Nuclear Spins in Quantum Dots and Interacting 2DEGs

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Outline

A) Motivation

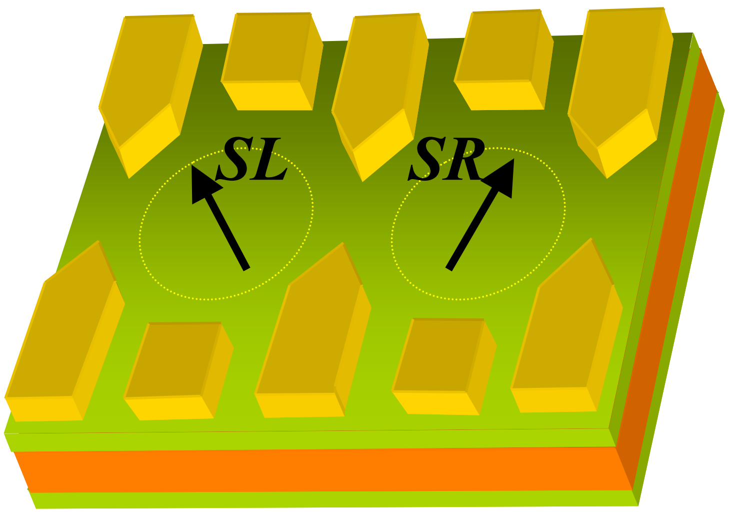
Spin decoherence in GaAs quantum dots
Nuclear spins and hyperfine induced decoherence
spin ½ in single dot and state narrowing in double dots

B) Ferromagnetic phase transition in nuclear spin system

- Kondo lattice model
- RKKY interaction
- Spin wave analysis and Curie temperature
- correlations in 2DEG

Spin-Qubits from Electrons

DL & DiVincenzo, PRA **57** (1998)



Spin 1/2 of electron = qubit

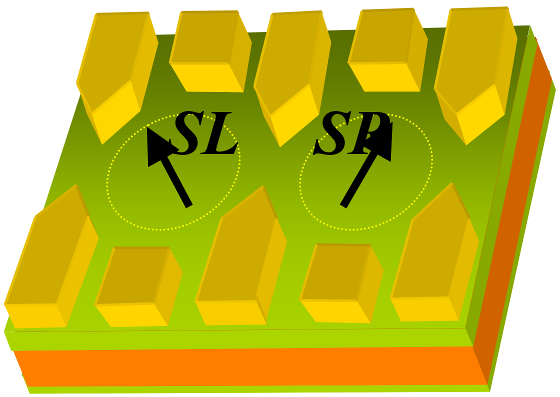
Quantum gates based on exchange interaction:

$$H(t) = J(t)\, \mathbf{S}_L \cdot \mathbf{S}_R$$

electrically controlled

Spin-Qubits from Electrons

DL & DiVincenzo, 1998



- qubit: Spin-1/2 g.s. of N (odd) electron system (Kramers doublet)
  'simplest' case: N=1, i.e. $|0\rangle = \uparrow, \ |1\rangle = \downarrow$

- single spin read out via spin-charge conversion ➔ Elzerman *et al.*, '04 (T_1=0.1s)

$$H = \sum_{\langle ij \rangle} J_{ij}(t)\vec{S}_i \cdot \vec{S}_j + \sum_i (g_i \mu_B \vec{B}_i)(t) \cdot \vec{S}_i$$

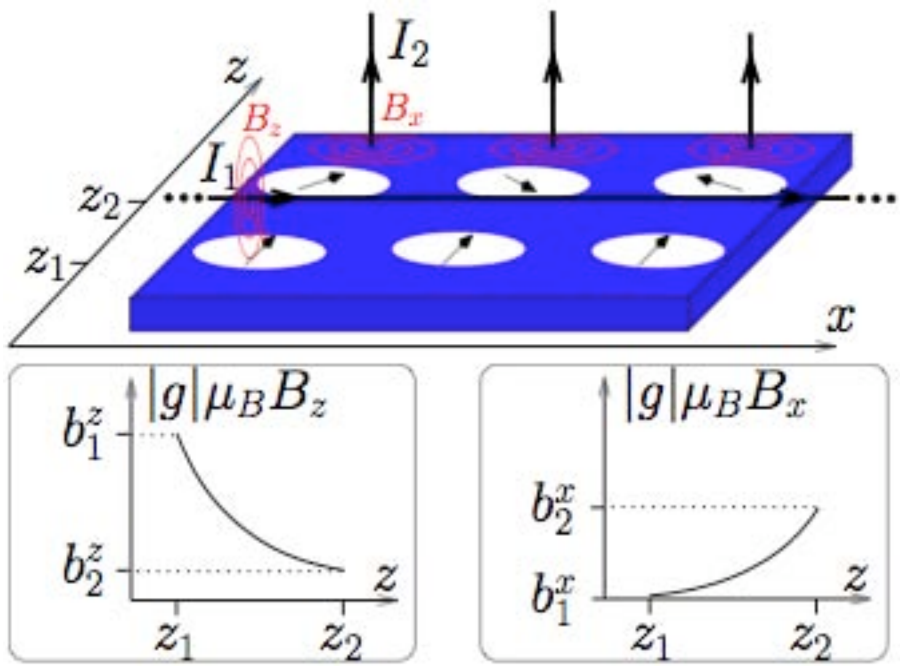
2-qubit gate
'sqrt of swap' ($\tau_s \approx$ 100 ps)
Petta *et al.*, Science '05

