

# Spin Hall Effects in HgTe Quantum Well Structures

Hartmut Buhmann

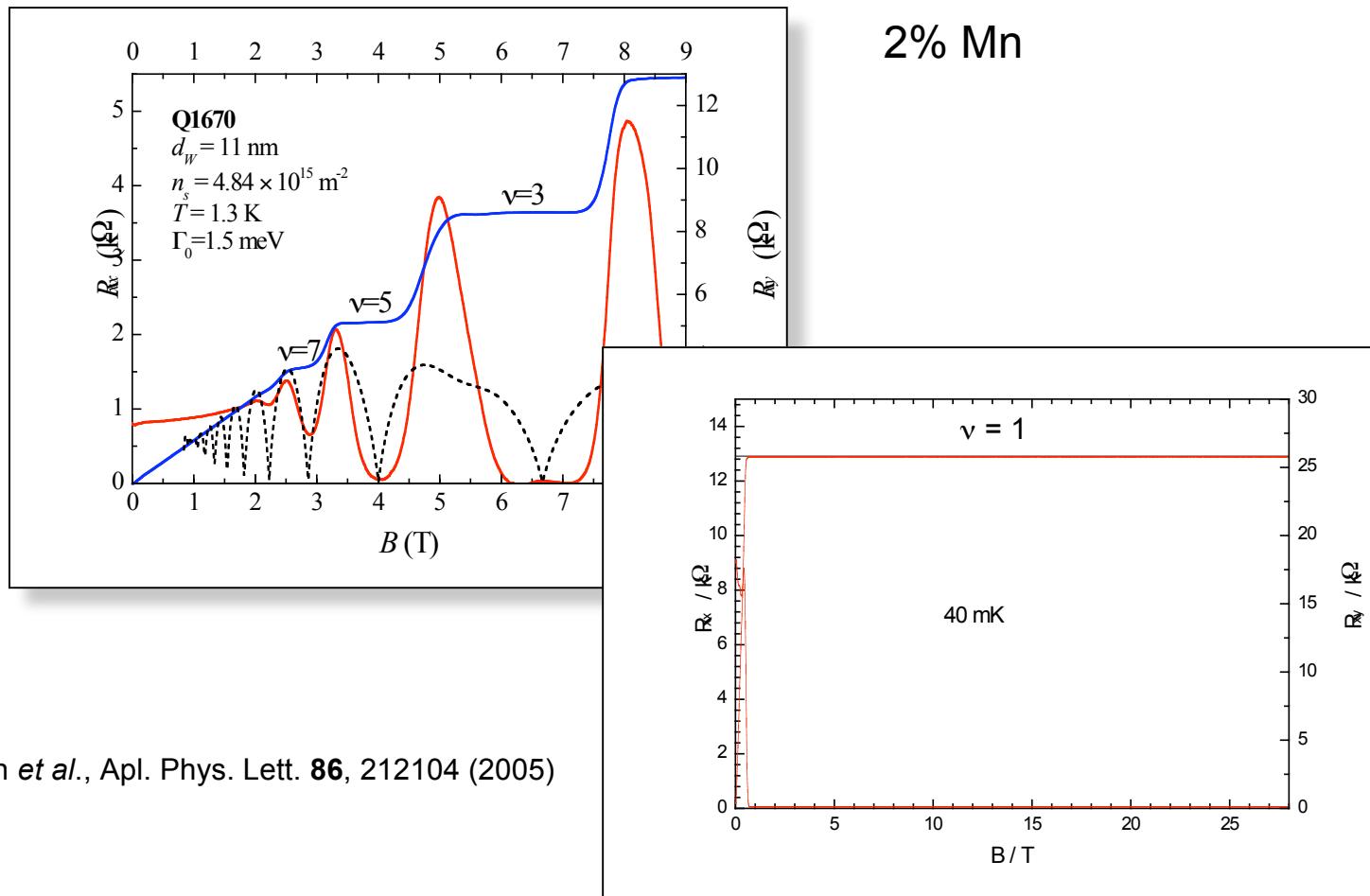
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Physikalisches Institut, EP3  
Universität Würzburg  
Germany

(slides with unpublished results have been removed!)

# Anomalous Quantum Hall Effect

in dilute magnetic semiconductor quantum wells

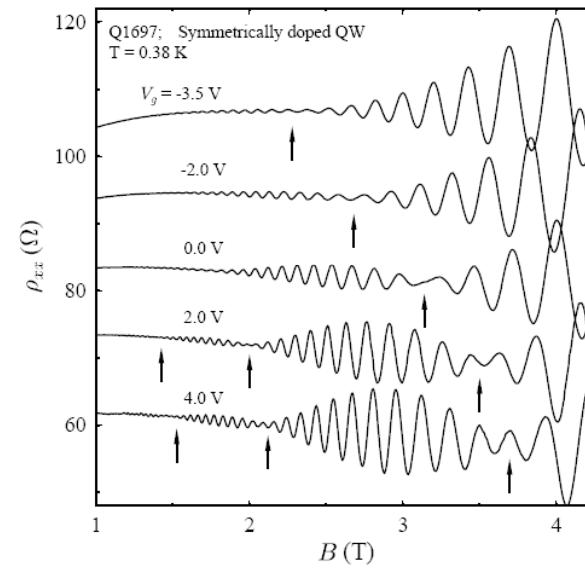
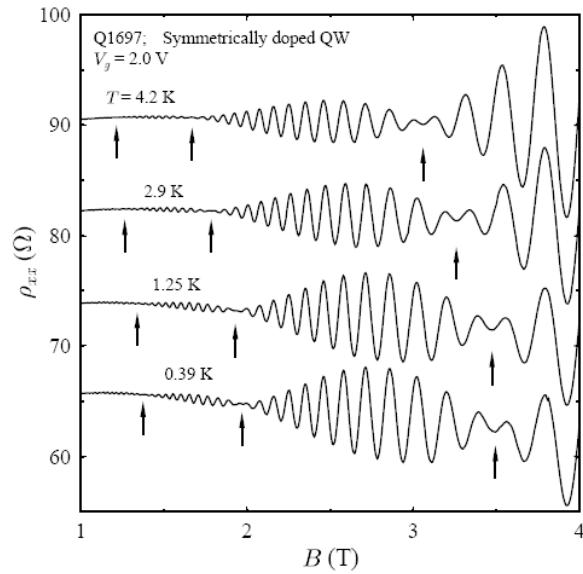


H. Buhmann *et al.*, *Apl. Phys. Lett.* **86**, 212104 (2005)

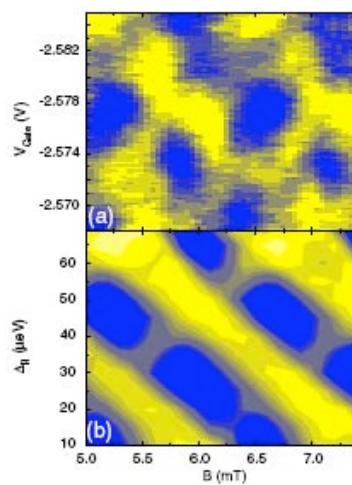
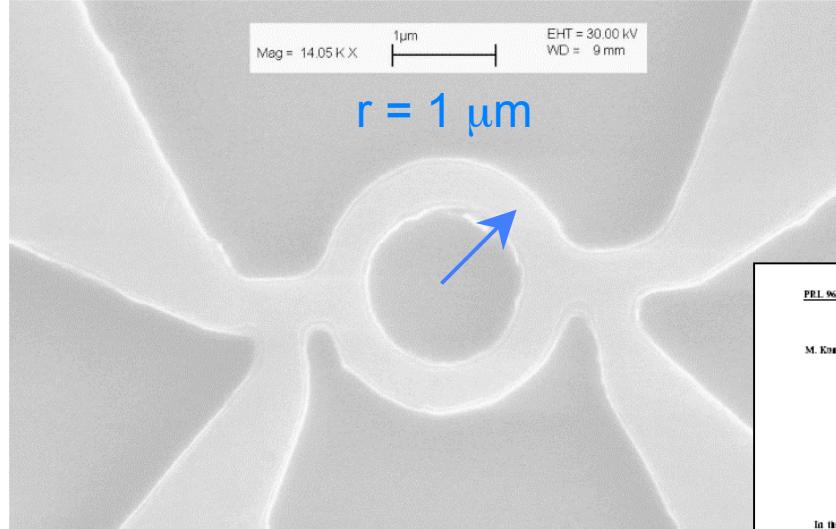
# Interplay of Rashba, Zeeman and Landau splitting in a magnetic two-dimensional electron gas

Y. S. GUI(\*), C. R. BECKER(\*\*), J. LIU, V. DAUMER, V. HOCK,  
 H. BUHMANN and L. W. MOLENKAMP

Europhys. Lett., 65, 393 (2004)



# Direct Observation of the Aharonov-Casher Phase

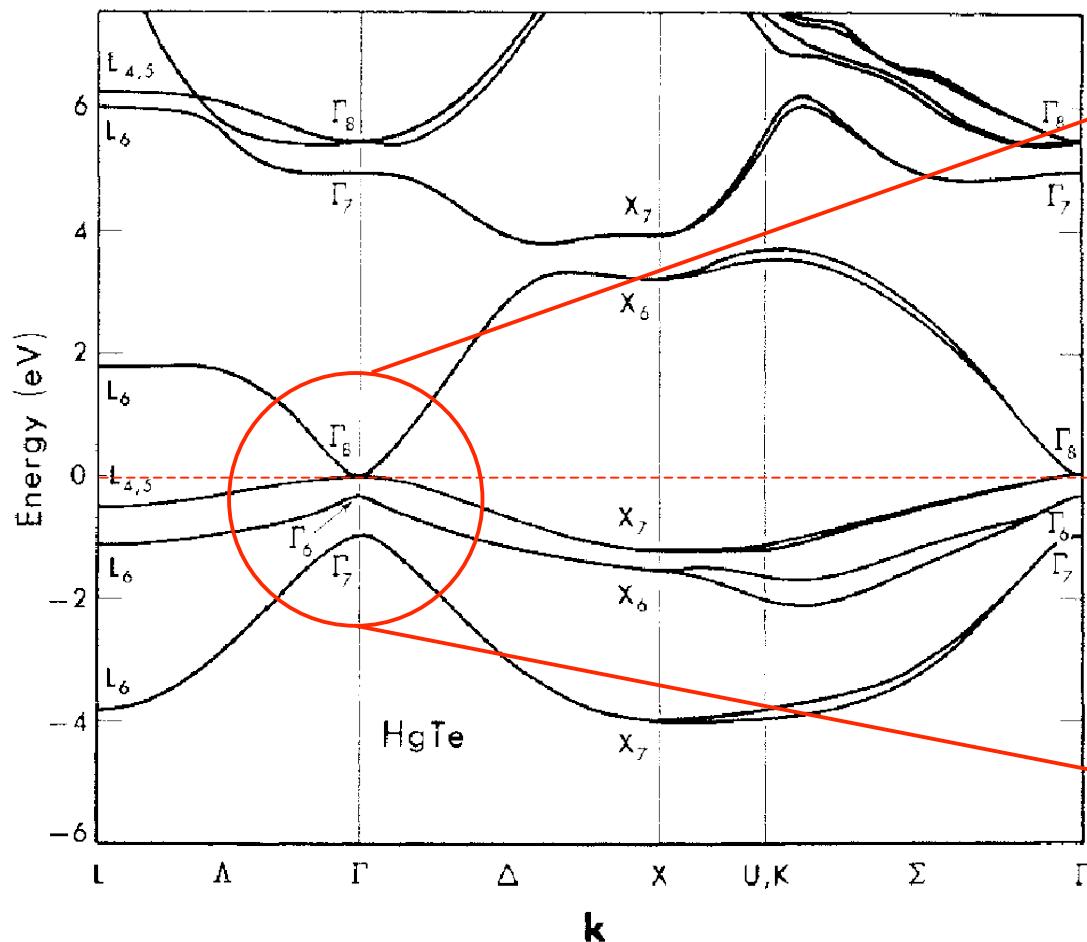


M.König, AT, EMH, JS, VH, VD, MS, CRB, **HB**, and LWM, PRL **96**, 076804 (2006).

PRL 96, 076804 (2006)	PHYSICAL REVIEW LETTERS	week ending 24 FEBRUARY 2006
Direct Observation of the Aharonov-Casher Phase		
M. König, <sup>1</sup> A. Tschettner, <sup>1</sup> E. M. Hartnacker, <sup>2,3</sup> Ján Slovák, <sup>3</sup> V. Heck, <sup>1</sup> V. Dommer, <sup>1</sup> M. Schäfer, <sup>1</sup> C. R. Becker, <sup>1</sup> H. Büttner, <sup>1</sup> and L. W. Molenkamp <sup>1</sup>		
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DOI: 10.1103/PhysRevLett.96.076804		
PACS numbers: 73.23.-b, 03.65.Vf, 71.70.Jd		
<p>In the early 1980s it was shown that a quantum mechanical system acquires a geometric phase for a cyclic motion in parameter space. This geometric phase under adiabatic motion is called the Berry phase [1], while its later generalization to include nonadiabatic motion is known as the Aharonov-Bohm phase [2]. A manifestation of the Berry phase is the Aharonov-Bohm effect (AB effect) [3]: a localized charge rotates twice around a magnetic flux. Aside from the AB effect, the first experimental observation of the Berry phase was reported in 1986 for photons in a wound optical fiber [4]. Another important Berry phase effect is the Aharonov-Casher (AC) effect [5], which has been proposed to occur when an electron propagates in a ring structure in an external magnetic field perpendicular to the ring plane in the presence of spin-orbit (SO) interaction [6].</p> <p>This AC effect can be seen when two partial waves move around the ring at different directions. They will acquire a phase difference which depends on the spin orientation with respect to the total magnetic field <math>B_{\text{ext}} = \vec{B} + \vec{B}_{\text{SO}}</math> and the parts of each partial wave. <math>\delta_{\text{AC}}</math> is the effective field induced by the SO interaction. The phase difference is approximately [6]</p>		
$\Delta \phi_{\pm \sigma_1 \sigma_2} = -2\pi \frac{\Phi}{\Phi_0} - \hbar \tau (1 - \cos \theta) \quad (1)$		
$\Delta \phi_{\pm \sigma_1 \sigma_2} = -2\pi \frac{\Phi}{\Phi_0} - \hbar \tau \frac{m^2 \pi}{k^2} \sin \theta, \quad (2)$		
<p>where <math>\sigma_1 = \pm 1</math> denotes spin-up and spin-down, <math>\theta</math> is the angle between the external field <math>\vec{B}_{\text{ext}}</math> and the axis along the ring, and the superscript <math>\pm</math> denotes a clockwise (counter-clockwise) evolution, respectively. In the above equations <math>\Phi</math> is the SO parameter, <math>r</math> the ring radius, <math>m</math> the effective electron mass, and <math>\hbar</math> the angle between the external field <math>\vec{B}_{\text{ext}}</math> and the total magnetic field <math>\vec{B}_{\text{ext}}</math>. For both equations, the first term on the right-hand side can be identified with the</p>		
<p>All phase and the second term of Eq. (1) with the geometric Berry or Aharonov-Bohm phase. The second term in Eq. (2) represents the dynamic part of the AC phase, i.e., the phase of a particle with a magnetic moment that moves around an electric field. From the expression above, it can be seen that an increase of <math>\tau</math> will lead to a phase change that increases proportionally, whereas the contribution due to the Aharonov-Casher phase results in a phase shift limited to <math>\Delta \phi_{\pm \sigma_1 \sigma_2} = \pi</math>.</p> <p>From the AC phase [7] and the geometric phase [8] depend on the SO interaction. As a result, one expects a complicated noncommutative interference pattern as a function of magnetic field and SO interaction strength. So far, no such interference patterns have been observed. The observation of phase-related effects in solid-state systems has been reported. Recently side bands in the Fourier transform of All oscillations have been interpreted as indications for the existence of a Berry phase [10–12]. However, these interpretations have been questioned [13–15]. Spin interference signals in open loop arrays have been reported recently by Molenkamp <i>et al.</i> [16] but their direct relationship to the AC effect is not easily established. It would be important to observe these phase-related effects directly.</p> <p>Here, we present experimental results on the magnetotransport properties of a HgTe ring structure. The strength of the SO interaction is controlled by varying the gate voltage <math>V_{\text{gate}}</math> by varying the asymmetry of the quantum well structure. We observe systematic variations in the conductance of the device as a function of both external field <math>B</math> and gate voltage. The gate-voltage dependent oscillation clearly exhibit a noncommutative phase change, which is related to the dynamic part of the AC phase. This interpretation is consistent with the calculations of a random walk model within the Landau-Bittermann formulation. The effects discussed here are observed in several samples of various dimension. For clarity, we present only data obtained from one sample.</p>		
<p>As displayed in Fig. 101, next with the ring radius of more detailed gate-voltage. For the displayed gate-voltage range the SO effect is diminished. Near the electric influence of the SO conductance oscillations of <math>\delta_{\text{AC}} = -2.988</math> V bias</p>		
<p>is significantly modified, which corresponds to <math>\delta_{\text{AC}} = 0</math>. The AC oscillations show an increasing gate voltage. Oscillation frequency and minima can observe conductance fluctuations of the gate voltage of the ring. For <math>V_{\text{gate}} = 0</math> and caused by the change in time the orientation of the ext to the ring plane. The magnetic field and with regarding around the symmetry axis, the field is reflected. The dynamic part of the AC phase contribution to increasing SO interaction, a discernible signature of the gate voltage studied here.</p> <p>Our data represent a direct proof of the phase-related effects in the Landau quantization [22]. The effective conductance with Rashba</p>		
<p>Figure 3(b) shows the influence of Rashba energy for small Rashba energy. The conductance pattern is almost identical to the simulation, similar to the theoretical, a computational data leads to the voltage of 10 mV leads to a energy, which is in obtained from the</p>		
<p>as, we performed the larger Rashba coupling, the appearance of oscillations in conductance [4,25]. In contrast the pattern for conductance with Rashba energy, this minimum conductance D coupling, as can be (Fig. 4(b)). For this was assumed the distance, where only y. This latter assumption</p>		
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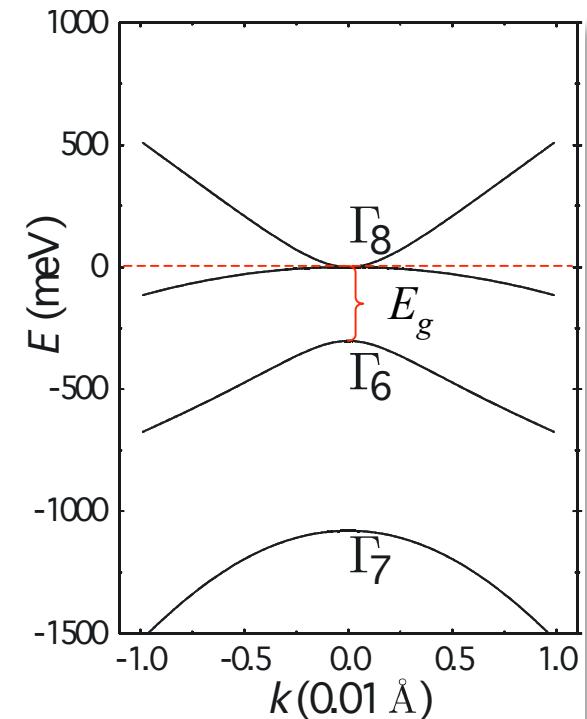
# HgTe-Quantum Well Structures

