Lorentz violation, causality, and black holes

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based on work with: Enrico Barausse, Jishnu Bhattacharyya, Mattia Colombo, Ted Jacobson, Ian Vega, Daniele Vernieri

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LV and causal structure

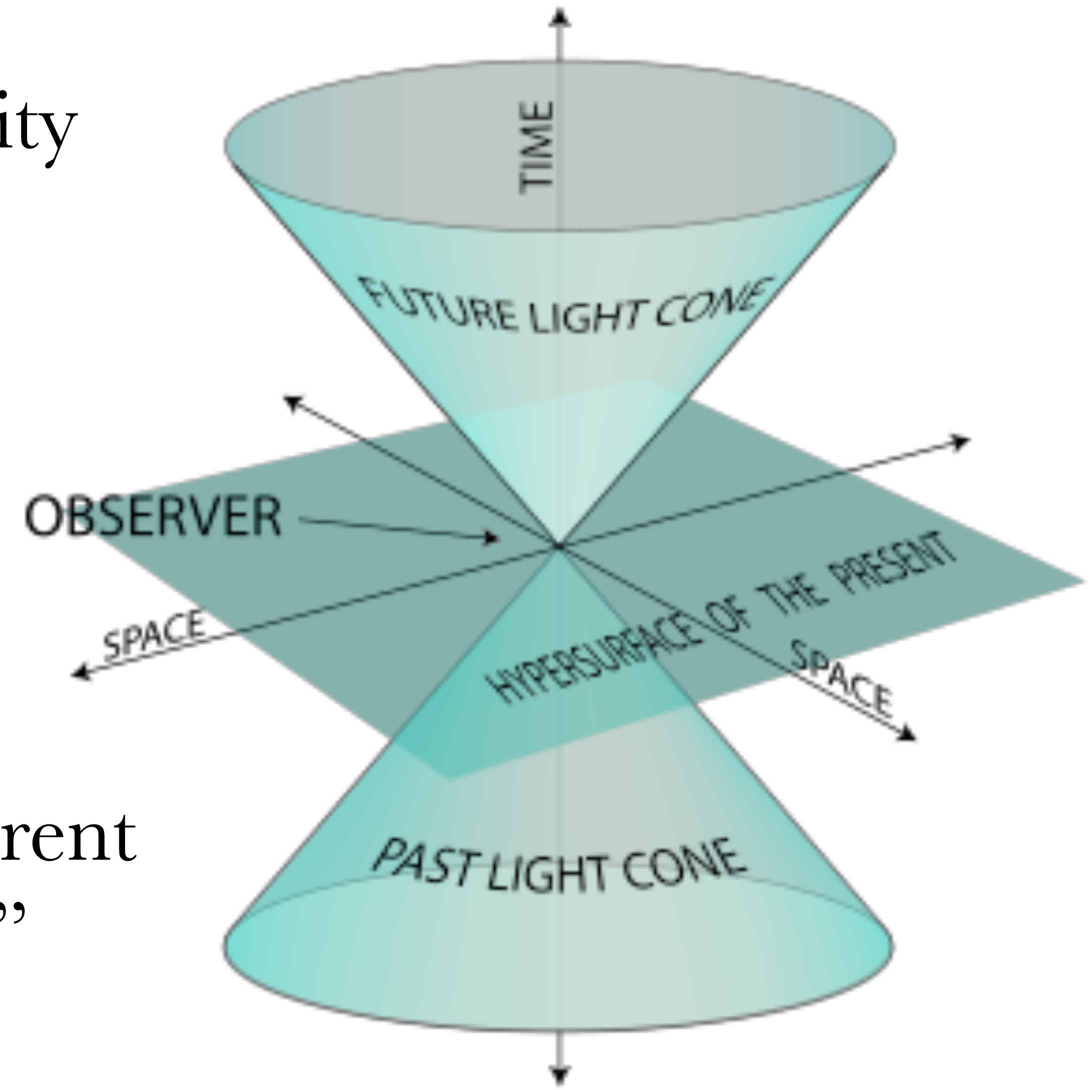
Causal structure in special relativity

- LV with linear dispersion relations

$$\omega \propto k$$

- Different modes have different speeds and different "light" cones

- But there are still "light" cones!



