Asilomar, CA, June 6th, 2007

Single Electron Spin in Interacting Nuclear Spin Bath — Coherence Loss and Restoration

Wang Yao

Department of Physics, University of Texas, Austin

Collaborated with: L. J. Sham (UC San Diego)
Renbao Liu (CUHK)
S. K. Saikin (UC San Diego)

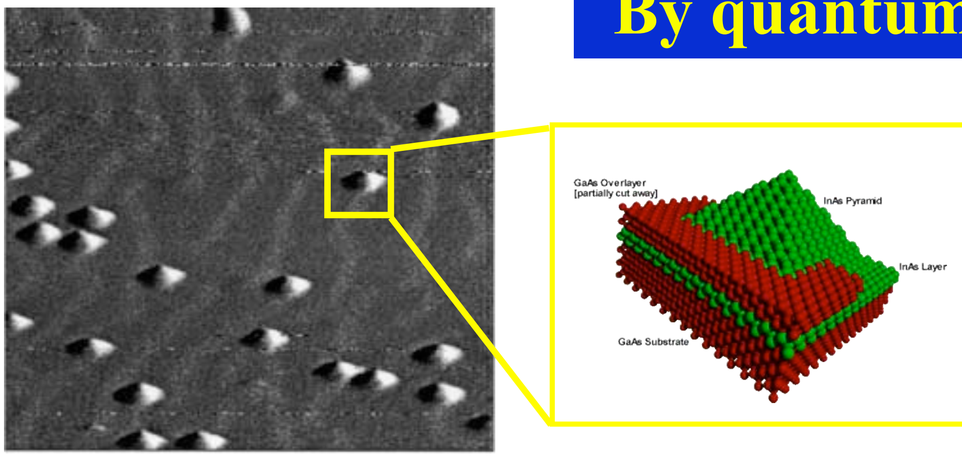
Supported by NSF DMR, ARDA/ARO, DARPA/AFOSR

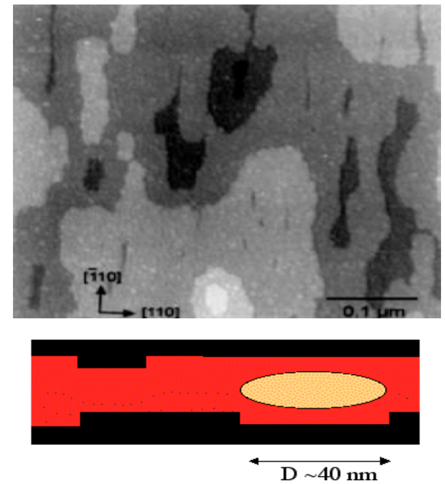
Issues and Problems

- Single spin in solids: decoherence and coherence protection

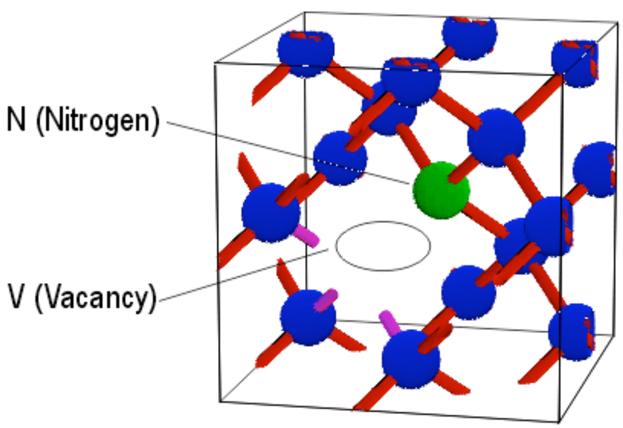
Single electrons localized in solids

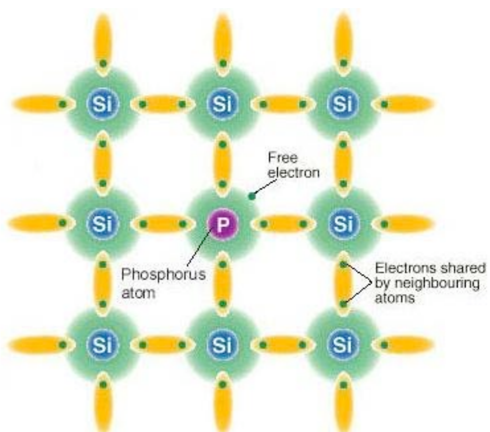
By quantum dot



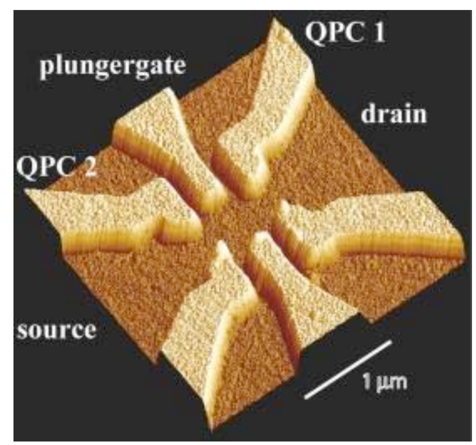


By impurities





By electrical gate



Issues and Problems

- Single spin in solids: decoherence and coherence protection
- A 2-level system + many interacting bath spins

Rep. Prog. Phys. **63** (2000) 669–726. Printed in the UK PII: S0034-4885(00)74677-X

Theory of the spin bath

N V Prokof'ev†‡§ and P C E Stamp §||

† Russian Science Centre 'Kurchatov Institute', Moscow 123182, Russia

‡ Physics and Astronomy, Hasbrouck Laboratory, University of Massachussetts, Amherst, MA 01003, USA

§ Physics Department, and Canadian Institute for Advanced Research, University of British Columbia, 6224 Agricultural Rd, Vancouver BC, Canada V6T 1Z1

|| Spinoza Institute, and Institute for Theoretical Physics, Minnaert Building, University of Utrecht, Leuvenlaan 4, 3508 TD Utrecht, The Netherlands

Received 28 September 1999

Abstract

The quantum dynamics of mesoscopic or macroscopic systems is always complicated by their coupling to many 'environmental' modes. At low T these environmental effects are dominated by *localized* modes, such as nuclear and paramagnetic spins, and defects (which also dominate the entropy and specific heat). This environment, at low energies, maps onto a 'spin bath' model. This contrasts with 'oscillator bath' models (originated by Feynman and Vernon) which describe *delocalized* environmental modes such as electrons, phonons, photons, magnons, etc. The couplings to N spin bath modes are *independent* of N (rather than the $\sim O(1/\sqrt{N})$ dependence typical of oscillator baths), and often strong. One cannot in general map a spin bath to an oscillator bath (or vice versa); they constitute distinct 'universality classes' of quantum environment.

Theories antecedent on spin bath

Spectral diffusion theory

- Herzog & Hahn, PR 56", Klauder & Anderson, PR 61", semi-classical theory

Electron nuclear hyperfine interation

- Schulten & Wolynes, J. Chem. Phys., 78" –frozen nuclear configuration
- Merkulov, Efros & Rosen, PRB 02"
- Khaetskii, Loss & Glazman, PRL 02", Coish & Loss, PRB 04"
- Schliemann, Khaetskii & Loss, PRB 02" – sys-bath entanglement
- Shenvi, De Sousa & Whaley, PRB 05" – small environment ~10 nuclei

Nuclear dipolar interaction plus diagonal e-n hyperfine coupling

- De Sousa & Das Sarma, PRB 03" – semiclassical stochastic solution
- De Sousa, Shenvi & Whaley, PRB 05" –semiclassical theory
- Witzel, De Sousa & Das Sarma, PRB 05" - ensemble study

Other low energy modes mapped to spin bath

- Prokov'ef & Stamp, Rep. Prog. Phys. 00" – magnets and superconducting systems

Sorry, not an exhaustive list.

Issues and Problems

- Single spin in solids: decoherence and coherence protection
- A 2-level system + many interacting bath spins
- Single electron in III-V semiconductor quantum dot

Spin decoherence in QD: Phonon Vs. Nuclei

Decoherence by phonon scattering

- Spin relaxation T_1 ~ ms – s, at ~ K in QD

  theories (Khaetskii and Nazarov; Woods, Reinecke and Lyanda-Gella); experiments (Tarucha group; Kouwenhoven group; Finley group; Steel group)

- Pure dephasing suppressed at low-temp

  theoretical estimation: $T_2 = 2T_1$ @ ~ K and below

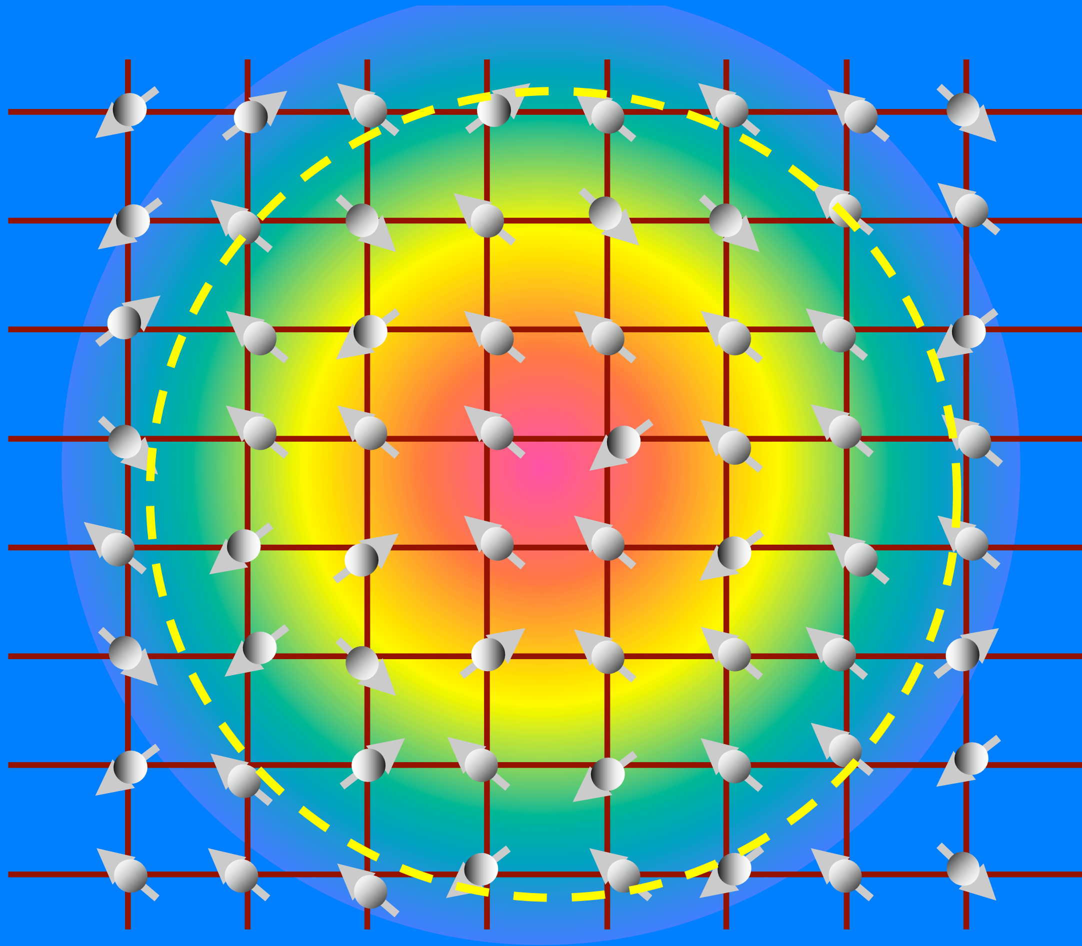
  (Golovach, Khaestskii and Loss, PRL 04'; Semenov & Kim, PRL 04')

Spin transverse decoherence time from exp

- Single spin T_2? not measurable with current capability
- T_2^*~1-10 ns >> T_1 gated dot in GaAs, SAD
- T_H~ 1.2 μs >> T_1 gated dot in GaAs (Petta *ea.*, Koppens *ea.*)

Nuclear spin is the dominant cause for transverse decoherence at low temp.