### band structure



D.J. Chadi et al. PRB, 3058 (1972)

### semi-metal or semiconductor

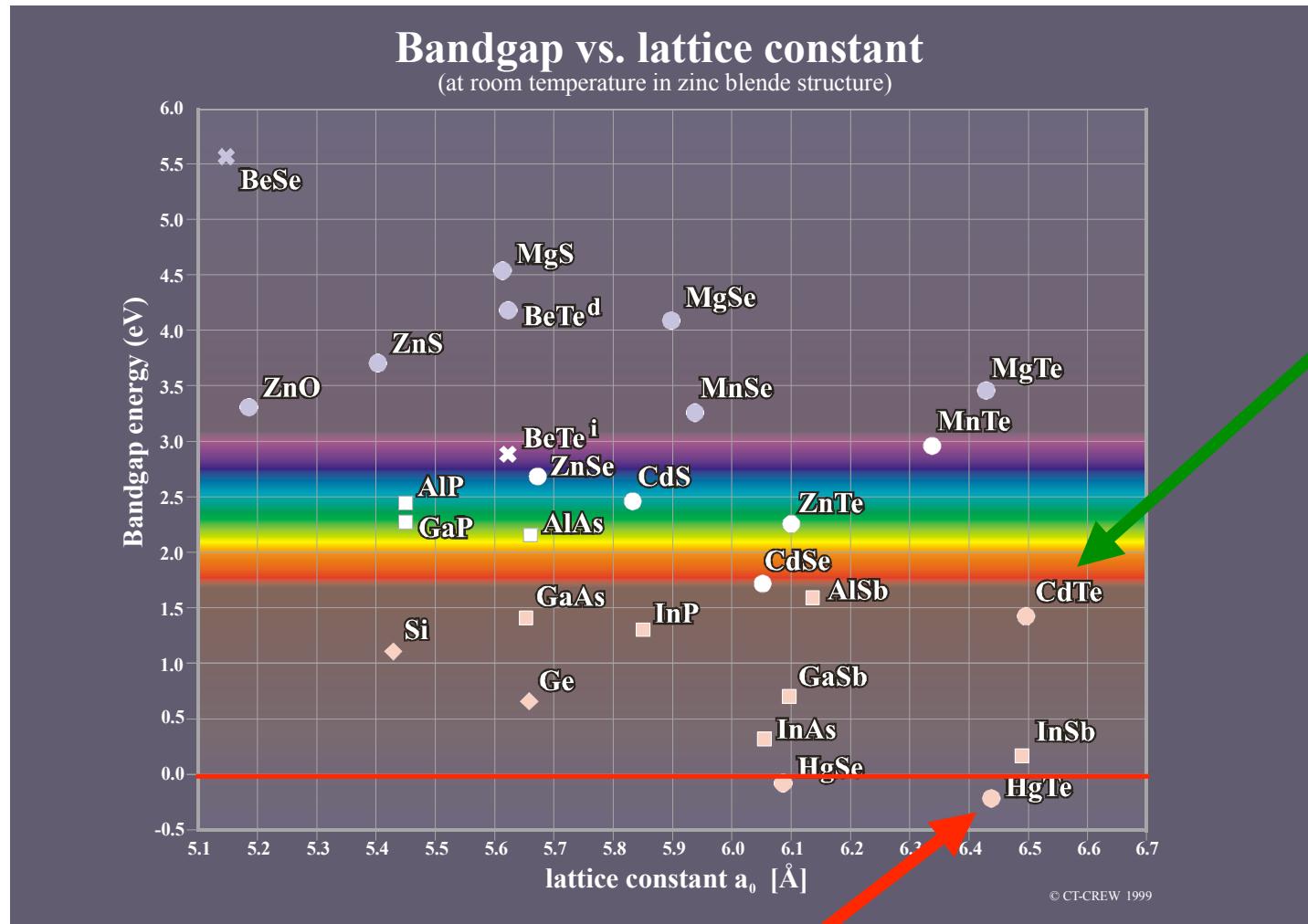


fundamental energy gap

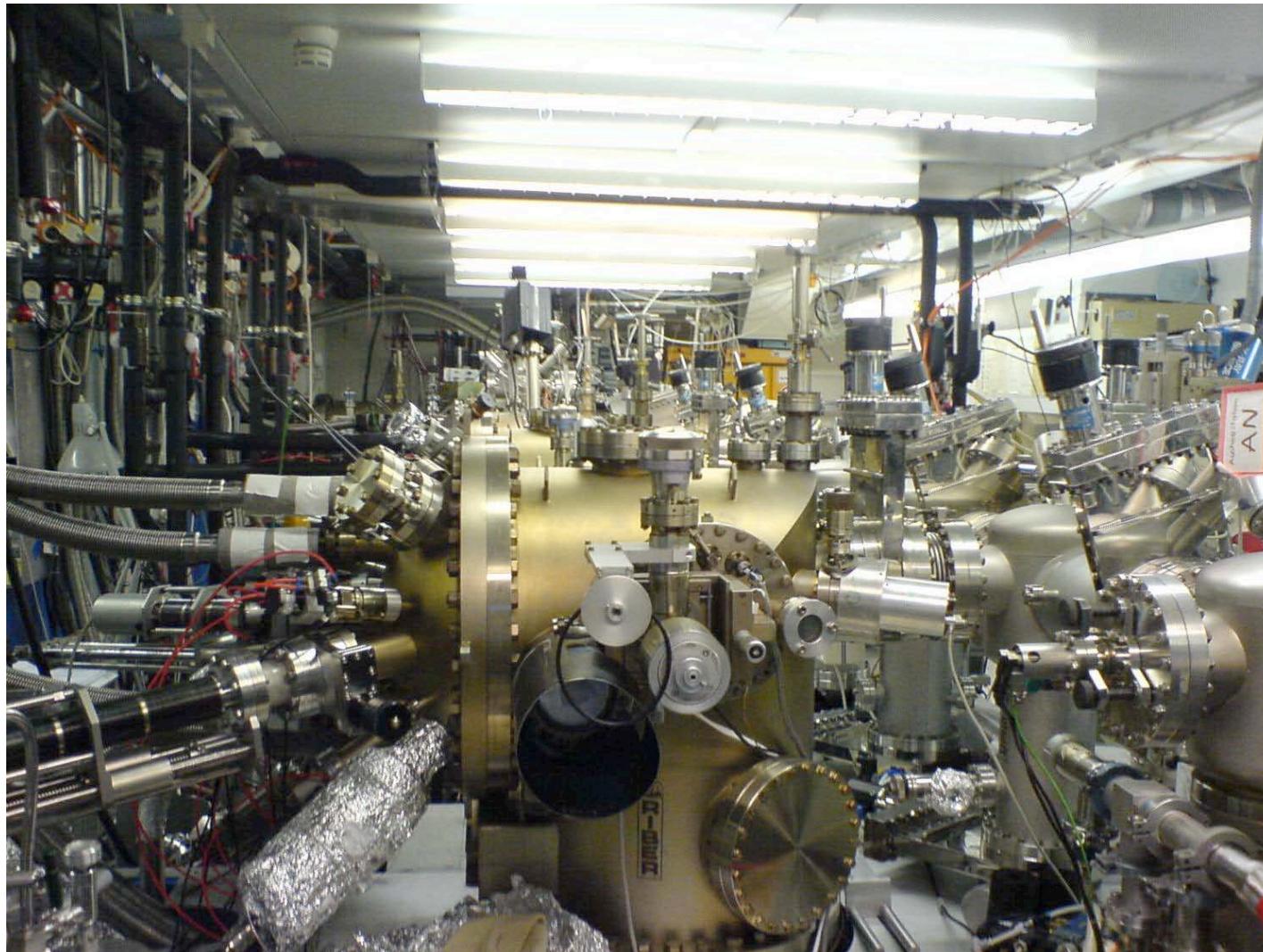
$$E^{\Gamma_6} - E^{\Gamma_8} \approx -300 \text{ meV}$$