Einstein-aether theory

The action of the theory is

$$S_{æ} = \frac{1}{16\pi G_{æ}} \int d^4x \sqrt{-g}(-R - M^{\alpha\beta\mu\nu} \nabla_\alpha u_\mu \nabla_\beta u_\nu)$$

where

$$M^{\alpha\beta\mu\nu} = c_1 g^{\alpha\beta} g^{\mu\nu} + c_2 g^{\alpha\mu} g^{\beta\nu} + c_3 g^{\alpha\nu} g^{\beta\mu} + c_4 u^\alpha u^\beta g_{\mu\nu}$$

and the aether is implicitly assumed to satisfy the constraint

$$u^\mu u_\mu = 1$$

- Most general theory with a unit timelike vector field which is second order in derivatives

T. Jacobson and D. Mattingly, Phys. Rev. D 64, 024028 (2001).

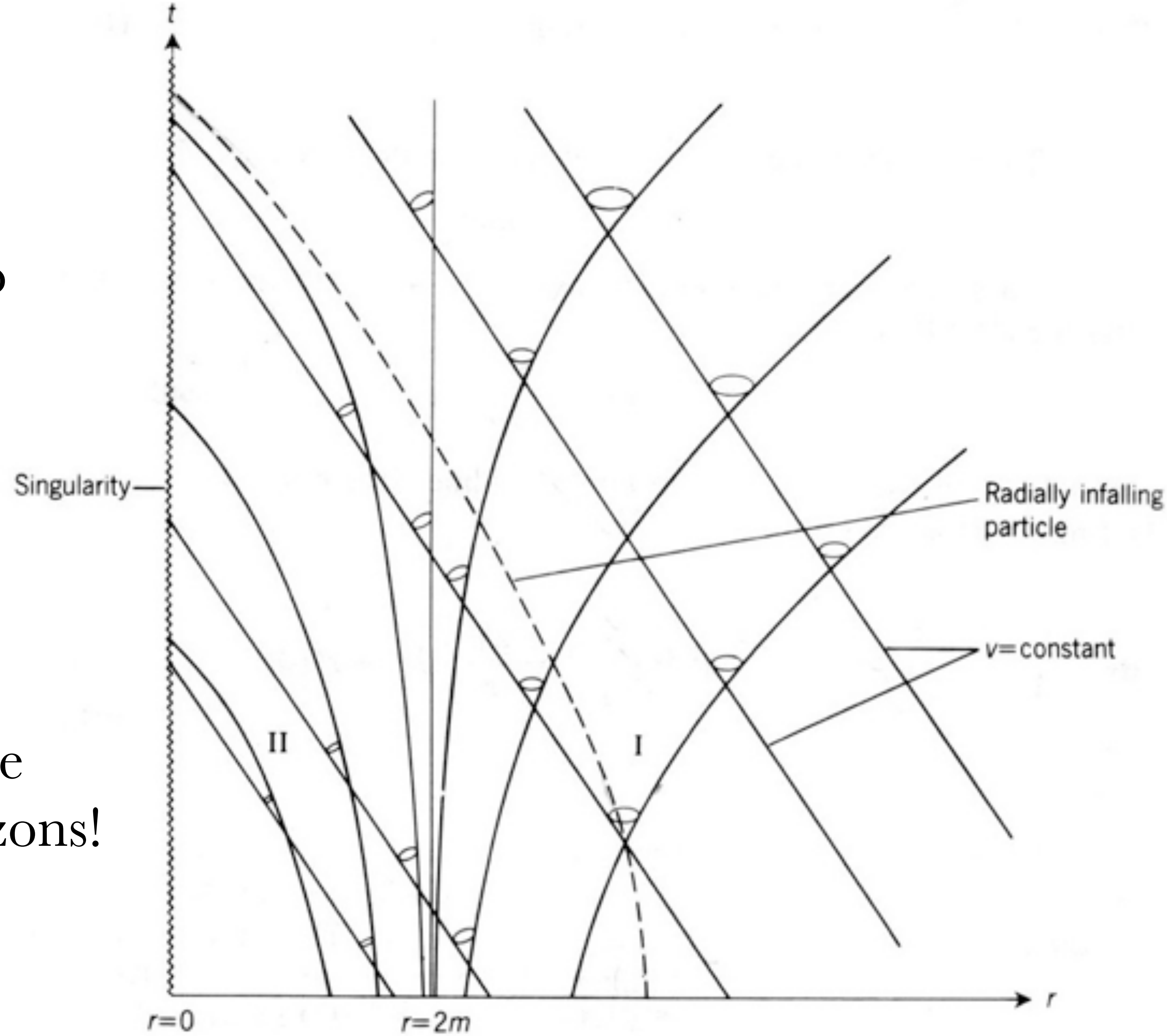
Einstein-aether theory

- Extensively tested and still viable
- It propagates a spin-2, a spin-1 and spin-0 mode.
- Linear dispersion relations.
- These modes travel at different speeds.
- We expect multiple horizons!

LV and black hole structure

What happens to black holes?

- They will have multiple horizons!



