Bootstrapping Time Dilation Decoherence

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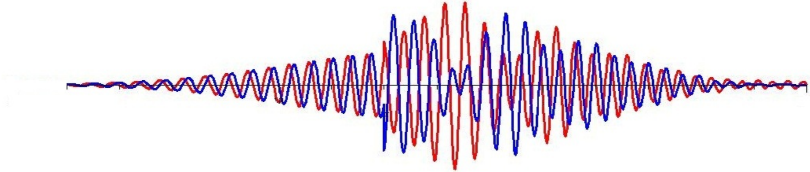
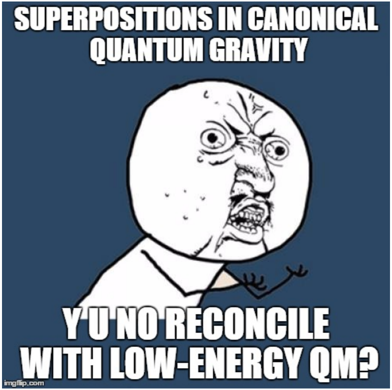


Preliminary Considerations

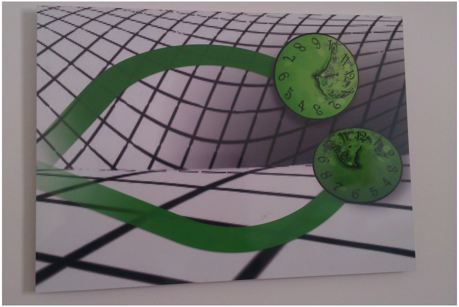


- Though we don't have a complete theory of QG, systems that involve both GR and QT are becoming more and more prominent
- Experimentalists are making progress towards detecting GR signatures (such as time dilation) in QM systems
- Conceptual hurdles to leap: GR tells us that matter curves space and time, while standard QM superposes matter states on a fixed spacetime

Outline

- Observation: Composite systems with dynamics that can be effectively decomposed into internal and external evolution are well-suited for probing interplay between GR and QT
- Example: Time dilation decoherence, as described by Pikovski et al. (arXiv:quant-ph/1311.1095), in the framework of QM in the fixed spacetime associated with the Earth
- Our question: how does general relativity affect the quantum properties of *self-gravitating* composite systems?
- Model system: Spherical self-gravitating shell with a "clock"
- Bootstrapping time dilation decoherence (arXiv:gr-qc/1503.05488v2)
- Denouement

Time Dilation Decoherence due to Earthly Gravitation



- Superpose single-particle states corresponding to paths with different heights above the Earth (and so different time dilations)
- Include internal degrees of freedom (d.o.f.) that evolve with respect to the proper time along each path
- Time dilation induces a coupling between the position and the internal d.o.f., and tracing out the internal d.o.f. effectively leads to decoherence (though there are revivals, eventually)

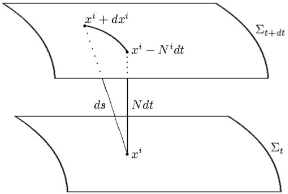
What about Self-gravitation?

- There once was a classical theory..

$$I = \frac{1}{16\pi}\int d^4x\sqrt{-g}\,\mathcal{R} + I_x$$

- Spacetime Metric in ADM (3+1) Form

$$g_{\mu\nu}dx^\mu dx^\nu = -N^2dt^2 + L^2\left(dr + N^r dt\right)^2 + R^2 d\Omega^2$$



- The shell action resembles that of a relativistic particle, but with a nonconstant mass:

$$I_x = -\int d\lambda\sqrt{\left|g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}\right|}M(R)$$

- The density and pressure are parametrized by $M(R)$:

$$\sigma = M(R)/4\pi R^2, \quad P_\sigma = -M'(R)/8\pi R$$

Internal Oscillation as a Local Clock

- We want our shell to have an internal degree of freedom (say, q) that evolves according to the proper time τ along the (external) shell trajectory:

$$I_q = \frac{1}{2}\int d\tau \left[m\left(\frac{dq}{d\tau}\right)^2 - kq^2\right]$$

$$\dot{\tau} = \int dr \sqrt{N^2 - L^2(N^r + \dot{x})^2}\delta(r - x) = \sqrt{\hat{N}^2 - \hat{L}^2(\hat{N}^r + \dot{x})^2}$$

- Then express everything in terms of the coordinate time, t:

$$\mathcal{L}_x = -\int dr \sqrt{N^2 - L^2(N^r + \dot{X})^2} M(R)\delta(r - X) = -\dot{\tau}\hat{M},$$

$$\mathcal{L}_q = \frac{1}{2}\left(m\frac{\dot{q}^2}{\dot{\tau}} - k\dot{\tau}q^2\right)$$

$$\implies I_s = \int dt\,\mathcal{L} = \int dt\,(\mathcal{L}_x + \mathcal{L}_q)$$

Hamiltonianization

- Legendre transformation of the internal and external shell variables:

$$\mathcal{H} = P\dot{X} + p\dot{q} - \mathcal{L} = \int dr\ (NH_t^s + N^r H_r^s),$$

$$H_t^s = \sqrt{L^{-2}P^2 + \tilde{M}^2}\delta(r - x),\ \ H_r^s = -P\delta(r - x),$$

with the definitions

$$\tilde{M} = \hat{M} + H_q,\ \ H_q = \frac{p^2}{2m} + \frac{1}{2}kq^2$$

- Transforming the gravitational variables as well:

$$I = \int dt\ \left(P\dot{X} + p\dot{q}\right) + \int dt\, dr\ \left(\pi_R\dot{R} + \pi_L\dot{L} - NH_t - N^r H_r\right),$$

for $H_t = H_t^s + H_t^G$ and $H_r = H_r^s + H_r^G$, such that

$$H_t^G = \frac{L\pi_L^2}{2R^2} - \frac{\pi_L\pi_R}{R} + \left(\frac{RR'}{L}\right)' - \frac{(R')^2}{2L} - \frac{L}{2},\ \ H_r^G = R'\pi_R - L\pi_L'$$

