Gravitational Cat State:

Quantum Information in the face of Gravity

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ongoing work with

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-- *PITP UBC - 2nd Galiano Island Meeting, Aug. 2015*

Based on C. Anastopoulos and B. L. Hu, *"Probing a Gravitational Cat State"* *Class. Quant. Grav. 32, 165022 (2015).* [*arXiv:1504.03103*]

DICE2014 *Castiglioncello, Italy Sept, 2014;* ***Peyresq 20****, France June 2015 (last slides courtesy CA)*

- *RQI-N (Relativistic Quantum Information) 2014* *Seoul, Korea. June 30, 2014*

COST *meeting on Fundamental Issues, Weizmann Institute, Israel Mar 24-27, 2014*

Five Parts: Confluence of Theories in the 80s-90s in Gravity and Quantum with new issues

Part I: Alternative Quantum Theories. **Diosi-Penrose Schemes**: (nonrelativistic) Schroedinger Newton/von-Neumann-Newton Eqs: Noise term put in by hand. 80s Nice review by Bassi et al Rev Mod Phys

Part II: Gravitational Decoherence: A Master Equation derived from General Relativity (GR) + Quantum Field theory (QFT) via quantum open systems (QOS) methods Recent Rise in attention

-- Peyresq Physics Meeting 16, Provence, France, June 23, 2011
-- Intrinsic Decoherence in Nature Galiano Island, Canada, May 22-25, 2013

- **[AH1]** ***"A Master Equation for Gravitational Decoherence:Probing the Textures of Spacetime"*** CQG ***30,*** *165007 (2013)* || M. Blencowe, PRL 111, 021302 (2013)

Part III. Problems with the Newton-Schrodinger Equations **[AH2]** *arXiv:1403.4921* ***New J. Physics 16 (2014) 085007 [Focus Issue on Gravitational Quantum Physics]; | "Newton-Schrodinger Equations are not derivable from General Relatvity + Quantum Field Theory"*** *arXiv:1402.3813*

Part IV. Four Levels of Semiclassical Gravity. **Mean-field** **80s**
Stochastic Gravity: Including Fluctuations **90s**

- **Review:** **B. L. Hu,** ***"Gravitational Decoherence, Alternative Quantum Theories and Semiclassical Gravity"*** invited talk at the Second International Conference on Emergent Quantum Mechanics, Austrian Academy of Science, Vienna, October 3-6, 2013 J. *Phys. Conf. Ser*. *arXiv:1402.6584*

Part V. Quantum information in Q systems interacting with gravity ← **This talk**

Three elements: Q I G
Quantum, Information and Gravity

- Quantum ← Quantum Mechanics ← **Quantum Field Theory**
  Schroedinger Equation |
- Gravity ← Newton Mechanics ← **General Relativity**
  GR+QFT= **Semiclassical Gravity** (SCG)
- **Laboratory** conditions: | **Strong Field** Conditions:
  Weak field, nonrelativistic limit: | Early Universe, Black Holes

Newton Schrodinger Eq ` | **Semiclassical Einstein Eq**
(NSE) | (SCE)

Two layers of theoretical construct:

(1 small surprise, 1 observation)

1) *Small Surprise?*:

NSE for single or multiple particles is not derivable from known physics

Newton-Schrodinger Eq <=/= Semiclassical Einstein Eq of Semiclassical Gravity

(this nomenclature is preferred over **Moller-Rosenfeld Eqn**)

Now bring in the most basic element in quantum information

Take the issue of **Quantum Entanglement**

Examine the **expectation value** not wrt a vacuum state (vev), but say a **cat state:**

| +- > = 1/*V*2 (|left> +- |right>) |…..0…..|

-x   +x

2) no Surprise:

One should know that **SCG is not sufficient for QI**, since it gives the **mean value** of the stress energy tensor Tmn, which predicts wrongly that the cat is at x=0.   No Superposition.

- Need to include contributions from the **fluctuations** in addition to the mean <Tmn> **(from SCG)**

- **Correlations of the stress energy tensor** <TmnTrs> is needed to address issues in
  quantum information with gravity (Relativistic RQI)

- There is such a theory, **Stochastic Semiclassical Gravity (SSG)**, **based solely on GR+QFT**.
  No new invention needed (or allowed).

- Just need to work things out carefully with experiments in mind. --- We are attempting this now:

Alternative Q Theories

- L **Diosi** (84, 87, 89) R. **Penrose**, *Phil. Trans. R. Soc. Lond.* A (1998) **356**, 1927-1939 / GRG (96)

-*Advocate gravity as the source of decoherence of quantum particles.*

-*Proposed different forms of* ***Newton-Schrodinger Equation*** *NSE*

- *But we find that* ***NSE cannot be derived from QFT + GR***

- **GRWP**: G.C. Ghirardi, R. Grassi, A. Rimini, Weber and Pearle Phys. Rev. A42, 1057 (1990).; Pearle. *Changing QM,* We view this class of theories as expressing a wish: That at a certain scale between the micro and macro, the wave function collapses: “ localization”. Less concerned with Why

Both classes of theories are ***Phenomenological, not Fundamental.***

- *Viewing QM as Emergent: Proposals of* ***sub-level theories***
  S. L. Adler’s book and recent papers, ‘t Hooft’s papers

Excellent Review by A. Bassi et al, Rev. Mod. Phys. 85, 471- 527 (2013)

Part I: Problems with the Newton-Schrodinger Equations

- "***Newton-Schrodinger Equations are not derivable from General Relativity + Quantum Field Theory***" *arXiv:1402.3813*

Newton-Schroedinger Equations

The NSE governing the wave function of a single particle $\psi(\mathbf{r}, t)$ is of the form

$$i\partial\psi/\partial t = -\frac{\hbar^2}{2m}\nabla^2\psi + m^2 V_N[\psi]\psi$$

Eq.(1), where $V_N(\mathbf{r})$ is the (normalized) gravitational (Newtonian) potential given by

$$V_N(\mathbf{r}, t) = -\int d\mathbf{r}' |\psi(\mathbf{r}', t)|^2 / |\mathbf{r} - \mathbf{r}'|.$$

It satisfies the Poisson equati

$$\nabla^2 V_N = 4\pi G\mu,$$

with the mass density $\mu = m|\psi(\mathbf{r}, t)|^2$
being the nonrelativistic limit of energy density $\varepsilon = T_{00}$.

Problems with NSE: (A) Nonlinearity

- In NSE the *gravitational self-energy introduces a* nonlinear term in the Schr¨odinger equation

  [In Diosi's theory, the gravitational self-energy introduces a stochastic term in the master equation.]

- With GR+QFT in the weak field (WF) limit gravitational self-energy only contributes to mass renormalization

- The Newtonian interaction term at the field level induces a divergent self-energy contribution to the single-particle Hamiltonian.

- It does not induce nonlinear terms to the Schr¨odinger equation for any number of particles.

Point (B). Wave Function in NSE not of one or many particles, but a Collective Variable for a system of N particles in the Hartree Approx.

B. The *one-particle NS equation* appears as the Hartree approximation for N particle states as $N \to \infty$. Consider the ansatz

$$|\Psi\rangle = |\chi\rangle \otimes |\chi\rangle \ldots \otimes |\chi\rangle$$

for a N-particle system. At the limit $N \to \infty$ the generation of particle correlations in time is suppressed and one gets an equation which reduces to the NS equation for χ [7, 8].
However, in the Hartree approximation, $\chi(\mathbf{r})$ is *not* the wave-function $\psi(\mathbf{r})$ of a single particle, but a *collective variable* that describes a system of N particles under a mean field approximation.