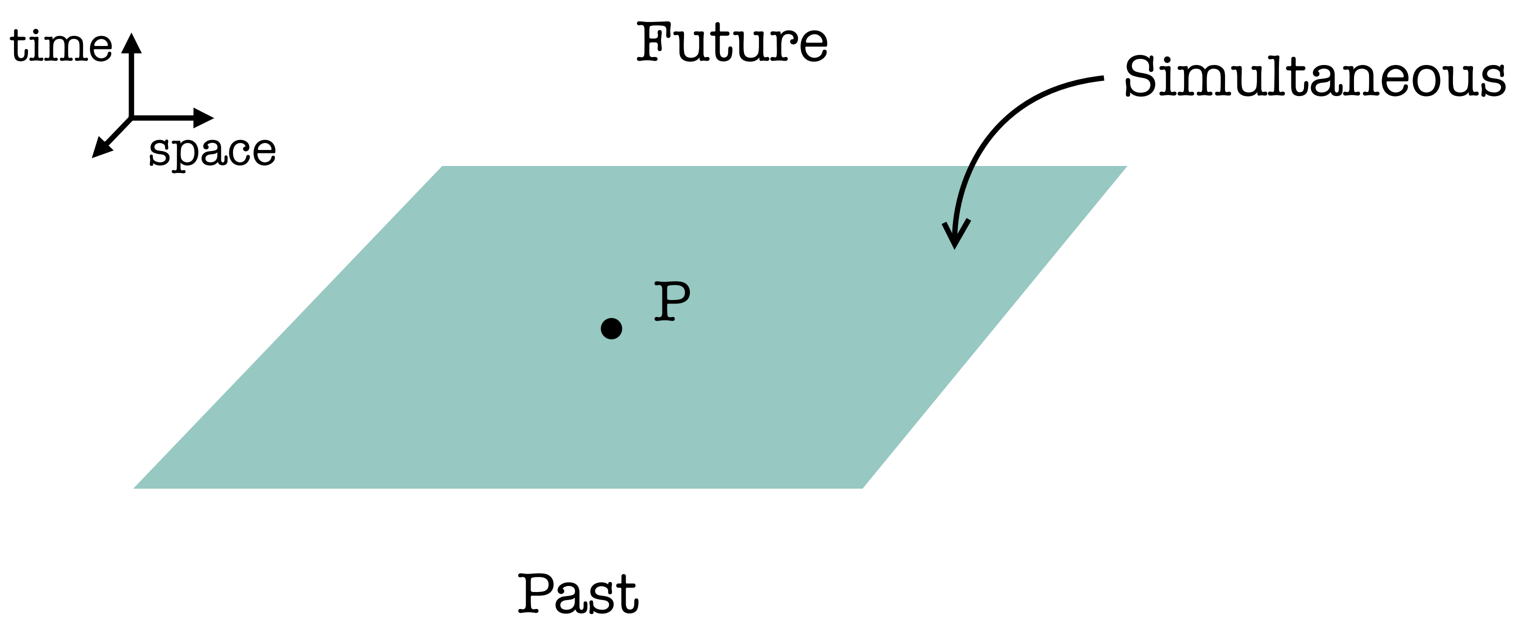
LV and black hole structure

- LV with non-linear dispersion relations

$$\omega^2 \propto k^2 + ak^4 + \ldots$$

- No light cones!

Causal structure without relativity



No black holes at all??

Hypersurface orthogonality

Now assume $$u_\alpha = \frac{\partial_\alpha T}{\sqrt{g^{\mu\nu}\partial_\mu T \partial_\nu T}}$$

and choose T as the time coordinate

$$u_\alpha = \delta_{\alpha T}(g^{TT})^{-1/2} = N\delta_{\alpha T}$$

Replacing in the action and defining one gets

$$S_{æ}^{ho} = \frac{1}{16\pi G_H}\int dT d^3x N\sqrt{h}\left(K_{ij}K^{ij} - \lambda K^2 + \xi\,{}^{(3)}R + \eta a^i a_i\right)$$

with $a_i = \partial_i \ln N$ and the parameter correspondence

$$\frac{G_H}{G_{æ}} = \xi = \frac{1}{1-c_{13}} \qquad \lambda = \frac{1+c_2}{1-c_{13}} \qquad \eta = \frac{c_{14}}{1-c_{13}}$$

T. Jacobson, Phys. Rev. D 81, 101502 (2010).

Horava-Lifshitz gravity

The action of the theory is

$$S_{HL} = \frac{1}{16\pi G_H} \int dT d^3x \, N\sqrt{h}\left(L_2 + \frac{1}{M_\star^2} L_4 + \frac{1}{M_\star^4} L_6 \right)$$

where

$$L_2 = K_{ij}K^{ij} - \lambda K^2 + \xi^{(3)}R + \eta a_i a^i$$

L_4 : contains all 4th order terms constructed with the induced metric h_{ij} and a_i