# HgTe-Quantum Wells

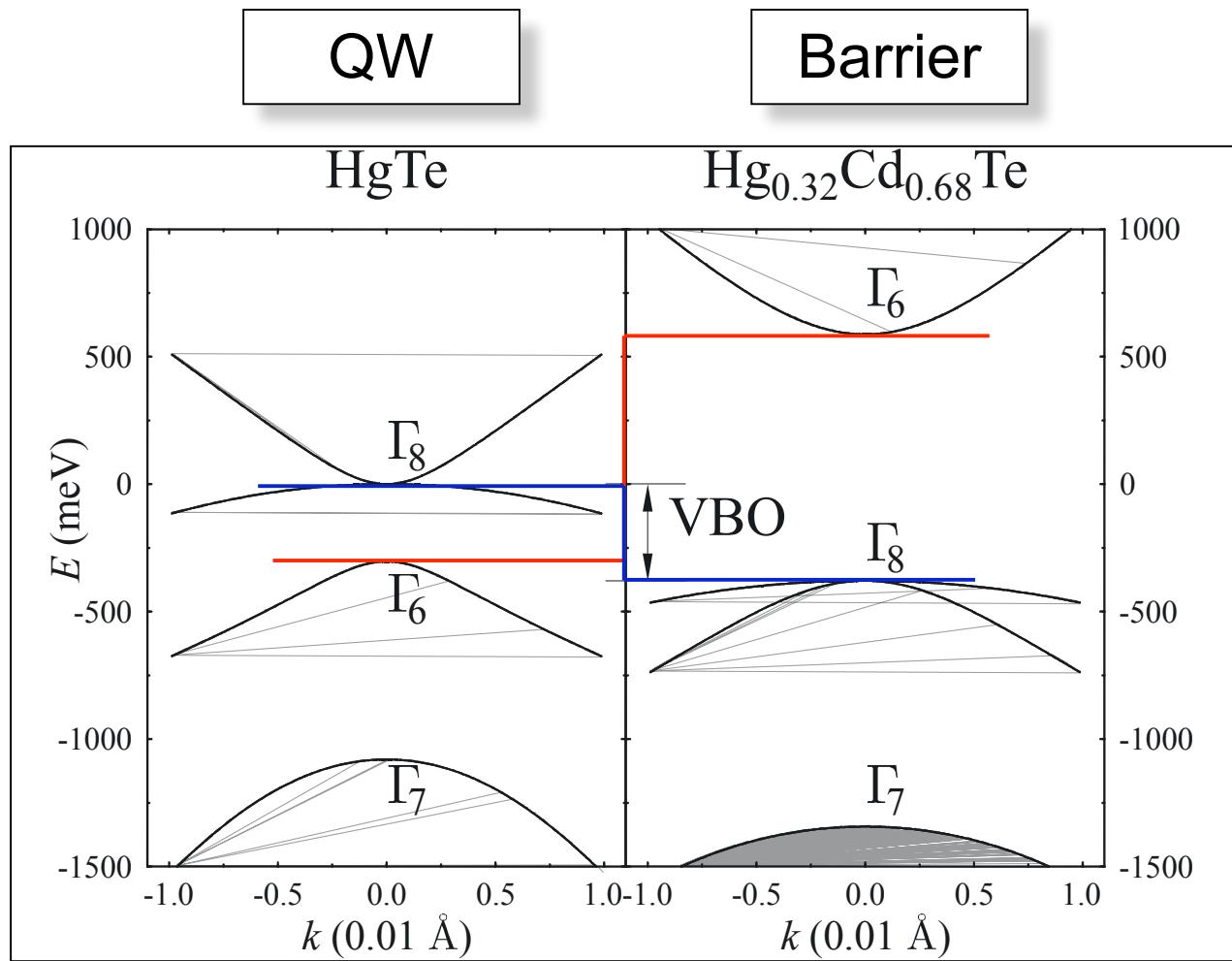
## MBE-Growth



## MBE-Growth Chamber



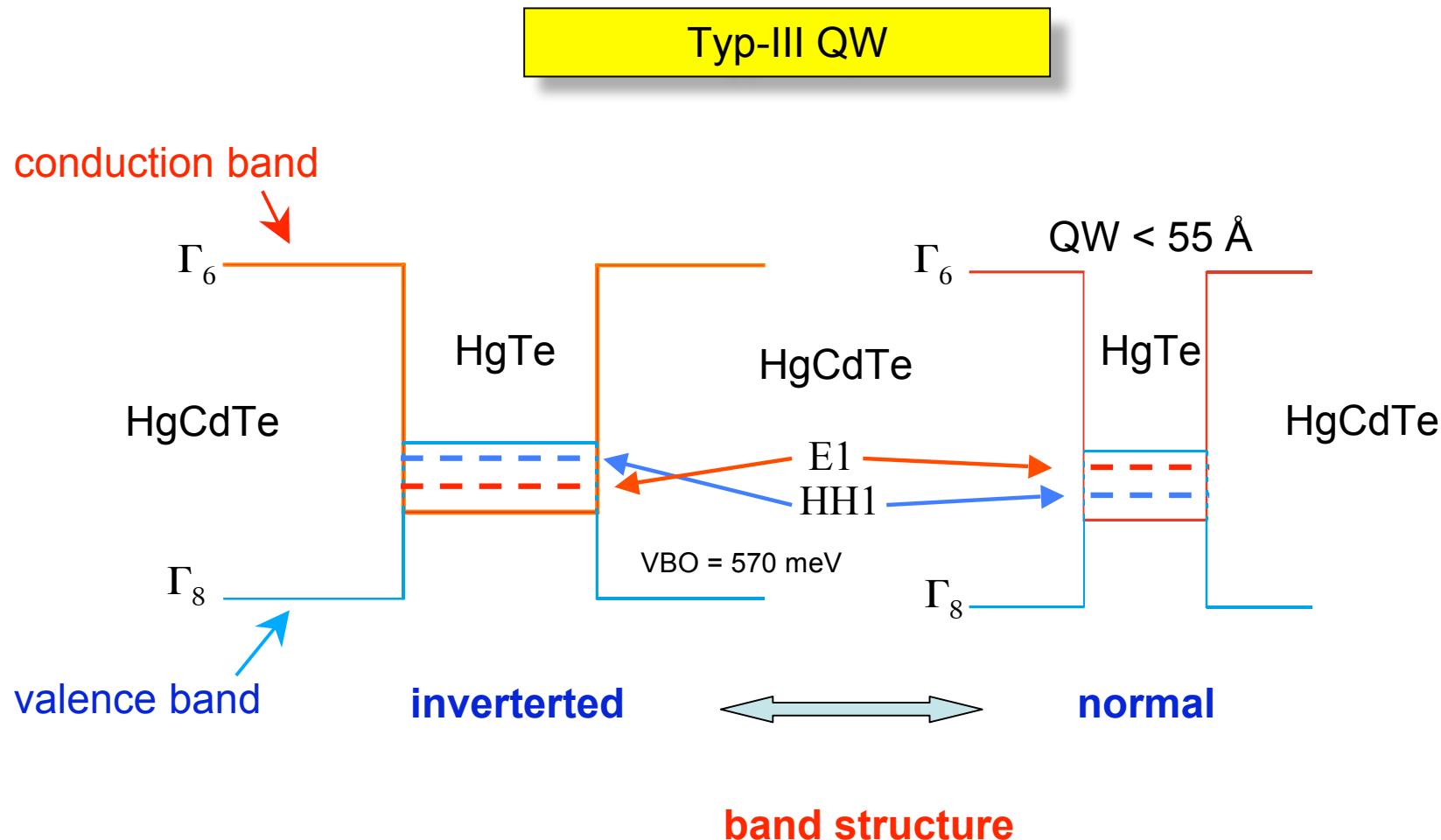
# HgTe-Quantum Wells



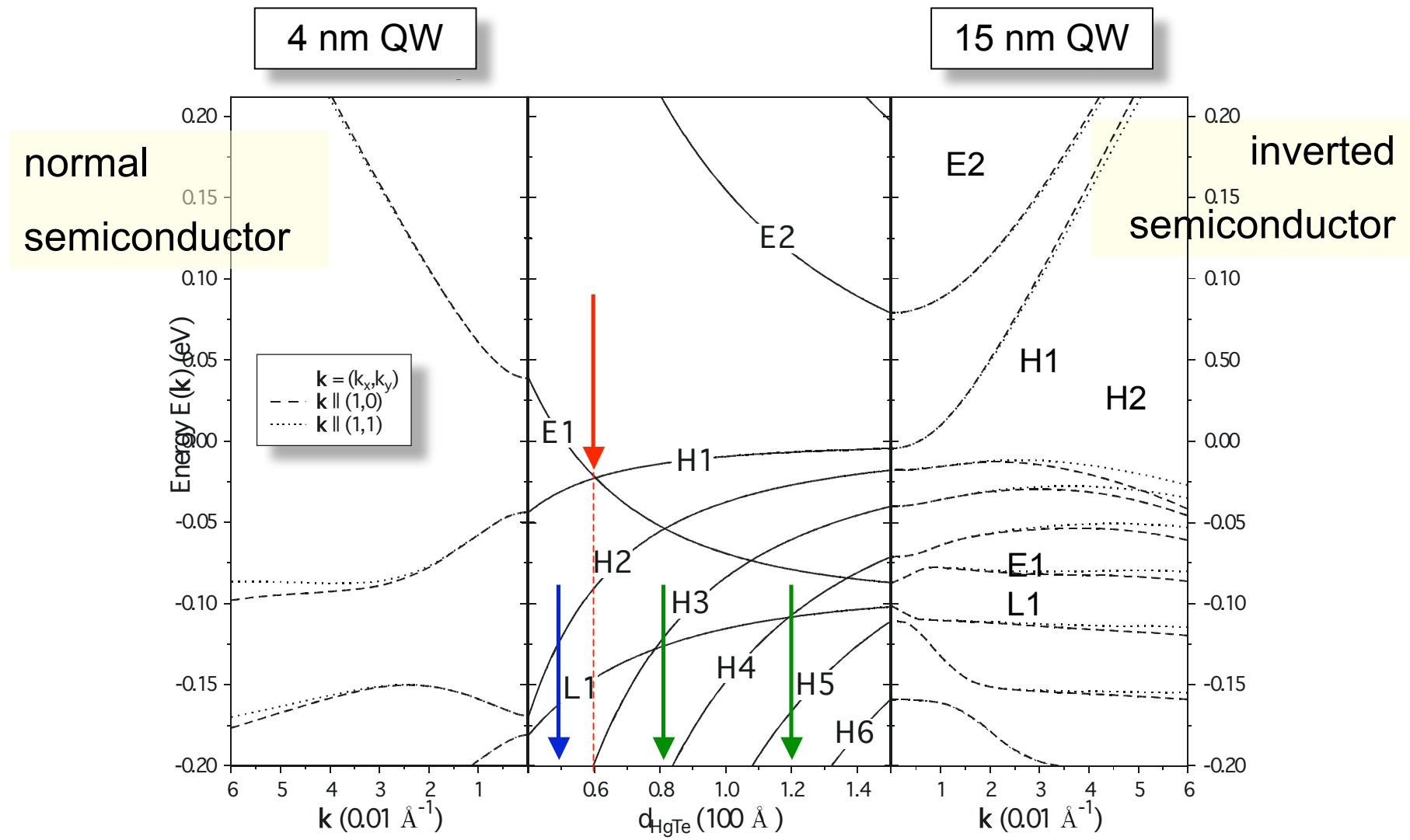
$$VBO = 570 \text{ meV}$$

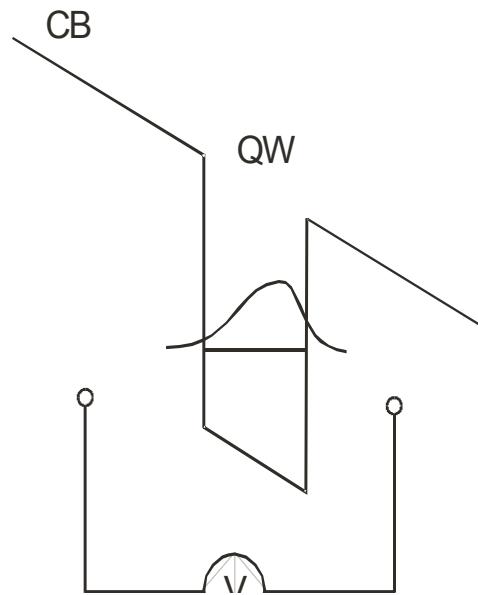
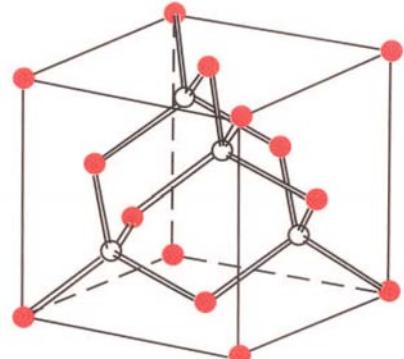
# HgTe-Quantum Wells

PD Dr. H. Buhmann



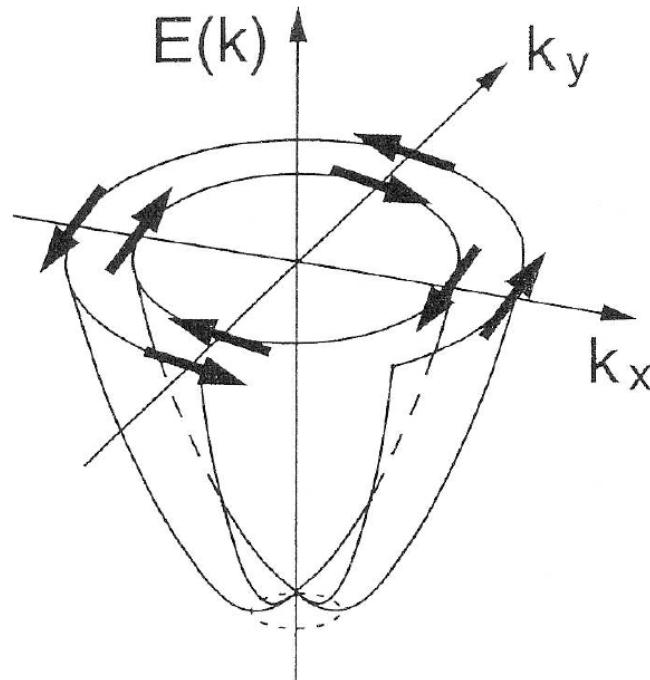
# Band Gap Engineering





structural inversion asymmetry (SIA)

$$H_R = \alpha_R (\sigma_x k_y - \sigma_y k_x)$$



$$E^\pm = E_i + \frac{\hbar^2 k_{\parallel}^2}{2m^*} \pm \alpha k_{\parallel}$$

(for electron and light hole bands)

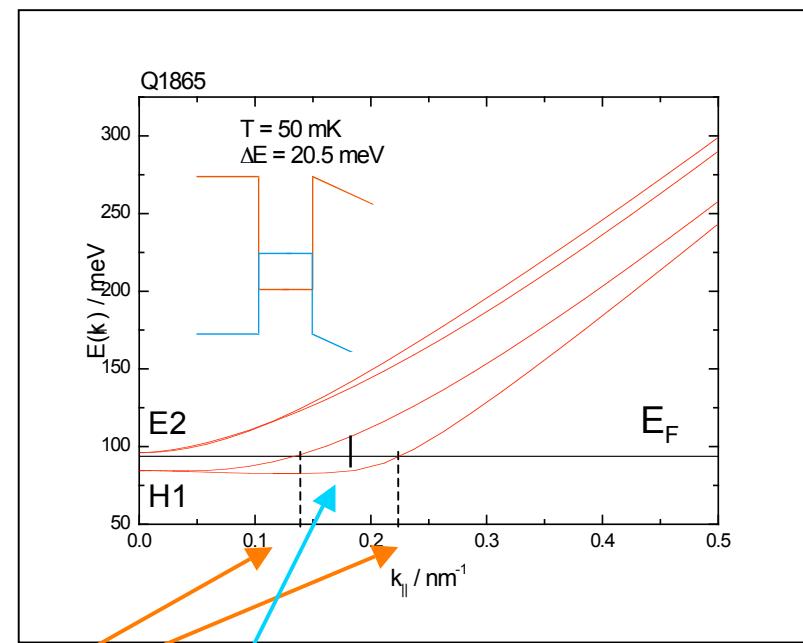
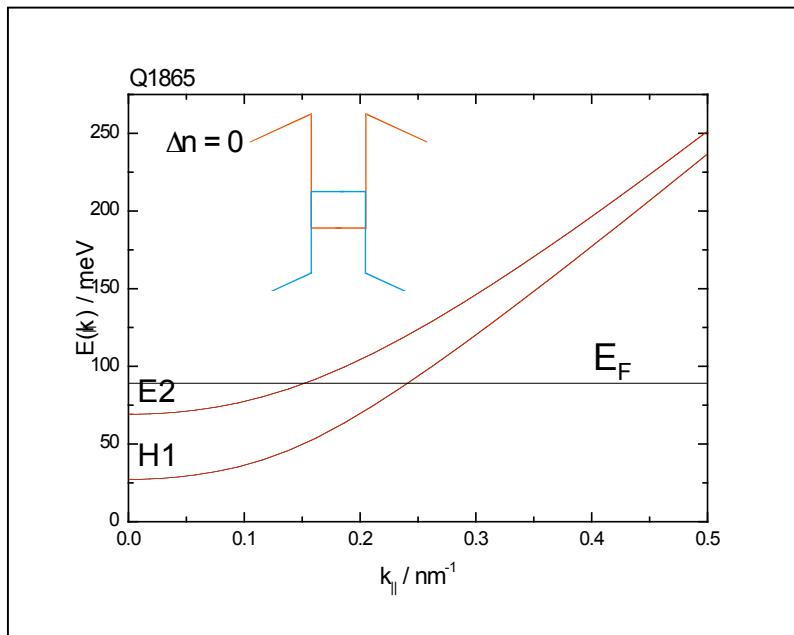
$$E^\pm = E_i + \frac{\hbar^2 k_{\parallel}^2}{2m^*} \pm \beta k_{\parallel}^3$$

(for heavy hole bands)

$8 \times 8 \mathbf{k} \cdot \mathbf{p}$  calculation

symmetric QW

asymmetric QW



E.G. Novik, HB, et al, PRB 72, 035321 (2005)

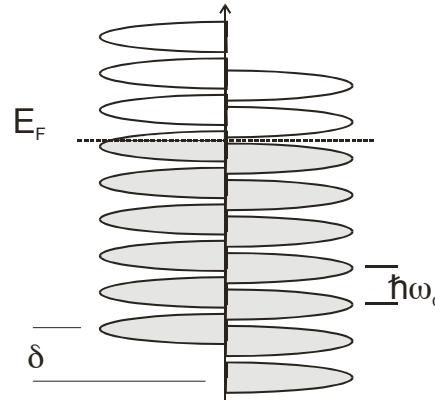
$\Delta n$

$\Delta R_{\max}$  up to 30 meV

Y.S. Gui, HB et al., PRB 70, 115328 (2004)

# Magneto-Transport

nodes in  
SdH:

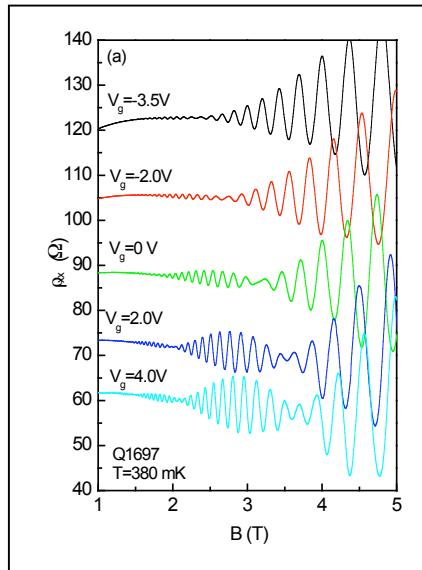


SdH-Amplitude:

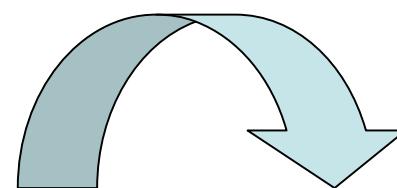
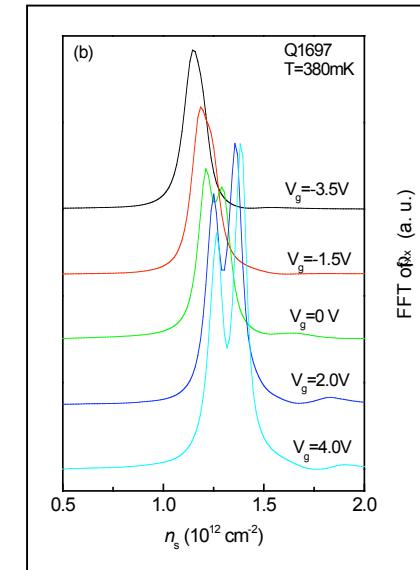
$$A \propto \cos(\pi\nu) \quad \nu = \frac{\delta}{\hbar\omega_c}$$

Y.S. Gui, HB et al., Europhys. Lett. **65**, 393 (2004)

