rad.} + \left(\frac{d\sigma}{d\Omega}\right)_{nonrad.} = \tag{29}$$

$$= \frac{\kappa^4 s^5}{2048\pi^2 t^2 u^2}\left\{1 + \frac{\kappa^2 s}{16\pi^2}\left[\ln\frac{-t}{s}\ln\frac{-u}{s} + \frac{tu}{2s^2} f\left(\frac{-t}{s}, \frac{-u}{s}\right)\right.\right.$$

$$\left.\left. - \left(\frac{t}{s}\ln\frac{-t}{s} + \frac{u}{s}\ln\frac{-u}{s}\right)\left(3\ln(2\pi^2) + \gamma + \ln\frac{s}{\Lambda^2} + \frac{\sum_{ij}\eta_i\eta_j \mathcal{F}^{(1)}(\gamma_{ij})}{\sum_{ij}\eta_i\eta_j \mathcal{F}^{(0)}(\gamma_{ij})}\right)\right]\right\}.$$

*

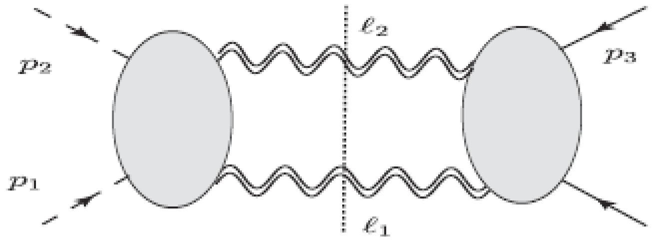
finite

Beautiful result:
-low energy theorem of quantum gravity

<u>**Light bending at one loop**</u>

Bjerrum-Bohr, JFD, Holstein
Plante, Vanhove

- Again using unitarity methods
- Gravity Compton as
  square of EM Compton
- Compare massless spin 0 and photon



$$
\begin{aligned}
i\mathcal{M}^{[\eta(p_1)\eta(p_2)]}_{[\phi(p_3)\phi(p_4)]} \simeq \frac{\mathcal{N}^\eta}{\hbar}(M\omega)^2 \Bigg[& \frac{\kappa^2}{t} + \kappa^4 \frac{15}{512}\frac{M}{\sqrt{-t}} + \hbar\kappa^4 \frac{15}{512\pi^2} \\
& \times \log\left(\frac{-t}{M^2}\right) - \hbar\kappa^4 \frac{bu^\eta}{(8\pi)^2}\log\left(\frac{-t}{\mu^2}\right) \\
& + \hbar\kappa^4 \frac{3}{128\pi^2}\log^2\left(\frac{-t}{\mu^2}\right) \\
& + \kappa^4 \frac{M\omega}{8\pi}\frac{i}{t}\log\left(\frac{-t}{M^2}\right)\Bigg], \qquad (11)
\end{aligned}
$$

$bu^\varphi = 3/40$ and $bu^\gamma = -161/120.$

bu^i is different coefficient for spin 0 and 1

Semiclassical method for calculating bending angle

$$\theta_\eta \simeq -\frac{b}{\omega}\int_{-\infty}^{+\infty}\frac{V'_\eta(b\sqrt{1+u^2})}{\sqrt{1+u^2}}du$$
$$\simeq \frac{4GM}{b}+\frac{15}{4}\frac{G^2M^2\pi}{b^2}+\frac{8bu^\eta+9+48\log\frac{b}{2r_o}}{\pi}\frac{G^2\hbar M}{b^3}$$

r_0 is IR cutoff
can turn into
energy dependence
depending on
resolution of IR
singularity

Recently reproduced using eikonal amplitude
- saddle point approximation

$$\mathcal{M}^{0+1}\left(\mathbf{\Delta}^\perp\right) = 2(s-M_\sigma^2)\int d^2\mathrm{b}^\perp e^{-i\mathbf{\Delta}^\perp\cdot\mathbf{b}^\perp}\left[e^{i(\chi_0-i\ln[1+i\chi_2])}-1\right]$$

Using
Akhoury
Ryo
Sterman

Massless particles deviate from null geodesics
- irreducible tidal effects from loops
- also non-universal – violation of some forms of EP
- perhaps energy dependence

<u>Other phenomena</u>?

Change in light bending is far too small to see

Are any other situations promising?
- want **lots** of gravitational field
- photon ring at Event Horizon Telescope?
- energy dependence of redshift, lensing? Mende

What could an ambitious experiment achieve?
Any tricks?