L_6 : contains all 6th order terms constructed in the same way

P. Hořava, Phys. Rev. D 79, 084008 (2009)

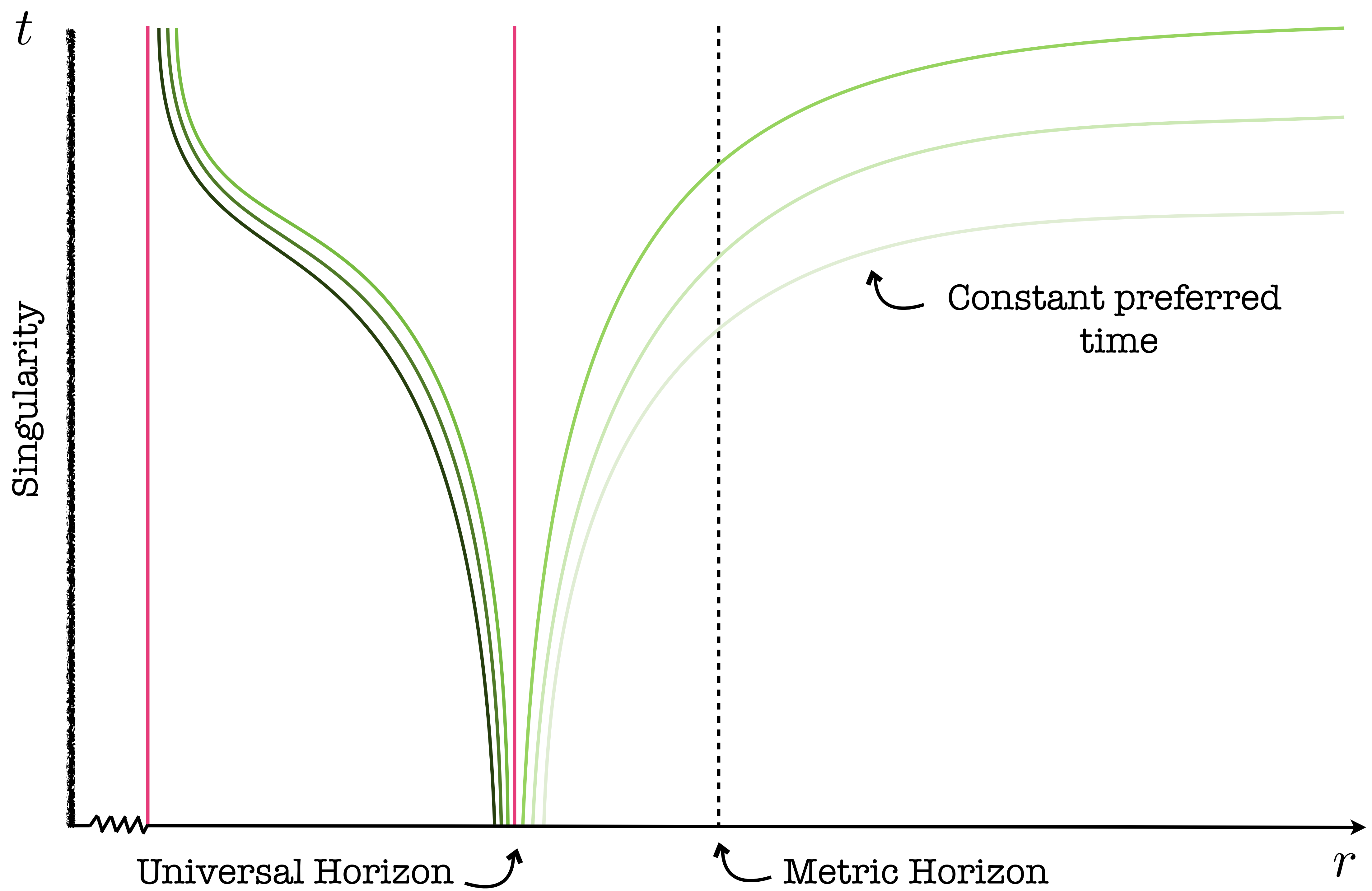
D. Blas, O. Pujolas and S. Sibiryakov, Phys. Rev. Let. 104, 181302 (2010)