Electron spin in a nuclear spin bath

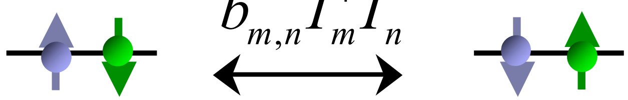


- E-N coupled through *contact* hyperfine interaction
- Mesoscopic size N ~ 10^6, *posterior* justification

Electron nuclear dynamics

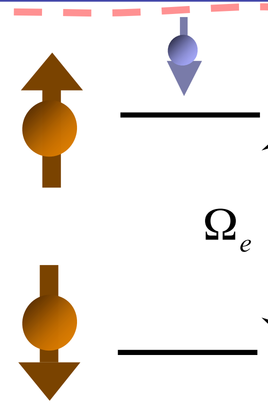
Intrinsic n-n interaction, e.g. dipolar

$b_{m,n} I_m^+ I_n^-$



e-n interaction diagonal

$a_n S_e^z I_n^z$

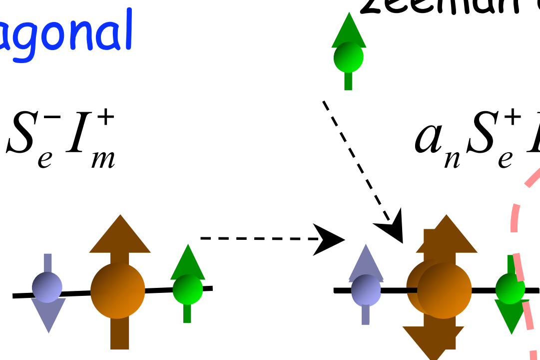


$\Omega_e = g \mu_B B$

Dynamical fluctuation of zeeman energy $a_n - a_m \neq 0$

e-n interaction off-diagonal

$a_m S_e^- I_m^+$ $\quad$ $a_n S_e^+ I_n^-$

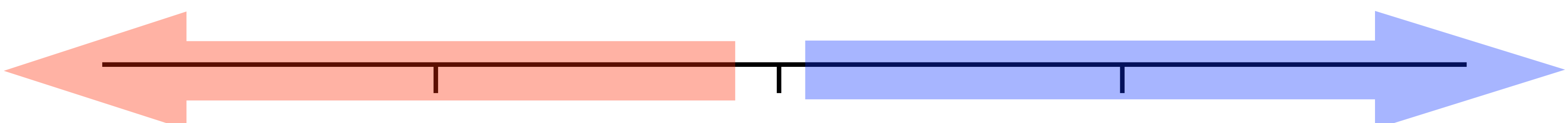


Inhomogeneous broadening by nuclear field $\rightarrow T_2^*$

Extrinsic n-n interaction

$$\frac{a_m a_n}{\Omega_e} S_e^z I_m^+ I_n^-$$

Zeeman energy	V.S.	e-n off-diagonal energy	
$g \mu_B B \sim 10$ GHz (1 T)		$\mathcal{A} \sim 1$ THz	Polarized
		$\mathcal{A}/\sqrt{N} \sim 0.1-1$ GHz	Unpolarized



Issues and Problems

- Single spin in solids: decoherence and coherence protection
- A 2-level system + many interacting bath spins
- Single electron in III-V semiconductor quantum dot
  - Low T and high field
  - Pure transverse decoherence $(T_1 \to \infty)$
  - Mescoscopic nuclear bath, N ~ 10^6, unpolarized
  - 1 + N well isolated from the rest universe
- Quantum mechanical origin of decoherence: entanglement

Decoherence of single quantum system

Pure system state initially factorized from bath

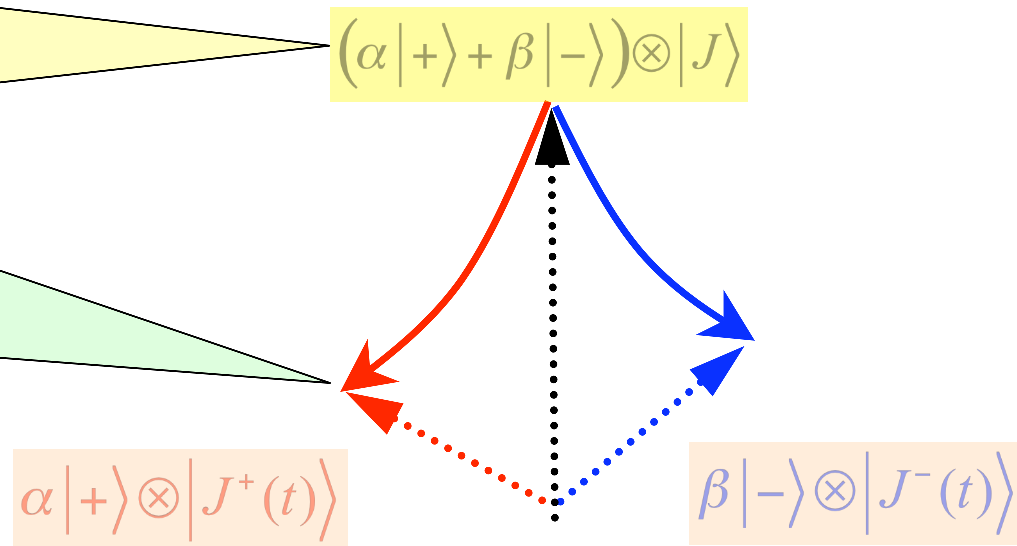
$$\rho^e(0) \equiv Tr[\rho_{tot}] = \begin{bmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{bmatrix}$$

System-bath entanglement

$$\alpha|+\rangle \otimes |J^+(t)\rangle + \beta|-\rangle \otimes |J^-(t)\rangle$$

$$\rho^e(t) = \begin{bmatrix} |\alpha|^2 & \alpha\beta^* \langle J^- | J^+ \rangle \\ \alpha^*\beta \langle J^+ | J^- \rangle & |\beta|^2 \end{bmatrix}$$

Zurek, RMP 03"; Schlosshauer, RMP 05"



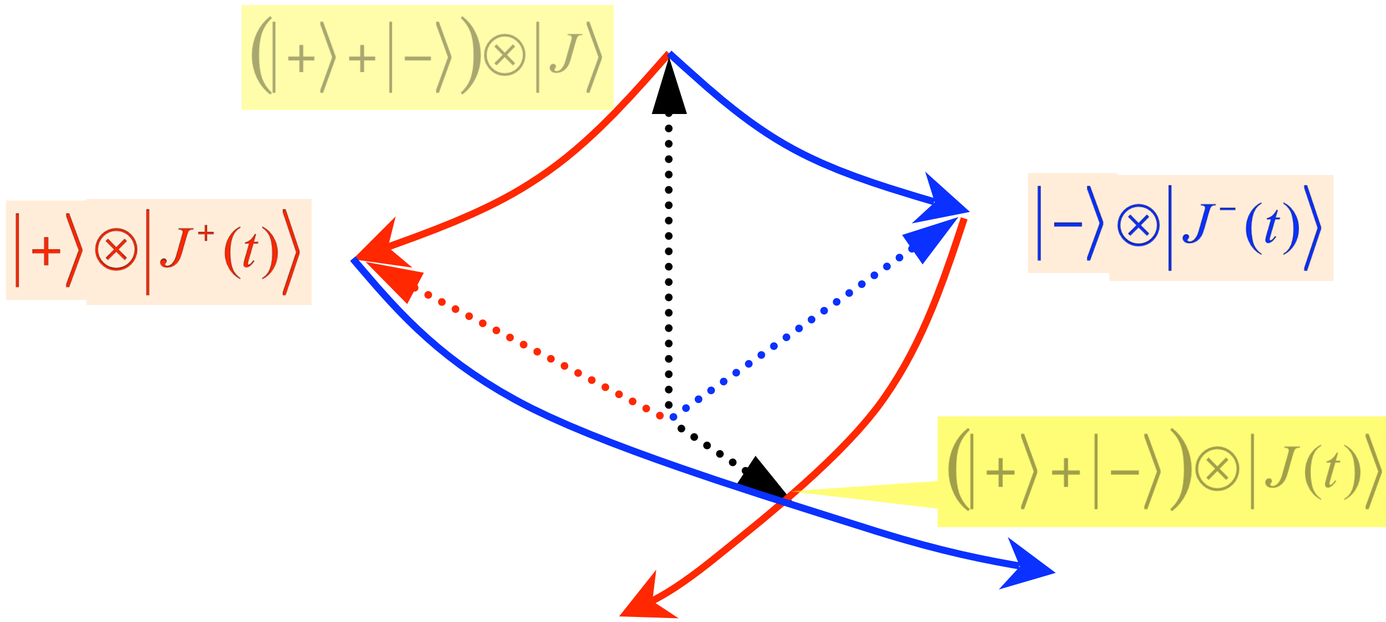
- Bath dynamics conditioned on system states
- Bath state bifurcates in the Hilbert space
- System (which-state) information measured by the bath
- Decoherence: decays of off-diagonal DM element (T_2)

Nuclear correlation func: $\mathcal{L}^s_{+,-}(t) \equiv \langle J^-(t) | J^+(t) \rangle$; $\rho^e_{+,-}(t) = \mathcal{L}^s_{+,-}(t)\rho^e_{+,-}(0)$

Issues and Problems

- Single spin in solids: decoherence and coherence protection
- A 2-level system + many interacting bath spins
- Single electron in III-V semiconductor quantum dot
  - Low T and high field
  - Pure transverse decoherence $(T_1 \to \infty)$
  - Mescoscopic nuclear spin bath, N ~ 10^6, unpolarized
  - 1 + N well isolated from the rest universe
- Quantum mechanical origin of decoherence: entanglement
- Coherence restoration by disentanglement

