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Loss of coherence due to the use of real clocks and rods to measure space-time

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THE PROBLEM OF TIME IN CANONICAL GRAVITY

There is by now extensive literature addressing the problem of time in classical and quantum gravity (e.g. Kuchař ‘s review). http://www.phys.lsu.edu/faculty/pullin/kvk.pdf

The heart of the problem lies in the fact that Einstein’s theory is a totally constrained system whose *Hamiltonian vanishes*, and since observable quantities are those that commute with the constraints (Dirac Observables) they therefore *do not evolve*.

We will discuss here two approaches to this problem.

Both have in common their relational character. In fact, one of the basic ingredients in the different proposals to describe evolution is the use of *relations* between different degrees of freedom in the theory .

- *Evolving Dirac observables.* (Bergmann, DeWitt, Rovelli, Marolf…)

- *Conditional probabilities approach* proposed by Page and Wootters.

We will see that both approaches present problems and do not provide a completely satisfactory solution to the issue of the evolution.

Problems are particularly acute when we try to compute propagators or assign probabilities to histories.

We will show that **a combination of both approaches** addresses most of the issues mentioned above. The resulting framework has an **intrinsic loss of coherence.**

1) Evolving Dirac Observables in totally constrained systems:

$$S = \int [p_a \dot{q}^a - \mu^\alpha \phi_\alpha(q,p)] d\tau$$

In the case of GR the constraints are first class

$$\phi_\alpha(q,p) = 0$$

$$\{\phi_\alpha(q,p), \phi_\beta(q,p)\} = C^\gamma_{\alpha\beta} \phi_\gamma(q,p)$$

$$H_T = \mu^\alpha \phi_\alpha(q,p)$$

The Hamiltonian vanishes: the generator of the evolution also generates gauge transformations

Dirac observables are gauge invariant quantities

$$\{O(q,p), \phi_\beta(q,p)\} \approx 0 \qquad \{O(q,p), H_T(q,p)\} \approx 0$$

Therefore, they are constants of the motion.

The issue of time: If the physically relevant quantities in totally constrained systems as general relativity are constants of the motion, how can we describe the evolution?

Evolving Dirac observables: Bergmann, DeWitt, Rovelli, Marolf … They are Dirac observables that depend on a parameter.

$$\{Q_i(t), \phi_\alpha\} \approx 0 \qquad Q_i(t, q^a, p_a)\big|_{t=q^1} = q_i$$

For instance, for the relativistic particle. $\phi = p_0^2 - p^2 - m^2$

Two independent observables:

$$p, X \equiv q - \frac{p}{\sqrt{p^2 + m^2}} q^0, \qquad Q(t, q^a, p_a) = X + \frac{p}{\sqrt{p^2 + m^2}} t$$

$$Q(t = q^0, q^a, p_a) = q$$

Notice that one needs to assume that there are variables as q^0 that are physically observable, even though they are not Dirac observables

The issue of the parameter t: does the proposal solve the problem of time?

Evolving observables depend on a real parameter t. That is we are assuming that there is an external quantity t, that is not represent by any quantum operator nor belongs to any physical Hilbert space.
One may wonder about the meaning of the condition $q^0 = t$ in the generic situation in which the clock variable q^0 is not defined in H_{phys}

$$q^0 |\psi>_{ph} \notin H_{ph}$$

In any generally covariant system as general relativity the clock will be associated to certain physical sub-system with dynamical variables that will not be well defined in H_{phys} . We don't have any external variable.

Summarizing, evolving constants are measurable quantities but, in the quantum realm, they depend on an external parameter, whose observation is not described by the theory.

2) Conditional probabilities.

The second alternative we want to consider is a description of the evolution in terms of conditional probabilities.

The idea is that one promotes all variables to quantum operators and computes conditional probabilities among them. This idea appears simple, natural and attractive in a closed system.

Unfortunately one runs into problems due to the totally constrained nature of gravity. Which variables to promote? Dirac observables? Page and Wootters proposed using kinematical variables, not Dirac observables. That way they had some form of evolution. **Phys.Rev.D27:2885,(1983)**

But Kuchař showed that if one used this proposal in model systems (two particles parameterized) and computes the propagator, one essentially gets that the particle does not propagate $\langle x',t'|x,t\rangle \sim \delta(t-t')\,\delta(x-x')$.

3) Our proposal: Conditional probabilities in terms of evolving Dirac observables.

As we have seen, both approaches require the use of variables which are not defined in the physical space.

Here we will elaborate upon a different approach where all reference to external parameters is abolished, and evolving constants are used to define correlations between Dirac observables in the theory.

We propose to revisit the Page-Wootters construction by **computing relational probabilities among evolving Dirac observables.** The latter are well defined on the physical space of states of the theory and are quantities that one can expect to observe and to be represented by well defined self-adjoint quantum operators.