Horava-Lifshitz gravity

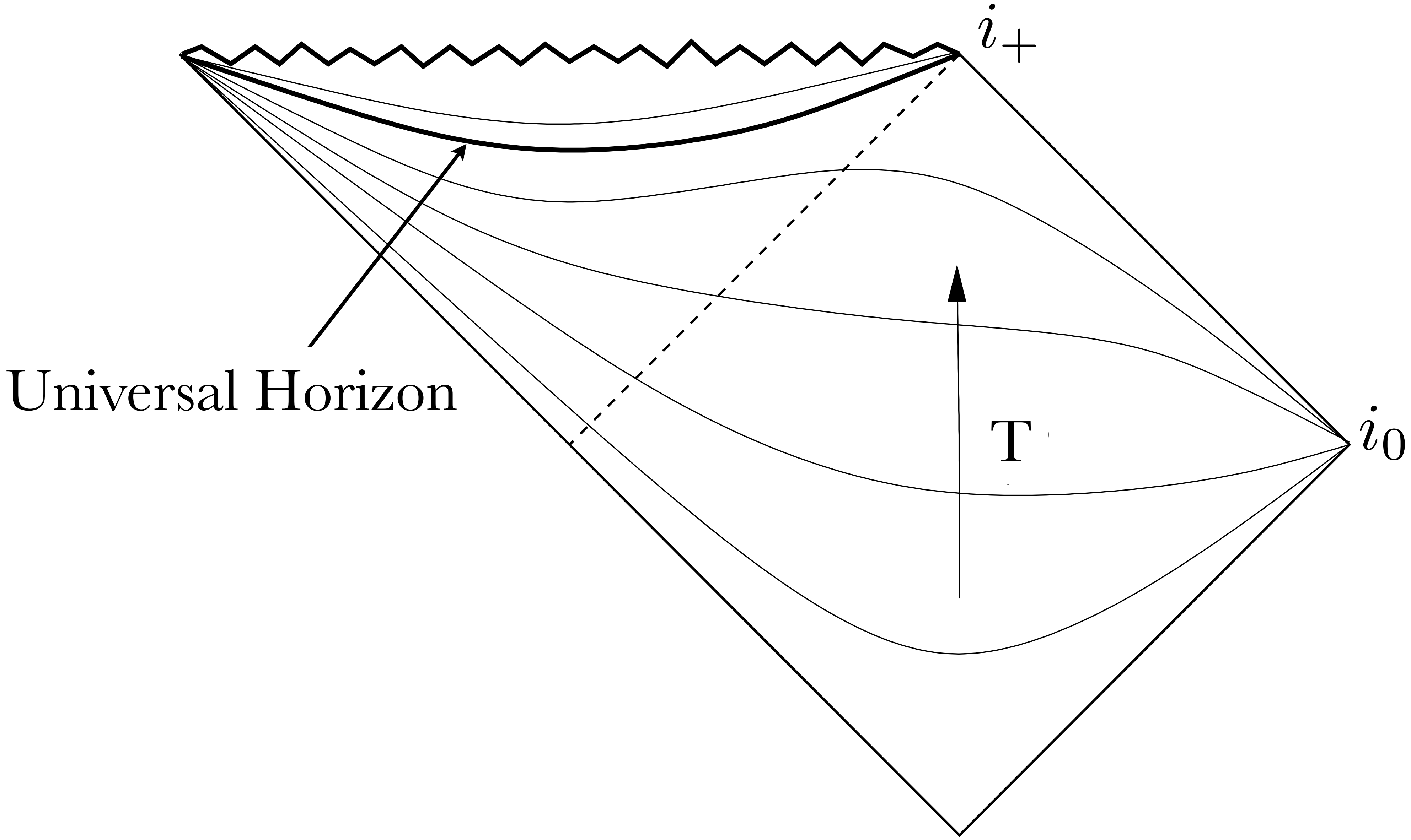
- Higher order terms contain higher order spatial derivatives: higher order dispersion relations!
- They modify the propagator and render the theory power-counting renormalizable
- All terms consistent with the symmetries will be generated by radiative corrections
- This version of the theory is viable so far
- "Low energy limit" is h.o. Einstein-aether theory!
- We expect no causal boundaries!

Spacetime diagram

E. Barausse, T. Jacobson and T.P.S., Phys. Rev. D 83, 124043 (2011)



Penrose diagram



Taken from D. Blas and S. Sibiryakov, Phys. Rev. D 84, 124043 (2011)

Rotating black holes

- Slowly rotating BHs in Einstein-aether theory do not have a preferred foliation.
- Slowly rotating BHs in Horava gravity have universal horizons.

T.P.S. and E. Barausse, Phys. Rev. Lett. 109, 181101 (2012)
T.P.S. and E. Barausse, Phys. Rev. D 87, 087504 (2013)
T.P.S. and E. Barausse, Class. Quant. Grav 30, 244010 (2013)

- 3d rotating black holes can have universal horizons even with flat asymptotics.
- Universal horizons can lie “outside” de Sitter horizons.

T.P.S., I. Vega and D. Vernieri, Phys. Rev. D 90, 044046 (2014)

What's next?

- A new "toolkit" is needed
- How do we define this horizon in full generality?
- Can we have a local definition when we have less symmetry?
- Is the universal horizon relevant to astrophysics?