Recoherence by disentanglement



- Flip of system states redirects bath evolution
- Bath states intersect in Hilbert space (which-state info erased)
- System disentangled from bath ➔ Coherence restored
- Possible in mesoscopic bath: sys+bath isolate from rest universe

Nuclear spin dynamics: pair wise flip-flop

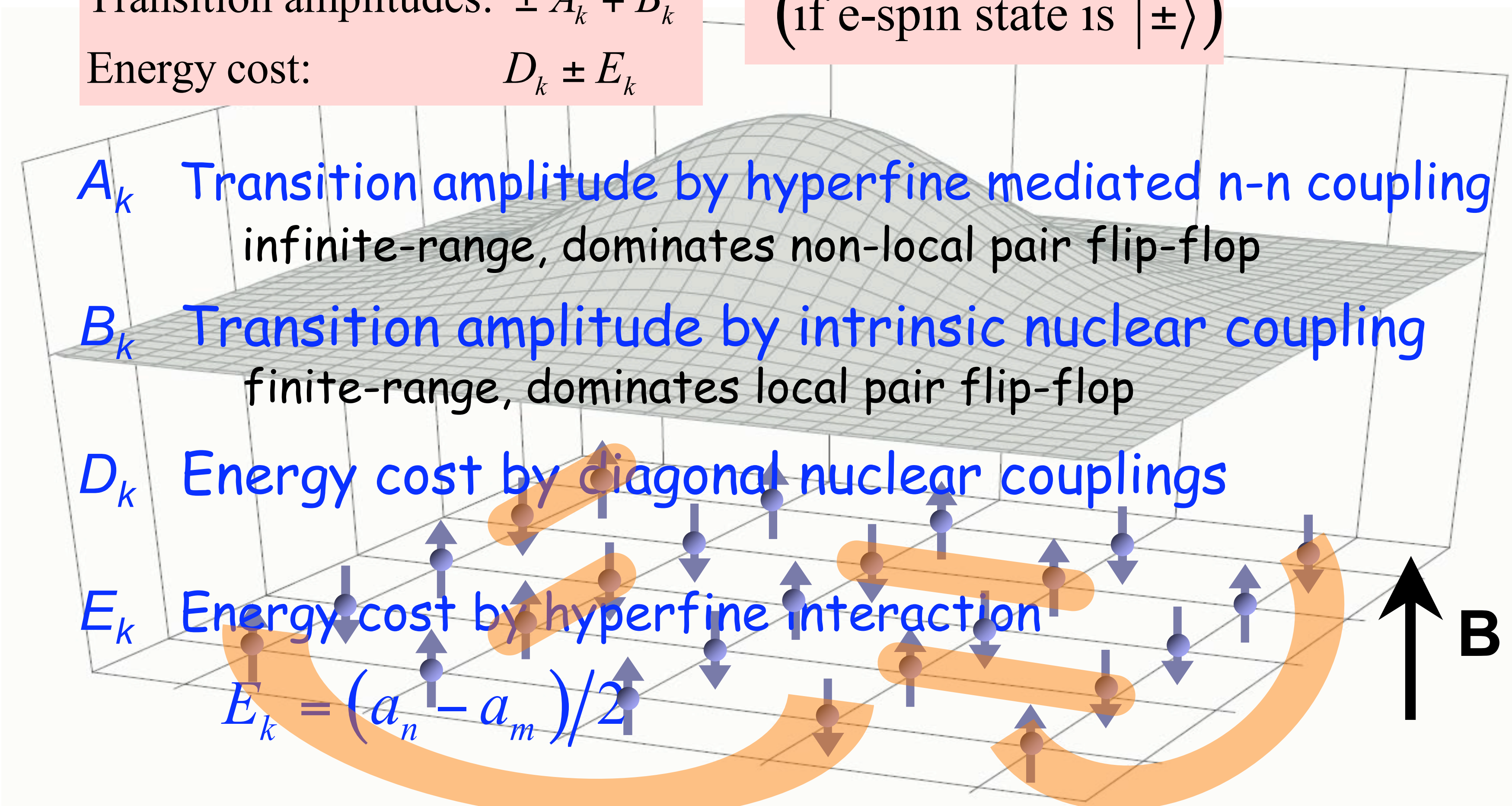
Elementary excitation:

$$|j_1\rangle \cdots |j_n\rangle \cdots |j_m\rangle \cdots |j_N\rangle \xrightarrow[|j_n+1\rangle|j_m-1\rangle\langle j_m|\langle j_n|]{k\text{th pair-flip}} |j_1\rangle \cdots |j_n+1\rangle \cdots |j_m-1\rangle \cdots |j_N\rangle$$

Transition amplitudes: $\pm A_k + B_k$

Energy cost: $D_k \pm E_k$

(if e-spin state is $|\pm\rangle$)



A_k Transition amplitude by hyperfine mediated n-n coupling

infinite-range, dominates non-local pair flip-flop

B_k Transition amplitude by intrinsic nuclear coupling

finite-range, dominates local pair flip-flop

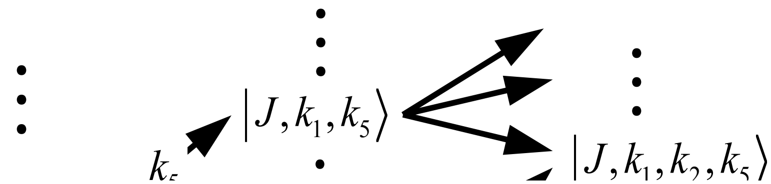
D_k Energy cost by diagonal nuclear couplings

E_k Energy cost by hyperfine interaction

$$E_k = (a_n - a_m)/2$$

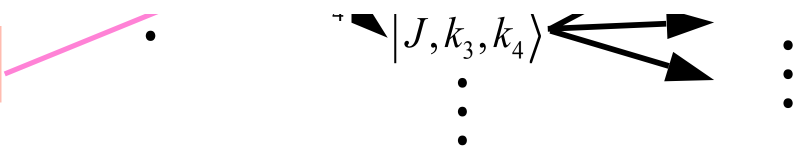
Nuclear spin dynamics: pair wise flip-flop

Hierarchy of dynamics:



In time scale of interest, the number of nuclear pair-flip excitations is small !

2 excitations

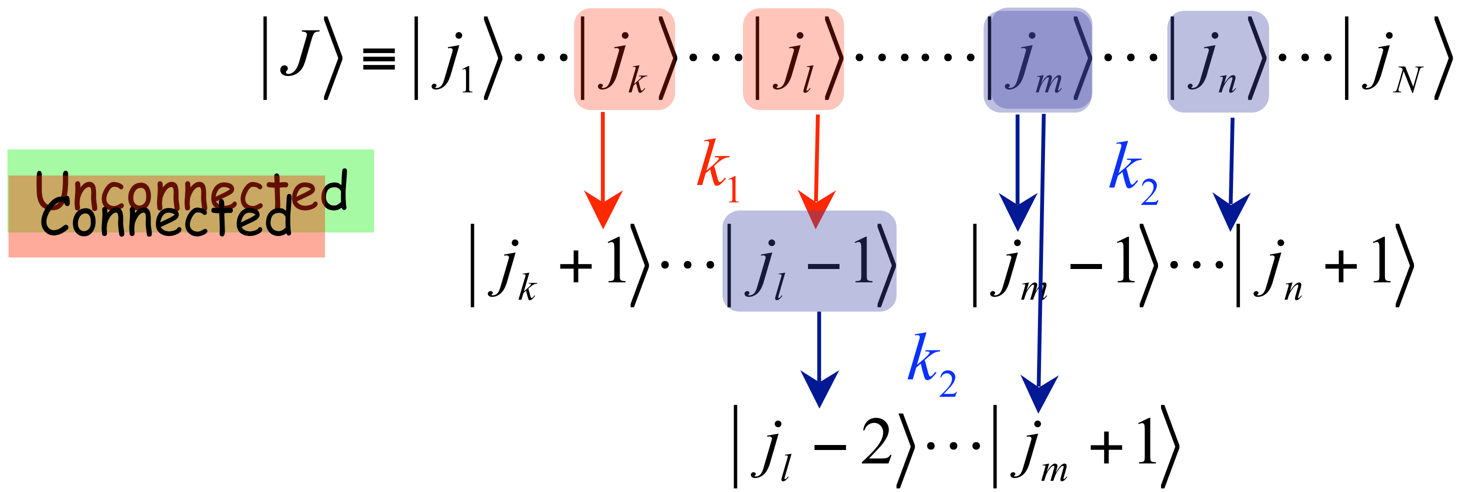


State at time t $\quad \left|J^{\pm}(t)\right\rangle = C_J^{\pm}(t)\left|J\right\rangle + \sum_{k_1} C_{J,k_1}^{\pm}(t)\left|J,k_1\right\rangle + \sum_{k_1,k_2} C_{J,k_1,k_2}^{\pm}(t)\left|J,k_1,k_2\right\rangle + \cdots$

Size of relevant Hilbert space $\quad \sim C_N^{N/4} C_{3N/4}^{N/4} C_{N/2}^{N/4}$

Nuclear spin dynamics: pair wise flip-flop

- **Unconnected pair-flips are independent**



- **Number of nuclear available for flip large**

  ~ N (~10^6) for typical $|J\rangle$

- **Number of flips much smaller than *N***

$$N_{\text{flip}} = \sum_{k_1} \left|C_{J,k_1}\right|^2 + 2\sum_{k_1,k_2} \left|C_{J,k_1,k_2}\right|^2 + 3\sum_{k_1,k_2,k_3} \left|C_{J,k_1,k_2,k_3}\right|^2 + \cdots \ll N$$

- **Probability of having connected pair-flips**

$$P_{\text{con}} \approx 1 - \exp\left(-N_{\text{flip}}^2 / N\right) \ll 1$$

Pair-Correlation-Method

- The two states connected by n-n pair-flip interactions is mapped into one two-state system (k[th] pseudo-spin).

Flip-pair to pseudo spin:
$$|j_n\rangle|j_m\rangle \Rightarrow |\uparrow\rangle_k, \quad |j_n+1\rangle|j_m-1\rangle \Rightarrow |\downarrow\rangle_k$$

flip-flop of nuclear pair mapped to single pseudo spin flip

Initial state:
$$|J\rangle \Rightarrow \bigotimes_k |\uparrow\rangle_k$$

- The dynamics of the pseudo-spins (pair-flips) is treated independent of each other.

Flip-flop dynamics to pseudo spin rotations:
$$|J^{\pm}(t)\rangle \Rightarrow \bigotimes_k |\psi_k^{\pm}\rangle, \quad |\psi_k^{\pm}\rangle \equiv e^{-i\mathcal{H}_k^{\pm}t}|\uparrow\rangle_k$$

Nuclear correlation func:
$$\mathcal{L}_{+,-}(t) = \left|\langle J^{-}(t)|J^{+}(t)\rangle\right| = \prod_k \left|\langle \psi_k^{-}(t)|\psi_k^{+}(t)\rangle\right|$$

WY, Liu and Sham, PRB 74, 195301, 2006

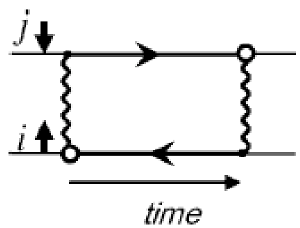
Pair-Correlation-Method

- Verified from a linked cluster expansion approach.