We have taken Three Routes to examine this issue

1) Weak Field (WF) nonrelativistic (NR) limit of Semiclassical Einstein Equation (SCE) from Relativistic Semiclassical Gravity

2) Work out a model from scratch. Perturbative gravity + matter field → quantize → NR limit [in AH1]
B. L Hu and Charis Anastopoulos, Class. Quant Grav. *30, 165007 (2013)*

3) Nonrelativistic limit of QED [details in AH2]
B. L Hu and Charis Anastopoulos "***Problems of the Newton-Schrodinger Equations***" arXiv:1403.4921 New J Phys. Focus Issue on grav q physics

2nd Route: Perturbative gravity + matter field → quantize → constraint → NR limit

B. L Hu and Charis Anastopoulos, Class. Quant Grav. *30, 165007 (2013)*

N quantum particles (described by a scalar field) **in a gravitational field**

1. Hamiltonian for a massive scalar field interacting with a gravitational field
2. 3+1 decomposition. Perturbation off a Minkowski space background.
3. Gauge choice, transverse-traceless components: physical degrees of freedom
   [The effect of self-gravity is fully taken into account.]
4. Hamiltonian -- Quantization → Hamiltonian operator
5. Tracing out the gravitational field. Technically possible for weak perturbations → Master eq for reduced density matrix of matter field [similar to QBM model]
6. Projecting to the one-particle subspace
7. Take the non-relativistic limit.

The Correct Schrodinger Equation obtained from GR+QFT for the quantum field matter state |Ψ> with gravitational interaction is [shown in Route 2 and 3]

Treating the matter degrees of freedom in terms of quantum fields $\hat{\psi}(\mathbf{r})$ and $\hat{\psi}^\dagger(\mathbf{r})$, we obtain the Schrödinger equation for the fields $i\partial|\Psi\rangle/\partial t = \hat{H}|\Psi\rangle$, with

$$\hat{H} = -\frac{\hbar^2}{2m}\int d\mathbf{r}\hat{\psi}^\dagger(\mathbf{r})\nabla^2\hat{\psi}(\mathbf{r})$$

$$-G\int\int d\mathbf{r}d\mathbf{r}'(\hat{\psi}^\dagger\hat{\psi})(\mathbf{r})(\hat{\psi}^\dagger\hat{\psi})(\mathbf{r}')/|\mathbf{r}-\mathbf{r}'|.$$

where ˆ ψ(**r**), ˆ ψ†(**r**) are respectively the non-relativistic field annihilation and creation operators

- This procedure is widely employed in condensed matter systems, with a Coulomb potential for electrostatic interaction replacing the Newtonian potential for gravitational interaction.

- Note that this equation obtained from GR+QFT is very different from the NSE when considering a single particle state.

- For single-particle states the gravitational interaction leads only to a mass renormalization term (similar to mass renormalization in QED). [This is point A made above.]

- Using the Hartree approximation to Eq. (4) leads to the same result as the NR WF limit of SCE, not NSE. [Point B above.]

Part II Semiclassical Gravity

Semiclassical Gravity: 4 levels

of theories describing quantum matter interacting with classical gravity

Level 0: nonrelativistic (NR) particle motion in weak gravitational field (WF): **Newton-Schrodinger Eqs.** belong to this level

- Note: **many versions of NSEq;** most are used as vehicles for the expressions of (not unreasonable) wishes. E.g., wave function collapse in coordinate basis for macroscopic objects.
- But grav decoherence according to GR is in the energy, not coordinate basis, as collapse models want it to be.
- Besides, NSEs are not the weak field (WF)-non relativistic (NR) limit of GR+QFT (as shown in e.g, C. Anastopoulos and B L Hu, *NJP 16 (2014) 085007*

Level 1: First quantized matter field in classical background geometry solved self-consistently: **Einstein-Klein-Gordon Eq.**

Level 2: Second quantized matter field: **particle creation** processes included.

A. Quantum field theory in curved spacetime
(test field in fixed background) 1970s

B. Relativistic Semiclassical Gravity
(backreaction of 2nd quantized matter field included) 1980s

Semiclassical Einstein Equation sourced by the expectation values of the stress energy tensor **=> Spacetime and quantum matter dynamically determined self-consistently.**

RSCG is a mean field theory [Hartle & Horowitz 80, large N; Hu, Peyresq 98. Roura and Verdaguer (unpublished)]

[Validity of SCG considered in Flanagan & Wald, Phys. Rev. D 54, 6233 (1996); Hu Roura and Verdaguer, Phys. Rev. D 70, 044002 (2004) Including the effects of quantum matter fluctuations and induced metric fluctuations]

Level 3: Fluctuations of quantum matter field included : Goes beyond the mean field theory of RSG. 1990s

Stochastic Semiclassical Gravity: Einstein-Langevin Equation.

Semiclassical Gravity

Semiclassical Einstein Equation (Moller-Rosenfeld):

$$\tilde{G}_{\mu\nu}(g_{\alpha\beta}) = \kappa\langle\hat{T}_{\mu\nu}\rangle_q + \kappa\,(T_{\mu\nu})_c$$

$\tilde{G}_{\mu\nu}$ is the Einstein tensor (plus covariant terms associated with the renormalization of the quantum field)

$\kappa = 8\pi G_N$ and G_N is Newton's constant

Free massive scalar field

$$(\Box - m^2 - \xi R)\hat{\phi} = 0.$$

$\hat{T}_{\mu\nu}$ is the stress-energy tensor operator

$\langle\,\rangle_q$ denotes the expectation value

Semiclassical Einstein Equation is the only known valid equation for quantum matter (QFT) interacting with classical gravity (GR)

A natural extension of well known and tested theories:

- Quantum field theory in curved spacetime (e.g., Hawking effect)
- Semiclassical gravity (e.g., inflationary cosmology)

Relativistic semiclassical gravity (RSCG) is a fully covariant theory based on GR+QFT with self-consistent backreaction of quantum matter on a classical spacetime dynamics.

- It has been applied to the backreaction of quantum matter field processes in strong gravitational fields such as in the early universe and black holes.
- Main advantage: Minimal speculative assumptions

The semiclassical Einstein equation is of the form

$$G_{\mu\nu} = 8\pi G \langle \Psi | \hat{T}_{\mu\nu} | \Psi \rangle$$

Eq.(2), where $\langle \hat{T}_{\mu\nu} \rangle$ is the expectation value of the stress energy density operator $\hat{T}_{\mu\nu}$ with respect to some quantum state $|\Psi\rangle$ of the field.
In the weak field limit the spacetime metric has the form $ds^2 = (1 - 2V)dt^2 - d\mathbf{r}^2$, and the non-relativistic limit of the SCE Eq becomes

$$\nabla^2 V = 4\pi G \langle \hat{\varepsilon} \rangle,$$

where $\hat{\varepsilon} = \hat{T}_{00}$ is the energy density operator.
This can be solved to yield

$$V(\mathbf{r}) = -G \int d\mathbf{r}' \langle \hat{\varepsilon}(\mathbf{r}') \rangle / |\mathbf{r} - \mathbf{r}'|$$

Eq.(3). Note $\langle \hat{\varepsilon}(\mathbf{r}') \rangle$ in general contains ultraviolet divergences and need be regularized via known procedures.

