Would Schrödinger's Cat Have Collapsed Its Own Wavefunction?

A Search For Gravitational Decoherence

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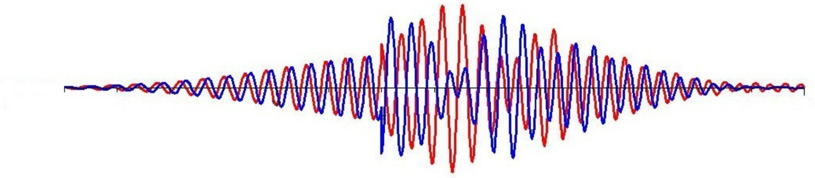


Why suspect a loss of coherence due to gravity?

- In the 50's, Feynman pointed out that quantum fluctuations of spacetime itself should affect the coherence of a system
- Standard quantum superpositions assume a fixed background spacetime, though an ambiguity arises if we take into account general relativity: the superposed states can correspond to arbitrarily different spacetimes!
- Penrose has been arguing since the 80's that this ambiguity leads to an instability of superpositions, characterized by a decay time that is inversely proportional to the difference in the (loosely-defined) gravitational self-energies:

$$\Delta t \simeq \hbar / E_\Delta, \quad E_\Delta \sim \int d^3x \left(\vec{g} - \vec{g}'\right)^2$$

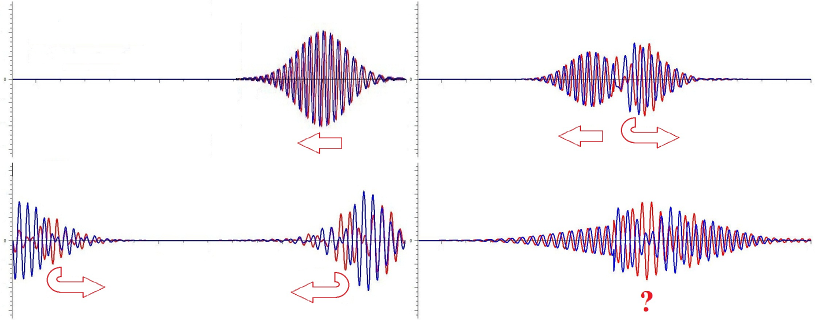
How can we show that general relativitic effects lead to a loss of quantum coherence?

- Ideally, we would like to quantize a fully general relativistic interferometer, and compare the possible coherence of the interferometer with that of the same type of interferometer in flat spacetime (easier said than done)
- We will consider a simple general relativistic model of a spherical thin-shell, use it to construct an analog of a Michelson interferometer
- The resulting interferometer can be analyzed with WKB modes in an optics-inspired approximation

Self-Gravitating Spherical Thin-Shell Interferometers

- In the minisuperspace approach, the shell radius $X(t)$ becomes the only physical degree of freedom
- A wavefunction can be defined for this degree of freedom, and we can build our interferometer by splitting and recombining travelling wave-packets

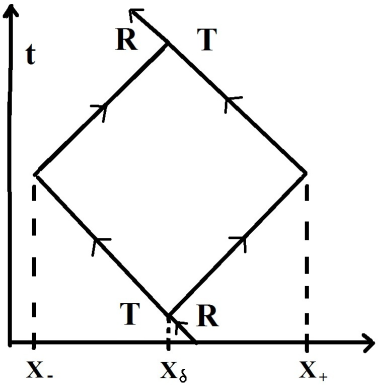
Sample Wavepacket Splitting (in the Newtonian Limit)



The blue curve is the real part of the shell wavefunction (as a function of X at successive values of t), while the red curve is the imaginary part

Interferometer Setup

- In flat spacetime, destructive interference in the final outgoing path can be made complete, which corresponds to full final transmission



- Our interferometer has two reflectors (at X_- and X_+), and a 'half-silvered mirror' somewhere in between (at X_δ)
- The effective beam-splitter has (mode-dependent) reflection and transmission coefficients R and T, respectively
- Our initial state will be ingoing, and localized just outside the splitter

Coherence and Probability Currents in the $\hbar \to 0$ Limit

- We seek to quantify coherence in our system by considering the final transmission coefficient, $T_f = \left|\frac{J_f}{J_i}\right|$, with J being the probability current. The continuity equation for probability is given by

$$\frac{\partial}{\partial t}\left(\|\psi\|^2\right) + \frac{\partial}{\partial X} J = 0$$