Phase Space Reduction

- Since there are 2 GR constraints ($H_t = 0$, $H_r = 0$) and 2 gravitational variables (L and R), we can solve the constraints for the gravitational momenta, in terms of the remaining variables
- Rather than working with equivalence classes of coordinate systems, it is sufficient to choose a representative; specifically, we use Painlevé-Gullstrand coordinates:

$$L = 1, \quad R = r \quad \text{(plus a deformation region near the shell)}$$

- We then insert the associated gravitational momenta solutions into the Liouville form $\mathcal{F}$ on the full phase space. This amounts to a pullback of $\mathcal{F}$ to the representative hypersurface defined by the coordinate choice:

$$\mathcal{F} = P\boldsymbol{\delta} X + p\boldsymbol{\delta} q + \int dr \left(\pi_L \boldsymbol{\delta} L + \pi_R \boldsymbol{\delta} R\right)$$

$$\rightarrow \tilde{\mathcal{F}} = P_c \boldsymbol{\delta} X + p\boldsymbol{\delta} q$$

Reduced Action in terms of an implicit Hamiltonian

- The gravitational contributions to the Liouville form pullback give rise to a new canonical momentum for the reduced system:

$$P_c = -\sqrt{2Hx} - x\ln\left(\frac{x+\beta-\sqrt{2Hx}}{x}\right)$$

$$\beta = \frac{H - \frac{\tilde{M}^2}{2x} \mp \sqrt{\left(H - \frac{\tilde{M}^2}{2x}\right)^2 - \tilde{M}^2\left(1-\frac{2H}{x}\right)}}{1+\sqrt{\frac{2H}{x}}}.$$

- We have thus determined our reduced action, and the Hamiltonian H is defined implicitly by the equation $P_c = P_c(H, x, q, p)$

$$I_{reduced} = \int dt\,(P_c\dot{x} + p\dot{q} - H)$$

Asymptotic Structure

- Flat spacetime limit (i.e. no external gravitational field, no self-gravitation):

$$P_c \to \pm\sqrt{H^2 - \tilde{M}^2} \implies H \to \sqrt{P_c^2 + \tilde{M}^2} = \sqrt{P_c^2 + \left(\hat{M} + H_q\right)^2}$$

- Nonrelativistic limit (including self-gravitation), for small clock energies ($H_q = \frac{p^2}{2m} + \frac{1}{2}kq^2 << \hat{M}$)

$$H = H_0 + H_{xq} = H_x + H_q + H_{xq},$$

$$H_x = \hat{M} + \frac{P_c^2}{2\hat{M}} + \left[\frac{P_c^2}{3x} - \frac{2}{3}\sqrt{\frac{2\hat{M}}{x}}P_c - \frac{\hat{M}^2}{18x}\right]$$

$$H_{xq} = -\frac{P_c^2}{2\hat{M}^2}H_q + \left[-\frac{\hat{M}}{9x}H_q - \frac{1}{3\hat{M}}\sqrt{\frac{2\hat{M}}{x}}H_q\right]$$

Bootstrapping Time Dilation Decoherence

- Unitary evolution of the full density matrix: $\dot{\rho} = -i\left[H, \rho\right]$
- Initially uncorrelated: $\rho(0) = \rho_x(0)\rho_q(0)$
- Change frame to primed variables: $\rho'(t) = e^{it(H_0+h)}\rho(t)e^{-it(H_0+h)}$

$$h(x, P_c) = Tr_q\left[H_{xq}\rho_q(0)\right] = \Gamma(x, P_c)\bar{E}_q$$

$$\Gamma\left(x, P_c\right) = -\frac{P_c^2}{2\hat{M}^2} + \left[-\frac{\hat{M}}{9x} - \frac{1}{3\hat{M}}\sqrt{\frac{2\hat{M}}{x}}P_c\right]$$

- One can then iterate and integrate the transformed evolution equation, and trace out the (internal) "clock" d.o.f.
- Next, use the Born part of Born-Markov, keeping up to 2nd order in the interaction H_{xq}, and replacing $\rho'(s)$ in the integral by $\rho_x'(s)\rho_q'(0)$. Also, define $\left(\Delta E_q\right)^2 = Tr_q\left[\left(H_q - \bar{E}_q\right)^2\rho_q\left(0\right)\right]$.

Nonunitary Evolution of the Reduced Density Matrix

- Transforming back to the unprimed frame leads to the reduced master equation for the (external) shell position:

$$\begin{aligned}\dot{\rho}_x(t) &= -i\left[H_x + \Gamma \bar{E}_q, \rho_x(t)\right] \\ &\quad - (\Delta E_q)^2 \int_0^t ds \left[\Gamma, e^{-isH_x}\left[\Gamma, \rho_x(t-s)\right] e^{isH_x}\right]\end{aligned}$$

- This parallels the reduced master equation given by Pikovski et al, though in this case the interpretation of the system variables is different: both the external shell motion and internal clock influence the state of their own geometry (the very ticking of our clock affects the manner in which it ticks)

Denouement

- The reduced dynamics of the external degree of freedom inherits an averaged geometry
- The decohering term is proportional to the energy variance of the clock state. Since the energy of the clock contributes to the ADM energy of the spacetime, the (effective) decoherence rate increases with increasing uncertainty in the geometry. Reminiscent of Penrose's proposal?

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