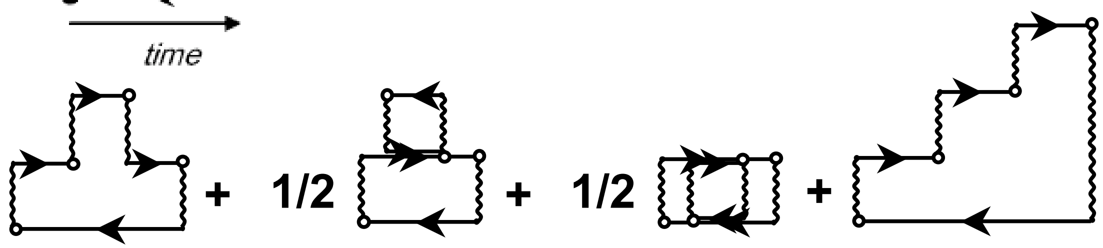
$$\langle J | T \exp\left(-i\int_0^t V(t')dt'\right) J\rangle = e^{\langle V_2(t)\rangle + \langle V_4(t)\rangle + \cdots}$$

Linked diagrams

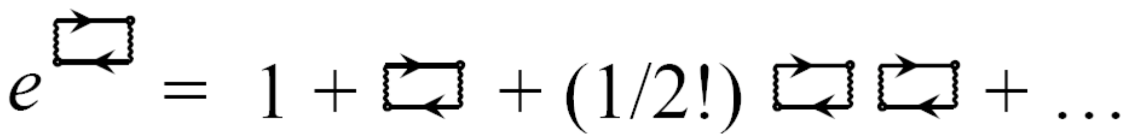
$\langle V_2(t)\rangle =$



$\langle V_4(t)\rangle =$



Independent pseudospin rotations:

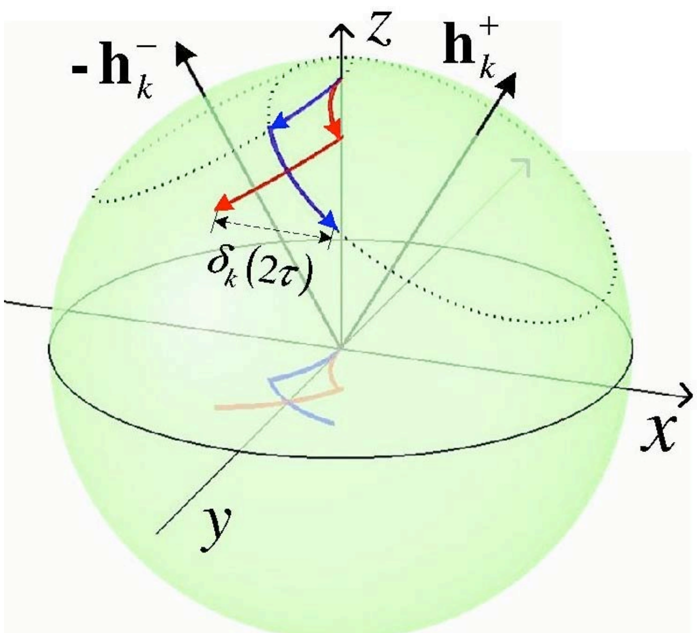


Saikin, WY and Sham, PRB 75, 125314, 2007

Geometrical picture for decoherence

$$\mathbf{h}_k^{\pm} = \left(B_k \pm A_k, 0, D_k \pm E_k\right) \qquad i\partial_t \left|\psi_k^{\pm}(t)\right\rangle = \left(\mathbf{h}_k^{\pm} \cdot \hat{\mathbf{o}}_k / 2\right)\left|\psi_k^{\pm}(t)\right\rangle$$

- Rotation of Bloch vector $\boldsymbol{S}_{\boldsymbol{k}}^{\pm} = \left\langle \psi_k^{\pm}(t) \middle| \hat{\mathbf{o}}_k \middle| \psi_k^{\pm}(t) \right\rangle$ in effective field $\mathbf{h}_k^{\pm}$.
- Distance $\delta_k = \left|\boldsymbol{S}_{\boldsymbol{k}}^{+} - \boldsymbol{S}_{\boldsymbol{k}}^{-}\right| = \sqrt{1 - \left|\left\langle \psi_k^{-}(t) \middle| \psi_k^{+}(t) \right\rangle\right|^2}$ is a direct measure of electron spin coherence.



Two mechanisms for pair-flip

Step 1: sort out all pseudo-spins (flip-pairs) from a given configuration

$$\mathcal{L}_{+,-}(t) = \left|\left\langle J^-(t)\middle| J^+(t)\right\rangle\right| = \prod_k \left|\left\langle \psi_k^-(t)\middle|\psi_k^+(t)\right\rangle\right|$$

Step 2: determine the pseudo-field from 1st-principle interactions

$$\mathbf{h}_k^\pm = \left(B_k \pm A_k, 0, D_k \pm E_k\right) \qquad i\partial_t\left|\psi_k^\pm(t)\right\rangle = \left(\mathbf{h}_k^\pm\cdot\hat{\boldsymbol{\sigma}}_k/2\right)\left|\psi_k^\pm(t)\right\rangle$$

D_k by diagonal nuclear couplings $\approx 10^2\ \mathrm{s}^{-1}$

Non-local pairs: Extrinsic n-n interaction dominant

E_k by diagonal hyperfine interaction $\approx 10^6\ \mathrm{s}^{-1}$

$\mathbf{h}_k^\pm = \left(\pm A_k, 0, \pm E_k\right)$ ~ N^2 pairs, magnetic field dependent

A_k by extrinsic hyperfine mediated nuclear coupling

infinite-range, $\propto \mathrm{B}^{-1}$, $\approx 1-10\ \mathrm{s}^{-1}$ (for $B \sim 40-1$ T)

Local pairs: Intrinsic n-n interaction dominant

$\mathbf{h}_k^\pm = \left(B_k, 0, \pm E_k\right)$ ~ N pairs, no field dependence

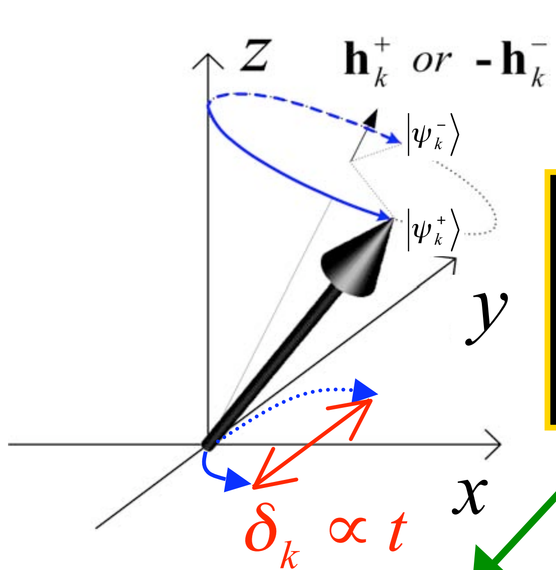
B_k by intrinsic nuclear coupling

finite-range, $\approx 10^2\ \mathrm{s}^{-1}$ for near neighbors

Two mechanisms for pair-flip

Extrinsic n-n interaction

$$\mathbf{h}_k^{\pm} = \left(\pm A_k, 0, \pm E_k\right)$$



$$\mathcal{L}_{+,-}(t) = \exp(-t^2 / T_{2,A}^2)$$

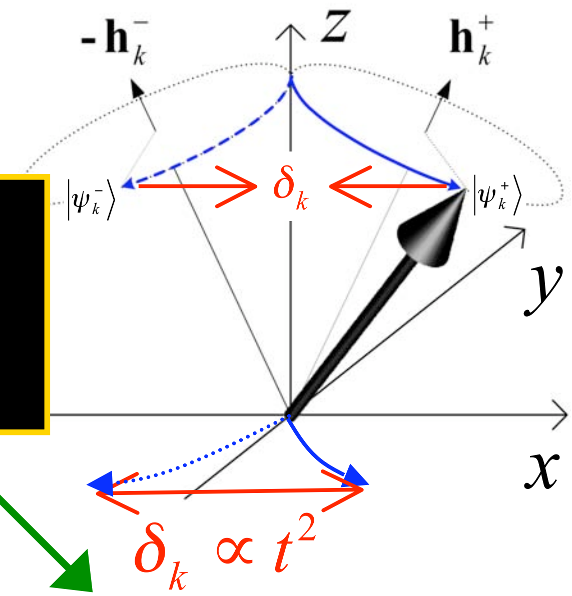
$$T_{2,A} \approx N \Omega_e \mathcal{A}^{-2}$$