- In the $\hbar \to 0$ limit, we may approximate our Hamiltonian by

$$H \sim H_0 + \left(\frac{\partial H}{\partial P}\right) P + \frac{1}{2}\left(\frac{\partial^2 H}{\partial P^2}\right) P^2 + \mathcal{O}(P^3)$$

This leads to a conserved probability current of the form

$$J \sim \left(\frac{\partial H}{\partial P}\right) \|\psi\|^2 + \frac{1}{2i}\left(\frac{\partial^2 H}{\partial P^2}\right)\left(\psi^* \psi' - \psi \psi^{*\prime}\right)$$

(Here derivatives are evaluated at $P = 0$)

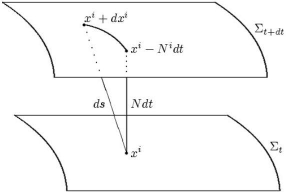
Now all we need is a general-relativistic Hamiltonian!

- There once was a classical theory..

$$I = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,\mathcal{R} + I_s$$

- Spacetime Metric in ADM (3+1) Form

$$g_{\mu\nu}dx^\mu dx^\nu = -N^2 dt^2 + g_{rr}\left(dr + N^r dt\right)^2 + g_{\theta\theta}d\Omega^2$$



- The shell action resembles that of a relativistic particle, but with a nonconstant mass:

$$I_s = -\int d\lambda\sqrt{\left|g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}\right|}M(\chi)$$

- The density and pressure are parametrized by $M(\chi)$, for $\chi = \sqrt{g_{\theta\theta}}$:

$$\sigma = M(\chi)/4\pi\chi^2, \quad p = -M'(\chi)/8\pi\chi$$

Hamiltonian Reduction and Coordinate Choices

- Since there are 2 constraints and 2 gravitational variables, we can solve the constraints and eliminate the gravitational degrees of freedom altogether, leaving a reduced action that depends only on the shell degree of freedom $X(t)$
- To reduce the system, we must impose coordinate conditions. We will choose coordinates that resemble flat-slice (Painlevé-Gullstrand)

$$g_{rr} = 1, \quad g_{\theta\theta} = r^2$$

- Strictly speaking, we must include a deformation region in $g_{\theta\theta}$ near the shell, to satisfy the constraints
- Connection between our time and Schwarzschild time (at the shell):

$$t_s = t - 2\sqrt{2XE} + 2E \ln\left(\frac{1+\sqrt{\frac{2E}{X}}}{1-\sqrt{\frac{2E}{X}}}\right)$$

Reduced Action in terms of an implicit Hamiltonian

- Because of gravitational contributions to the action, there is a new canonical momentum for the reduced system:

$$P_c = -\sqrt{2HX} - X \ln \left(\frac{X + \beta - \sqrt{2HX}}{X} \right)$$

$$\beta = \frac{H - \frac{\hat{M}^2}{2X} \mp \sqrt{\left(H - \frac{\hat{M}^2}{2X}\right)^2 - \hat{M}^2 \left(1 - \frac{2H}{X}\right)}}{1 + \sqrt{\frac{2H}{X}}}.$$

Here, $\hat{M} \equiv M(\chi$ (evaluated on the shell)) $\equiv M(X)$ (in our coordinates)

- This gives us the reduced action, and the Hamiltonian H is defined implicitly by the equation $P_c = P_c(H, X)$