Aside: Quantum corrections do not organize into running G(E)

Basic reason: Loops generate effects at order R^2 , not renormalizing G

Indicators:

1) Kinematic $G q^2$ not even sign definite in Lorentzian signatures
2) Process dependence – no universal definition possible

$$Amp_i \sim Amp_i^{(0)} \left[1 + a_i G m \sqrt{-q^2} \;+\; b_i G q^2 \ln \frac{-q^2}{\mu^2} \;+\; c_i(\mu) G q^2 + ... \right]$$

True even with cutoff or momentum shells:

$G\Lambda^2$ disappears from physical processes

- not indicator of true energy dependence

Anber, Donoghue
Toms

Many "running coupling" papers are wrong
Relationship to Asymptotic Safety is subtle (discussion?)

Beyond scattering amplitudes

Gravity much more than scattering
- but QFT techniques less developed

<u>Non-local effective actions:</u>
- most work done by Barvinsky, Vilkovisky and collab.
- covariant
- expansion in curvature

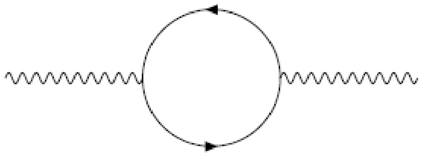
Others:
Starobinsky
Hartle, Hu

Note: This is a different expansion from EFT derivative expansion

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots \right\}$$ EFT

$$S_{curv} \sim \int d^4x \sqrt{-g} \; ... + c(\mu) R^2 + dR \log(\Box/\mu^2) R + R^2 \frac{1}{\Box} R + ... + R^{n+1} \frac{1}{\Box^n} R + ...$$ BV

<u>Example: Non local action for massless QED:</u>



Vacuum polarization contains divergences but also log q^2
Integrate out **massless** matter field and write effective action:

$$S = \int d^4x \ -\frac{1}{4}F_{\rho\sigma}\left[\frac{1}{e^2(\mu)} - b\log\left(\nabla^2/\mu^2\right)\right]F^{\rho\sigma} \qquad +\mathcal{O}(F^4)$$

Displays running of charge

$$b = \frac{1}{12\pi}$$

Really implies a non-local effective action:

$$\langle x|\ln\left(\frac{\nabla^2}{\mu^2}\right)|y\rangle \equiv L(x-y) = \int \frac{d^4q}{(2\pi)^4}e^{-iq\cdot(x-y)}\ln\left(\frac{-q^2}{\mu^2}\right)$$

Connection: Running and non-local effects

Perturbation with gravity

JFD, El-Menoufi

Non-local effective action for gravitational coupling

$$\Gamma^{(1)}[A,h] = -\frac{1}{2}\int d^4x\, h^{\mu\nu}\left[b_s \log\left(\frac{\Box}{\mu^2}\right) T^{cl}_{\mu\nu} - \frac{b_s}{2}\frac{1}{\Box}\tilde{T}^s_{\mu\nu}\right]$$

$$\tilde{T}^s_{\mu\nu} = 2\partial_\mu F_{\alpha\beta}\partial_\nu F^{\alpha\beta} - \eta_{\mu\nu}\partial_\lambda F_{\alpha\beta}\partial^\lambda F^{\alpha\beta}$$

Stress tensor is then non-local

Can be causal

$$T^i_{\mu\nu}(x) = T^{cl}_{\mu\nu}(x) - e^2 b_i \int d^4y \left[L(x-y)T^{cl}_{\mu\nu}(y) + i\frac{1}{2}\Delta_F(x-y)\tilde{T}^i_{\mu\nu}(y)\right], \quad i = s, f$$

Also would lead to EP violating light-bending

$$\theta_{non-flip} \approx \frac{4GM_\odot}{b} + \frac{8\beta GM_\odot}{eb}(\ln mb + \gamma_E - \ln 2) - \frac{4\beta GM_\odot}{eE^2b^3}$$

Full expansion in the curvature:

Most interesting are the $1/\Box$ terms

$$\Gamma_{anom.}[g,A] = \int d^4x\,\sqrt{g}\left[n_1 F_{\rho\sigma}F^{\rho\sigma}\frac{1}{\Box}R + n_3 F^{\rho\sigma}F^{\gamma}_{\lambda}\frac{1}{\Box}C_{\rho\sigma\gamma}{}^{\lambda}\right]$$