1-qubit gate
via ESR for single spin ($T_2 \approx 1\mu s$)
Koppens *et al.*, Nature '06

➔ $T_2/\tau_s \sim 10^4$

Single-Spin Rotations by Exchange

Coish & DL, cond-mat/0606550



Requires auxiliary spins, Zeeman gradient & exchange

➔ fast switching times (1ns) with high fidelity (< 10^{-3})

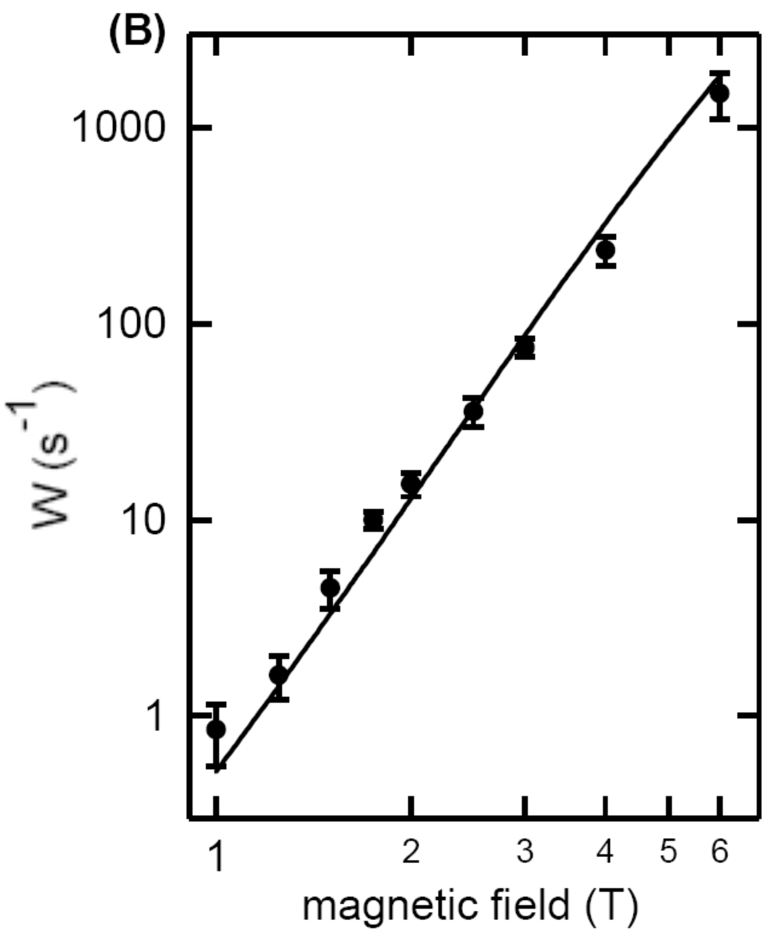
Relaxation of spin in GaAs quantum dots dominated by spin-orbit & phonons with ultra-long relaxation times T_1:

$T_1 \sim O(s)$ for $B \sim 1T$

Amasha *et al.*, cond-mat/0607110

Current record: T_1 > 1 s (B ≈ 1 T)