N: QD size

Ω_e: electron zeeman energy

Intrinsic n-n interaction

$$\mathbf{h}_k^{\pm} = \left(B_k, 0, \pm E_k\right)$$



$$\mathcal{L}_{+,-}(t) = \exp(-t^4 / T_{2,B}^4)$$

$$T_{2,B} \approx N^{1/4} \mathcal{A}^{-1/2} b^{-1/2}$$

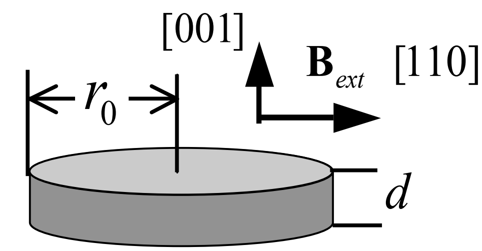
$\mathcal{A}$: hyperfine constant

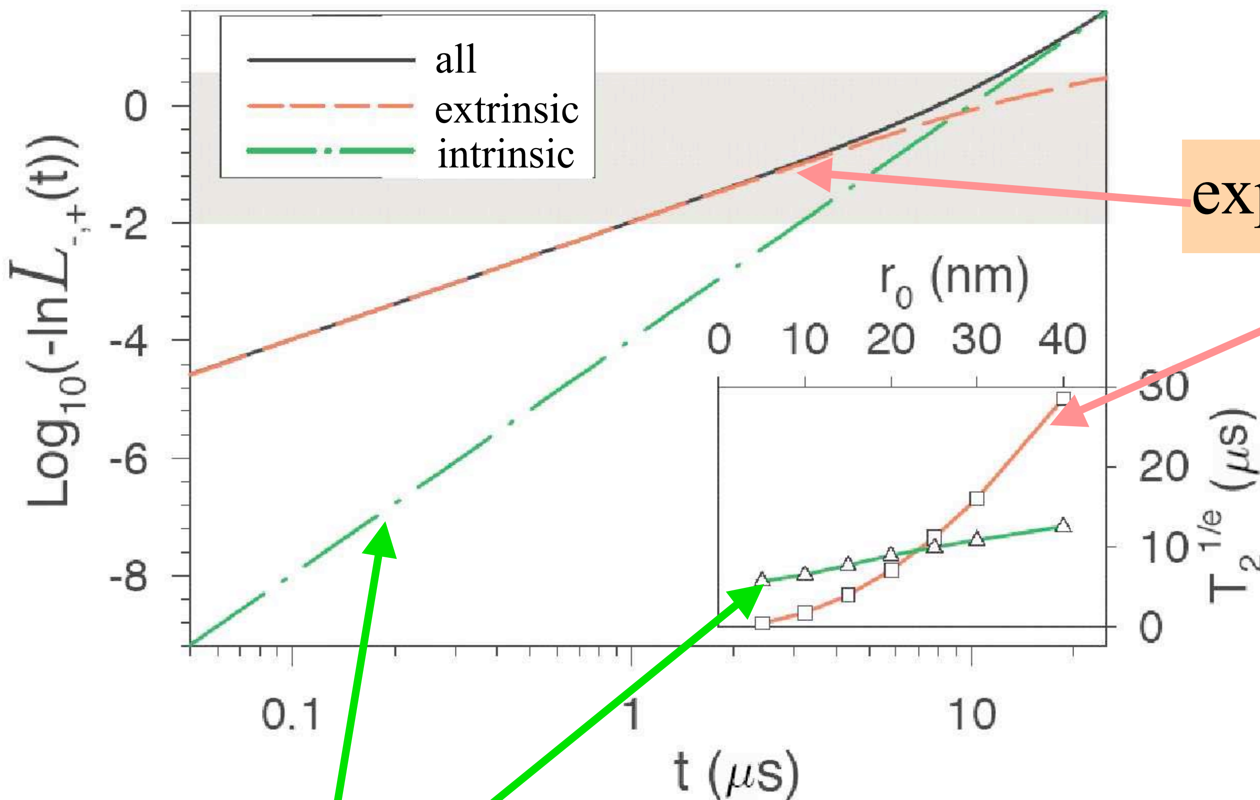
b: intrinsic n-n coupling strength

Single spin FID: intrinsic vs extrinsic

Nuclear bath begins on a pure state of random configuration

GaAs, d=6 nm, r_0=25 nm, B = 12T





$\exp[-(t/T_{2,A})^2]$

$\exp[-(t/T_{2,B})^4]$

Extrinsic interaction dominates small dot

Intrinsic interaction dominates large dot

WY, Liu and Sham, PRB 74, 195301, 2006

Single System FID: field dependence

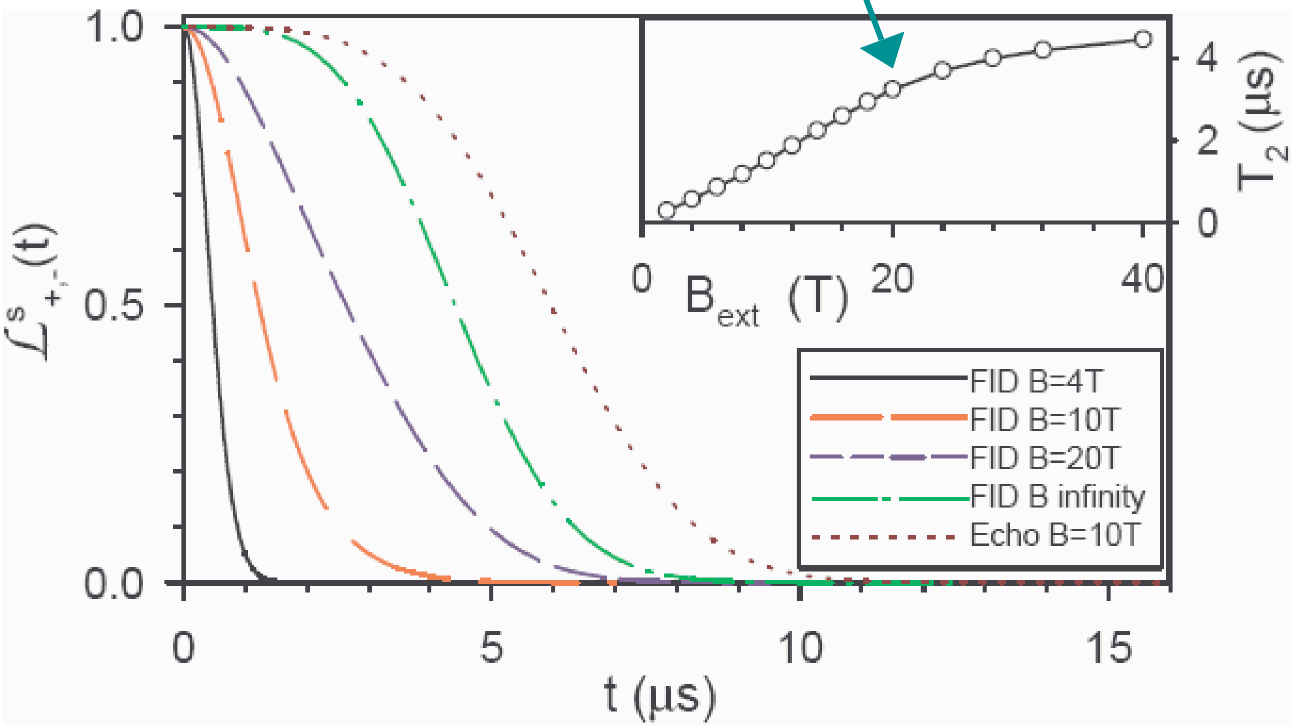
Nuclear bath begins on a pure state of random configuration

Strong field dependence shows extrinsic n-n interaction important

GaAs

d=3 nm

r_0=15 nm



Ensemble dynamics

Single system (t=0): $\left|\psi^{e}(0)\right\rangle \otimes \left|J\right\rangle$

Ensemble (t=0): $\rho(0) = \rho^{e}(0) \otimes \sum_{J} P_J \left|J\right\rangle\left\langle J\right|$

Ensemble correlation function:

nuclear Overhauser field $\mathcal{E}_J = \Sigma_n a_n j_n$

by the spectrum of flip-pairs of $\left|J\right\rangle$

$$\rho^{e}_{+,-}(t) = \mathcal{L}^{en}_{+,-}(t)\rho^{e}_{+,-}(0) \qquad \mathcal{L}^{en}_{+,-}(t) = \sum_{J} P_J e^{-i\mathcal{E}_J t} \left|\left\langle J^{-}(t) \middle| J^{+}(t)\right\rangle\right|$$

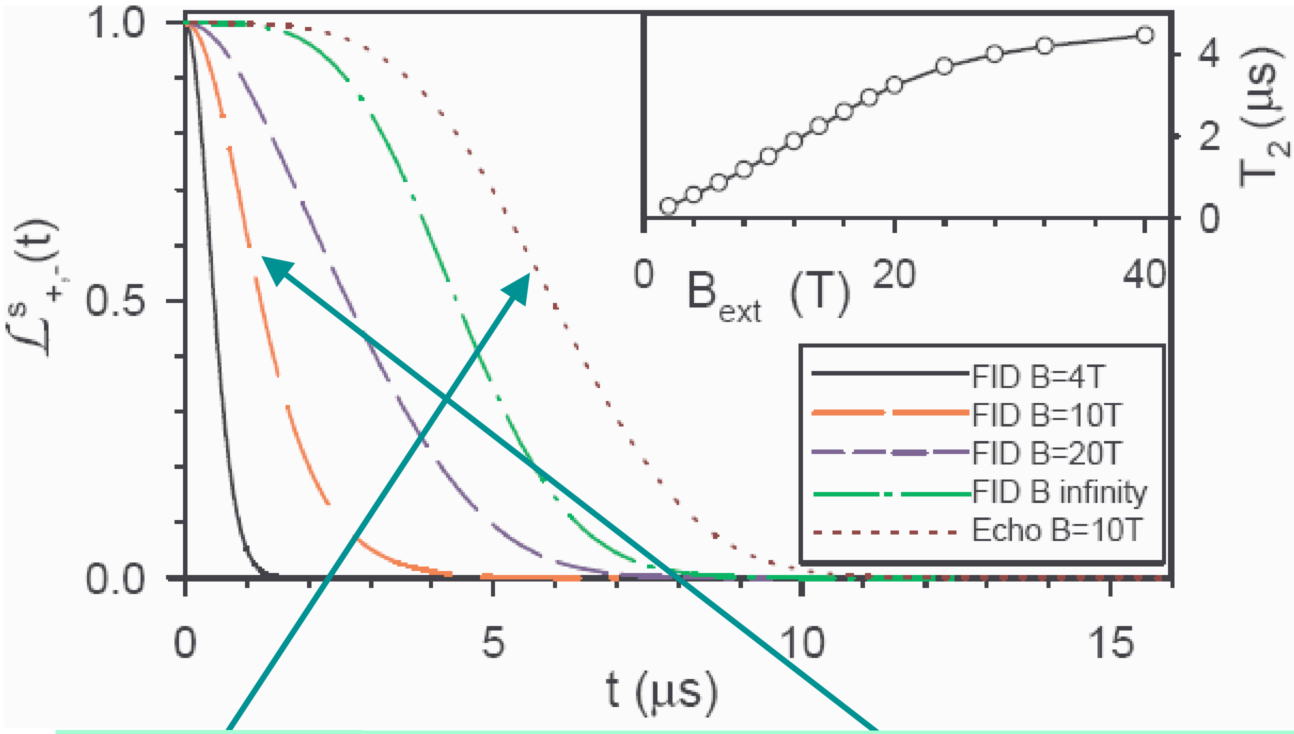
- Number of flip-pairs *M* ~ *N* (or N^2) for most $\left|J\right\rangle$
- Identical excitation spectrum up to an error of □ $1/\sqrt{M}$
- Identical temporal behavior for $\left|\left\langle J^{-}(t) \middle| J^{+}(t)\right\rangle\right|$
- $\mathcal{L}^{en}_{+,-}(t) = \left|\left\langle J_0^{-}(t) \middle| J_0^{+}(t)\right\rangle\right| \sum_{J} P_J e^{-i\mathcal{E}_J t}$

System-bath entanglement

Inhomogeneous broadening
$\sum_{J} P_J e^{-i\mathcal{E}_J t} = e^{-(t/T_2^*)^2}, T_2^* \sim 10$ ns