But $\log\Box$ (with covariant Box) proves most problematic

$$\begin{aligned}\Gamma_{log} =& \frac{b_i}{4}\int d^4x\sqrt{g}\Big\{F_{\alpha\beta}\ln\left(\nabla^2/\mu^2\right)F^{\alpha\beta} - \frac{1}{3}F_{\alpha\beta}F^{\alpha\beta}\frac{1}{\nabla^2}R \\ &+ 4R^{\mu\nu}\frac{1}{\nabla^2}\Big[\log(\nabla^2)\left(-F_{\mu\sigma}F_{\nu}{}^{\sigma} + \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right) \\ &+ F_{\mu\sigma}\log(\nabla^2)F_{\nu}{}^{\sigma} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}\log(\nabla^2)F^{\alpha\beta}\Big] \\ &+ \frac{1}{3}RF_{\alpha\beta}\frac{1}{\nabla^2}F^{\alpha\beta} - C^{\alpha}{}_{\beta\mu\nu}F_{\alpha}{}^{\beta}\frac{1}{\nabla^2}F^{\mu\nu}\Big\}\end{aligned}$$

dangerous

Is this a useful path for phenomenology?

Not the usual energy expansion

Interpreting ln ∇^2

∇^2 involves propagation in full background spacetime
Also need tractable approximation
Non-local representation

$$S = \int d^4x \sqrt{g(x)} R(x) \int d^4y \sqrt{g(y)} \langle x| \log\left(\frac{\Box}{\mu^2}\right)|y\rangle R(y)$$

where states are normalized

$$\langle x|y\rangle = \frac{\delta^{(4)}(x-y)}{\left(\sqrt{g(y)}\sqrt{g(x)}\right)^{1/2}}$$

We use locally flat approximation

$$\langle x| \log\left(\frac{\Box}{\mu^2}\right)|y\rangle = \left(\sqrt{g(y)}\sqrt{g(x)}\right)^{-1/2} \int \frac{d^4k}{(2\pi)^4} \log\left(\frac{-k^2}{\mu^2}\right) e^{-ik\cdot(x-y)}$$

Results seem insensitive to long-time tail where this is incorrect

Non-local action for gravity - 2nd order in curvature:

$$\left(R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + RR\right)$$ is a total derivative

$$\left(R_{\mu\nu\alpha\beta}\log\left(\Box\right)R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}\log\left(\Box\right)R^{\mu\nu} + R\log\left(\Box\right)R\right)$$ is not

Calculationally simplest basis:

$$S_{QL} = \int d^4x\sqrt{g}\left(\alpha R\log\left(\frac{\Box}{\mu_\alpha^2}\right)R + \beta R_{\mu\nu}\log\left(\frac{\Box}{\mu_\beta^2}\right)R^{\mu\nu} + \gamma R_{\mu\nu\alpha\beta}\log\left(\frac{\Box}{\mu_\gamma^2}\right)R^{\mu\nu\alpha\beta}\right)$$

Conceptually better basis:

$$S_{QL} = \int d^4x\,\sqrt{g}\,\Big[\bar{\alpha}R\log\left(\frac{\Box}{\mu_1^2}\right)R + \bar{\beta}C_{\mu\nu\alpha\beta}\log\left(\frac{\Box}{\mu_2^2}\right)C^{\mu\nu\alpha\beta} + \bar{\gamma}\big(R_{\mu\nu\alpha\beta}\log\left(\Box\right)R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}\log\left(\Box\right)R^{\mu\nu} + R\log\left(\Box\right)R\big)\Big]\,.$$

$$C_{\mu\nu\alpha\beta} = R_{\mu\nu\alpha\beta} - \frac{1}{2}\left(g_{\mu\alpha}R_{\nu\beta} - g_{\mu\beta}R_{\nu\alpha} + g_{\nu\alpha}R_{\mu\beta} - g_{\nu\beta}R_{\mu\alpha}\right) + \frac{1}{6}R\left(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}\right)$$

Last term has no scale dependence
Second term (Weyl tensor) vanishes in FLRW
First term vanishes for conformal fields

Renormalize R² parameters and generate non-local terms:

Barvinsky, Vilkovisky, Avrimidi

Perturbative running is contained in the R^2 terms

$$S_4 = \int d^4x \sqrt{g}\,\left[c_1(\mu)R^2 + c_2(\mu)R_{\mu\nu}R^{\mu\nu}\right]$$
$$+\left[\bar{\alpha}R\log(\nabla^2/\mu^2)R + \bar{\beta}C_{\mu\nu\alpha\beta}\log(\nabla^2/\mu^2)C^{\mu\nu\alpha\beta}\right.$$
$$\left. + \bar{\gamma}\big(R_{\mu\nu\alpha\beta}\log(\nabla^2)R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}\log(\nabla^2)R^{\mu\nu} + R\log(\nabla^2)R\big)\right] \quad +\mathcal{O}(R^3)$$

Again running can all be packaged in non-local terms:

$$S_{tot} = \int d^4x \sqrt{g}\,\frac{2}{\kappa^2}R$$
$$+\left[\bar{\alpha}R\log(\nabla^2/\Lambda_1^2)R + \bar{\beta}C_{\mu\nu\alpha\beta}\log(\nabla^2/\Lambda_2^2)C^{\mu\nu\alpha\beta}\right.$$
$$\left. + \bar{\gamma}\big(R_{\mu\nu\alpha\beta}\log(\nabla^2)R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}\log(\nabla^2)R^{\mu\nu} + R\log(\nabla^2)R\big)\right] + \ldots.$$

	α	β	γ	$\bar{\alpha}$	β	$\bar{\gamma}$
Scalar	$5(6\xi-1)^2$	-2	2	$5(6\xi-1)^2$	3	-1
Fermion	-5	8	7	0	18	-11
Vector	-50	176	-26	0	36	-62
Graviton	430	-1444	424	90	126	298

Coefficients of different fields. All numbers should be divided by $11520\pi^2$

Non-local FLRW equations:

Quantum memory

$$\frac{3a\dot{a}^2}{8\pi} + N_s\left[6(\sqrt{a}\,\ddot{a})_t \int dt'\,L(t-t')\mathcal{R}_1 + 6\left(\frac{\dot{a}^2}{\sqrt{a}}\right)\int dt'\,L(t-t')\mathcal{R}_2 + 12(\sqrt{a}\dot{a})_t \int dt'\,L(t-t')\frac{d\mathcal{R}_3}{dt'}\right] = a^3\rho$$

with

$$\mathcal{R}_1 = -\sqrt{a}\ddot{a}(6\alpha+2\beta+2\gamma) - \frac{\dot{a}^2}{\sqrt{a}}(6\alpha+\beta)$$

$$\mathcal{R}_2 = -\sqrt{a}\ddot{a}(12\alpha+\beta-2\gamma) - \frac{\dot{a}^2}{\sqrt{a}}(12\alpha+5\beta-6\gamma)$$

$$\mathcal{R}_3 = \sqrt{a}\ddot{a}(6\alpha+2\beta+2\gamma) + \frac{\dot{a}^2}{\sqrt{a}}(6\alpha+\beta)$$

and the time-dependent weight:

$$L(t-t') = \lim_{\epsilon\to 0}\left[\frac{\theta(t-t'-\epsilon)}{t-t'} + \delta(t-t')\log(\mu_R\,\epsilon)\right]$$

For scalars: $\alpha = \frac{1}{2304\pi^2} \quad \beta = \frac{-1}{5760\pi^2}, \quad \gamma = \frac{1}{5760\pi^2}$