$$I_{reduced} = \int dt \left(P_c \dot{X} - H \right)$$

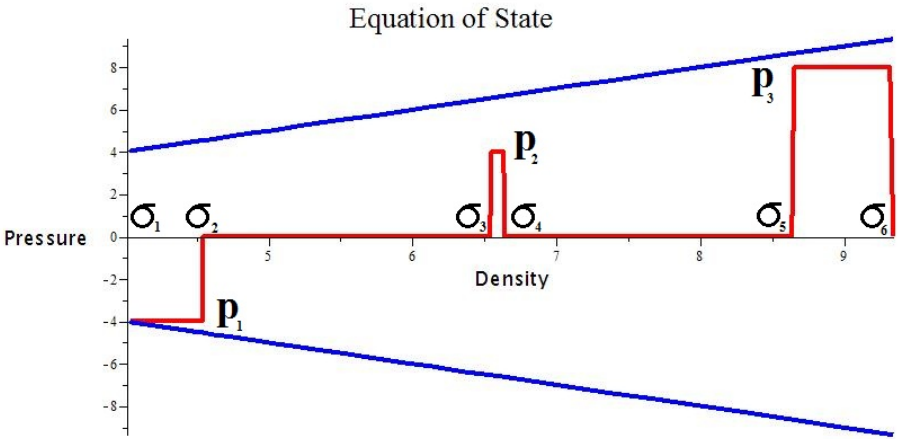
Weak-Field Asymptotics

- The reduced phase space is defined by an expression for the canonical momentum as a function of the Hamiltonian and the shell radius
- Finding the Hamiltonian as a function of X and P_c involves solving a transcendental equation
- We expect to see effects in the weak-field as well, so as well as working in the semiclassical limit ($\hbar \to 0$) we will take $G \to 0$ and work perturbatively:

$$H_0 \sim \hat{M} - \frac{\hat{M}^2}{18X}, \quad \left(\frac{\partial H}{\partial P}\right) \sim -\frac{2}{3}\sqrt{\frac{2\hat{M}}{X}}, \quad \left(\frac{\partial^2 H}{\partial P^2}\right) \sim \frac{1}{\hat{M}} + \frac{2}{3X}$$

- So we have an explicit Hamiltonian to work with, as well as a conserved probability current. Now, what function $\hat{M} = M(X)$ will describe our interferometer?

Equation of State for Interferometry

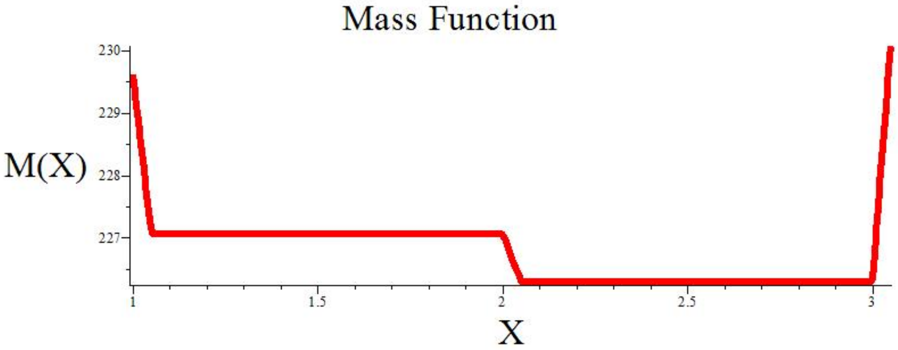


$$
\begin{aligned}
p = \quad & p_1 \quad (\Theta(\sigma - \sigma_1) - \Theta(\sigma - \sigma_2)) + p_2\,(\Theta(\sigma - \sigma_3) - \Theta(\sigma - \sigma_4)) \\
+ \quad & p_3 \quad (\Theta(\sigma - \sigma_5) - \Theta(\sigma - \sigma_6))
\end{aligned}
$$

Mass Function Parametrization

$$\hat{M} \equiv M(X) = M_0 + 4\pi \sum_i \tilde{p}_i \left(X_i^2 - X^2\right) \Theta(X - X_i)$$

$$\tilde{p}_2 = -\tilde{p}_1 = p_1 < 0, \tilde{p}_4 = -\tilde{p}_3 = p_2 > 0, \tilde{p}_6 = -\tilde{p}_5 = p_3 > 0$$



$$\sigma = M(X)/4\pi X^2, \quad p = -M'(X)/8\pi X$$

Quantum State Preparation

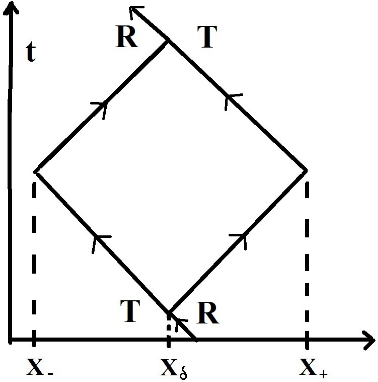
- With our interferometer setup complete, we can form an initial wave-packet and propagate it through the system
- Rather than solve the full eigenvalue problem, let us make use of WKB modes and a continuous-spectrum approximation:

$$\psi_0(X) \approx \int N\, e^{\frac{(E-E_0)^2}{4\sigma_E^2}}\, e^{i\int P_- dX} dE$$

- This integral can be evaluated if the ingoing WKB phase is linearized in energy. Because of square-roots, the resulting function of X is difficult to localize..