Differences between NSE and the NR limit of SCE:

Two key differences between the NR limit of SCE and NSE are:
i) the energy density $\hat{\varepsilon}(\mathbf{r})$ is an operator, not a c-number. The Newtonian potential is not a dynamical object in GR, but subject to constraint conditions.
ii) the state $|\Psi\rangle$ of a field is a N-particle wave function. Quantum matter is coupled to classical gravity as a mean-field theory, well defined only when N is sufficiently large.

The (misplaced) procedure leading one from SCE to a NS equation is the treatment of $m|\psi(\mathbf{r},t)|^2$ as a mass density for a single particle, while in fact the mass density is a quantum operator $\hat{\epsilon}(\mathbf{r}) = \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r})$ in the QFT Hilbert space. Not treating these quantities as operators bears the consequences A and B.

One particle. We first consider a single particle state

$$|\phi\rangle = \int d\mathbf{r}\hat{\psi}^\dagger(\mathbf{r})\phi(\mathbf{r})|0\rangle, \tag{9}$$

where $\phi(\mathbf{r})$ is the single-particle wave-function. e. The expectation value $\langle\phi|\hat{\mu}_{reg}(\mathbf{r})|\phi\rangle$ indeed coincides with $m|\phi(\mathbf{r})|^2$. However, the substitution of an operator with its mean value is a good approximation *only if the system is presupposed to behave classically*. In the context of the SCE equation, such an approximation is meaningful only at the mean field description of a many particle system. When considering a single particle, the mass-density ought to be treated as an operator in the evolution equations.

Two particles. Next, we consider a 2-particle state

$$|\phi_1, \phi_2\rangle = \frac{1}{\sqrt{2}}\int d\mathbf{r}_1 d\mathbf{r}_2 \phi_1(\mathbf{r}_1)\phi_2(\mathbf{r}_2)\hat{\psi}^\dagger(\mathbf{r}_1)\hat{\psi}^\dagger(\mathbf{r}_2)|0\rangle.$$

Stochastic Gravity

Einstein- Langevin Equation (schematically):

$$\tilde{G}_{\mu\nu}(g_{\alpha\beta}) = \kappa\left(T^{\mathrm{c}}_{\mu\nu} + T^{\mathrm{qs}}_{\mu\nu}\right)$$

$T^{c}_{\mu\nu}$ is due to classical matter or fields

$$T^{\mathrm{qs}}_{\mu\nu} \equiv \langle \hat{T}_{\mu\nu} \rangle_{\mathrm{q}} + T^{\mathrm{s}}_{\mu\nu}$$

$T^{\mathrm{qs}}_{\mu\nu}$ is a new stochastic term
related to the quantum fluctuations of $T_{\mu\nu}$

NOISE KERNEL

- Exp Value of 2-point correlations of stress tensor: bitensor

- Noise kernel measures quantum flucts of stress tensor

It can be represented by (shown via influence functional to be equivalent to) a classical stochastic tensor source $\xi_{ab}[g]$

$$\langle \xi_{ab} \rangle_s = 0 \qquad \langle \xi_{ab}(x) \xi_{cd}(y) \rangle_s = N_{abcd}(x, y)$$

- **Symmetric, traceless** (for conformal field), **divergenceless**

Einstein-Langevin Equation

- Consider a weak gravitational perturbation h off a background g $g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu},$ The ELE is given by (The ELE is Gauge invariant)

$$G_{ab}[g+h] + \Lambda(g_{ab}+h_{ab}) - 2(\alpha A_{ab} + \beta B_{ab})[g+h]$$
$$= 8\pi G\left(\left\langle \hat{T}^{R}_{ab}[g+h]\right\rangle + \xi_{ab}[g]\right).$$

- Nonlocal dissipation and colored noise

Nonlocality manifests with **stochasticity**

because the gravitational sector is an open system

Stochastic Semiclassical Gravity

Noise and fluctuations in quantum field *induced metric fluctuations* spacetime (foam) microstructure described by *Einstein-Langevin Eq*.

B. L. Hu and E. Verdaguer, [Review]: *Liv. Rev. Rel. 11 (2008) 3*
----- [Monograph]: ***Semiclassical and Stochastic Gravity***
(Cambridge University Press, in preparation)

Semiclassical Stochastic Gravity for early universe and black holes

- For problems in the early universe and black holes, one is interested in quantum processes related to the vacuum, e.g., particle creation, vacuum fluctuations, vacuum polarization (Hawking Effect).
- In analogous laboratory settings, with moving detectors mirrors,e.g. Unruh Effect, dynamical Casimir Effect,

- Vacuum Expectation Values of Tmn or Tmn Trs taken wrt a vacuum state.

Part III: *Now turn attention to* **Quantum Information Issues in gravitational quantum physics**

- New emphasis: not vacuum state, but one particle and n particle states: That's OK

  [Squeezed states can be handled. In fact, cosmological expansion is squeezing.

But for quantum superposition states

- Bell states, etc. SCG cannot handle

- Consider a wave function composed of 2 Gaussian packets located at +x and –x

Hu Paz Zhang PRD 92
Paz Habib Zurek PRD 93

$$\Psi(x,t=0)=\Psi_1(x)+\Psi_2(x)\ ,$$

where

$$\Psi_{1,2}(x)=N\exp\left[-\frac{(x\mp L_0)^2}{2\delta^2}\right]\exp(\pm iP_0x)\ ,$$

-

$$N^2\equiv\frac{\overline{N}^{\,2}}{\pi\delta^2}=\frac{1}{2\pi^2\delta^2}\left[1+\exp\left[-\frac{L_0^2}{\delta^2}-\delta^2P_0^2\right]\right]^{-1}$$

Decoherence in QBM models:

1 HO System– nHO bath

$$S[x,q_n]=\int_0^t ds\left[\frac{1}{2}M(\dot{x}^2-\Omega_0^2x^2)+\sum_n\frac{1}{2}m_n(\dot{q}_n^2-\omega_n^2q_n^2)\right.$$
$$\left.-\sum_n C_n x q_n\right], \tag{3}$$

$$\Psi(x,t=0)=\Psi_1(x)+\Psi_2(x),$$

where

$$\Psi_{1,2}(x)=N\exp\left[-\frac{(x\mp L_0)^2}{2\delta^2}\right]\exp(\pm iP_0x),$$

$$N^2\equiv\frac{\overline{N}^2}{\pi\delta^2}=\frac{1}{2\pi^2\delta^2}\left[1+\exp\left[-\frac{L_0^2}{\delta^2}-\delta^2P_0^2\right]\right]^{-1}$$