Collapsing universe – singularity avoidance

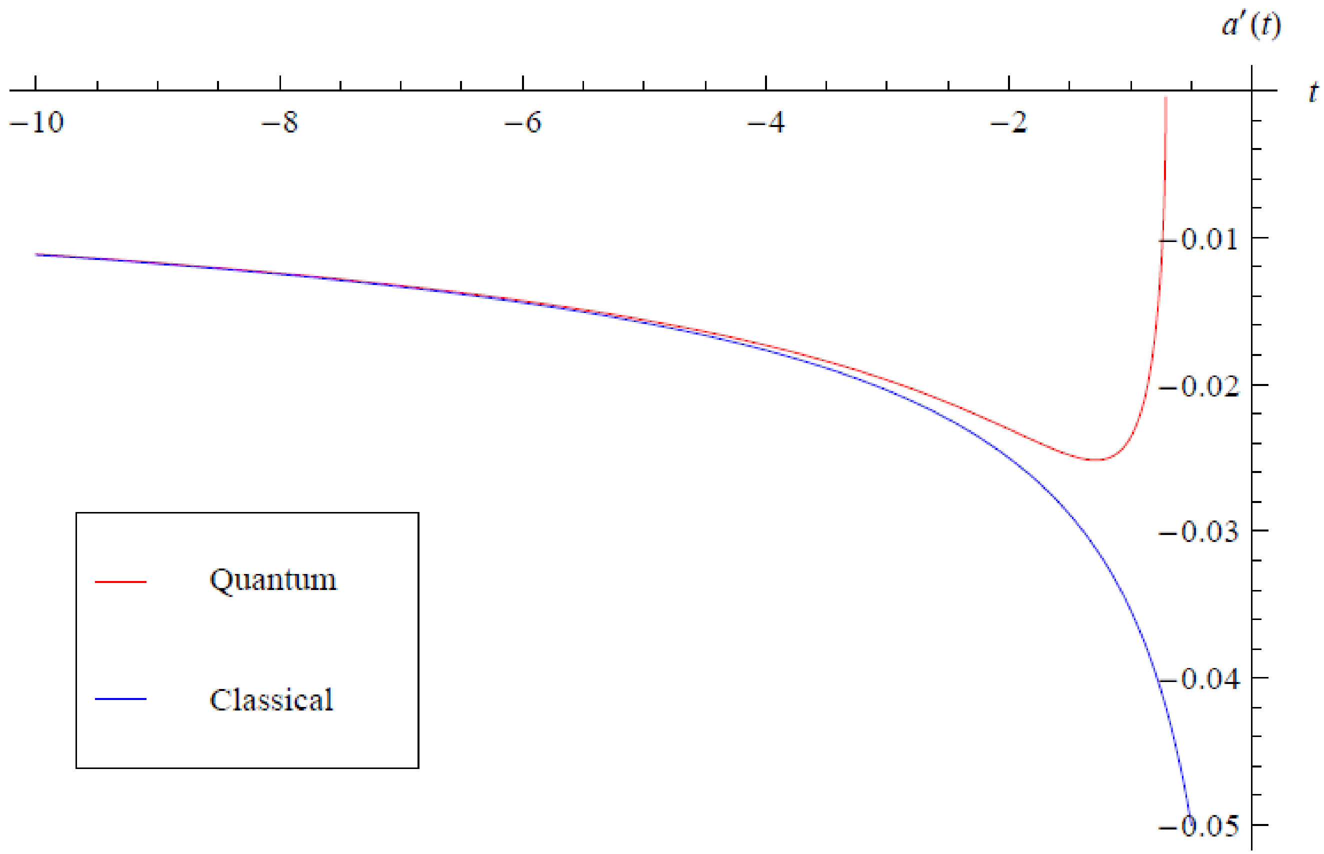


FIG. 12: Collapsing radiation-filled universe with gravitons only considered.

No free parameters in this result

With all the standard model fields:

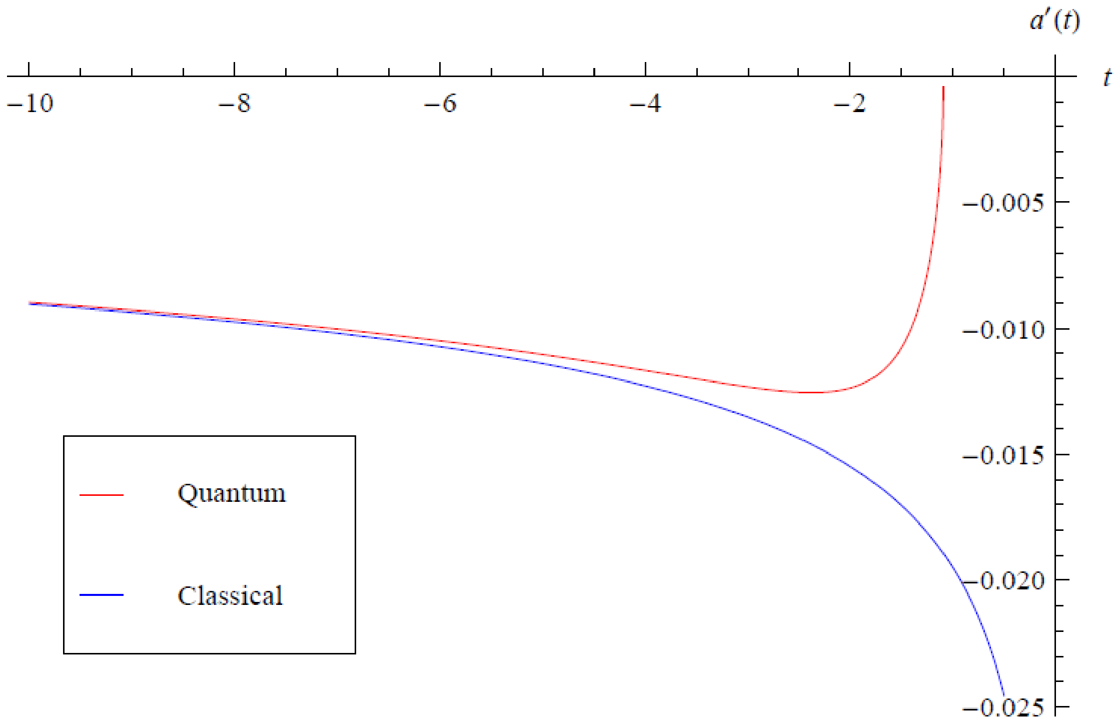
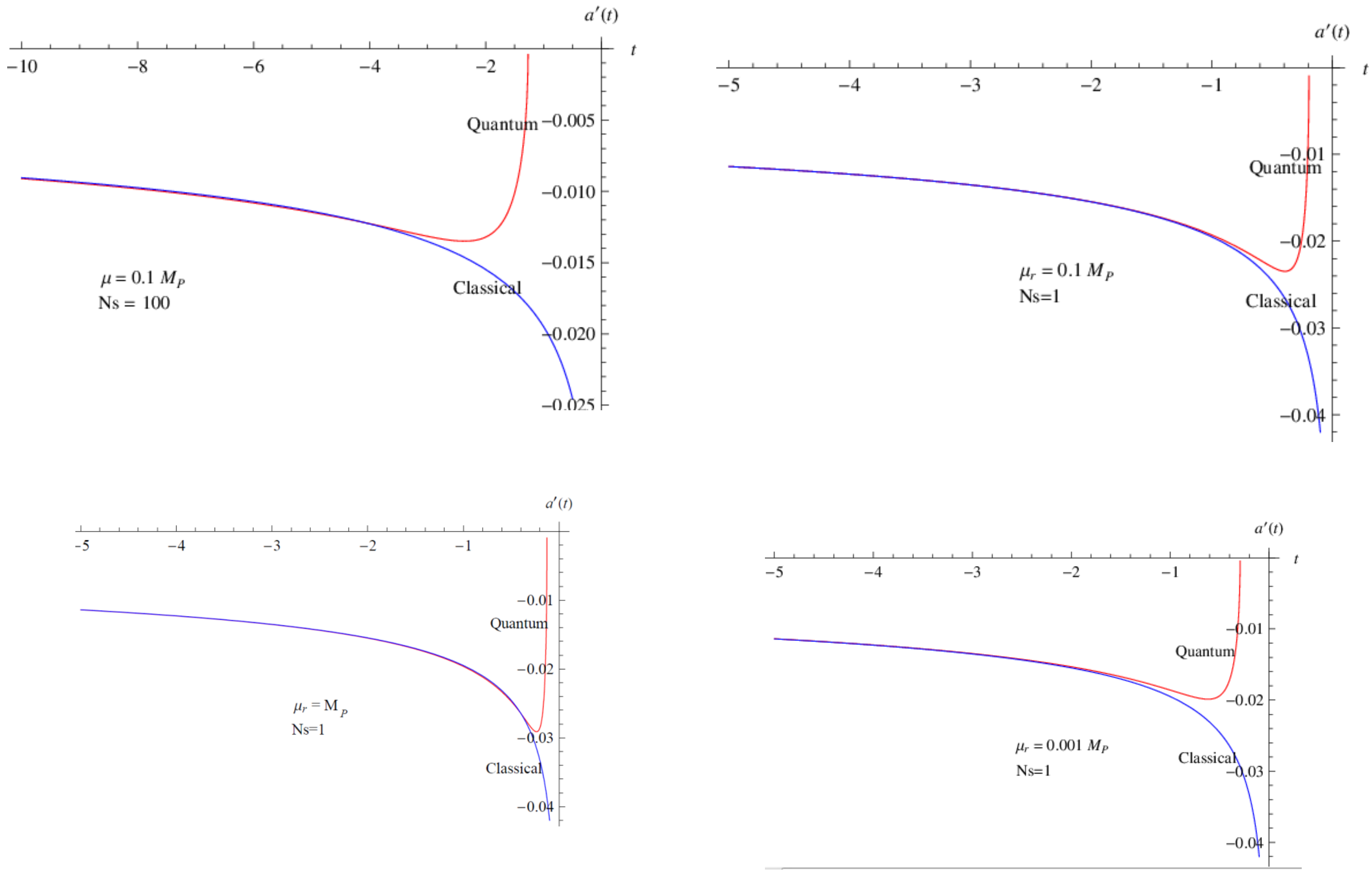