Mode-by-mode Propagation through the Interferometer

- As long as the width of our wave-packet is much less than the distance between optics, we may treat each component separately



- The initial splitting involves reflection and transmission coefficients given by

$$R_{\leftarrow} = \frac{P_{-+} - P_{--}}{P_{--} - P_{++}}, \, T = \frac{P_{-+} - P_{++}}{P_{--} - P_{++}}$$

- Each arm segment picks up a factor $e^{i \int P_{\pm} dX}$
- The recombination leading to our desired output involves the same T, and $R_{\rightarrow} = -R_{\leftarrow}$

Single-Mode Results

- Approximate Probability Current

$$J \sim \frac{1}{2\hat{M}i}\left(\psi^*\psi' - \psi\psi^{*\prime}\right)\left(1 + \frac{2\hat{M}}{3X}\right) - \frac{2}{3}\sqrt{\frac{2\hat{M}}{X}}\|\psi\|^2$$

- Initial state/current:

$$\psi_i = A, \quad J_i \sim |A|^2\left[\frac{P_{-+}}{M_+}\left(1 + \frac{2M_+}{3X_\delta}\right) - \frac{2}{3}\sqrt{\frac{2M_+}{X_\delta}}\right]$$

- Final current:

$$J_f \sim 4|A|^2 R^2 T^2\left[\frac{(P_{+-} + P_{-+})}{2M_-}\left(1 + \frac{2M_-}{3X_\delta}\right) - \frac{2}{3}\sqrt{\frac{2M_-}{X_\delta}}\right]\sin^2\omega$$

$$\omega \equiv \left(\int_{X_-}^{X_\delta} - \int_{X_\delta}^{X_+}\right)\left(\frac{P_+ - P_-}{2}\right)$$

Optimizing the Final Transmission

- Keeping the multi-mode problem in mind, let us tune the outer arm length $L_+ \equiv X_+ - X_\delta$ such that the travel times are classically the same (for the peak energy E_0)
- We can then tune M_+ so that the oscillatory term is optimized, for a given M_- and peak energy E_0
- In flat spacetime, these conditions imply

$$L_+ = L_- \sqrt{\frac{E_0^2 - M_+^2}{E_0^2 - M_-^2}} > L_-$$

$$L_- \sqrt{E_0^2 - M_-^2} - L_+ \sqrt{E_0^2 - M_+^2} = \left(n + \frac{1}{2}\right)\pi, \quad n \in \mathbb{Z}$$

- For the multi-mode problem, we can also tune the inner arm length L_- so that the propagation time through the system is less than the coherence time of the wave-packet ($\sim 1/\sigma_E$)

Tuning the Multi-Mode System

- The amplitude multiplying the oscillatory term in the single-mode transmission (T_f) can be optimized by tuning E_0
- This can produce complete final transmission ($T_f = 1$) for single modes, yielding no evidence of decoherence
- However, the single-mode tuning guides our multi-mode analysis; for instance, the wave-packet width σ_E should be chosen such that $E \to E_0 + \delta E$ produces a minimal reduction of the optimized amplitude, for $\delta E \sim \sigma_E$
- Remaining freedom (i.e. choice of X_-, M_-) can be used to further optimize both the oscillatory term and its amplitude
- For wave-packets, the structure of T_f is considerably more complicated! Further analysis is still required to determine whether this system exhibits evidence of gravitational decoherence

Outlook

- We are currently finishing up the analysis for wave-packets made of general-relativistic WKB modes
- To check coordinate dependence of the results, the analysis is also being performed with a one-parameter family of similar coordinates
- Still, important questions remain: is either the reduced phase space or the WKB approximation throwing out the baby with the bathwater?
- Thanks to Bill Unruh for guidance, NSERC for financial support, and Friedemann Queisser for fruitful discussions