Single System FID and Ensemble Echo

Nuclear bath begins on a pure state of random configuration



GaAs

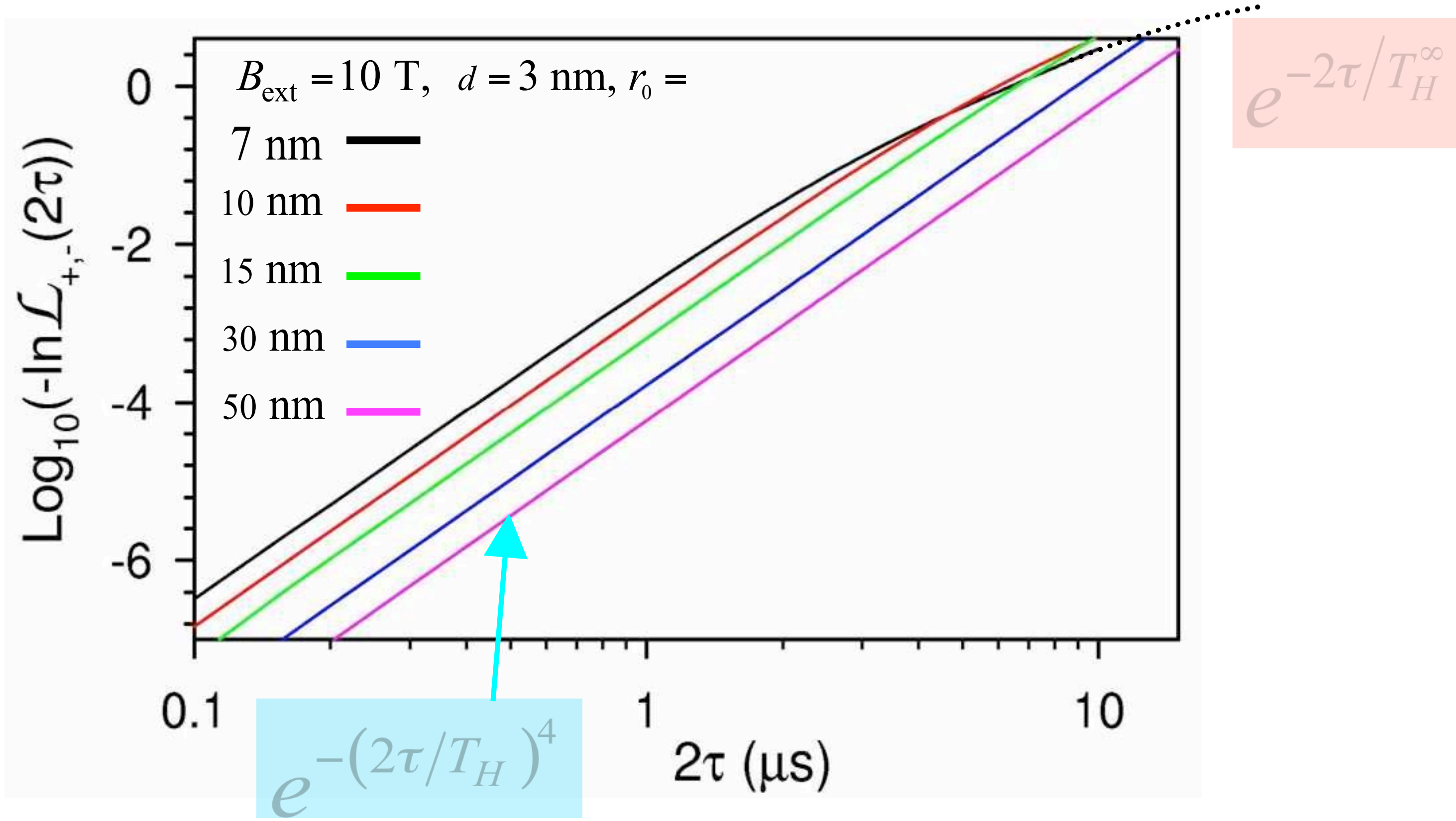
d=3 nm

r_0=15 nm

Spin echo (T_H) does NOT measure single spin FID (T_2)

WY, Liu and Sham, PRB 74, 195301, 2006

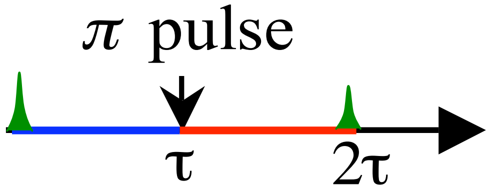
Ensemble Spin Echo



Decoherence due to hyperfine mediated nuclear interactions is removed in spin echo!

(also in simulation with ~10 bath spin, Shenvi, deSousa and Whaley, PRB 05")

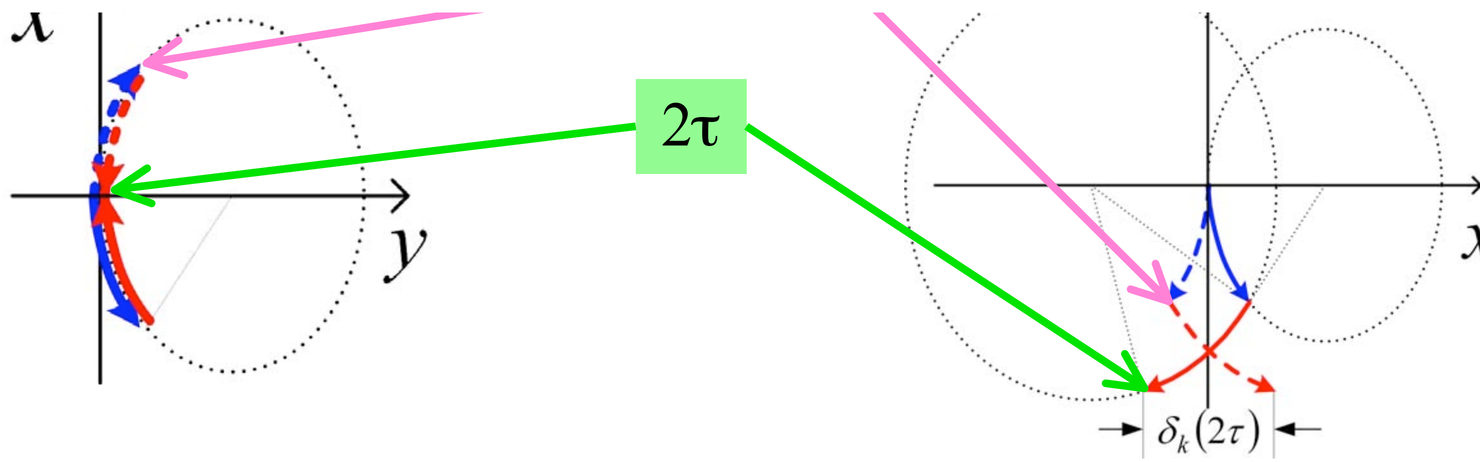
Pulses on electron and effects on nuclear spin



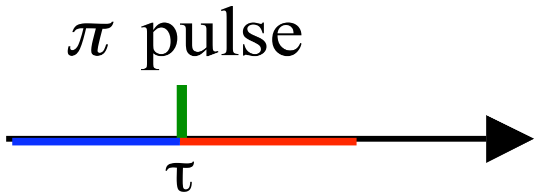
$$\text{e spin: } \left|+\right\rangle \xrightarrow{\pi \text{ pulse}} \left|-\right\rangle$$

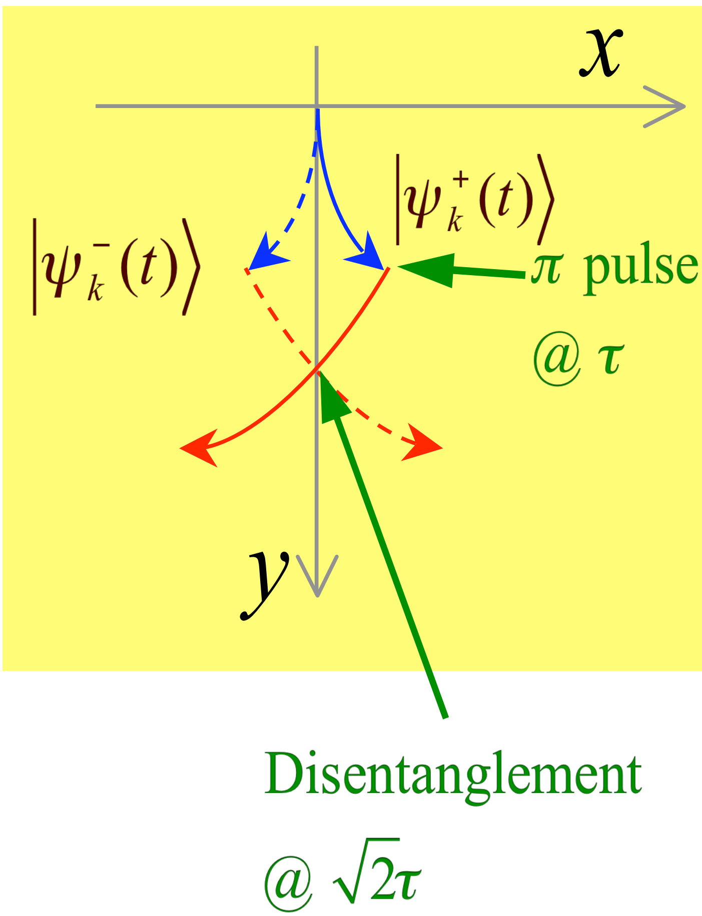
$$\text{pseudo spin: } \left|\psi_k^{\pm}(2\tau)\right\rangle = \exp(-i\mathcal{H}_k^{\mp}\tau)\exp(-i\mathcal{H}_k^{\pm}\tau)\left|\uparrow\right\rangle_k$$

Spin echo measures the single spin decoherence induced by intrinsic nuclear interactions only ! $T_H = \sqrt{2}T_{2,B}$



Control of Bath Evolution for Disentanglement





- Flip of electron redirects evolution of bath
- Pseudo-spin paths re-intersects
- Universally at $\sqrt{2}\tau$ for all pseudo-spins
- System disentangled from bath

$$\left(\alpha|+\rangle+\beta|-\rangle\right)\otimes\left|J(\sqrt{2}\tau)\right\rangle$$