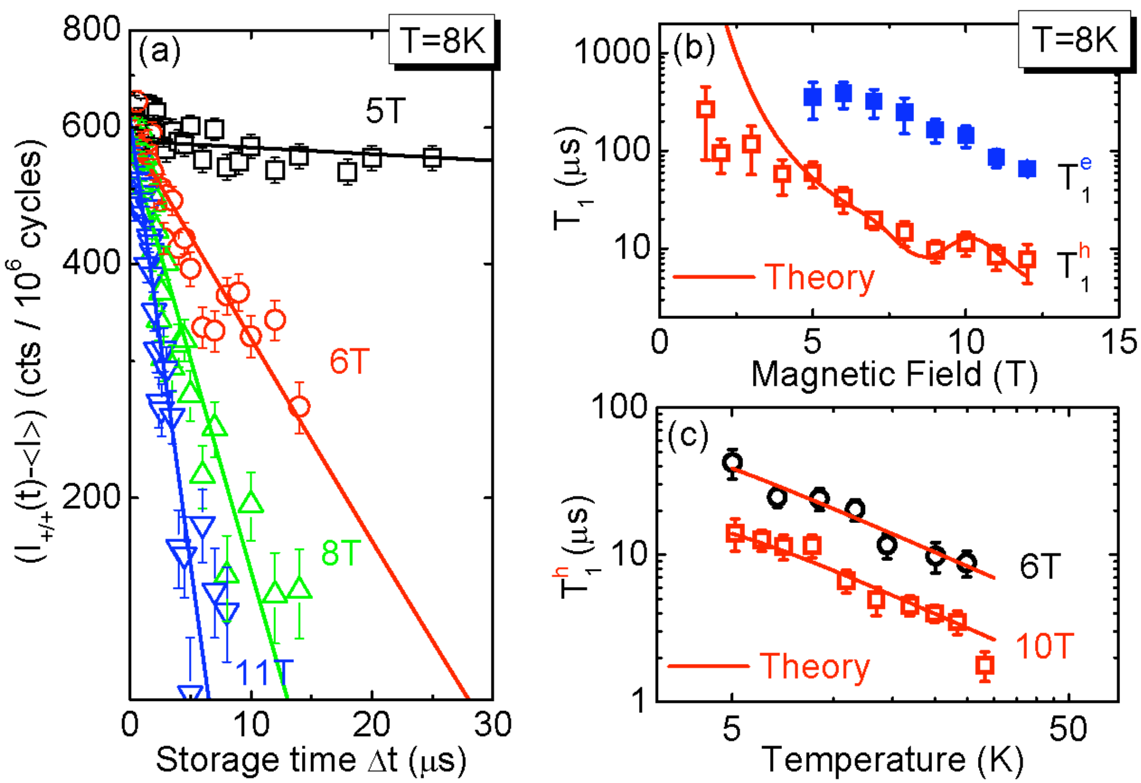
Amasha, Kastner & Zumbühl, '07



➔ data in good agreement with theory
Golovach, Khaetskii, DL, PRL 93 (`04)

Hole Spin Relaxation: T_1~200 μs

Theory: Bulaev & D. Loss, Phys. Rev. Lett. 95, 076805 (2005)
Experiment: Abstreiter & Finley group, cond-mat/0705.1466



Relaxation of spin in GaAs quantum dots dominated by spin-orbit & phonons with ultra-long relaxation times T_1:

$T_1 \sim O(s)$ for B ~ 1T

Amasha *et al.*, cond-mat/0607110

From SOI we expect $T_2 = 2T_1$ Golovach et al., PRL '04

But measured spin decoherence times are much shorter: $T_2 \sim 1$- 10 μs Petta et al. '05; Koppens et al. '06/'07

Thus, spin decoherence in GaAs must be dominated by other effects ➔ hyperfine interaction with nuclear spins

Burkard, DL, DiVincenzo, PRB ’99

Hyperfine Interaction in *Single* Quantum Dot

$$H = \sum_i A_i \vec{S} \cdot \vec{I}_i + g\mu_B B\, S^z + H_{dd}$$

(the H_{dd} term is crossed out with a red arrow)

hyperfine interaction is non-uniform:

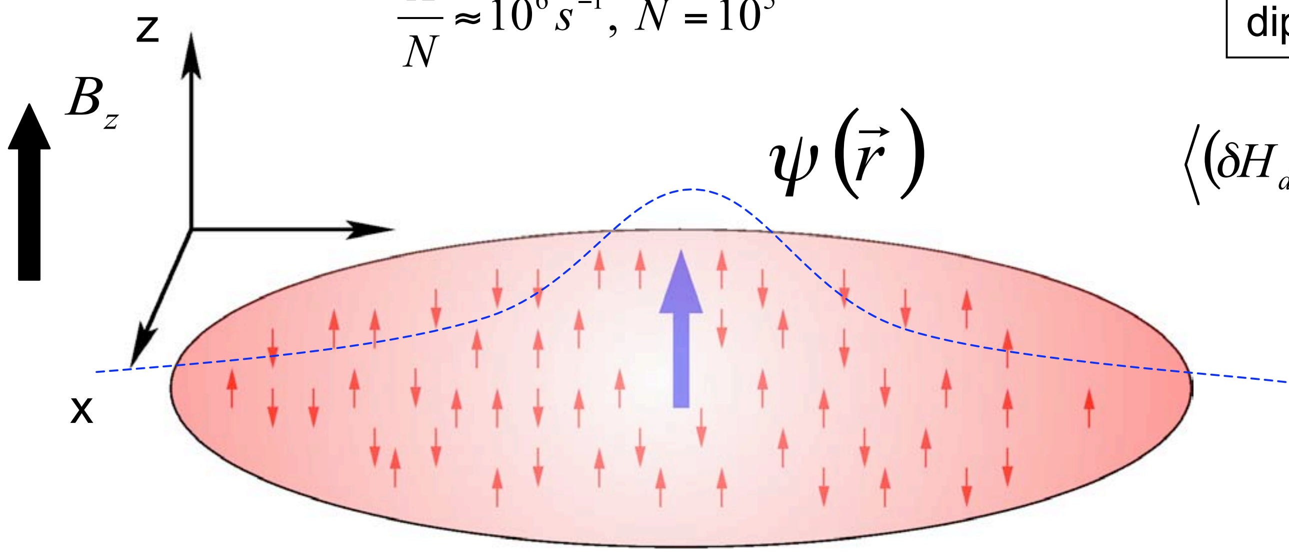
$$A_i \propto A \left|\psi(\vec{r}_i)\right|^2$$

$$\frac{A}{N} \approx 10^6 s^{-1},\ N = 10^5$$

electron Zeeman energy

nuclear spin dipole-dipole interaction

$$\left\langle (\delta H_{dd})^2 \right\rangle^{1/2} \approx 10^4 s^{-1} \approx 100 nK$$



Khaetskii, DL, Glazman, ’02; Coish & DL, ’04; De Sousa ea, ’05; Sham ea,’06; Altshuler ea, ’06;

Separation of the Hyperfine Hamiltonian

Hamiltonian: $$H = g\mu_B B S_z + \vec{S}\cdot\vec{h} = H_0 + V$$

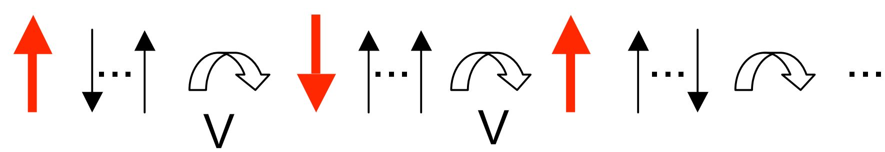
Note: nuclear field $\vec{h} = \sum_i A_i \vec{I}_i$ is a quantum operator

Separation:

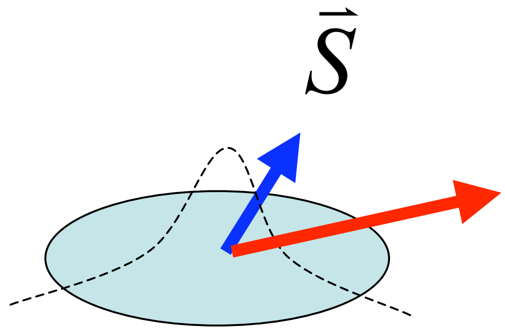
$$H_0 = (g\mu_B B + h_z) S_z$$ longitudinal component

$$V = \frac{1}{2}\left(h_+ S_- + h_- S_+\right)$$ flip-flop terms

$$h_\pm = h_x \pm i h_y$$



Nuclear spins provide hyperfine field *h* with quantum fluctuations seen by electron spin:



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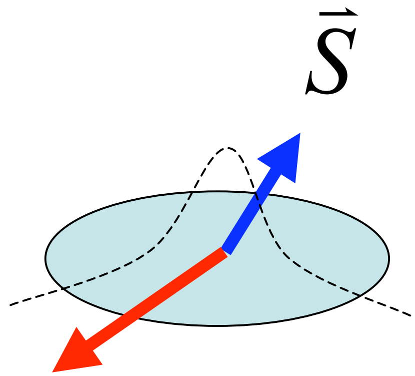

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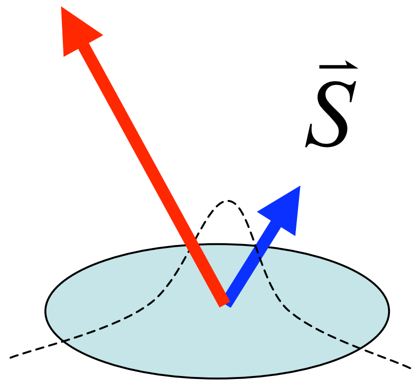


With mean $\langle h \rangle = 0$ and quantum variance δh:

Initial conditions for nuclear spins

Coish &DL, PRB 70, 195340 (2004)

Spin dynamics for V=0*):

- Superposition (1) or mixture (2) of h_z-eigenstates:

Rapid Gaussian decay!