M. Colombo, J. Bhattacharyya, and T.P.S., arXiv:1508.???? [gr-qc]

Causally preferred foliation

Consider a manifold with a preferred foliation $(\mathscr{M}, \Sigma, \mathrm{g})$

Definition: Ordered foliation

- Every event in $\mathscr{M}$ lies on a unique leaf
- Every pair of events has a unique causal relation

No preferred labeling implies invariance under $T \to \tilde{T}(T)$

Definition: Causal and acausal curves

Continuous, piecewise differentiable curve with tangent t^μ

causal, future directed if $u_\mu t^\mu > 0$

causal, past directed if $u_\mu t^\mu < 0$

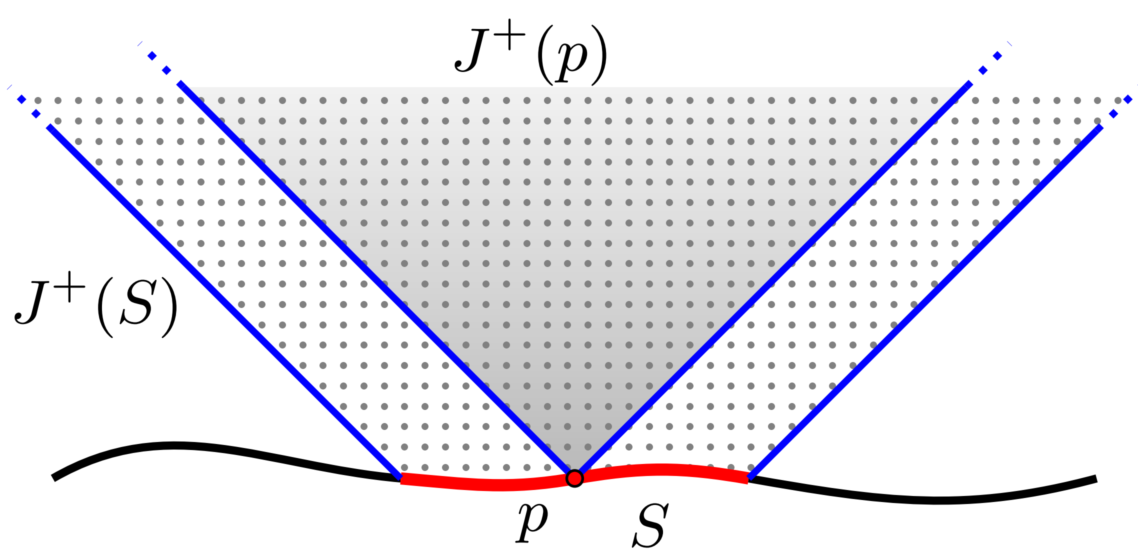
acausal if $u_\mu t^\mu = 0$

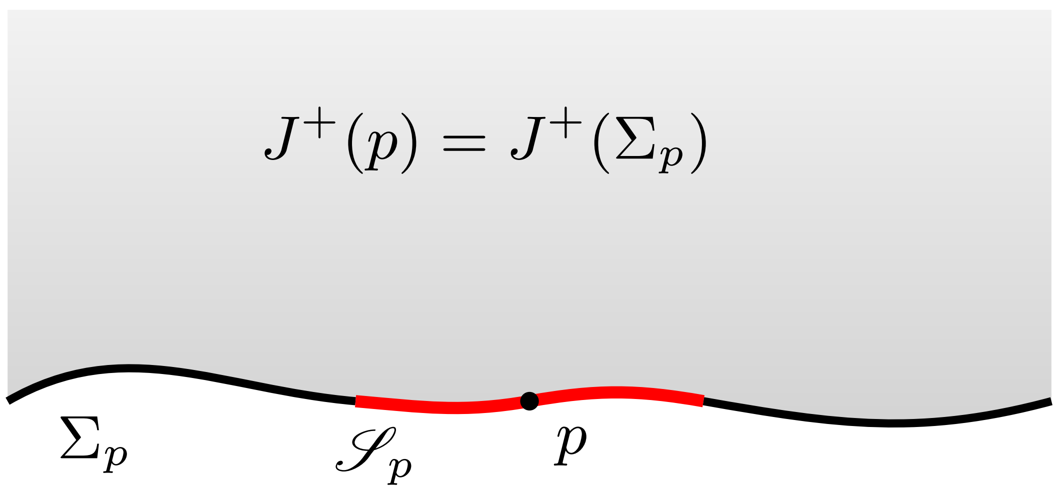
Future and Past

Definition: Future and Past

The future $J^+(p)$ of an event p is the set of all events that can be reached from p by a future directed causal curve.