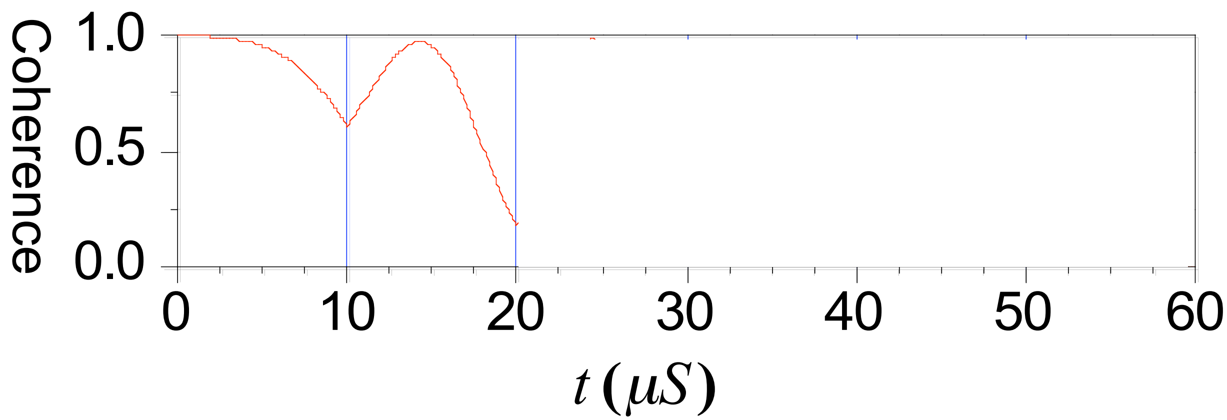
WY, Liu and Sham, PRL 98, 077602, 2007

Liu, WY and Sham, cond-mat/0703690

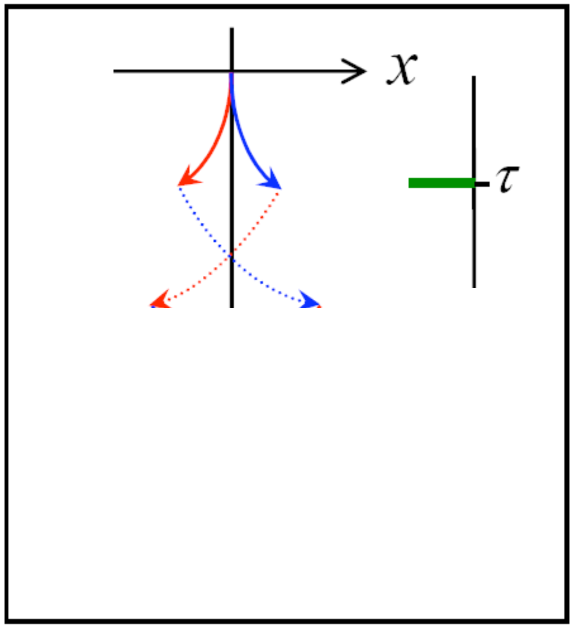
Coherence Echo in Single System Dynamics

Nuclear bath begins on a pure state of random configuration

$d = 6$ nm, $r_0 = 40$ nm, $B = 10$ T, intrinsic n-n interaction dominant



coherence echo @ $\sqrt{n(n+1)}\tau$



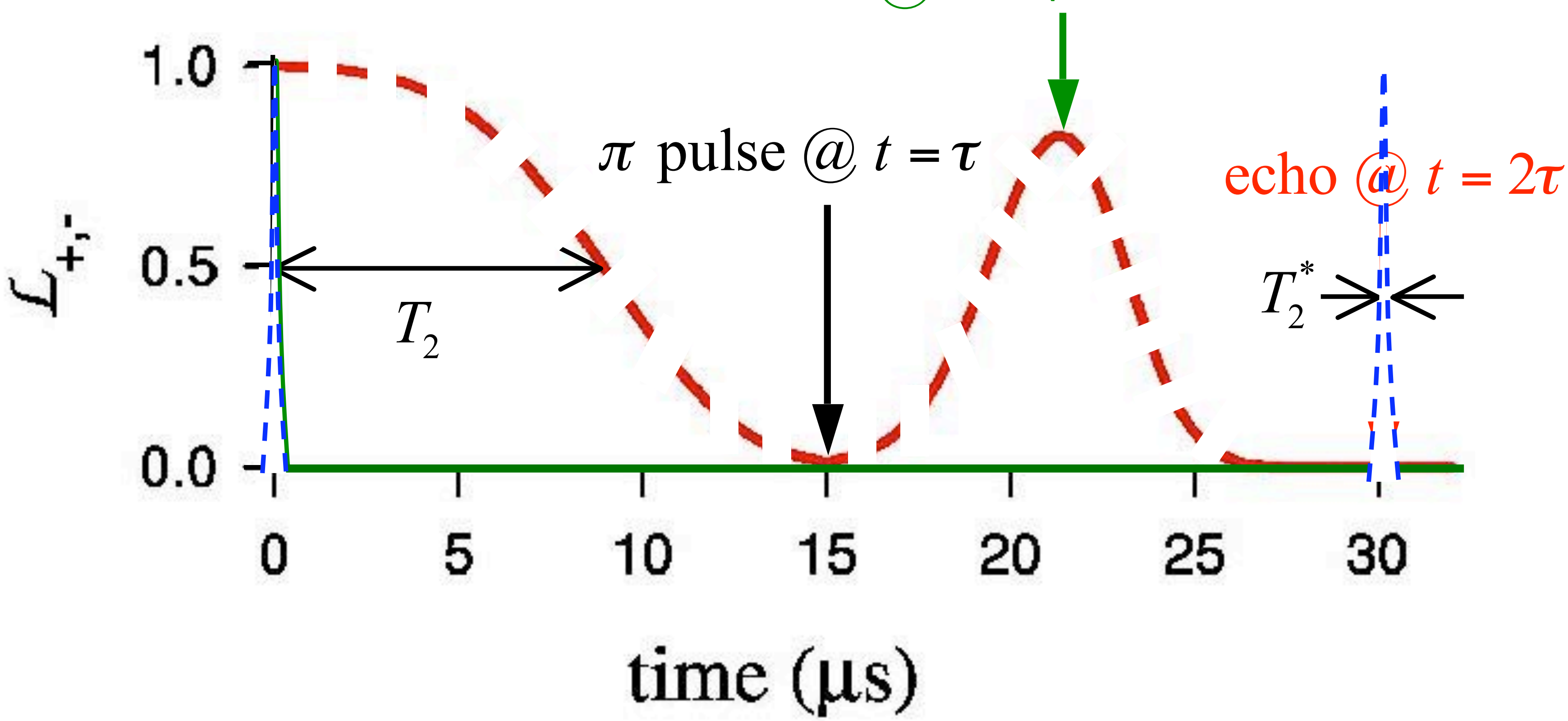
- Different from dynamical decoupling, sys-bath coupling does not vanish

$$e^{-iH_{\text{eff}}\sqrt{2}\tau} \equiv e^{iH^-\left(\sqrt{2}-1\right)\tau} e^{-iH^+\tau} \Rightarrow H_{\text{eff}} \neq 0$$

Disentanglement Concealed in Ensemble Dynamics

$d = 6$ nm, $r_0 = 40$ nm, $B = 10$ T, intrinsic n-n interaction dominant

$$\mathcal{L}^{en}_{+,-}(t) = \left|\left\langle J_0^-(t) \middle| J_0^+(t) \right\rangle\right| \times \sum_J P_J e^{-i\mathcal{E}_J t}$$



Disentanglement Concealed in Ensemble Dynamics

To observe quantum disentanglement in ensemble dynamics

a. Filter inhomogeneous broadening by measurement projection

Schemes to be realized in near future:

Klauser, Coish & Loss, PRB 2006

Giedke, Taylor, D'Alessandro, Lukin & Imamoglu, PRA 2006

Stepanenko, Burkard, Giedke & Imamoglu, PRL 2006

or

b. Arrange disentanglement time to coincide ensemble echo time

WY, Liu and Sham, PRL 98, 077602, 2007

Liu, WY and Sham, cond-mat/0703690

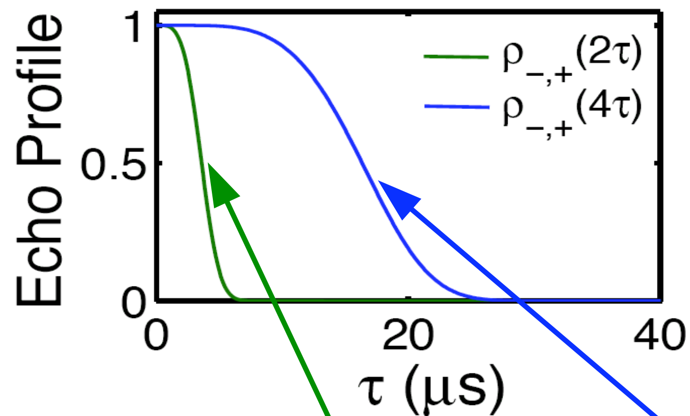
Two-pulse control: Disentanglement at echo time

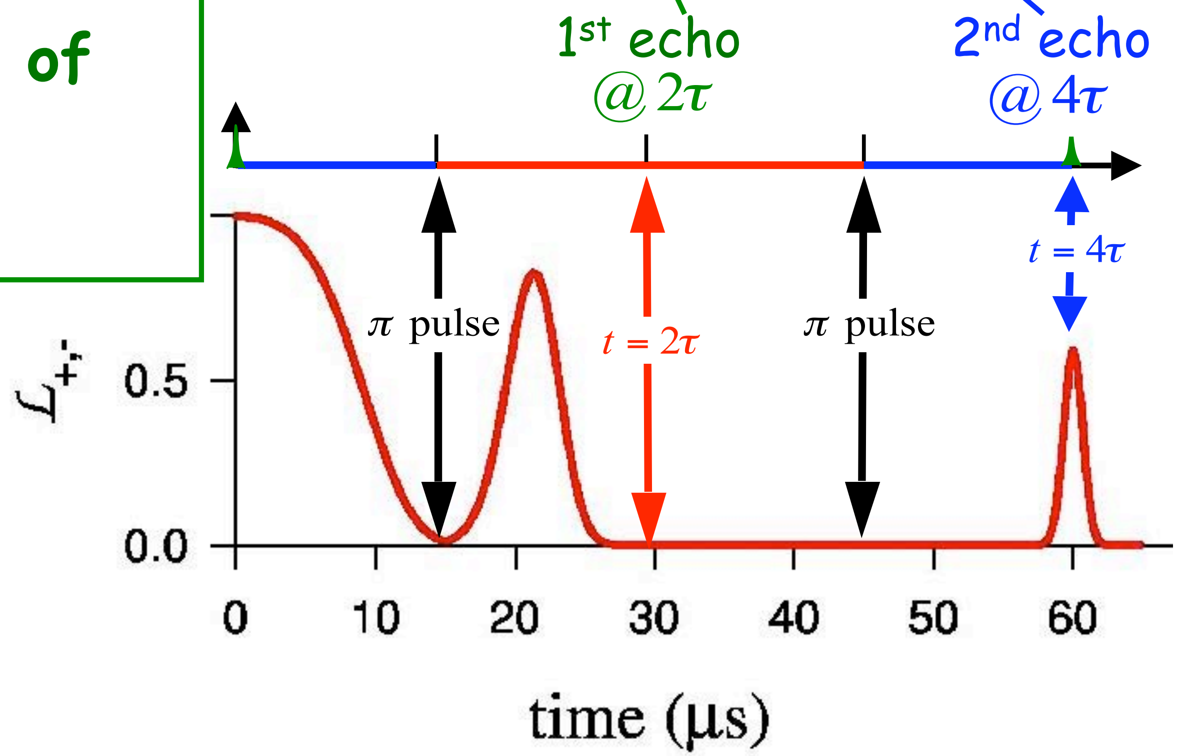
π pulse @ τ

Absences of spin echo

≠

Irreversible loss of coherence





Disentanglement @ 2nd echo time

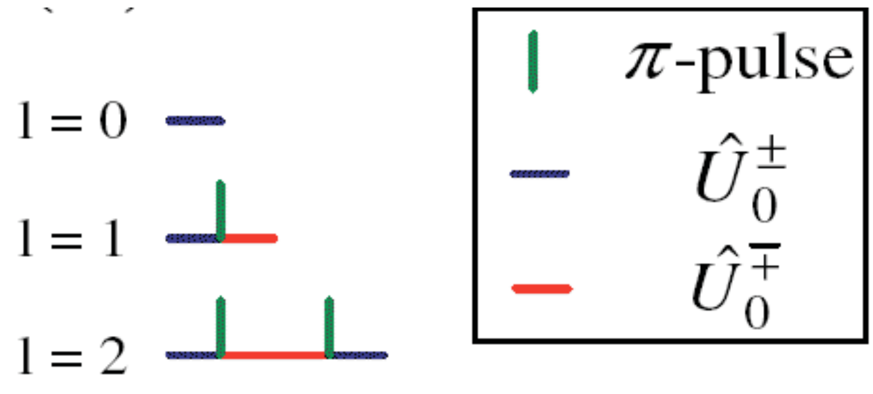
Concatenated Dynamical Disentanglement

Concept of concatenation on dynamical decoupling, Khodjasteh and Lidar, PRL 05"