Pointer Basis: Interaction Hamiltonian

left: xq
right: pp

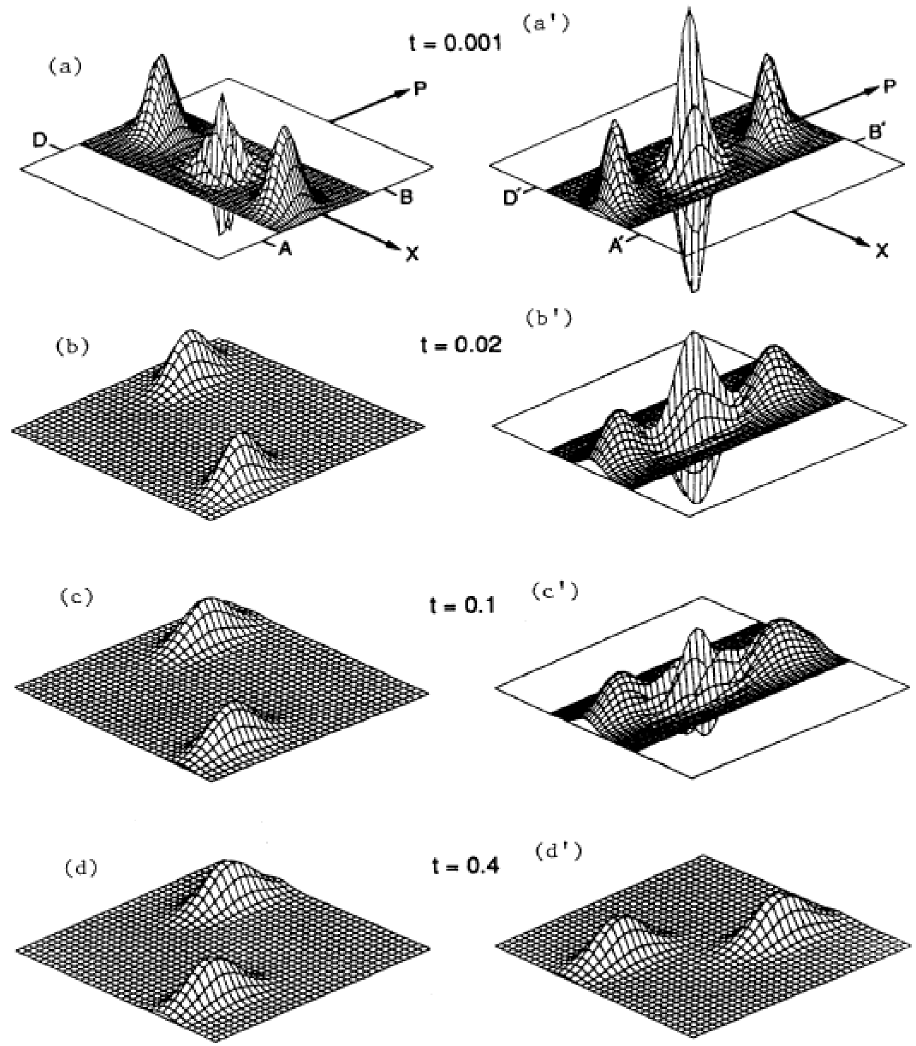


FIG. 2. The time evolution of initial conditions A and A'. The oscillations disappear faster in the first case since the environment can distinguish between the two peaks. In the second case, the interference is damped over a dynamical time scale.

Semiclassical theory can’t cope

- SCG being a ***mean field theory would wrongly*** *predict*

  peaking at the average value x=0 [Ford 82]

- Likewise ***it does not admit cat states*.**

  It gives only the mean value (between alive and dead – *the drunken tipsy turvy cat in the “twilight zone”),* ***Not*** *-- the clear minded cat in*

  ***a coherent quantum superposition*.**

Look for the **Gravitational Quantum Cat** from the **fluctuations of energy density**, or the **correlator of the stress energy tensor**: the **Noise Kernel**

(Not the full Schrodinger cat – some quantum tributes of the cat)

Consider a scalar quantum field $\hat{\phi}(\mathbf{r})$ and its conjugate momentum $\hat{\pi}(\mathbf{r})$ of the creation and annihilation operators $\hat{a}_{\mathbf{k}}$ and $\hat{a}^{\dagger}_{\mathbf{k}}$

$$\hat{\phi}(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3\sqrt{2\omega_{\mathbf{k}}}} \left[\hat{a}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} + \hat{a}^{\dagger}_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{r}}\right]$$

$$\hat{\pi}(\mathbf{r}) = i\int \frac{d^3k}{(2\pi)^3}\sqrt{\frac{\omega_{\mathbf{k}}}{2}} \left[-\hat{a}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} + \hat{a}^{\dagger}_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{r}}\right].$$

For a free field, the Hamiltonian operator is

$$\hat{\mathrm{H}} = \int \frac{d^3k}{(2\pi)^3} \omega_{\mathbf{k}} \hat{a}^{\dagger}_{\mathbf{k}} \hat{a}_{\mathbf{k}},$$

where $\omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + m^2}$.

In the non-relativistic regime we define the fields

$$\hat{\psi}(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3} \hat{a}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}, \qquad \hat{\psi}^{\dagger}(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3} \hat{a}^{\dagger}_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{r}},$$

and to leading order in $|\mathbf{k}|/m$,

$$\hat{\phi}(\mathbf{r}) = \frac{1}{\sqrt{2m}} \left[\hat{\psi}(\mathbf{r}) + \hat{\psi}^{\dagger}(\mathbf{r})\right], \qquad \hat{\pi}(\mathbf{r}) = -i\sqrt{\frac{m}{2}} \left[\hat{\psi}(\mathbf{r}) - \hat{\psi}^{\dagger}(\mathbf{r})\right]$$

Mass density operator of a *non-relativistic N-Particle System*

The Hamiltonian then becomes

$$\hat{\mathrm{H}} = m \int d\mathbf{r} \hat{\psi}^{\dagger}(\mathbf{r}) \hat{\psi}(\mathbf{r}) - \frac{1}{2m} \int d\mathbf{r} \hat{\psi}^{\dagger}(\mathbf{r}) \nabla^2 \hat{\psi}(\mathbf{r}). \tag{11}$$

We will denote the second term in Eq.(11) as $\hat{H}_0$ because it corresponds to the Hamiltonian for N non-relativistic particles. The first term in Eq.(11) corresponds to Nm, for an N-particle state. Hence, the number operator $\hat{N}$ is

$$\hat{N} = \int d\mathbf{r} \hat{\psi}^{\dagger}(\mathbf{r}) \hat{\psi}(\mathbf{r}) \tag{12}$$

This suggests that $m\hat{\psi}^{\dagger}(\mathbf{r})\hat{\psi}(\mathbf{r})$ can be identified as the mass-density operator $\hat{\mu}(\mathbf{r})$.

We include the effect of a confining potential $V(\mathbf{r})$, by modifying the field Hamiltonian

$$\hat{\mathrm{H}} = m \int d\mathbf{r} \hat{\psi}^{\dagger}(\mathbf{r}) \hat{\psi}(\mathbf{r}) + \int d\mathbf{r} \hat{\psi}^{\dagger}(\mathbf{r}) \left[-\frac{1}{2m} \nabla^2 + V(\mathbf{r}) \right] \hat{\psi}(\mathbf{r}). \tag{13}$$

Mass Density Correlations

2.2. Mass-density correlations, noise kernel

In the non relativistic limit, the dominant component of the stress tensor $T_{\mu\nu}$ is the energy density, which is dominated by the mass density, namely

$$T_{\mu\nu}(\mathbf{r},t) = \delta^0_\mu \delta^0_\nu \mu(\mathbf{r},t) \tag{14}$$

Thus, it suffices to calculate the correlation functions of the Heisenberg-picture operator

$$\hat{\mu}(\mathbf{r},t) = e^{i\hat{H}t}\hat{\mu}(\mathbf{r})e^{-i\hat{H}t}. \tag{15}$$

We assume an one-particle state

$$|\phi\rangle = \int d\mathbf{r}\phi(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r})|0\rangle, \tag{16}$$

where $\phi(\mathbf{r})$ is the one-particle wave-function.