- But: Single h_z eigenstate (3):

No decay! (if flip-flop V is neglected)

*) corresponds to B>>h

Initial conditions for nuclear spins

Coish &DL, PRB 70, 195340 (2004)

- Superposition or mixture of h_z-eigenstates:

Rapid Gaussian decay!

- But: Single h_z eigenstate :

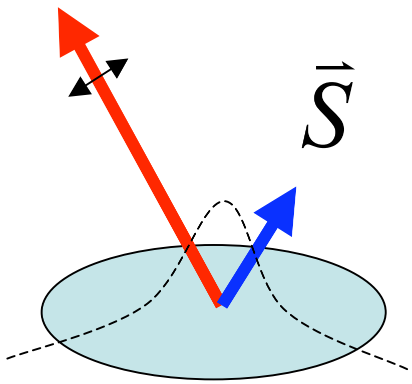
No decay! (if flip-flop V is neglected)

➔ It is advantageous to prepare the nuclear spin system with a von Neumann measurement on the Overhauser field (operator!):

➔

[via ESR, see Klauser, Coish & DL, PRB **73**, 205302 (2006)]

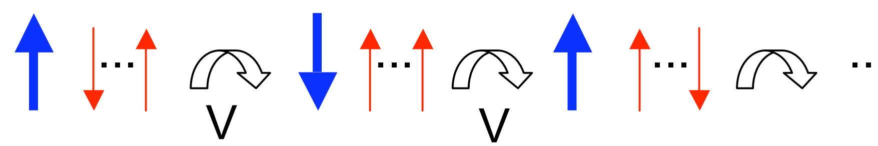
Sharp initial nuclear spin state: δh=0 at t=0



➔ back action of ***S*** on ***h***

flip-flops

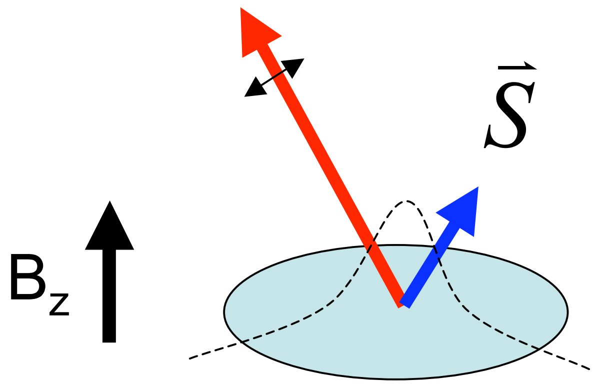
t>0: quantum dynamics



changes hyperfine field in time by 1/N ➔ spin precesses in fluctuating hyperfine field ➔ spin dephases (power law decay)

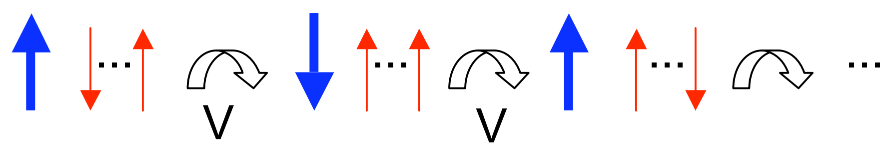
Khaetskii, DL, Glazman, PRL ’02 & PRB ’03
Coish &DL, PRB 70, 195340 (2004)

Sharp initial nuclear spin state ➔ δh=0 at t=0



➔ back action of ***S*** on ***h***

Dynamics (flip-flops):



E.g.

$$S_z(t) - S_z(0) \propto \frac{A^2}{4N(b+pIA)^2} \frac{e^{itA/N}}{(At/N)^{3/2}}$$

power law decay

Time scale is N/A = 1μs (GaAs) and **decay is bounded**

Summary: Nuclear spins in quantum dot

Dephasing due to 'random hyperfine fields' yields Gaussian decay on a scale:

$T_2^* = \sqrt{N} / A = 10$ ns dephasing *)

Note: for times $t > T_2^* = \sqrt{N} / A = 10$ ns (and $B>0$)
classical (mean field) and quantum dynamics differ!