SdH



FFT



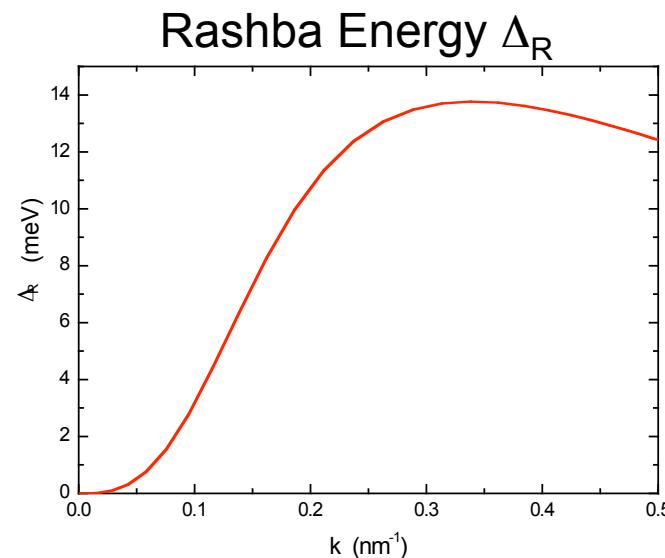
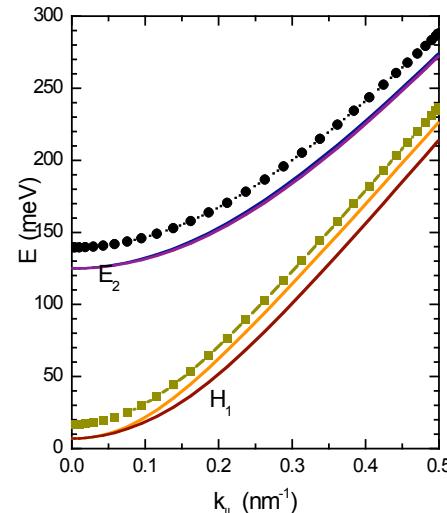
# Rashba Spin-Splitting Energy

$$E^\pm = E_i + \frac{\hbar^2 k_{\parallel}^2}{2m^*} \pm \alpha k_{\parallel}$$

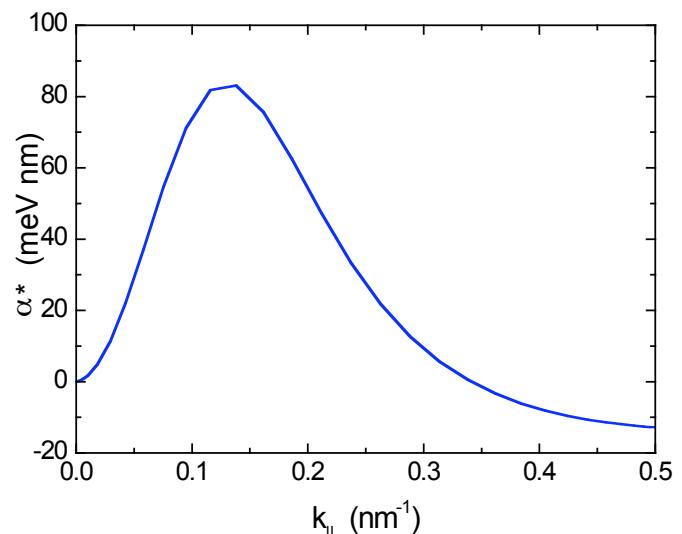
(for electron and light hole bands)

$$E^\pm = E_i + \frac{\hbar^2 k_{\parallel}^2}{2m^*} \pm \beta k_{\parallel}^3$$

(for heavy hole bands)

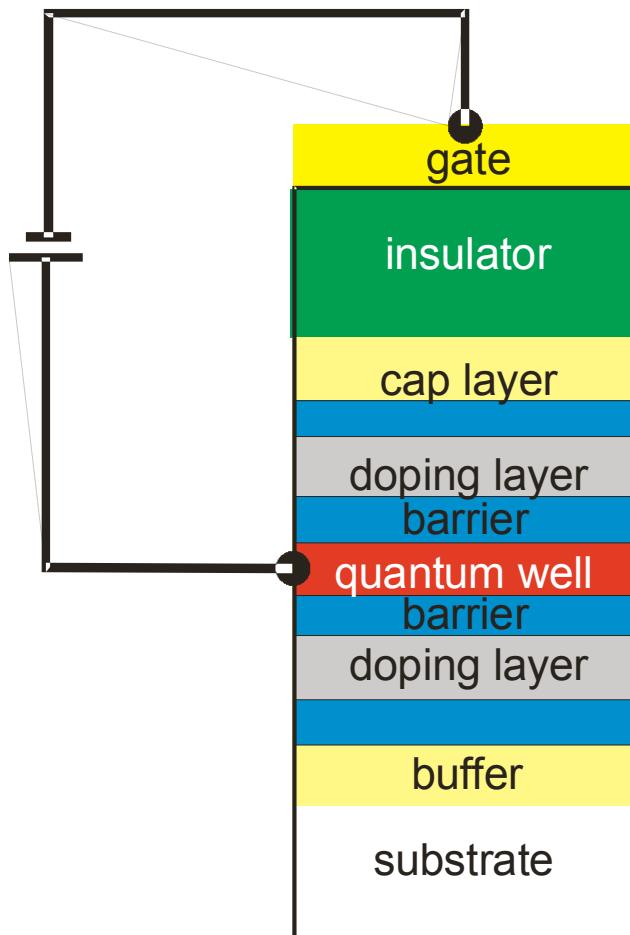


Rashba Coefficient  $\alpha$



Y.S. Gui, HB et al., PRB 70, 115328 (2004)  
E.G. Novik, HB, et al, PRB 72, 035321 (2005)

# Layer Structure



Carrier densities:  $n_s = 1 \times 10^{11} \dots 2 \times 10^{12} \text{ cm}^{-2}$

Carrier mobilities:  $\mu = 1 \times 10^5 \dots 1 \times 10^6 \text{ cm}^2/\text{Vs}$

Au

100 nm  $\text{Si}_3\text{N}_4/\text{SiO}_2$

25 nm CdTe

10 nm HgCdTe  $x = 0.7$

9 nm HgCdTe with I

10 nm HgCdTe  $x = 0.7$

4 - 12 nm HgTe

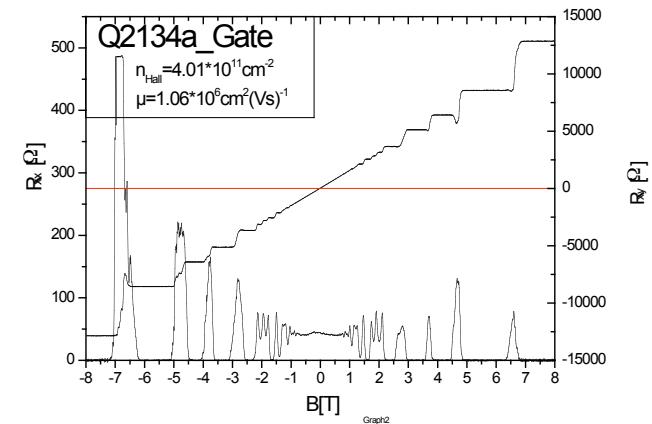
10 nm HgCdTe  $x = 0.7$

9 nm HgCdTe with I

10 nm HgCdTe  $x = 0.7$

25 nm CdTe

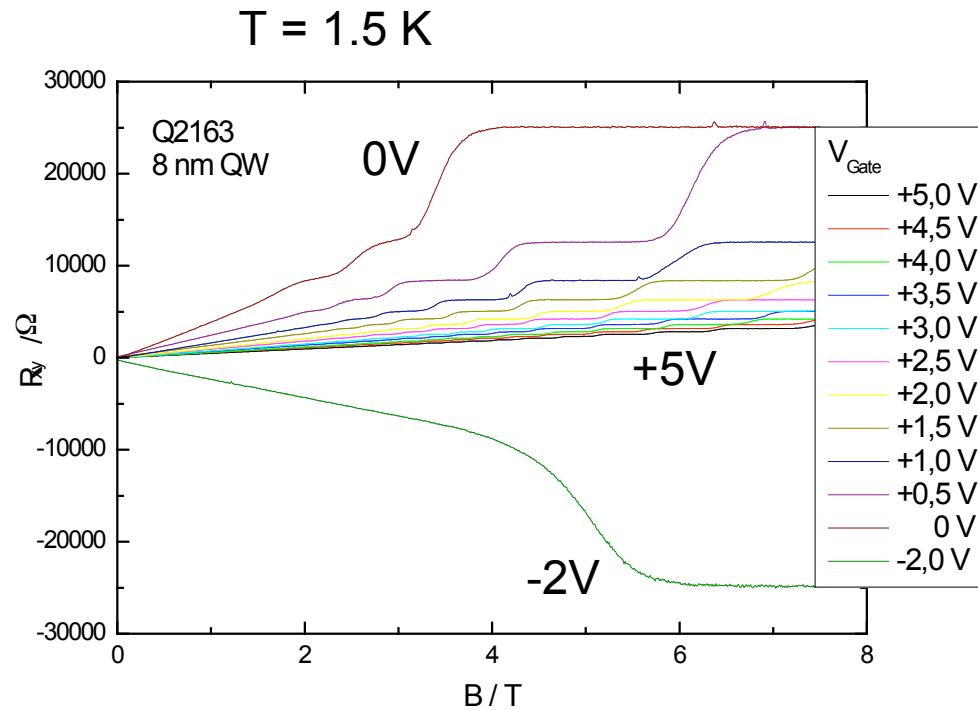
CdZnTe(001)