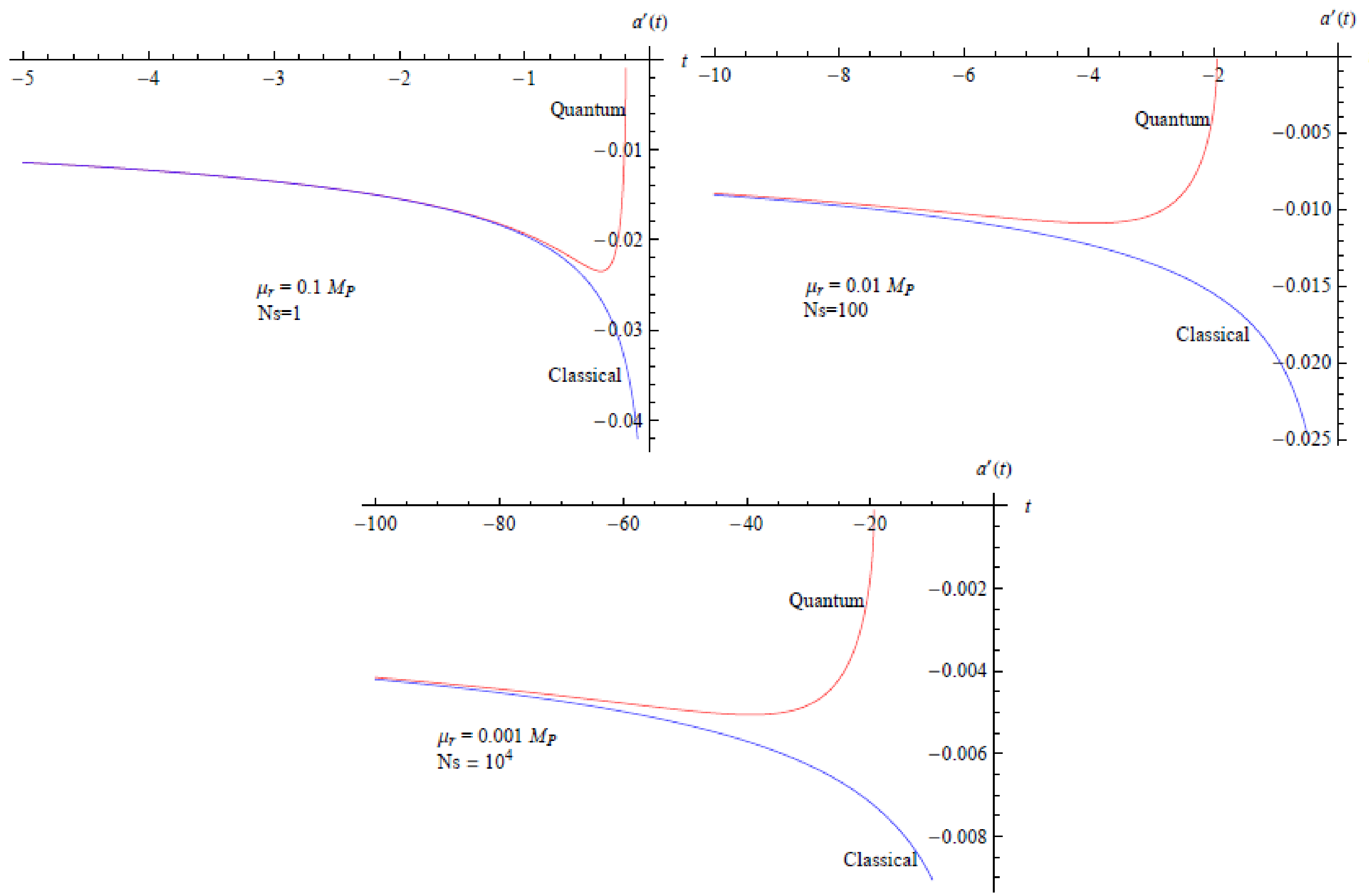