Coish, Yuzbashyan, Altshuler, and DL, cond-mat/0610633

Dephasing removable by state preparation and/or spin echo, and remaining decay is purely quantum (power law) on a scale:

$" T_2 " = N/A = 1\ \mu s$

and amount of decay is strongly suppressed by factor $(A/B)^2\ (1/p^2N) \ll 1$

i.e. for large magnetic fields B (>4T) and/or high polarization p

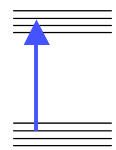
*) ensemble of dots: Merkulov, Efros & Rosen, PRB '02

Narrowing of nuclear spins in double dots with ESR

Klauser, Coish & DL, PRB **73**, 205302 (2006)

- ESR: oscillating exchange $J(t)=J_0+j\cos(\omega t)$ leads to Rabi oscillations:

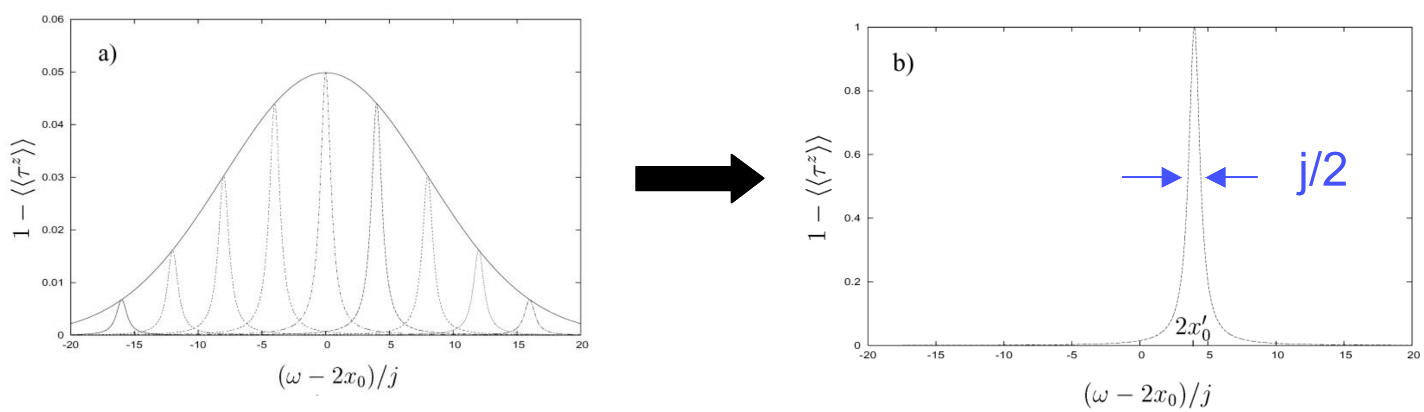
$|\downarrow\uparrow\rangle = |+\rangle$



$g\mu_B B + \delta h_n^z$

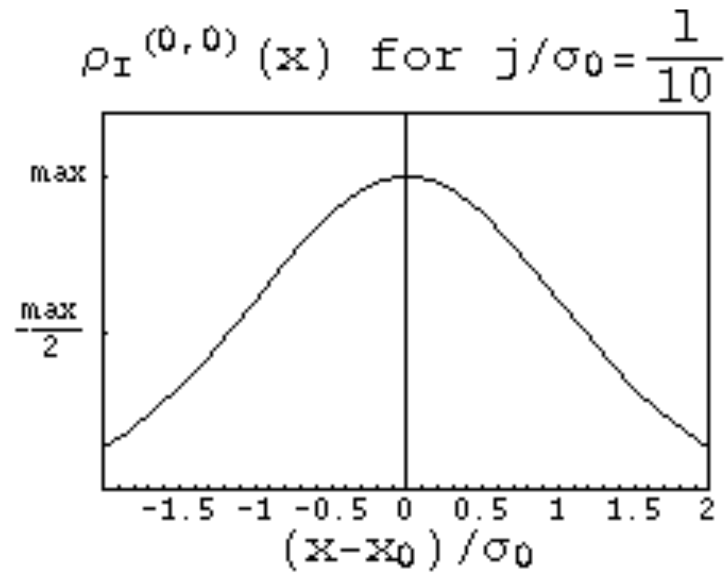
ESR at frequency $\omega = g\mu_B B + \delta h_n^z$ measures eigenvalue
➔ nuclear spins projected into corresponding eigenstate |n>

If quantum measurement is ideal, then Gaussian superposition collapses to a single Lorentzian (ESR linewidth):



Quantum control of many-body system through transport measurement

Klauser, Coish & DL, PRB (2006)



Optical scheme: see Stepanenko, Burkard, Imamoglu , `06

Polarization of nuclear spins

1. Dynamical polarization

- optical pumping: <65%, Dobers et al. '88, Salis et al. '01, Bracker et al. '04
- transport through dots: 5-60%, Ono & Tarucha, '04/ '07, Koppens et al., '06,...
- projective measurements: experiment?

2. Thermodynamic polarization
   i.e. ferromagnetic phase transition? Simon & Loss, PRL '07

Q: Is it possible in a 2DEG? What is Curie temperature?