We find

$$\langle\mu(\mathbf{r},t)\rangle = \langle\phi|\hat{\mu}(\mathbf{r},t)|\phi\rangle = m\phi^*(\mathbf{r},t)\phi(\mathbf{r},t) \tag{17}$$

$$\langle\mu(\mathbf{r},t)\mu(\mathbf{r}',t')\rangle = \phi^*(\mathbf{r},t)\phi(\mathbf{r}',t')G(\mathbf{r},t;\mathbf{r}',t') \tag{18}$$

where $\phi(\mathbf{r},t)$ is the time-evolved single particle wave function and $G(\mathbf{r},t;\mathbf{r}',t')$ is the one-particle propagator,

$$G(\mathbf{r},t;\mathbf{r}',t') = \langle\mathbf{r}'|e^{-i\hat{H}(t'-t)}|\mathbf{r}\rangle. \tag{19}$$

Noise Kernel

For a free particle,

$$G(\mathbf{r}, t; \mathbf{r}', t') = \left(\frac{m}{2\pi i t}\right)^{3/2} \exp\left[\frac{im(\mathbf{r}-\mathbf{r}')^2}{2(t'-t)}\right] \tag{20}$$

We note that the two-point correlation function is complex valued. In general, it does not define a stochastic process. However, the real part,

$$\xi(\mathbf{r}, t; \mathbf{r}', t') = Re\langle\mu(\mathbf{r}, t)\mu(\mathbf{r}', t')\rangle, \tag{21}$$

known as the noise kernel, corresponds in some cases to the two-point correlation function of a stochastic process.

Of importance is also the connected two-point correlation function for the mass densities

$$\eta(\mathbf{r}, t; \mathbf{r}', t') = \langle\mu(\mathbf{r}, t)\mu(\mathbf{r}', t')\rangle - \langle\mu(\mathbf{r}, t)\rangle\langle\mu(\mathbf{r}', t')\rangle. \tag{22}$$

Smeared Mass-Density Function

- In realistic systems the mass density is not defined at a sharp spacetime point but smeared over a finite spacetime region. In actual experiments, the particles under consideration (atoms) have a finite size d and it is meaningless to talk about mass densities at scales smaller than d, unless one has a detailed knowledge of the particle's internal state.

- For this reason, rather than the exact mass density function, we consider a smeared mass density function:

$$\hat{\mu}_s(\mathbf{r}, t) = \int d\mathbf{r}' f(\mathbf{r} - \mathbf{r}')\hat{\mu}(\mathbf{r}', t), \tag{23}$$

for some smearing function $f(\mathbf{r})$ of dimension $[\text{length}]^{-3}$, centered around $\mathbf{r} = 0$. The smearing scale ℓ is defined by the condition $\ell^3 = 1/f(0)$.

We define the positive operator

$$\hat{P}_{\mathbf{r}} = \int d\mathbf{r}' g(\mathbf{r} - \mathbf{r}')|\mathbf{r}'\rangle\langle\mathbf{r}'|, \tag{24}$$

where $g(\mathbf{r}) := f(\mathbf{r})/f(0)$.

If the sampling function g is a characteristic function of some set (i.e., if $g^2 = g$), then $\hat{P}_{\mathbf{r}}$ is a projection operator. Here, we will consider Gaussian functions of the form $g(\mathbf{r}) = e^{-\frac{\mathbf{r}^2}{2s_x^2}}$, where s_x is the width of the sampling. In this case, $\hat{P}_{\mathbf{r}}$ is an approximate projector. The corresponding smearing function is

$$f(\mathbf{r}) = \frac{1}{(2\pi)^{3/2} s_x^3} e^{-\frac{\mathbf{r}^2}{2s_x^2}} \tag{25}$$

and corresponds to $\ell = \sqrt{2\pi} s_x$.

The correlation functions of the mass density become

$$\langle \mu_s(\mathbf{r}, t) \rangle = \frac{m}{\ell^3} \langle \phi | \hat{P}_{\mathbf{r},t} | \phi \rangle \tag{26}$$

$$\langle \mu_s(\mathbf{r}, t) \mu_s(\mathbf{r}', t') \rangle = \frac{m^2}{\ell^6} \langle \phi | \hat{P}_{\mathbf{r}t} \hat{P}_{\mathbf{r}'t'} | \phi \rangle, \tag{27}$$

where $\hat{P}_{\mathbf{r}t} = e^{i\hat{H}t} \hat{P}_{\mathbf{r}} e^{-i\hat{H}t}$ is the Heisenberg-picture evolution of $\hat{P}_{\mathbf{r}}$.

The expectation value of the smeared mass density is proportional to the probability of a position measurement at time t. The two point correlation function is proportional to the decoherence functional

$$\mathcal{D}(\mathbf{r}, t; \mathbf{r}', t') := \langle \phi | \hat{P}_{\mathbf{r}t} \hat{P}_{\mathbf{r}'t'} | \phi \rangle \tag{28}$$

for a pair of histories one corresponding to a position record $\mathbf{r}$ at time t and the other to a position record $\mathbf{r}'$ at time t'. As explained by the decoherent histories approach to quantum

Wigner function representation

- For a free particle, we can express the correlation functions in terms of the Wigner function $W_0(\mathbf{r}, \mathbf{p})$ of the initial state.

- For scales of observation much larger than *l*, we have

$$
\begin{aligned}
\langle \mu_s(\mathbf{r}, t) \rangle \qquad\qquad &= \frac{m}{(2\pi)^3} \int d\mathbf{p} W_0(\mathbf{r} - \frac{\mathbf{p}t}{m}, \mathbf{p}) \\
\langle \mu_s(\mathbf{r}, t) \mu_s(\mathbf{r}', t') \rangle &= \frac{m^5}{(2\pi)^3 (t-t')^3} W_0(\frac{\mathbf{r}+\mathbf{r}'}{2} - \frac{(\mathbf{r}-\mathbf{r}')(t+t')}{2(t-t')}, m\frac{\mathbf{r}-\mathbf{r}'}{t-t'})
\end{aligned}
$$

- For an initial state with vanishing mean momentum, we obtain a stationary process.

$$
\begin{aligned}
&\langle \mu_s(\mathbf{r}, t) \rangle = m |\psi_0(\mathbf{r})|^2. \\
&\langle \mu_s(\mathbf{r}, t) \mu_s(\mathbf{r}', t') \rangle = m^2 |\psi_0(\mathbf{r})|^2 \delta^3(\mathbf{r} - \mathbf{r}').
\end{aligned}
$$

Quantum feature: A classical charge distribution would involve $|\psi_0(r)|^4$.

Key features of correlations of quantum systems

Mass density fluctuations are

- Of the same order of magnitude with the mean mass density

  This property seems to be generic in stress-energy fluctuations (Kuo+Ford 93, Phillips+Hu 97,00).
- Highly **non-Markovian.** They are unlike any classical stochastic process.

Fluctuations of the mass density generate fluctuations of the Newtonian force through Poisson's equation.