FIG. 11: Collapsing dust-filled universe with the Standard Model particles and a conformally coupled Higgs. The result is purely non-local and hence independent of any scale μ_R.

Collapsing phase – singularity avoidance



Physics does scale like $M_P/\sqrt{N}$



Some future issues

1) **Resolution of IR divergences**
   - Passarino-Veltman - bubbles, triangles and boxes
   - how are these resolved in gravitational settings?

2) **Gravity effects build up**
   - small effects can become large
   - any summation of quantum effects?

3) **Extreme infrared**
   - differences from other EFTs
   - patching together EFTs?

IR divergences in flat space:

Passarino-Veltman decomposition (boxes, triangles, bubbles)

$$I_4(s,t) = r_\Gamma \frac{1}{st}\left\{ \frac{2}{\epsilon^2}\left[(-s)^{-\epsilon} + (-t)^{-\epsilon}\right] - \ln^2\left(\frac{-s}{-t}\right) - \pi^2 \right\}$$

$$= r_\Gamma \frac{1}{st}\left\{ \frac{4}{\epsilon^2} - \frac{2\ln(-s) + 2\ln(-t)}{\epsilon} + 2\ln(-s)\ln(-t) - \pi^2 \right\}$$

IR

$$I_3(s) = \frac{r_\Gamma}{\epsilon^2}(-s)^{-1-\epsilon} = -\frac{r_\Gamma}{s}\left(\frac{1}{\epsilon^2} - \frac{\ln(-s)}{\epsilon} + \frac{\ln^2(-s)}{2}\right)$$

$$I_2(s) = \frac{r_\Gamma}{\epsilon(1-2\epsilon)}(-s)^{-\epsilon} = r_\Gamma\left(\frac{1}{\epsilon} - \ln(-s) + 2\right)$$

UV

We know how to resolve these in scattering

<u>But in curved space??</u>

- is there a PV reduction?
- what resolves IR?

Extreme Infrared:

Consider horizons:

- locally safe – we could be passing a BH horizon right now
  - local neighborhood should make a fine EFT
  - can be small curvature - no curvature singularity
  - locally flat coordinates in free fall through horizon

Firewalls?

- but cannot pass information to spatial infinity
  - EFT cannot be continued to very long distances (!)

- also, when far away, horizon appears source of thermal radiation
  - incoherent, non-unitary

simple EFT has some failure at long distance

- but long distance is where the EFT is supposed to work

- what is the parameter governing the problem?

Singularities could be even more problematic

- can you consider wavelengths past the nearest singularity?

Diagnosis: Consider Riemann normal coordinates

Taylor expansion in a local neighborhood:

$$g_{\mu\nu}(y) = \eta_{\mu\nu} + \frac{1}{3}R_{\mu\alpha\nu\beta}(y_0)y^\alpha y^\beta - \frac{1}{6}R_{\mu\alpha\nu\beta;\gamma}(y_0)y^\alpha y^\beta y^\gamma$$
$$+ \left[\frac{1}{20}R_{\mu\alpha\nu\beta;\gamma\delta}(y_0) + \frac{2}{45}R_{\alpha\mu\beta\lambda}(y_0)R^\lambda{}_{\gamma\nu\delta}(y_0)\right] y^\alpha y^\beta y^\gamma y^\delta + \mathcal{O}(\partial^5)$$

Even for small curvature,
there is a limit to a perturbative treatment of long distance:

$$R_{\mu\alpha\nu\beta}(y_0)y^\alpha y^\beta \ll 1$$

Hawking-Penrose tell us that this is not just a bad choice of coordinates

But this is not the usual EFT expansion:

$$R\, y^2 \gg R / q^2$$

- gets worse at long distance

Integrated curvature qualitatively explains extreme IR issues:

- curvature builds up between horizon and spatial infinity
- singularities due to evolution of any curvature to long enough distance

But how do we treat this in EFT?

Maybe singularities can be treated as gravitational sources

- excise a region around the singularity
- include a coupling to the boundary
- analogy Skyrmions in ChPTh

But distant horizons?

- perhaps non-perturbatively small??

Summary:

We understand some aspects of quantum general relativity
- focus on the IR
- modifications to classical GR

Challenge is moving on beyond scattering processes

Non-local effective action
- how useful is the curvature expansion?
- alternate covariant representations?

Need better understanding of IR limit of the theory