Similarly for the Past.





Asymptotics

What's the analogue of asymptotic flatness?

projector: $p_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$

Conformally extended manifold: $\tilde{\mathscr{M}}$, $\tilde{p}_{\mu\nu} = \Omega^2 p_{\mu\nu}$

point as spatial infinity of Σ_p: i_p $\tilde{\Sigma}_p = \Sigma_p \cup i_p$

One also needs conditions for the foliation!

Key concept: 'trivially foliated asymptotically flat end'

$\langle\langle \mathscr{M} \rangle\rangle$: open region in which every leaf has trivially foliated asymptotically flat end

$$\mathscr{I} = \bigcup_{p \in \langle\langle \mathscr{M} \rangle\rangle} i_p$$

Universal horizons

Future and past event horizons:

$$\mathcal{H}^+ \equiv \partial J^-(\mathscr{I}) \qquad\qquad \mathcal{H}^- \equiv \partial J^+(\mathscr{I})$$

It follows that

- $\langle\langle \mathscr{M} \rangle\rangle = J^-(\mathscr{I}) \cap J^+(\mathscr{I})$
- $\mathcal{H}^\pm$ are leaves and boundaries of $\langle\langle \mathscr{M} \rangle\rangle$

Black hole: $\mathcal{B}(\mathscr{I}) \equiv \mathscr{M} \setminus J^-(\mathscr{I})$

White hole: $\mathcal{W}(\mathscr{I}) \equiv \mathscr{M} \setminus J^+(\mathscr{I})$

Key point: $\mathcal{H}^\pm(\mathscr{I}) \cap \mathscr{I} = 0$

Local characterisation

Definition: Stationary spacetime (simplified)

There exist a killing vector that is causal in an open set $\mathcal{X}$ that overlaps with $\langle\langle \mathscr{M} \rangle\rangle$ and 'aligns' with the aether asymptotically.

Theorem

$(u \cdot \chi) = 0, \quad (a \cdot \chi) \neq 0$ form a set of necessary and sufficient conditions for a hypersurface to be a universal horizon

Key points:

- $\langle\langle \mathscr{M} \rangle\rangle = \mathcal{X}$
- χ is unique
- analogy with Killing horizon, $(a \cdot \chi) = \text{constant}$

Astrophysical relevance

Assume also axisymmetry

$$(u \cdot \varphi) = (a \cdot \varphi) = 0$$

Suppose there is a universal horizon

- Does there need to be a Killing horizon?
- Does is cloak the universal horizon?

$$V^\mu \equiv \chi^\mu + W\varphi^\mu \qquad W \equiv -(\chi \cdot \varphi)/(\varphi \cdot \varphi) \qquad (V \cdot \varphi) = 0$$

Iff $\qquad \chi_{[\mu}\varphi_\nu \nabla_\kappa \varphi_{\lambda]} = 0 \qquad \varphi_{[\mu}\chi_\nu \nabla_\kappa \chi_{\lambda]} = 0$

then by Carter's rigidity theorem

$(V \cdot V) = 0$ is a null hypersurface where $W =$constant

Perspectives

- Black holes are of great interest in Lorentz-violating theories. New notion: “universal horizon”
- Non-trivial causal structure
- Is this horizon stable?
- Does it form from collapse?

D. Blas and S. Sibiryakov, Phys. Rev. D 84, 124043 (2011)

M. Saravani, N. Afshordi and R. B. Mann, Phys. Rev. D 89, 084029 (2014)