- Temporal correlation functions of quantum systems are **highly** c**ontextual** (Anastopoulos 04,05).

- A characteristic feature of quantum correlations exemplified by the Leggett-Gard inequality (or temporal Bell inequalities).

Measured values of correlations

• By **contextual**, we mean that the measured values depend strongly on the measurement context, i.e., on the specific set-up through which the correlations are measured.

• To compare, all samplings of position correspond to probabilities that closely approximate the **ideal distribution** $|\psi(\mathbf{r})|^2$.

• **There are no ideal distributions** for generic multi-time measurements. Probabilities are highly sensitive to the details of the sampling.

• Hence, there is no **intrinsic** stochastic process that describes the mass density fluctuations of a particle, but

• Any stochastic process that describes the experimental data depends on the specific procedure through which the measurement is carried out.

Gravitational Cat State:

a consequence of the intrinsic conflicts of Q + G

Penrose (1996) "**On gravity's role in quantum state reduction**". *Gen. Rel. Grav.* **28**, 581-600 [*just read the letters in red below*:]

Addresses the question of the *stationarity* of a quantum system which consists of a linear superposition $|\psi\rangle = |\alpha\rangle + |\beta\rangle$ of two well-defined states $|\alpha\rangle$ and $|\beta\rangle$, each of which would be stationary on its own, and where we assume that each of the two individual states has the same energy E

$$\mathrm{i}\frac{\partial|\alpha\rangle}{\partial t} = E|\alpha\rangle, \quad \mathrm{i}\frac{\partial|\beta\rangle}{\partial t} = E|\beta\rangle.$$

Just QM alone: If gravitation is ignored, then the quantum superposition $|\psi\rangle = a|\alpha\rangle + b|\beta\rangle$ would also be stationary, with the same energy *E* and this is the normal supposition.

$$\mathrm{i}\frac{\partial|\psi\rangle}{\partial t} = E|\psi\rangle,$$

With Gravity: However, when the gravitational fields of the mass distributions of the states are taken into account, we must ask what the Schroedinger operator $\partial/\partial t$ actually *means* in such a situation.

Let us consider that each of the stationary states $|\alpha>$ and $|\beta>$ takes into account whatever the correct quantum description of its gravitational field might be, in accordance with Einstein's theory.

Then, to a good degree of approximation, there will be a classical spacetime associated with each of $|\alpha>$ and $|\beta>$, and the operator $\partial/\partial t$ would correspond to the action of the Killing vector representing the time displacement of stationarity, in each case.

Stationary state makes demand of spacetime properties.

Clash between QM and GR

Now, the problem that arises here is that these two Killing vectors are *different* from each other. They could hardly be the same, as they refer to time symmetries of two different spacetimes.

It could only be appropriate to identify the two Killing vectors with one another if it were appropriate to identify the two different spacetimes with each other point-by-point.

But such an identification would be at variance with the principle of general covariance, a principle which is fundamental to Einstein's theory. According to standard quantum theory, unitary evolution requires that there be a Schr¨odinger operator that applies to the superposition just as it applies to each state individually; and its action on that superposition is precisely the superposition of its action on each state individually.

There is thus a certain tension between the fundamental principles of these two great theories, and one needs to take a position on how this tension is to be resolved.

Penrose's position is (provisionally) to take the view that an *approximate* pointwise identification may be made between the two spacetimes, and that this corresponds to a slight error in the identification of the Schr¨odinger operator for one spacetime with that for the other. This error corresponds, in effect, to a slight ***uncertainty in the energy*** of the superposition.

One can make a reasonable assessment as to what this energy uncertainty E_G might be, at least in the case when the amplitudes a and b are about equal in magnitude.

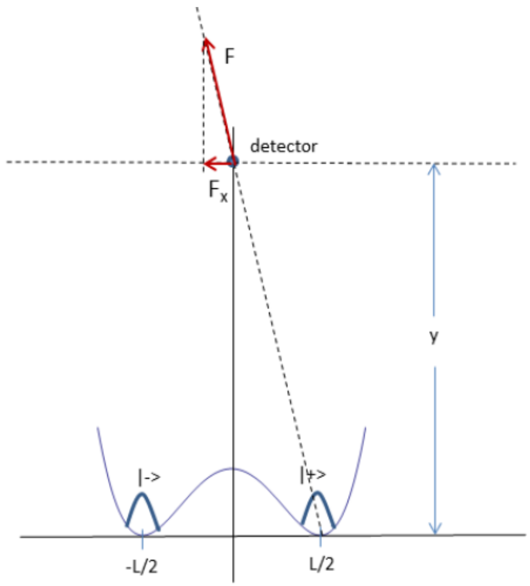
This estimate (in the Newtonian approximation) turns out to be the gravitational self-energy of the difference between the mass distributions of the two superposed states. This energy uncertainty E_G is taken to be a fundamental aspect of such a superposition and, in accordance with Heisenberg's uncertainty principle, the reciprocal hbar/E_G is taken to be a measure of the *lifetime* of the superposition (as with an unstable particle).

The two decay modes of the superposition $|\psi\rangle = a\,|\alpha\rangle + b\,|\beta\rangle$ would be the individual states $|\alpha\rangle$ and $|\beta\rangle$, with relative probabilities $|a|^2 : |b|^2$.

Gravitational Cat State:

Direct bearing of Quantum Optomechanics

(read Markus A's review, talk to Yanbei Chen and M Romero-Isart)



A particle of **mass m** in a confining potential with two minima. Assume that the **distance L** between the minima is much larger than the width of the localization region.

The system can be approximated by a qubit with defining states |+> and |->.

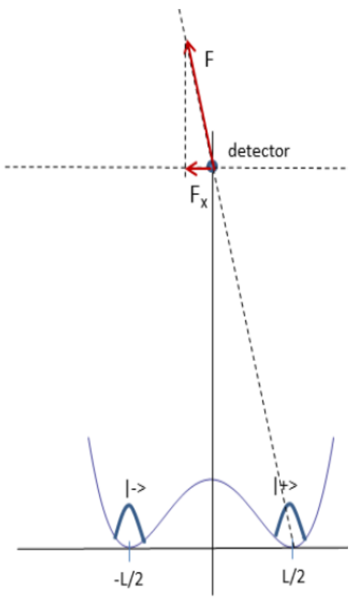
Hamiltonian $\hat{H} = \nu\, \hat{\sigma}_1$, where v is the tunneling rate between the two minima.

- The famous ***atomic*** cat of Wineland et al (1996) had L = 80nm and m = 8 amu. The cattiness record seems to come from the Ardnt 2012 diffraction experiment with L = 100nm and m =1300 amu. Bassi's review has more recent data Record for weakest force measured from CalTech ? (2014), ~ 4 x 10^{-23} N. Yanbei

- **Recent experiments** on **entanglement between massive objects**: *Ask Markus* A. Indirect (entanglement with third party measured); Direct (Calvendish expt)

Measurement by a classical probe

Consider a particle of mass m_0 near the particle of mass m that was prepared in a cat state.