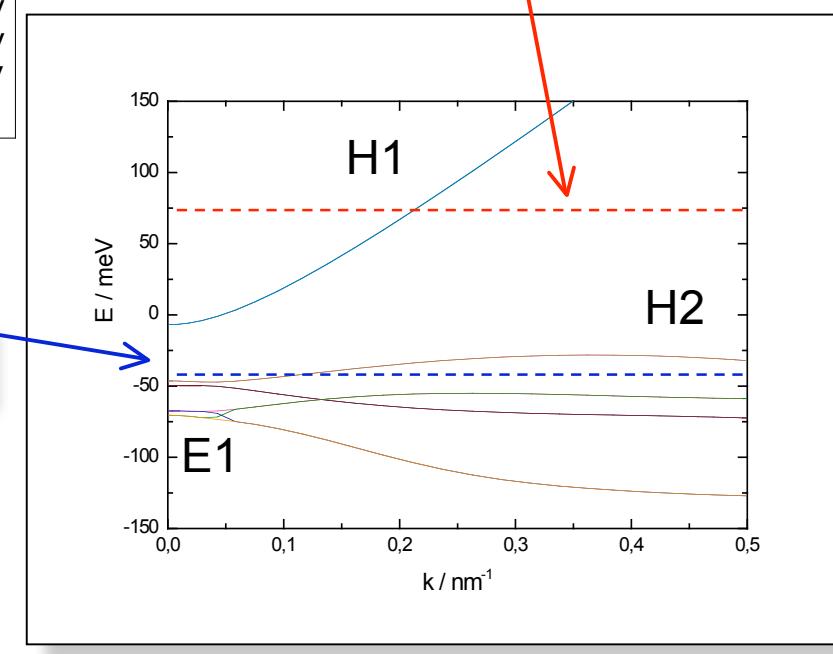
symmetric or asymmetric  
doping

# Gated Low Carrier Densities Samples

# n to p Transitions

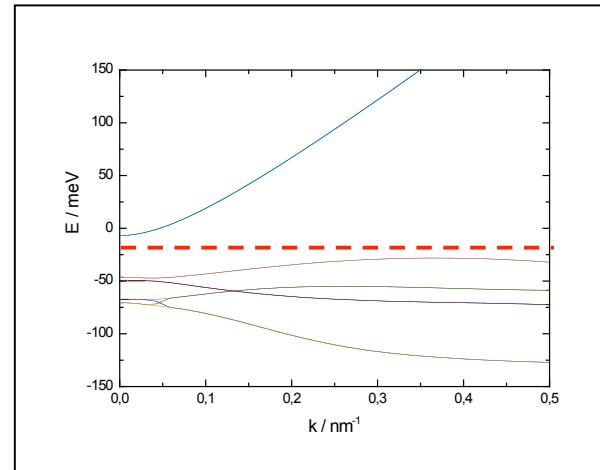


$$n_{\max} = 1.35 \times 10^{12} \text{ cm}^{-2}$$

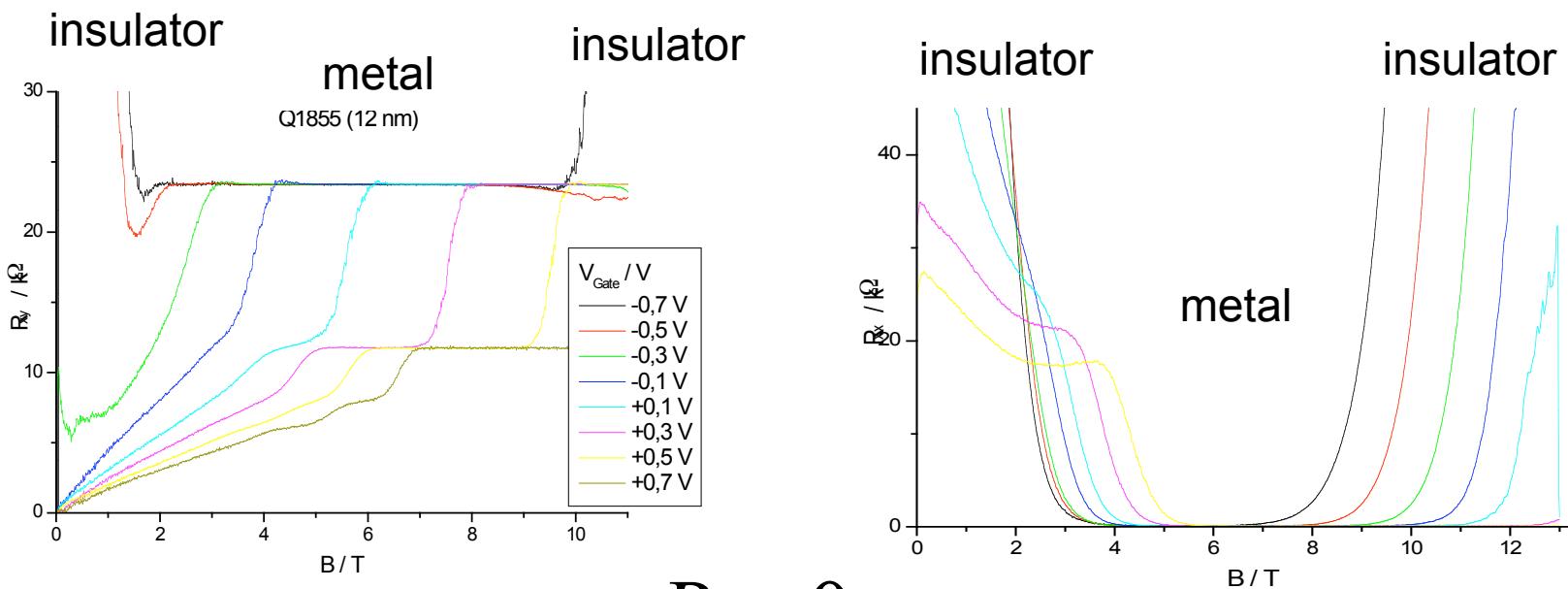


# Magneto-Resistance in the insulating regime

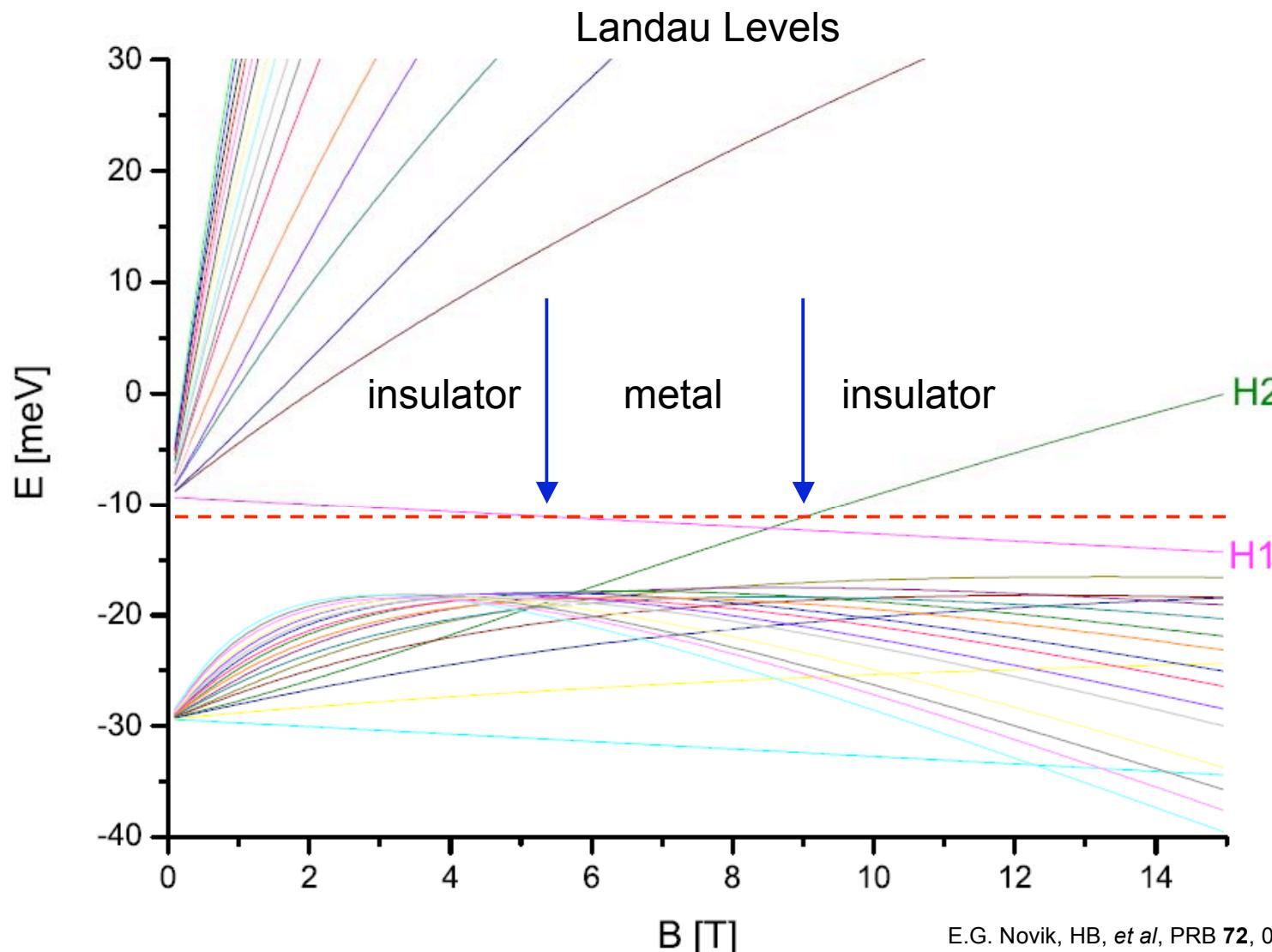
Hall



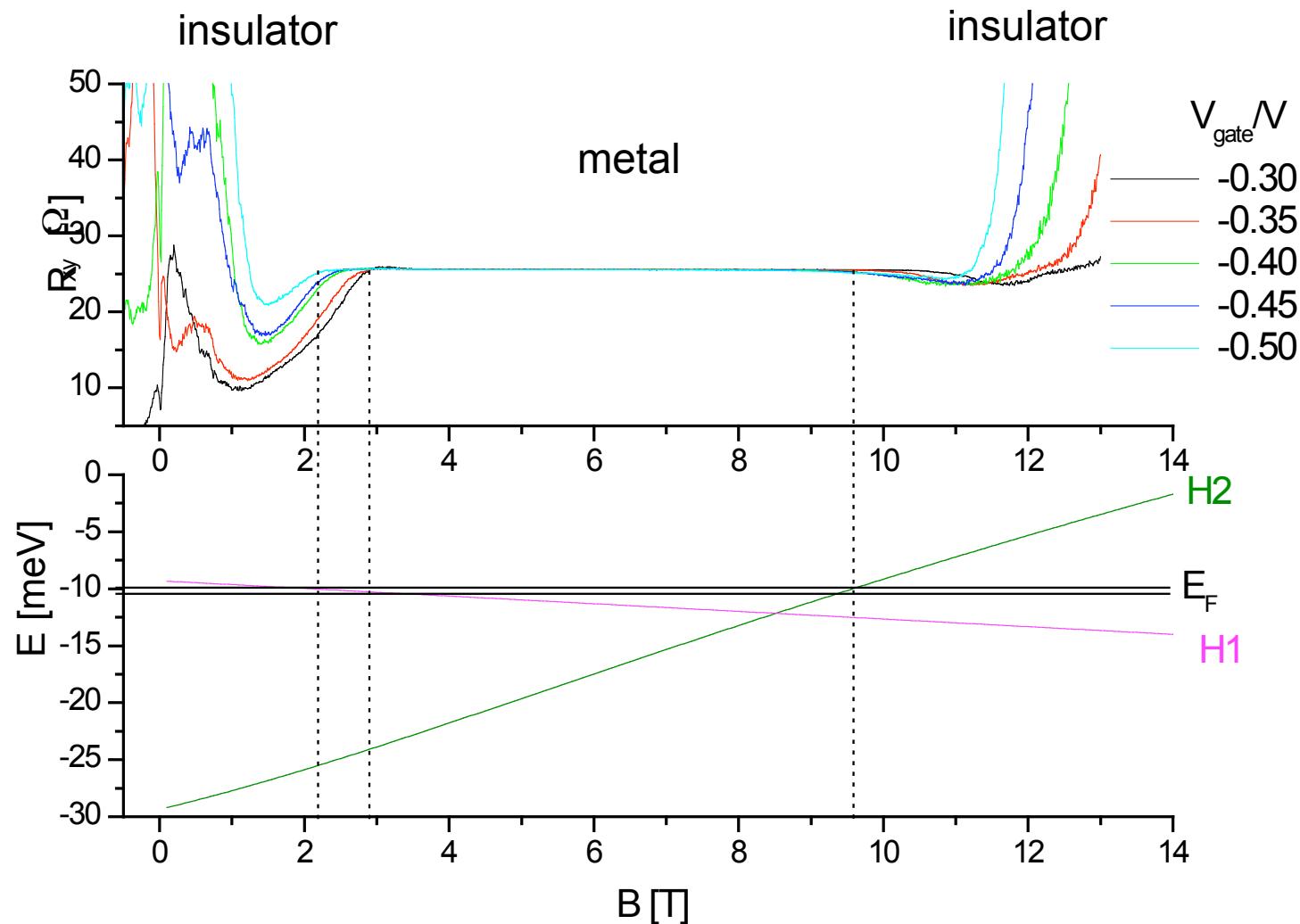
SdH



# Insulator-Metal-Insulator Transition



# Insulator-Metal-Insulator Transition

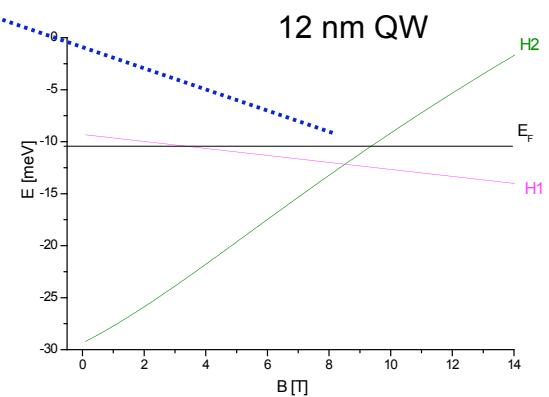
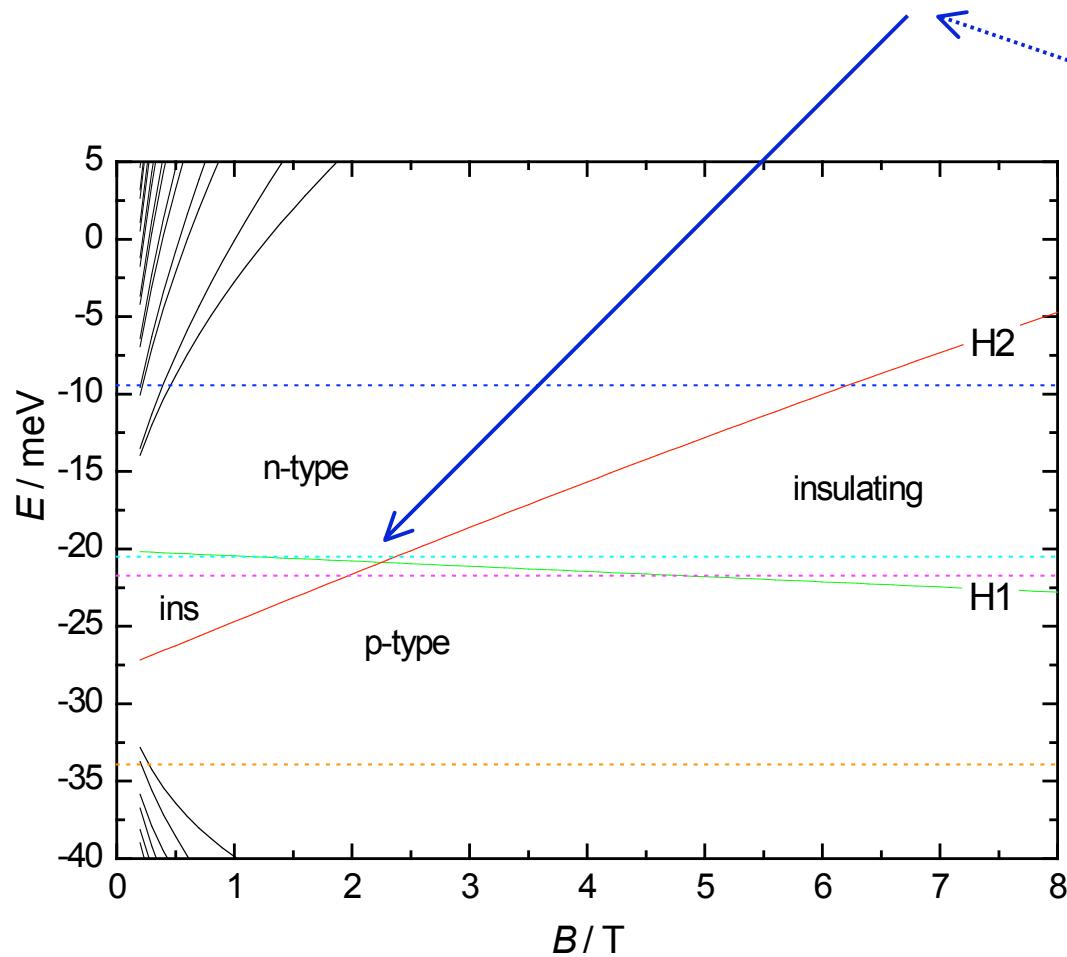


E.G. Novik, HB, *et al*, PRB **72**, 035321 (2005)

# Insulator-Metal-Insulator Transition

6.5 nm QW

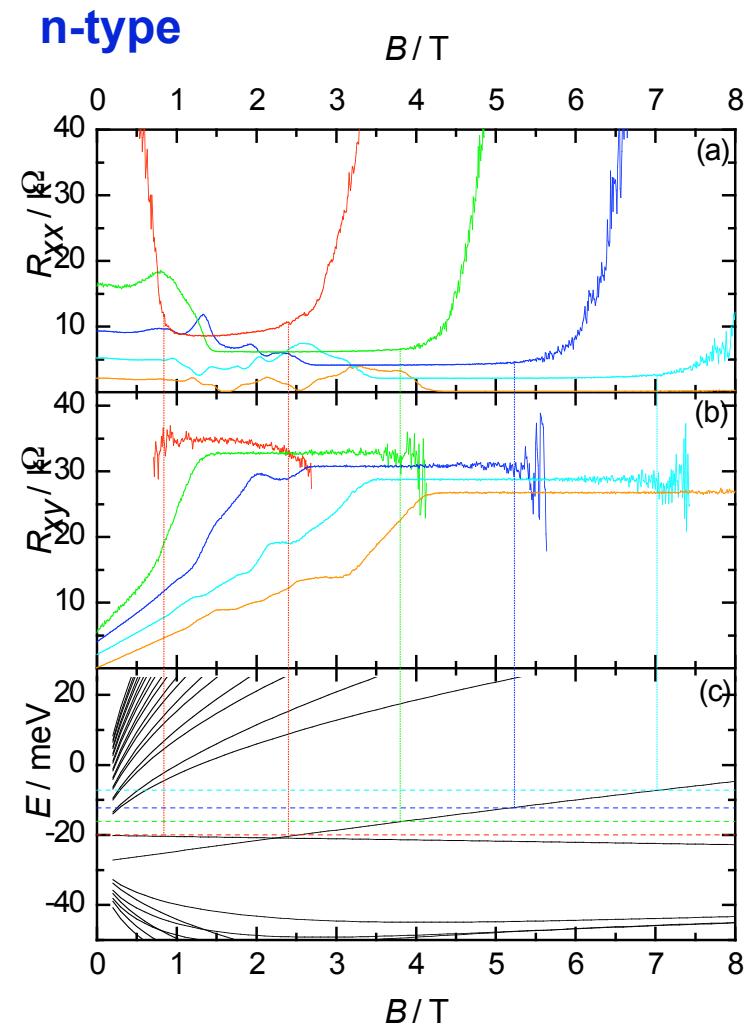
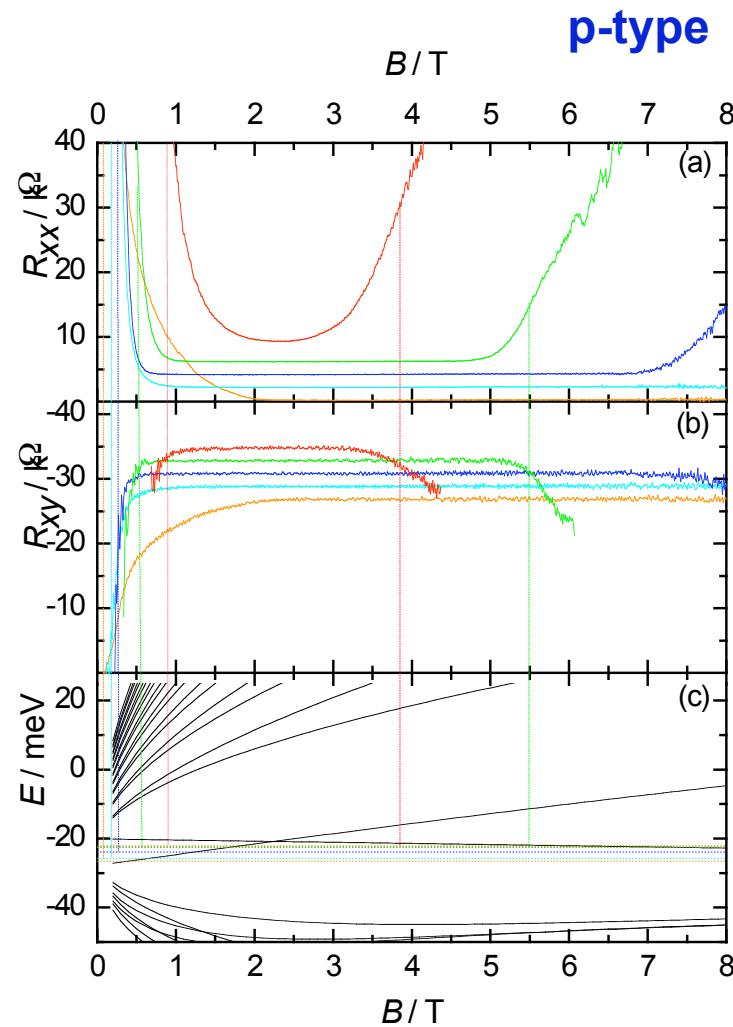
(shifts the LL crossing to lower magnetic fields)



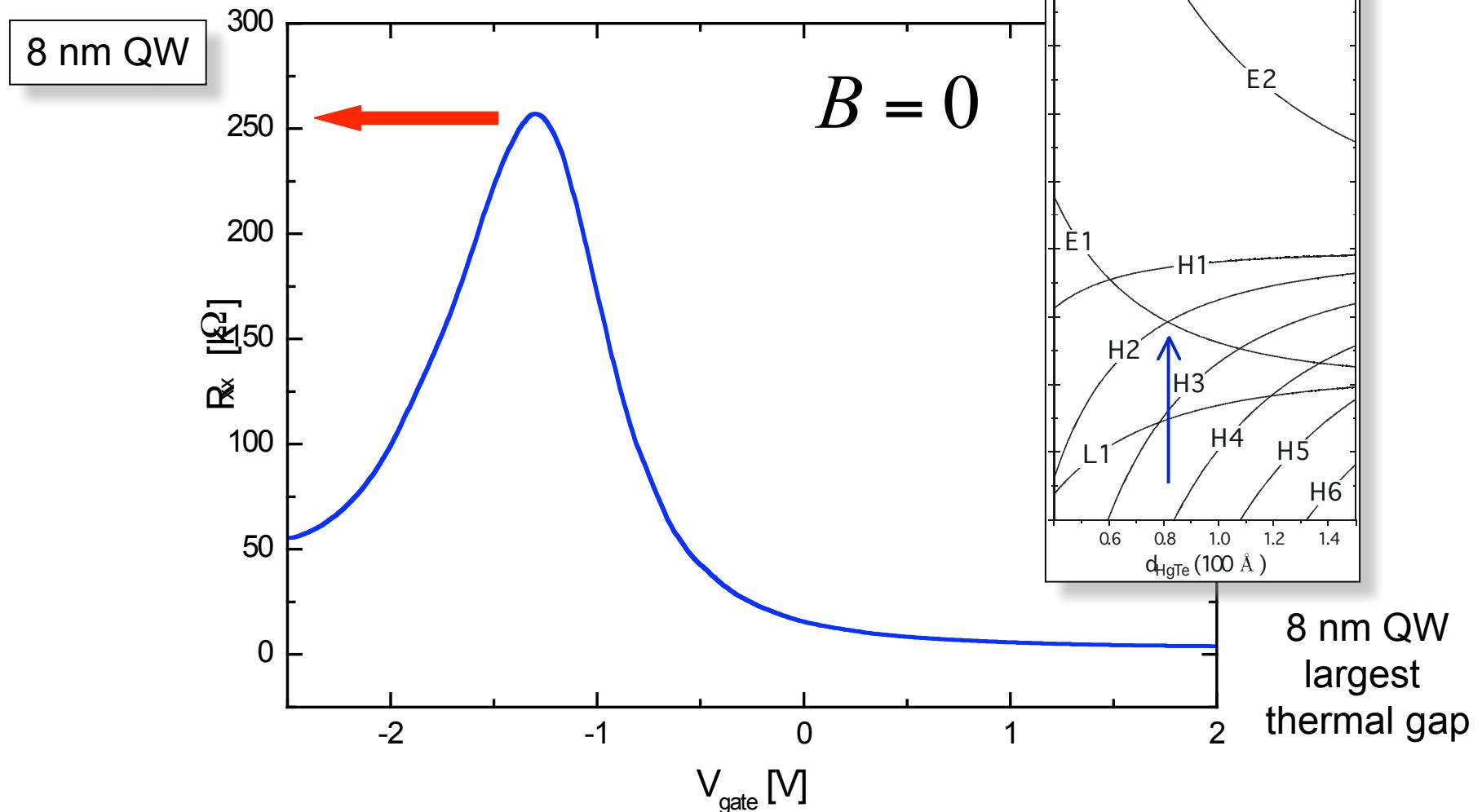
E.G. Novik, HB, et al, PRB **72**, 035321 (2005)