$$U_0^\pm \equiv e^{-i\mathcal{H}_k^\pm \tau}$$

$$U_1^\pm \equiv U_0^\mp U_0^\pm$$

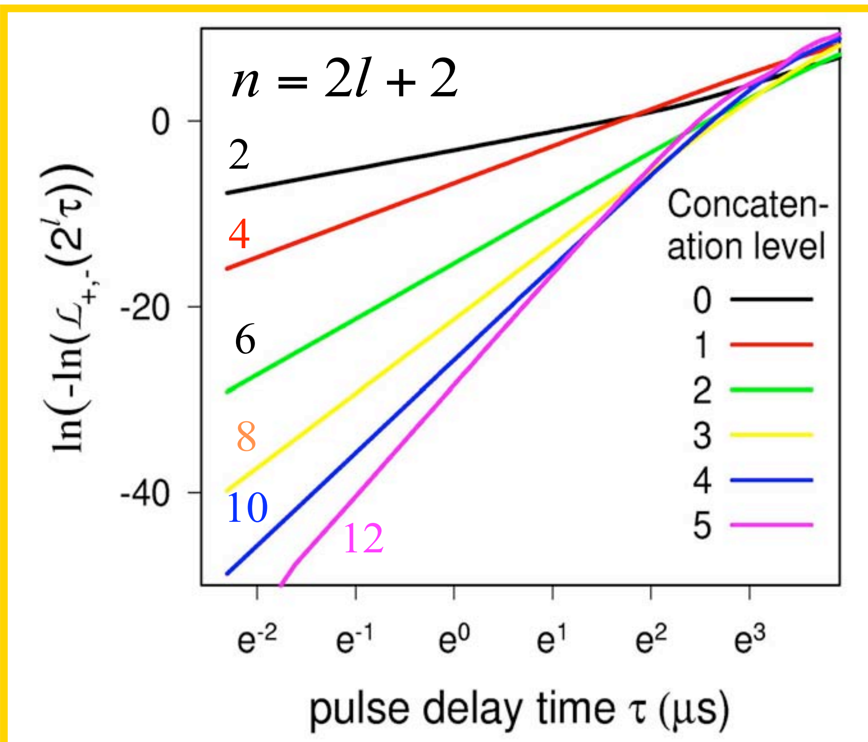
$$U_2^\pm \equiv U_1^\mp U_1^\pm$$

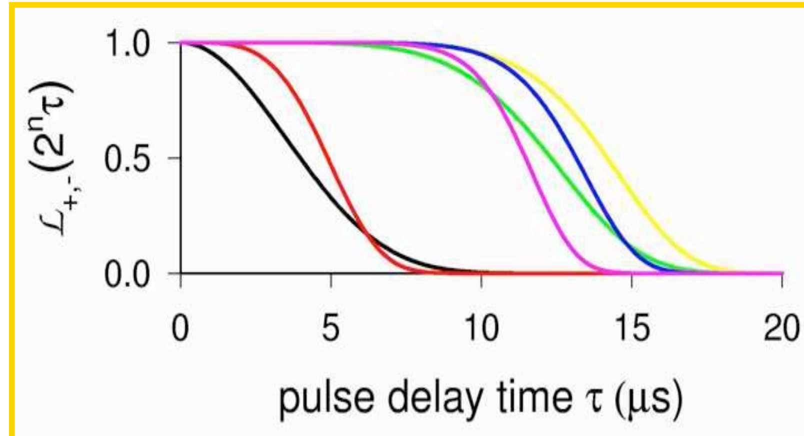


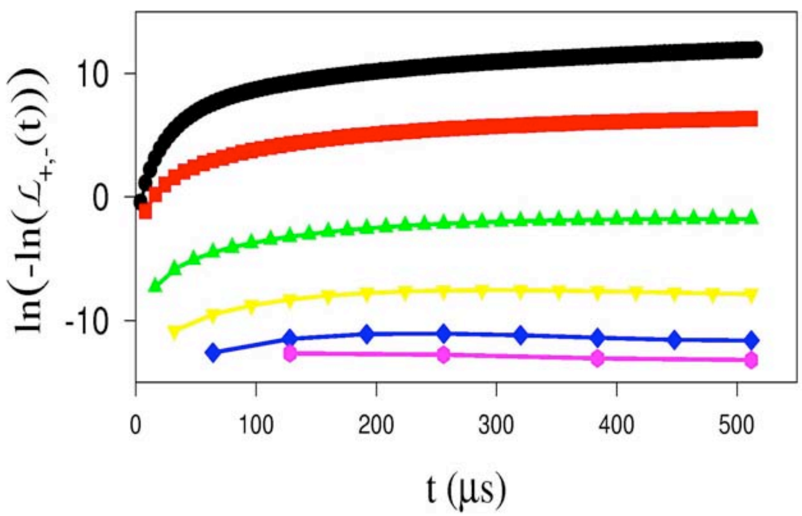
- Disentanglement occur at $\tau_l = 2^l \tau$
- Decoherence reduced by $(b\tau_l)^2$ with each level of concatenation
- Controlling small parameter: bath spin interaction $b \sim 10^2 s^{-1}$

Concatenated Dynamical Disentanglement

$$\exp\left(-t^{2l+2}/T_{c,l}^{2l+2}\right)$$







WY, Liu and Sham, PRL 98, 077602, 2007

Liu, WY and Sham, cond-mat/0703690

Stabilization with repeated # of Concatenated units

Summary

- ✓ **1+N isolated from rest universe in relevant timescale**
- ✓ **Quantum theory of decoherence: bath evolution conditioned on system state**
  - Decoherence by sys-bath entanglement
  - Operations on system affects bath evolution
  - Spin echo removes more than inhomogenesous dephasing
  - Echo decay time T_H NOT equivalent to single spin T_2
- ✓ **Slow bath dynamics: pair-correlations dominates**
- ✓ **Sys-bath disentanglement by pulse sequences on electron.**
  - Concatenation design more efficient then periodic sequence.

Thank You!

Canonical transformation:

$$\hat{W} \equiv \exp\left(\sum_n \frac{a_n}{2(\Omega_e - \omega_n)} (\hat{S}_e^+ \hat{J}_n^- - \hat{S}_e^- \hat{J}_n^+) \right)$$

Effective Hamiltonian:

$$\hat{H}_{\text{red}} = \hat{W} \hat{H} \hat{W}^{-1}$$

Rotation of wavefunction:

$$\hat{W} | \pm \rangle \otimes |\mathcal{J}\rangle \approx \left(1 - \frac{1}{2} \sum_n |w_n^{\pm}|^2 \right) | \pm \rangle \otimes |\mathcal{J}\rangle \mp \sum_n w_n^{\pm} | \mp \rangle \otimes |j_n \pm 1\rangle \bigotimes_{m \neq n} |j_m\rangle$$

Visibility loss: $(\Omega_e \sqrt{N} / \mathcal{A})^{-2} \ll 1$ for B > 1T

Effective dynamics: $e^{-i\hat{H}t} | \pm \rangle \otimes |\mathcal{J}\rangle \cong e^{-i\hat{H}_{\text{red}}t} | \pm \rangle \otimes |\mathcal{J}\rangle$

Description of 2 level quantum system

Single quantum system

$$|\varphi\rangle = \alpha|+\rangle + \beta|-\rangle \qquad \rho \equiv |\varphi\rangle\langle\varphi| = \begin{bmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{bmatrix}$$

Ensemble of identical quantum systems

$$\rho = \sum_{i=1}^{N} P_i |\varphi_i\rangle\langle\varphi_i| = \begin{bmatrix} \sum_i P_i|\alpha_i|^2 & \sum_i P_i\alpha_i\beta_i^* \\ \sum_i P_i\alpha_i^*\beta_i & \sum_i P_i|\beta_i|^2 \end{bmatrix}$$

Reduced density matrix for open quantum system

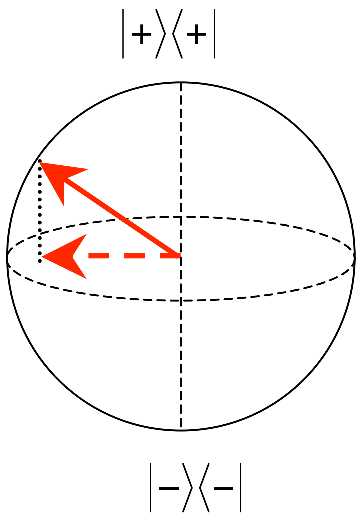
$$|\psi_{tot}\rangle = \sum_i |\alpha_i\rangle_{sys} \otimes |\beta_i\rangle_{en} \quad \Rightarrow \quad \rho_{sys} = Tr_{en}[|\psi_{tot}\rangle\langle\psi_{tot}|]$$

Decoherence: relaxation and pure dephasing

Geometric representation

$$\rho = \frac{1}{2}(I + \vec{a}\cdot\vec{\sigma}),$$

Bloch vector : $\frac{1}{2}\vec{a} = Tr\left[\rho\vec{S}\right]$



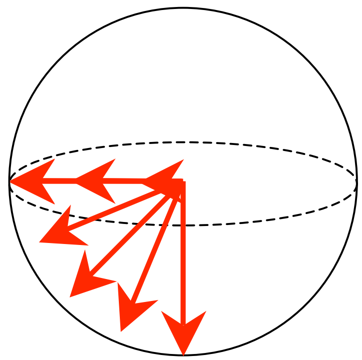
$$a_x = 2\,\mathrm{Re}\,\rho_{21} \qquad a_y = 2\,\mathrm{Im}\,\rho_{21} \qquad a_z = \rho_{11} - \rho_{22}$$

Longitudinal relaxation

$T_2^{rl} = 2T_1$ Decay of coherence as a result of decay of population

Pure dephasing

T_2^{deph} No decay of population. Usually much faster than T_1



Inhomogeneous broadening & spin echo