We evaluate the x component of the Newtonian force exerted by the two-level system on the test particle. Again denote $a = \pm$. If the system lies on the minimum of the potential at $x = aL/2$, the force $F_x(a)$ exerted on the test particle in the x direction is

$$F_x(a) = -\frac{Gmm_0L}{2(y^2 + L^2/4)^{3/2}}a = -f_0 a \tag{62}$$

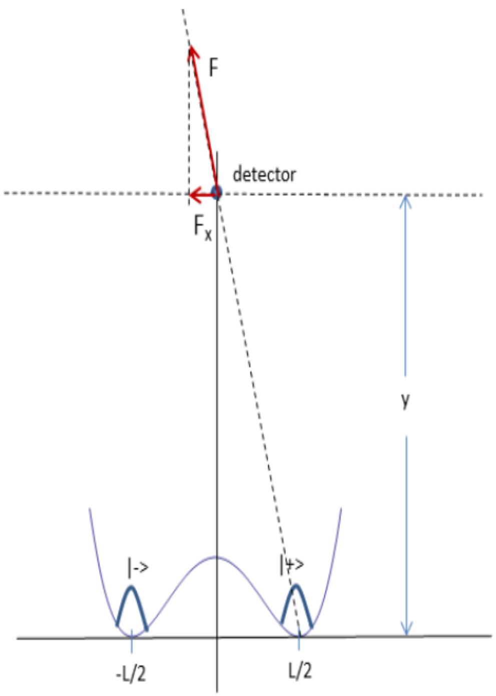
Assuming that the test mass is not allowed to move, the force F_x takes only two values f_0 and $-f_0$. These values are correlated with the projectors $\hat{P}_a$, Eq. (48). Thus F_x corresponds to a self-adjoint operator

$$\hat{P}_+ = \int_0^\infty dx \int_\infty^\infty dy \int_\infty^\infty dz |x,y,z\rangle\langle x,y,z|$$

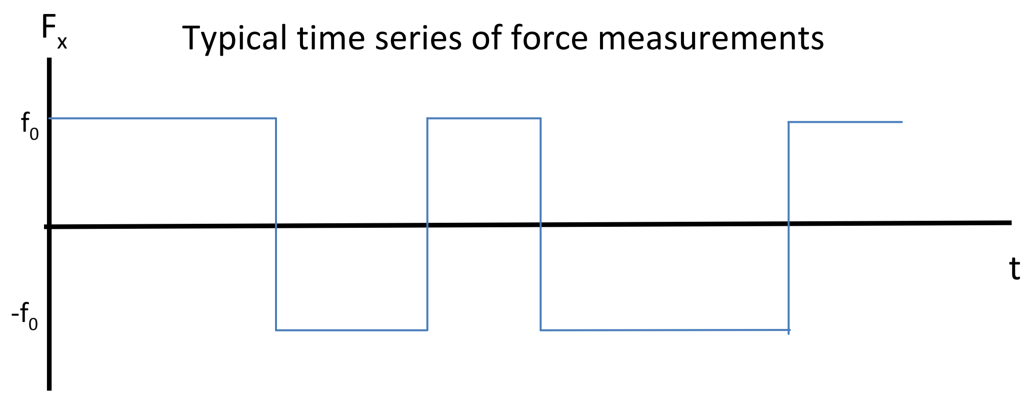
$$\hat{F} = -f_0\hat{P}_+ + f_0\hat{P}_- = -f_0\hat{\sigma}_3, \qquad \hat{P}_+ = \int_{-\infty}^0 dx \int_\infty^\infty dy \int_\infty^\infty dz |x,y,z\rangle\langle x,y,z|. \tag{63}$$

on the 2-state system's Hilbert space. Thus, the gravitational force behaves as a quantum variable, its probabilities and correlations determined by quantum mechanics.

Measurement by a classical probe



Since Newton's law is instantaneous, a force will be recorded by the macroscopic probe at all times. Thus we have a **continuous-time measurement** for a qubit.



Essentially similar to the **quantum jump** expts of Dehmelt et al (86).

Calculate the correlation functions of the force from the quantum probabilities for a continuous-time measurement

$$\langle F(t)\rangle = -f_0 e^{-\Gamma t}$$
$$\langle F(t')F(t)\rangle = f_0^2 e^{-\Gamma|t'-t|}.$$

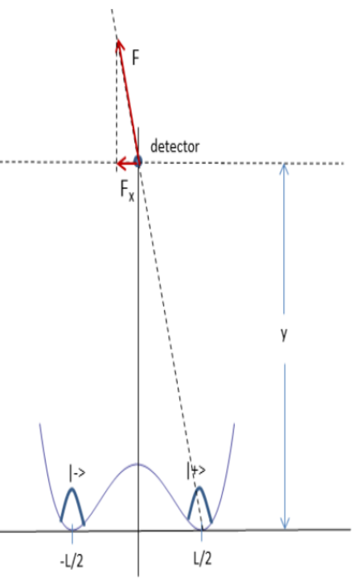
$$\Gamma = \frac{\nu^2 \tau}{2},$$

τ is the temporal resolution of the probe.

Non-Markovian, obtained for ντ << 1.

Measurement by a quantum probe 1

- Coupling through the Newtonian force to a quantum harmonic oscillator constrained to move along the x-axis.



Now consider a quantum probe made of a harmonic oscillator of frequency ω that is constrained to move along the horizontal axis as in Fig. 1. The Hamiltonian of the harmonic oscillator probe is

$$H_P = \omega \hat{a}^\dagger \hat{a} \tag{68}$$

If the amplitude of the oscillations is much smaller than L, the length scale of the cat state, the force acted upon the oscillator along the x direction is approximately constant and equal to Eq. (62). This corresponds to an interaction Hamiltonian

$$\hat{H}_I = -f_0 \hat{\sigma}_3 \hat{x} = -\frac{f_0}{\sqrt{2m_0\omega}} \hat{\sigma}_3 (\hat{a} + \hat{a}^\dagger), \tag{69}$$

Thus, the total Hamiltonian of the two state system interacting with the oscillator probe is

$$\hat{H} = H_S + H_P + H_I = \nu\hat{\sigma}_1 + \omega\hat{a}^\dagger\hat{a} + g\hat{\sigma}_3(\hat{a} + \hat{a}^\dagger), \tag{70}$$

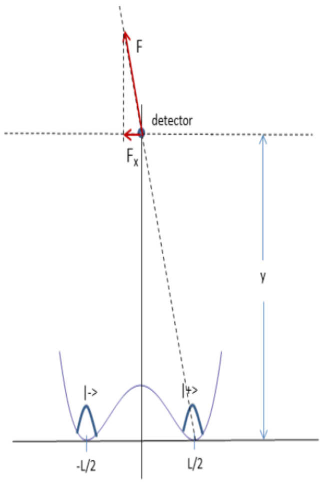
where the system Hamiltonian H_S is given by Eq. (56) specialized to $\chi = 0$, i.e., it is equivalent to the Hamiltonian of a single-mode Jaynes-Cummings model with a coupling constant

Eq. (56) $\hat{H}_S := \nu(\cos\chi\hat{\sigma}_1 + \sin\chi\hat{\sigma}_2)$

$$g = -\frac{f_0}{\sqrt{2m_0\omega}}. \tag{71}$$

Equivalent to the Jaynes-Cummings (JC) model of quantum optics.

quantum probe 2



- If the oscillator is to act as a measurement, the coupling term should be strong, it cannot be treated as a small perturbation.
- Thus we cannot use the commonly employed Rotating Wave Approximation.
- JC model was recently shown to be integrable (Braak 11), but the solution is not helpful in finding time evolution.