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6.5 nm QW

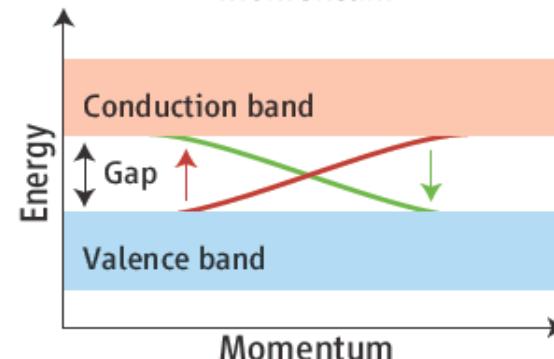
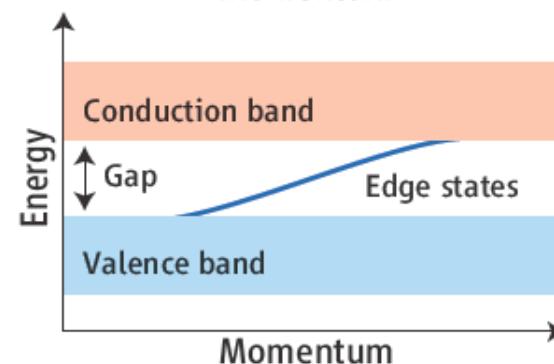
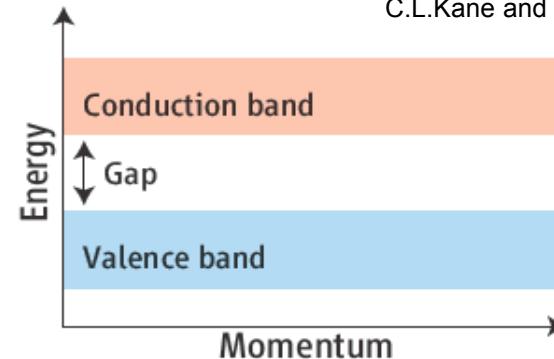
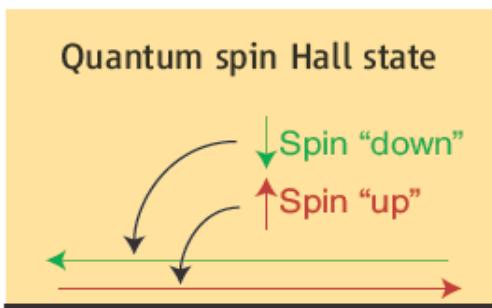
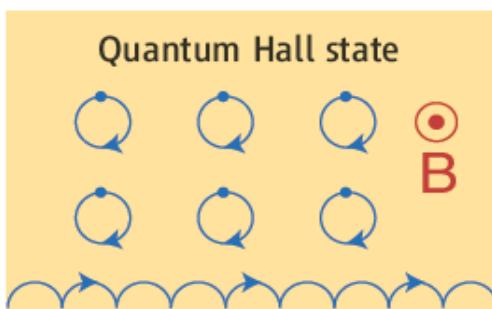
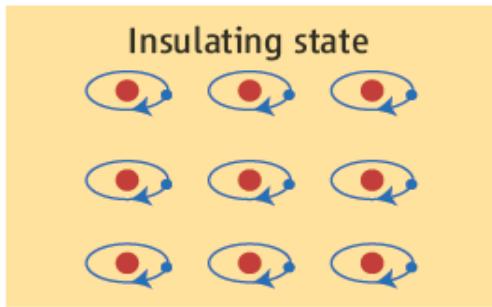


# Finite conductance in the insulating regime



# QSH insulator

# Topological Quantization

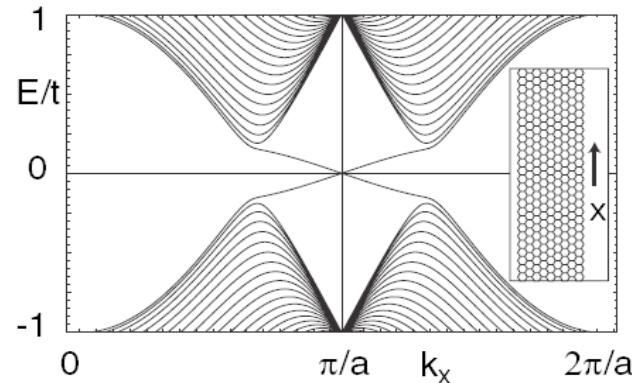


C.L.Kane and E.J.Mele, PRL **95**, 146802 (2005)  
C.L.Kane and E.J.Mele, PRL **95**, 226801 (2005)

C.L.Kane and E.J. Mele, Science **314**, 1692 (2006)

# Topological Quantization

## Graphene edge states



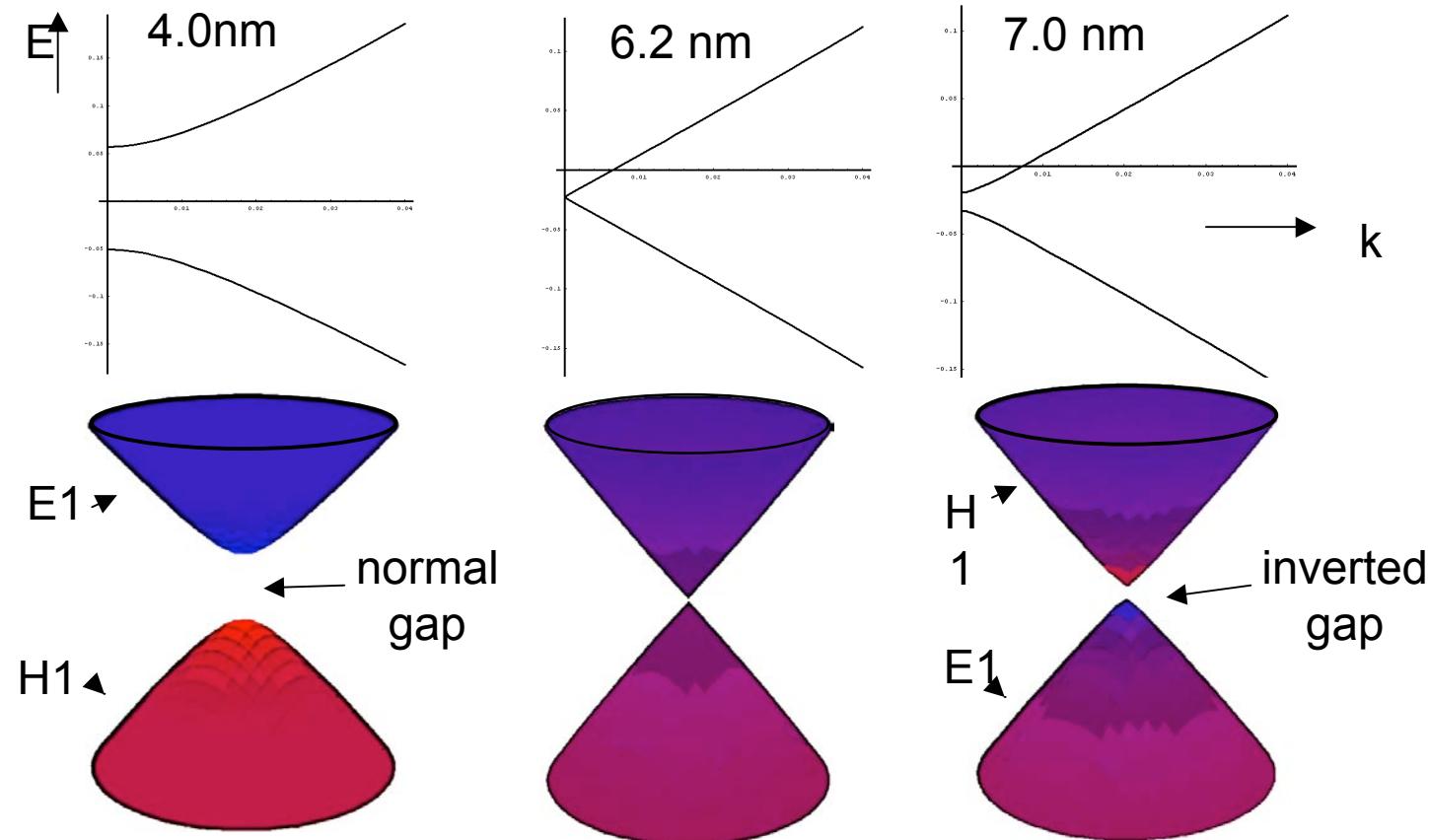
C.L.Kane and E.J.Mele, PRL **95**, 146802 (2005)  
 C.L.Kane and E.J.Mele, PRL **95**, 226801 (2005)



Conductance:  $G = \frac{2e^2}{h}$

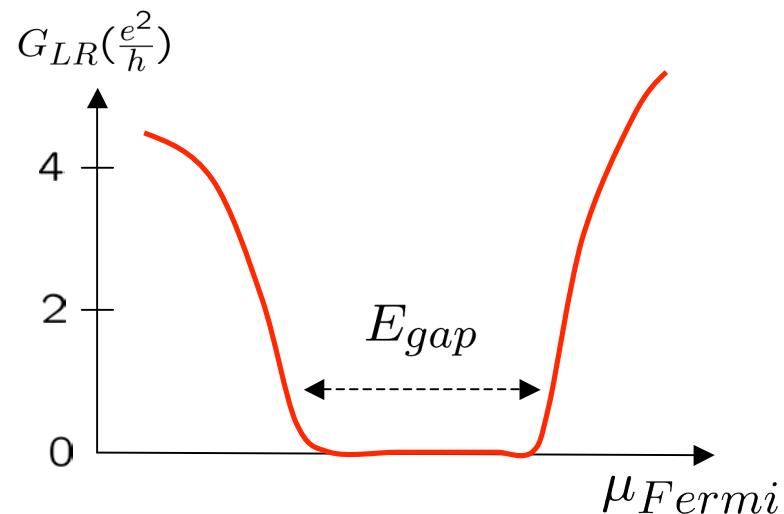
# Bandstructure HgTe

B.A Bernevig, T.L. Hughes, S.C. Zhang, Science **314**, 1757 (2006)

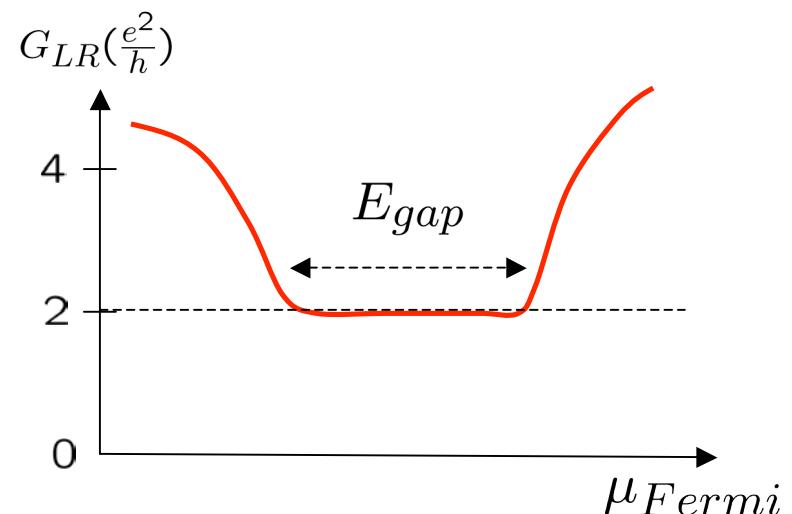
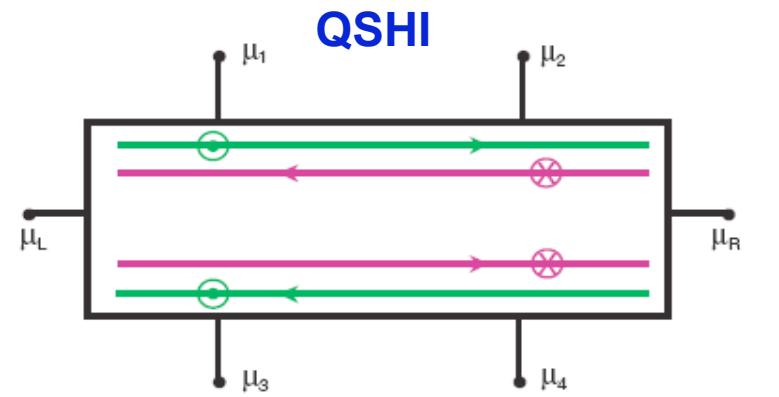