Problem is quite old and was first studied in 1940
by Fröhlich & Nabarro for bulk metals!

Hyperfine interaction in tight-binding formulation

P. Simon & DL, Phys. Rev. Lett. 98, 156401 (2007)

on d-dimensional lattice

Kondo Lattice formulation

is the electron spin operator at lattice site $\vec{r}_j$

e-e interactions

NB: For a single electron in a strong confining potential, we recover quantum dot description by projecting the hyperfine Hamiltonian in the electronic ground state

alternative approach for numerics on dot-spin dynamics ?

Effective nuclear spin Hamiltonian (RKKY)

Strategy: **A** (hyperfine) is the smallest energy scale ➔
integrate out electronic degrees of freedom
including e-e interactions (e.g. via Schrieffer-Wolff trafo):

Pure spin-spin Hamiltonian for nuclear spins only:

'RKKY interaction'

where is the electronic spin susceptibility in the static limit (ω=0)
(justified since nuclear spin dynamics is much slower than electron dynamics)

Assuming no electronic polarization:

Effective nuclear spin Hamiltonian (RKKY)

where 'RKKY interaction'

and

is the electronic longitudinal spin susceptibility in the static limit (ω=0).

Free electrons: J_r is standard RKKY interaction Ruderman & Kittel, 1954
Note that result is also valid in the presence of electron-electron interactions

Curie-Weiss mean field theory (1)

Consider a particular nuclear spin at site $\vec{r}$

Mean field approx.:

Effective magnetic field:

with:

If we assume , we obtain the MFT eq.

Curie temperature for **FM** phase

Curie-Weiss mean field theory (2)

electron DOS (2D)

Simon &DL, PRL '07

For a **3D metal** with one electron per lattice site:

Fröhlich & Nabarro, 1940

with

Naïve use of MFT for a **2D semiconductor** with low electronic density $n_e << n$:

GaAs:

Curie temperature is quite low

2D: What about the Mermin-Wagner theorem?

The Mermin-Wagner theorem states that there is no finite temperature phase transition in 2D for a Heisenberg model provided that

$$\sum_{\vec{r}} r^2 |J(r)| < \infty$$

For non-interacting electrons, $J(r)$ reduces to the long range RKKY interaction:

$$J(r) \sim \frac{\cos(2k_F r)}{r^2}$$

➔ nothing can be inferred from the MW-theorem !

Nevertheless, due to the oscillatory character of the RKKY interaction, one may expect some extension of the Mermin-Wagner theorem, and, indeed it was conjectured that in 2D $T_c = 0$ (P. Bruno, PRL 87 ('01)).

FM phase and spin waves in 2D

The mean field calculation suggests a ferromagnetic phase a low temperature. Let us assume such a FM phase and analyze its stability against spin wave excitations:

Energy of a magnon:

The magnetization per site:

magnon occupation number

The Curie temperature T_c is then defined by

1. Non-interacting electron gas in 2D

In the continuum limit the condition m(T_c)=0 becomes:

with

For non-interacting electrons (2D):

with the electronic DOS in 2D

But: for any finite T_c

1. Non-interacting electron gas in 2D

In the continuum limit the condition m(T_c)=0 becomes:

with

For non-interacting electrons (2D):

Thus:

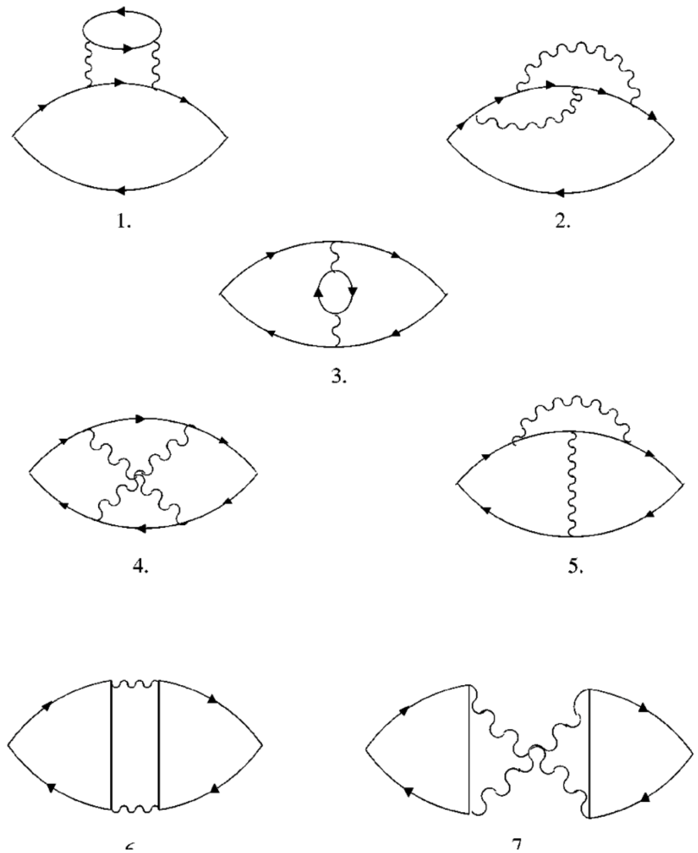
➔ **not** consistent with Curie-Weiss MFT, but rather as expected!?