Consider **adiabatic regime** v=0 (vanishing tunneling). Then for the oscillator probe initially in the vacuum and the cat particle in $c_+|+\rangle + c_-|-\rangle$,

$$|\Psi(t)\rangle = e^{i\frac{g^2}{\omega^2}[\omega t - \sin(\omega t)]} \begin{pmatrix} c_+|\zeta(t)\rangle \\ c_-|-\zeta(t)\rangle \end{pmatrix}, \tag{74}$$

where the path

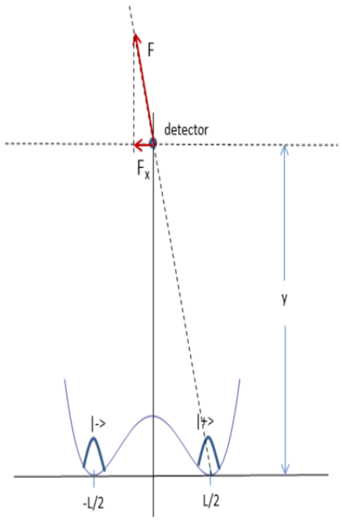
$$\zeta(t) = -\frac{g}{\omega}(1 - e^{-i\omega t}), \qquad \text{Coherence state representation} \tag{75}$$

describes an oscillation centered around $\zeta_0 = -\frac{g}{\omega} = \frac{f_0}{\sqrt{2m_0\omega^3}}$. The center of the oscillation corresponds to position $x_0 = \frac{f_0}{m_0\omega^2}$ and momentum $p_0 = 0$.

We obtain a superposition of two oscillations around different centers. The centers are distinguished only if $|\langle\zeta_0|-\zeta_0\rangle| << 1$, or

$$\omega^3 << \left(\frac{f_0}{m_0}\right)^2 m_0.$$

quantum probe 3



Treat small values of ν as perturbations of the adiabatic solution.

4.2.2. Rabi oscillations A finite value of ν allows for transitions between the two gravitational quantum states, which induce transitions among the phase space paths of the oscillator. While the model is not exactly solvable, we can estimate the rate of such transitions using perturbation theory with respect to the tunneling rate ν. In Appendix B, we show that to leading order in ν, $e^{-i\hat{H}t} = e^{-i\hat{H}_0 t}\hat{O}_t$, where

$$\hat{O}_t = \begin{pmatrix} \cos\nu t & -i\sin\nu t\hat{D}(2\zeta_0) \\ -i\sin\nu t\hat{D}(-2\zeta_0) & \cos\nu t \end{pmatrix}. \tag{79}$$

As an estimate of the transition between the two gravitational quantum states, we compute the amplitude $\langle -\zeta_0, -|\hat{O}_t|\zeta_0, +\rangle$, between the stationary states $|\zeta_0, +\rangle$ and $|-\zeta_0, -\rangle$. We find

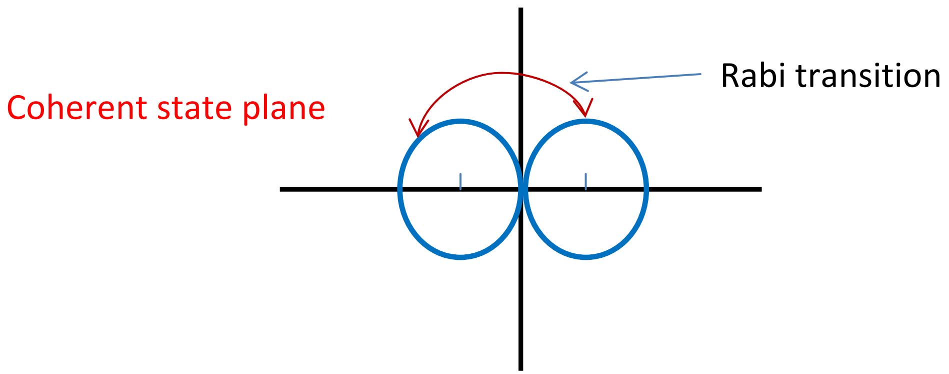
$$\langle -\zeta_0, -|\hat{O}_t|\zeta_0, +\rangle = -i\sin\nu t, \tag{80}$$

and thus the associated probability

$$p(t) = |\langle -\zeta_0, -|\hat{O}_t|\zeta_0, +\rangle|^2 = \sin^2\nu t, \tag{81}$$

exhibits Rabi-type oscillations, with frequency ν.

Then we obtain Rabi oscillations of frequency ν between the two centers ζ_0 and $-\zeta_0$



Implications

Since the gravitational field is slaved to matter, the gravitational force is represented by an operator on the Hilbert space of the matter field.

Thus, the standard operational procedures in QM can be invoked for measuring a gravitational force. But what does this mean?

Standard interpretation: weak field

- Once we measure a force **F** on a test particle of mass m, we can calculate the field strength **g** = **F**/m.
- The field strength corresponds to a gravitational potential φ.
- In the weak field limit of GR, the potential appears in the g_{00} component of the metric tensor.

From the vantage point of GR:
Spacetime & quantum matter intimately linked

Do quantum fluctuations of the force define quantum fluctuations of the spacetime geometry? [stochastic gravity addresses this issue]

Operational definitions of spacetime geometry seem to agree on that.

If this is true, the state we considered here is a **genuine gravcat,** a quantum superposition of two spacetime geometries.

Discussions

- *Does the gravitational force remain slaved to the mass density as classical GR dictates*, even if the latter behaves quantum mechanically? (it has fluctuations, it is subject to quantum measurements, etc.)

We can only answer this questions by attempting to construct gravcats, or other non-classical states for Macroscopic systems. Optomechanical systems seems to be the most promising route.

- In principle, we can construct probes that record quantum jumps of the gravitational force. Can we talk about q jumps on the gravitational potential? And then about **jumps of (not just in) the induced quantum spacetime**?

Does this idea even make sense?

The conceptual tension between GR and QM Such as spelled out by Penrose, already Manifest in the Newtonian regime.

- Invoking gravitational decoherence (grav field as environment to quantum systems) to kill gravcats may solve the problem above, but the intrinsic tension between GR +QM remains.

The interface between macroscopic quantum phenomena and gravitational quantum physics is of fundamental significance from this perspective.

- In view of advances in AMO,CMP and Optomechanics **precision experiments in weak gravitational fields**

  -- *Gravitational Quantum physics (e.g., focus issue in NJP)*

- it pays to reexamine the WF-NR limit of

  1) semiclassical Einstein Equation, (in relation to NSEq etc)

  2) Noise kernel, or stress energy density correlators (new)

  Bringing gravity into consideration of issues in

- **quantum foundations** such as the Born Rule; and
- **quantum information** such as the Cat State with gravity

> *Can we infer attributes of spacetime fluctuations from quantum experiments even at the level of Newtonian gravity without appealing to new theories of QM or GR?*

Conclusion: *Investigation of Q Information issues of gravitational systems using quantum probes*

- *Quantum Gravity (theories for the micro-scopic structures of spacetime) is not needed.*

- *Focus on systems under laboratory conditions: nonrelativistic systems, weak gravitational field.*
- *Semiclassical gravity is inadequate.*

- *Focus on fluctuations and correlations of mass density: Stochastic Semiclassical Gravity*

Thank YOU for your attention!

& the Organizers for their hard work!