# Experimental Signature

normal insulator state

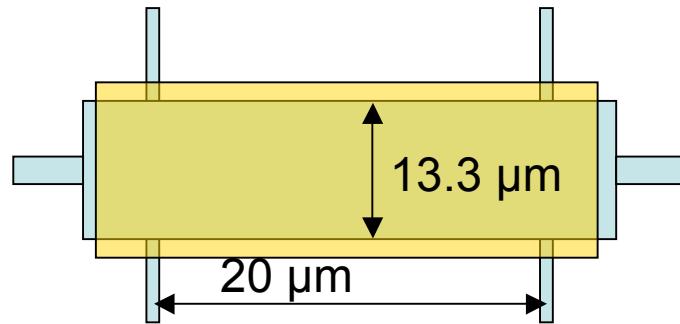


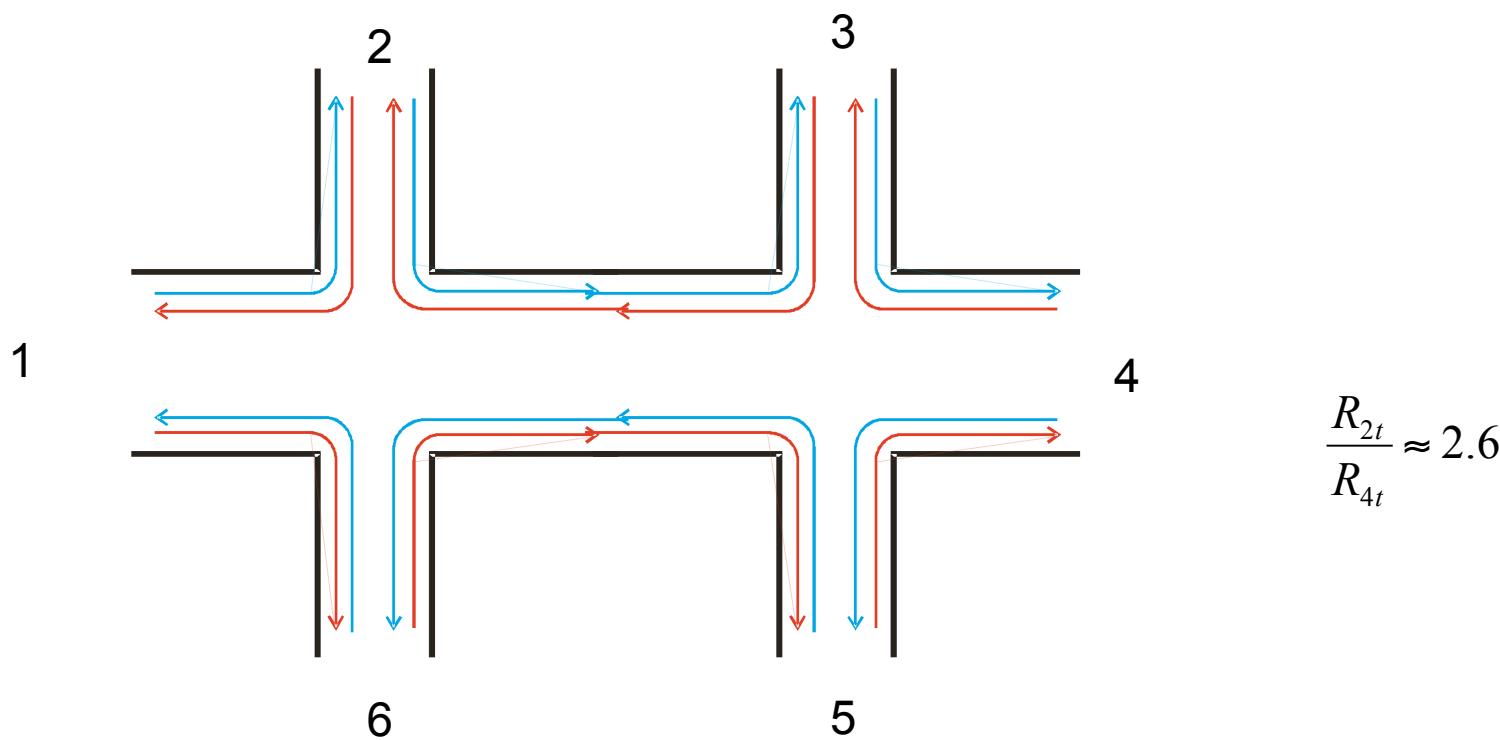
$d < d_c$ , normal regime



$d > d_c$ , inverted regime

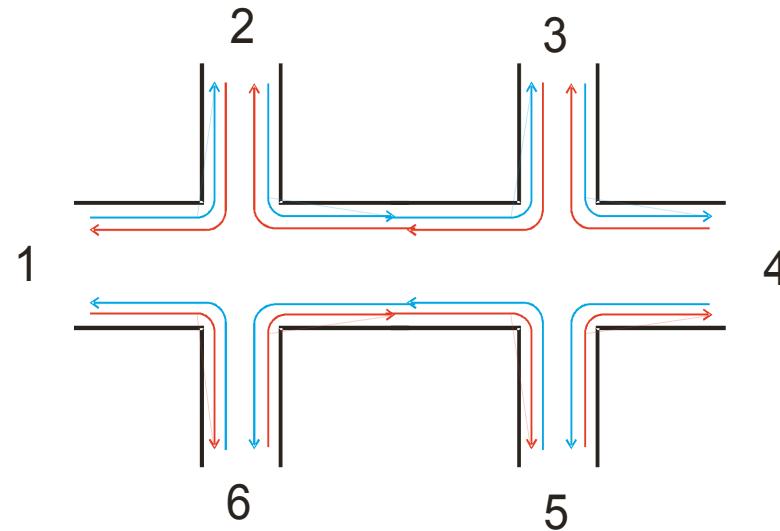
sample layout





measurements show that 4-terminal conductance is comparable to 2-terminal

# Multi-Terminal Probe



$$T = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

X.L. Qi (Stanford Univ.)

$$\Rightarrow \begin{cases} G_{2t} = \frac{I_{14}}{\mu_4 - \mu_1} = \frac{2e^2}{3h} \\ G_{4t} = \frac{I_{14}}{\mu_3 - \mu_2} = \frac{2e^2}{h} \end{cases}$$

$$\left( G_{4t,\text{exp}} \approx 1.9 \frac{e^2}{h} \right)$$

generally

$$R_{2t} = \frac{(n+1)h}{2e^2}$$

$$\Rightarrow \frac{R_{2t}}{R_{4t}} = 3 \quad (\approx 2.6)_{\text{exp}}$$

# QSHI

## Summary 1:

signatures for QSH insulator state  
for inverted HgTe QW  
micro-structures

multi-terminal resistance  
corresponds to expected values for  
helical edge states

# SHE

# Spin-Hall Effect

VOLUME 83, NUMBER 9

PHYSICAL REVIEW LETTERS

30 AUGUST 1999

## Spin Hall Effect

J. E. Hirsch

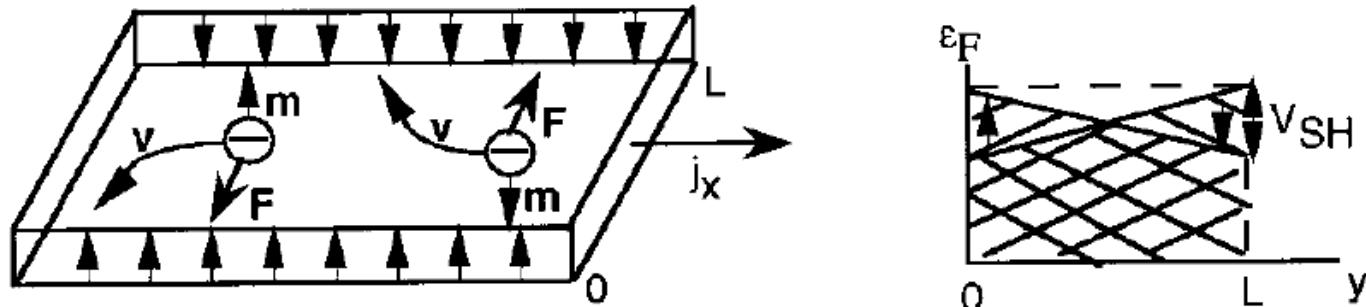
*Department of Physics, University of California, San Diego, La Jolla, California 92093-0319*

(Received 24 February 1999)

It is proposed that when a charge current circulates in a paramagnetic metal a transverse spin imbalance will be generated, giving rise to a "spin Hall voltage." Similarly, it is proposed that when a spin current circulates a transverse charge imbalance will be generated, giving rise to a Hall voltage, in the absence of charge current and magnetic field. Based on these principles we propose an experiment to generate and detect a spin current in a paramagnetic metal.

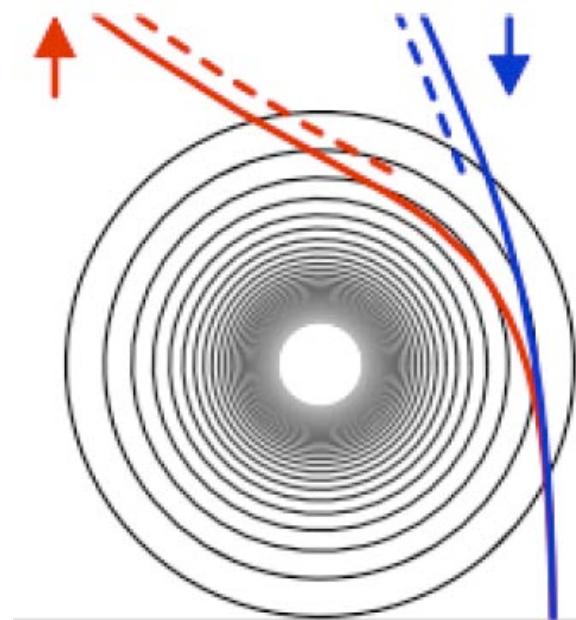
PACS numbers: 72.15.Gd, 73.61.At

## Spin Hall effect



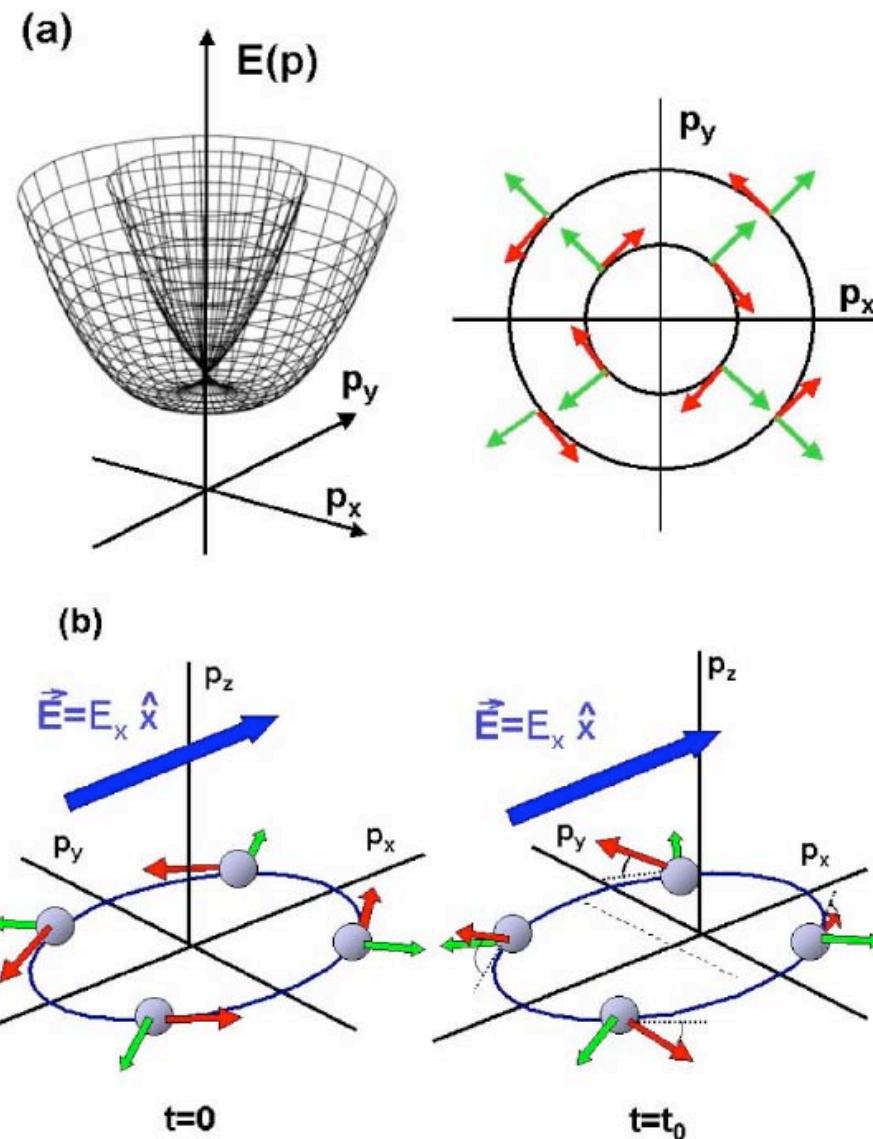
extrinsic

- skew scattering
- side jump effect

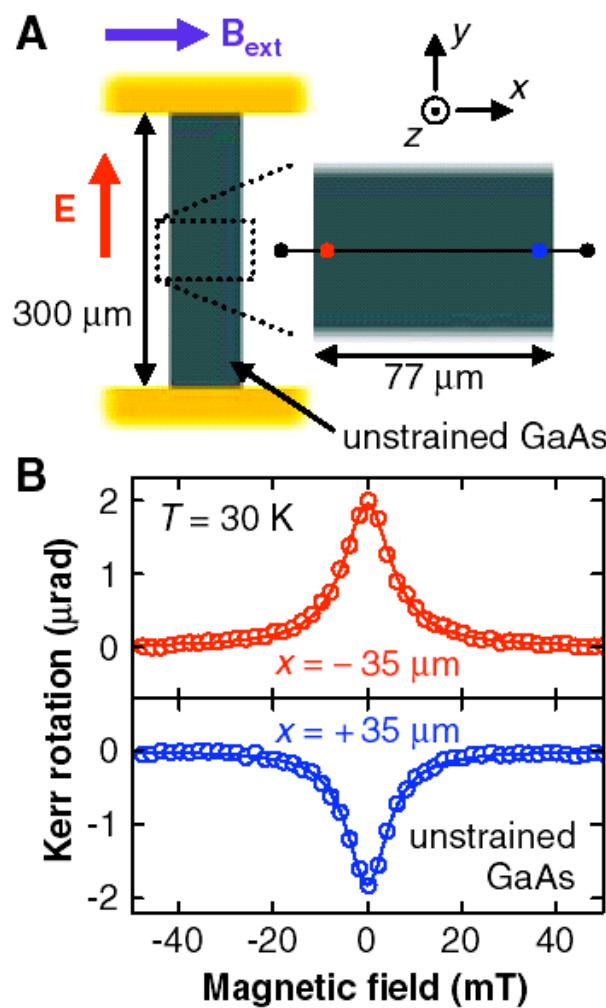


## 2. Intrinsic SHE

Rashba effect

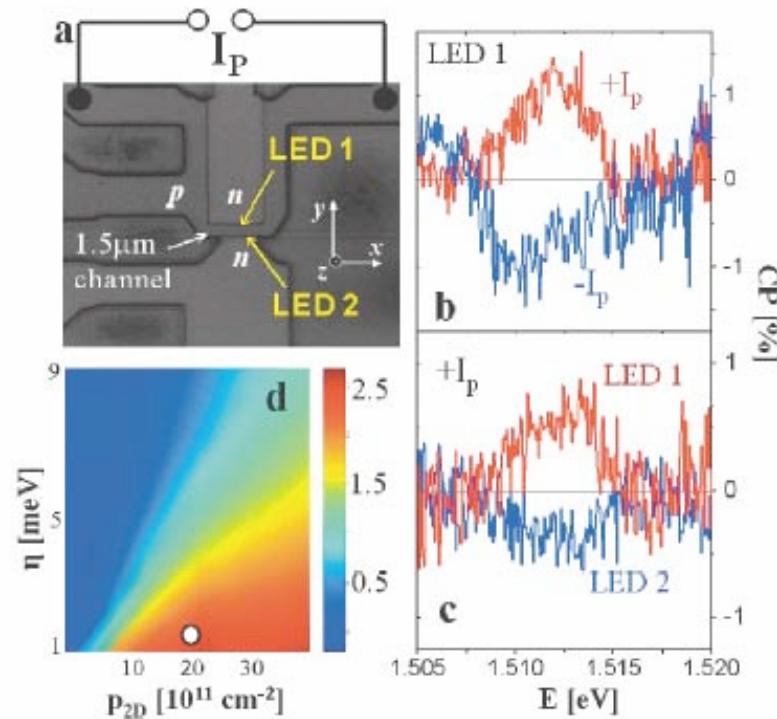


J.Sinova et al.,  
Phys. Rev. Lett. **92**, 126603 (2004)



Kato et al. Science 306, 1910 (2004)

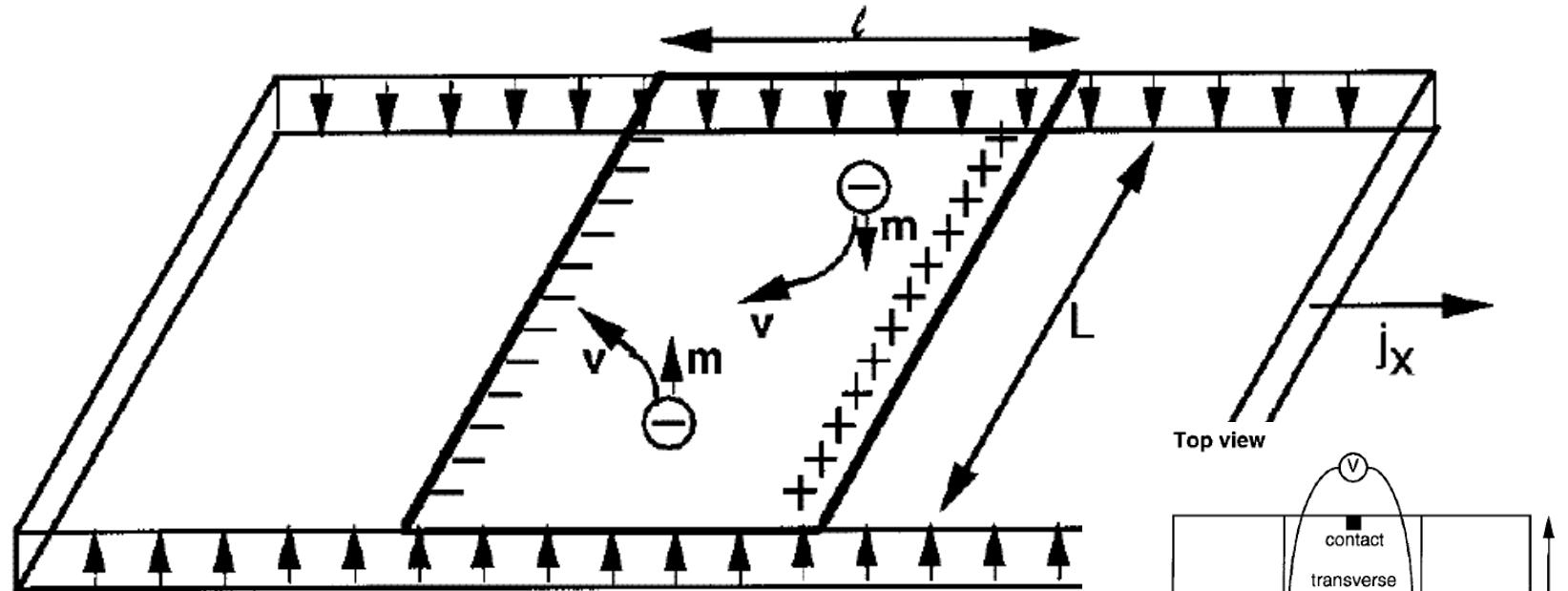
## optical detection



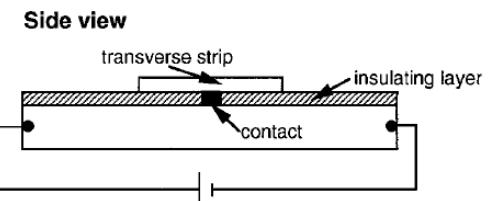
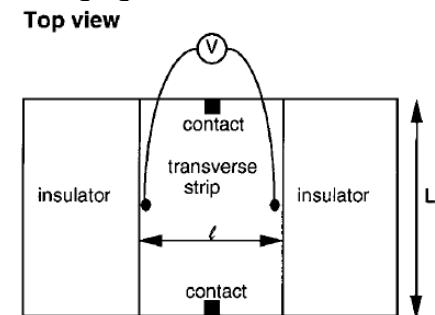
Wunderlich et al. PRL 94, 47204 (2005)