Include now electron-electron interactions

Perturbative calculation of spin susceptibility in 2DEG

Consider **screened** Coulomb *U* and 2[nd] order pert. theory in *U*:

Chubukov, Maslov, PRB **68**, 155113 (2003)

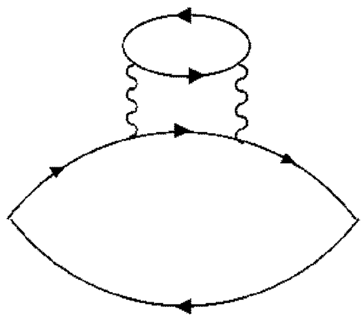


➔ give singular corrections to spin and charge susceptibility due to non-analyticity in polarization propagator Π (sharp Fermi surface)

➔ non-Fermi liquid behavior in 2D

Perturbative calculation of spin susceptibility in 2DEG

Consider **screened** Coulomb *U* and 2nd order pert. theory in *U*:



Chubukov, Maslov, PRB **68** ('03)

i.e. in the low q limit

where $\Gamma_s \sim -Um/4\pi$ denotes the backscattering amplitude

This linear -dependence (non-analyticity*) permits ferromagnetic order with finite Curie temperature!

*) See also Belitz et al., PRB B 55 (1997)

Nuclear magnetization at finite temperature

Magnon spectrum ω_q becomes now **linear** in q due to e-e interactions:

with spin wave velocity (GaAs: c~20cm/s)

What about q > $2k_F$? ➔ such q's are not relevant in $m(T)$ for temperatures T with

since then $\beta\omega_q>1$ for all $q>2k_F$

Nuclear magnetization at finite temperature

where T_c is the 'Curie temperature':

➔ finite magnetization at finite temperature in 2D!

estimate for GaAs 2DEG: $T_c \sim 25\ \mu K$

Note that self-consistency requires

since $a\pi/a_B \sim 1/10$ in GaAs

➔ temperatures are finite but still very small!

Beyond simple perturbation theory (1)

P. Simon & DL, PRL 98, 156401 (2007)

vertex

Γ is the exact electron-hole scattering amplitude
and G the exact propagator

Γ obeys Bethe-Salpether equation as function of p-h--irreducible vertex Γ_{irr}

➔ solve BS in lowest order in Γ_{irr}

Beyond simple perturbation theory (2)

P. Simon & DL, PRL 98, 156401 (2007)

Lowest approx. for vertex: ➔ can derive simple formula:

as before use Maslov-Chubukov

onset of Stoner instability for

This leads to a dramatic enhancement of

and therefore also of Curie temperature $T_c \sim \delta\chi_s$

Estimate:

'Stoner factor'

The local field factor approximation

with long history: see e.g. Giuliani & Vignale*, '06

Consider unscreened 2D-Coulomb interaction

Idea (Hubbard): replace the average electrostatic potential seen by an electron by a local potential:

Determine 'local spin field factor' $G_{-}(q)$ semi-phenomenologically*:

Thomas-Fermi wave vector, and g_0=g(r=0) pair correlation function

Note: $G_{-}(q) \sim q$ for $q<2k_F$ ➔ this is in agreement also with Quantum Monte Carlo (Ceperley et al., '92,'95)

The local field factor approximation

➔

i.e. again strong enhancement through correlations:

for

Giuliani & Vignale, '06

strong enhancement of the Curie temperature:

for $r_s \sim 5$

Conclusions

- Spin decoherence in GaAs quantum dots dominated by hyperfine interaction ➔ increase nuclear polarization

- Kondo lattice formulation of hyperfine interaction in 2DEG (➔ useful for numerics in quantum dots?)

- non-Fermi liquid correlations in 2DEG permit ferromagnetic phase in 2d-Kondo lattice at finite temperature

- Electron correlations increase Curie temperature:

$$T_c \approx O(mK) \text{ for } r_s \sim 5$$

- Many open questions:
  Disorder, nuclear spin glas? Spin decoherence in ordered phase? Experimental signature?