# Spin-Hall Effect

electrical detection



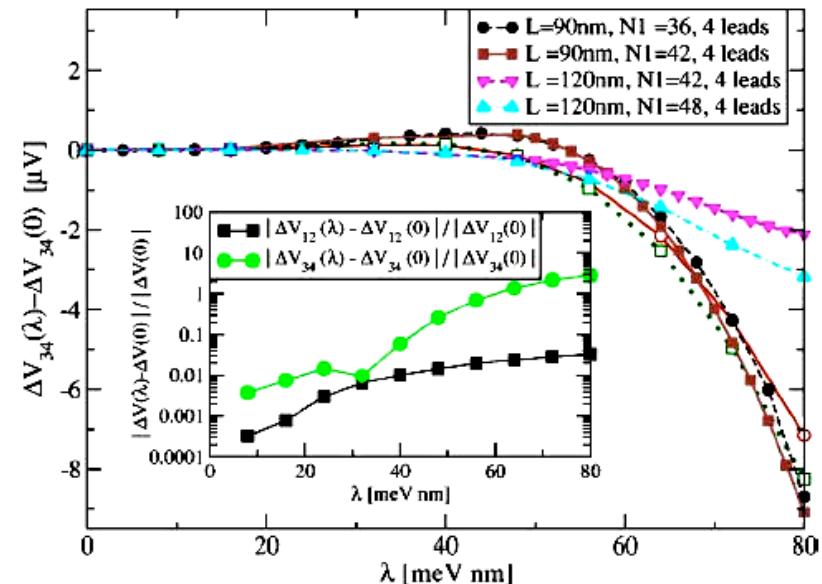
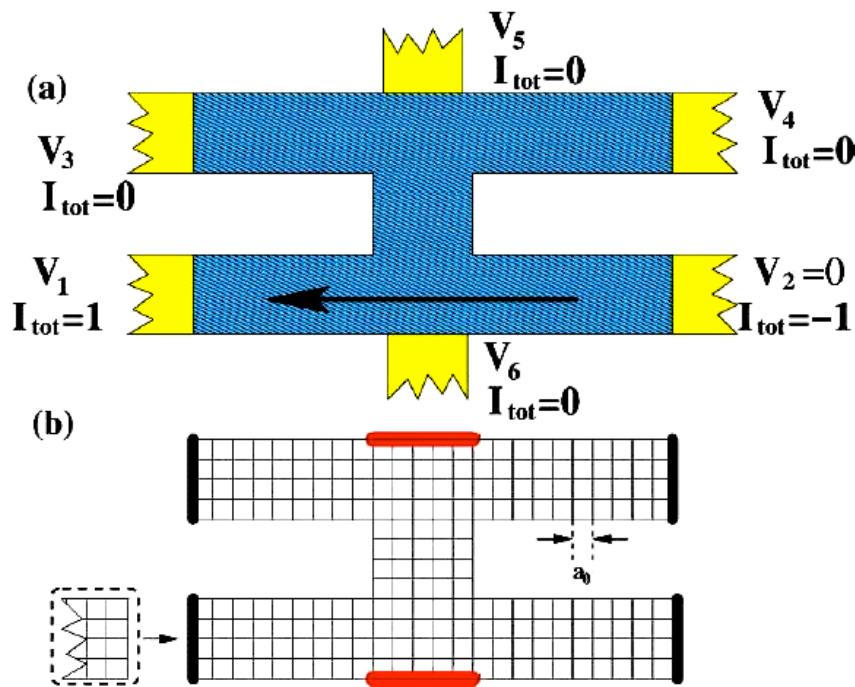
**SHE-1**



Hirsch PRL 83, 1834 (1999)

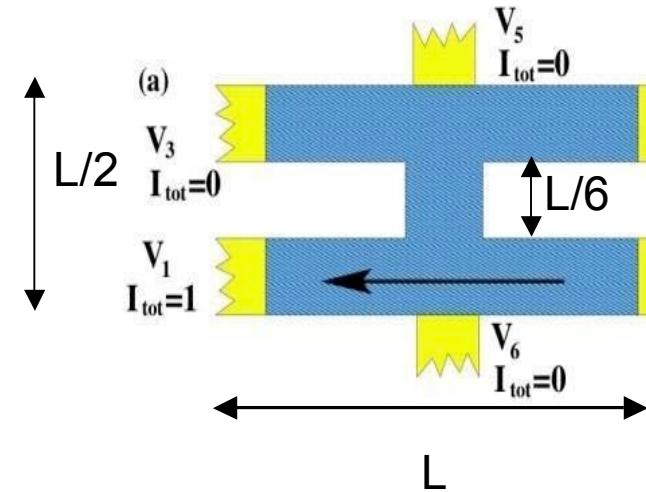
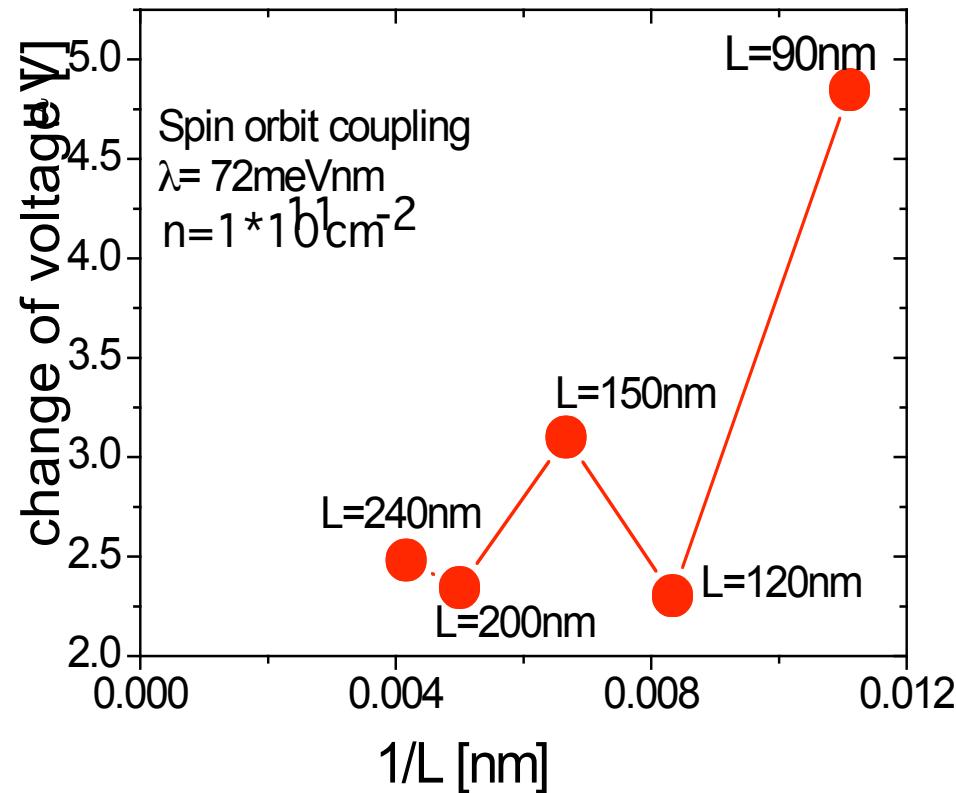
# Spin-Hall Effect

electrical detection

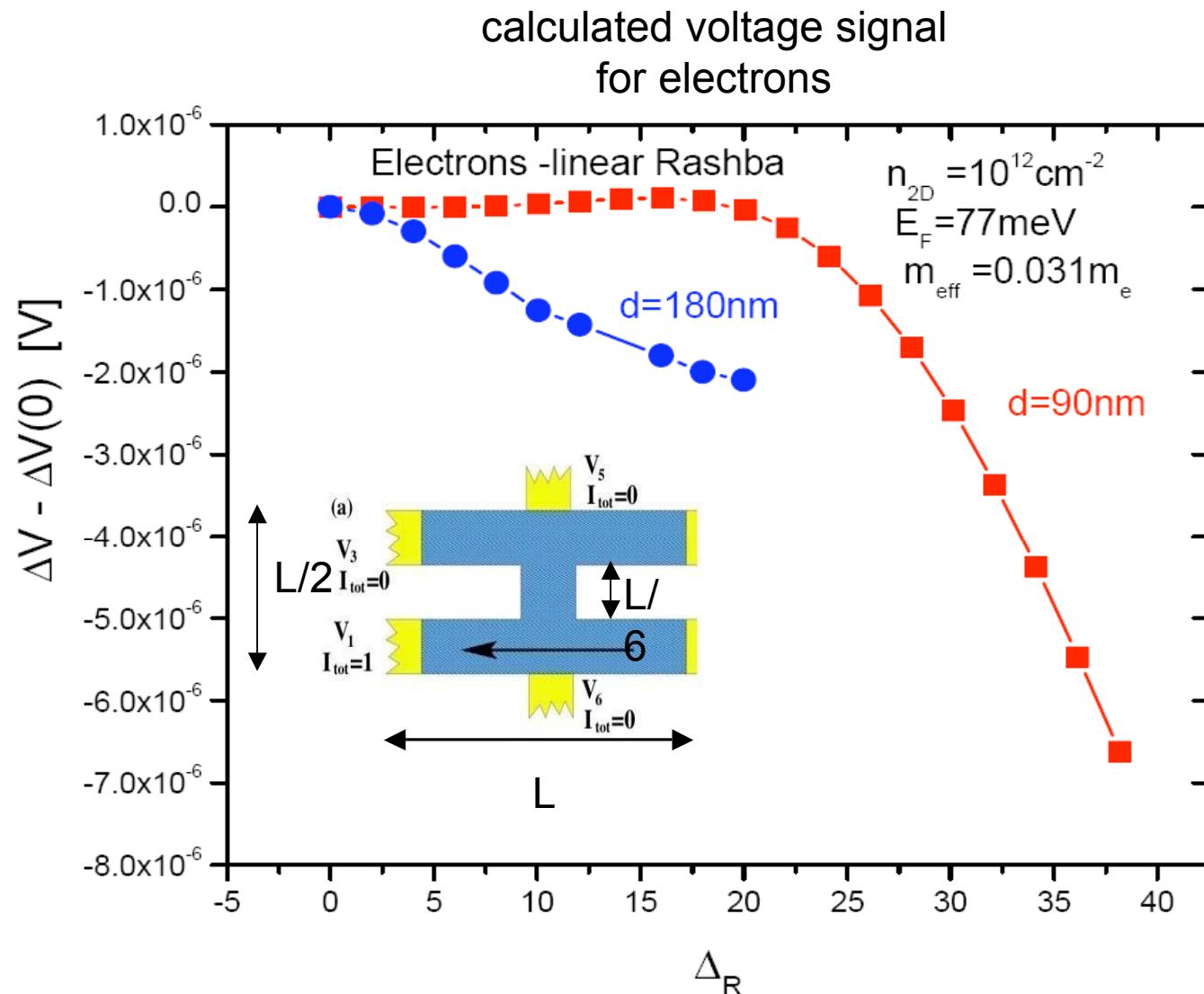


E.M. Hankiewicz et al . PRB 70, 241301 (2004)

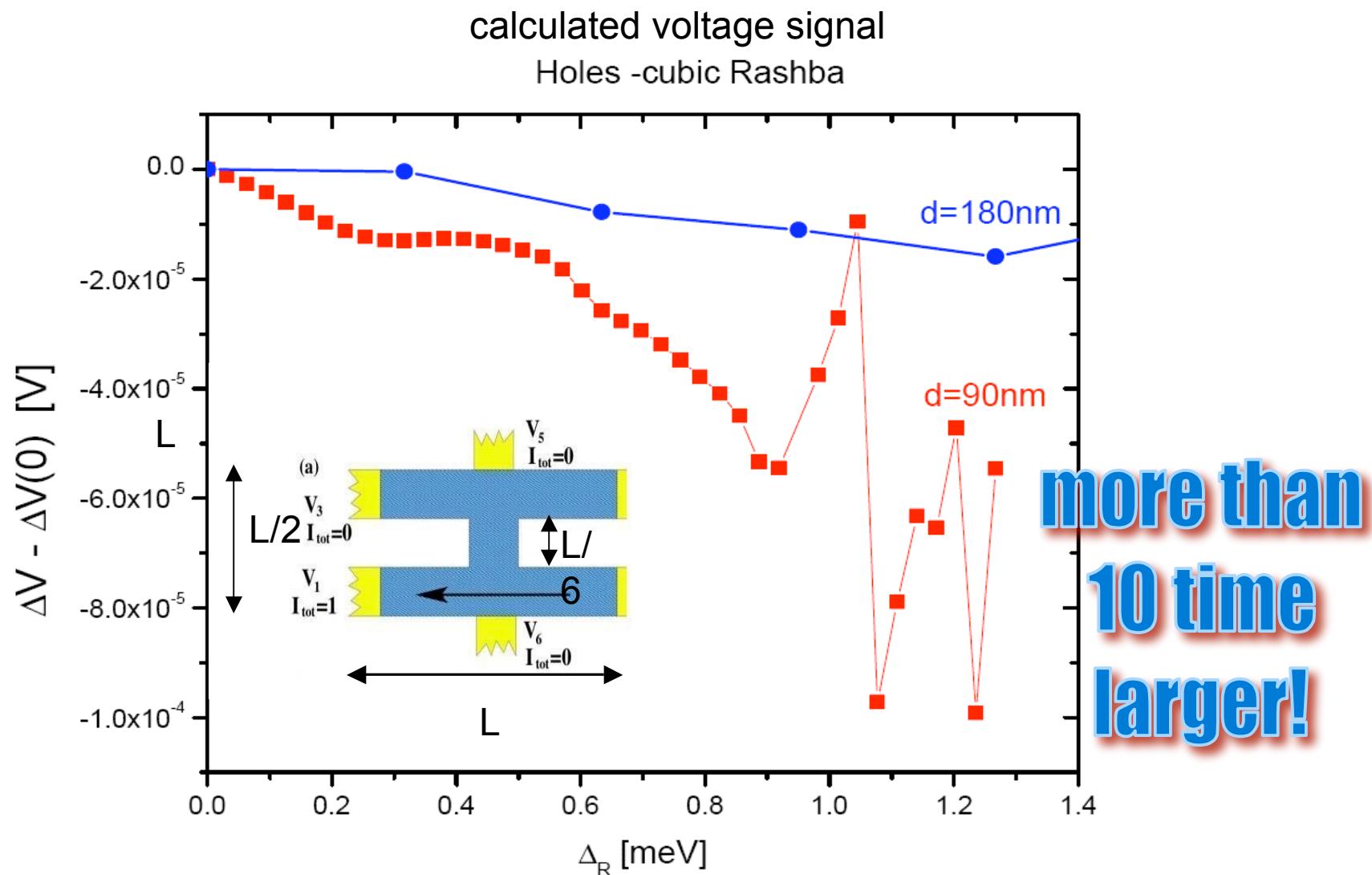
## Scaling of H-samples with the system size



Oscillatory character of voltage difference with the system size.



# Measurement



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S.C. Zhang (Stanford Univ.)  
X.L. Qi (Stanford Univ.)