First you choose an evolving observable as your clock, let us call it *T(t)* .
Then one identifies the set of observables $O_1(t)\ldots O_N(t)$ that commute with T and describes the physical system whose evolution one wants to study and compute:

$$\mathcal{P}\left(O \in [O_0 - \Delta O, O_0 + \Delta O] | T \in [T_0 - \Delta T, T_0 + \Delta T]\right) = \lim_{\tau \to \infty} \frac{\int_{-\tau}^{\tau} dt \, \mathrm{Tr}\left(P_{O_0}(t) P_{T_0}(t) \rho P_{T_0}(t)\right)}{\int_{-\tau}^{\tau} dt \, \mathrm{Tr}\left(P_{T_0}(t) \rho\right)}$$

Notice that this expression is a proposal.
t is the parameter associated to the variable used to define the evolving observables. This variable is treated as an ideal quantity that we do not need to observe (it is integrated over).

A simple example.

One considers the constrained system:

$$H = \frac{p_1^{\ 2}}{2m_1} + \frac{p_2^{\ 2}}{2m_2}$$

$$\phi = p_0 + H(q^a, p_a) = 0$$

We have two free particles and one can define:

$$X_1(t) = q^1 - \frac{p_1}{m_1} q^0 + \frac{p_1}{m_1} t$$

$$X_1(t)\,|_{t=q^0} = q^1$$

$$X_2(t) = q^2 - \frac{p_2}{m_2} q^0 + \frac{p_2}{m_2} t$$

We are using q^0 as unobservable parameter

and compute

$$P(X_2 \,|\, X_1)$$

This simple problem is completely solvable: one can find the space of states that is annihilated by the constraint, find the common eigenstates of the evolving constants and compute the probabilities explicitly.

We can then write the conditional probabilities that yield the propagators,

$$P(X_2^f \,|\, T_2 = X_1^f, X_2^i, T_1 = X_1^i, \rho) \equiv$$

$$\lim_{\tau\to\infty} \frac{\int_{-\tau}^{\tau} dt \int_{-\tau}^{\tau} dt' \, Tr(P_{X_2^f, X_1^f}(t) P_{X_2^i, X_1^i}(t') \rho P_{X_2^i, X_1^i}(t'))}{\int_{-\tau}^{\tau} dt \int_{-\tau}^{\tau} dt' \, Tr(P_{X_1^f}(t) P_{X_2^i, X_1^i}(t') \rho P_{X_2^i, X_1^i}(t'))}$$

This expression would be associated with the propagator for the system to move from

$$X_2^i, X_1^i \qquad \text{to} \qquad X_2^f, X_1^f$$

Notice that in particular no assumption about the relative ordering of the unobservable variables *t* and *t'* is needed.

One can show that the previous expression yields the correct propagator, with suitable assumptions. Namely:

a) The clock and system under study don't interact. (Interactions can be added, they lead to additional effects, Asher Peres studied this in detail).
b) The clock is in a coherent state behaving semiclassically, with a well defined position and velocity and evolves monotonously without recurrences.
c) One does not demand too much accuracy in the measurement of times (otherwise one "breaks the clock")

In fact there are a lot of parallels between the work of Peres in the context of non-relativistic quantum mechanics and the consequences of our proposal. A. Peres "Measurement of time with quantum clocks" AJP 48, 552 (1980).

> It thus seems that the Schrödinger wave function $\psi(t)$, with its continuous time evolution given by $i\hbar\dot{\psi} = H\psi$, is an idealization rooted in classical theory. It is operationally ill defined (except in the limiting case of stationary states) and should probably give way to a more complicated dynamical formalism, perhaps one nonlocal in time. Thus, in retrospect, the Hamiltonian approach to quantum physics carries the seeds of its own demise.

Real clocks and loss of unitarity.

Under the assumptions stated, one gets the result:

$$P\left(x'_2|x'_1,x_2,x_1,\rho_0\right) \sim \lim_{\tau\to\infty}\int_0^\tau dt'\,|\langle x'_2,t'|x_2,t(x_1)\rangle|^2\mathcal{P}_{x'_1}(t')\Delta x_2$$

$$\mathcal{P}_{x'_1}(t') \equiv \operatorname{Tr}\left(P_{x'_1}(t')\rho_0\right) / \int_{-\infty}^{\infty} dt \operatorname{Tr}\left(P_{x'_1}(t)\rho_0\right)$$

$$\int \mathcal{P}_{x'_1}(t')dt' = 1$$

And $\mathcal{P}_{x'_1}(t')$ can be interpreted as the probability that the external unobservable time q^0 is t' when the variable taken as a clock reads x'_1

This probability will be controlled by the position of the peak and the width of the wave packet of the particle 1. If $\mathcal{P}_{x'_1}(t')$ were a Dirac delta we would recover the exact ordinary non-relativistic propagator.

The use of real clocks may lead to a loss of quantum coherence and therefore to corrections to the standard propagator.

$$P(x'_2 \mid x'_1, x_2, x_1) = \int Tr[\rho_{x_2x_1} P_{x'_2}(t')]\mathcal{P}_{x'_1}(t')dt' = \int Tr[\rho_{x_2x_1}(t')P_{x'_2}]\mathcal{P}_{x'_1}(t')dt' = Tr[\rho_{x_2x_1}(x'_1)P_{x'_2}]$$

$$\rho(T = x'_1) = \int dt'\mathcal{P}_{x'_1}(t')U(t',t(x_1))\rho_{x_2x_1}U^+(t',t(x_1))$$

We have therefore ended with the standard probability expression with an "effective'' density matrix in the Schrödinger picture given by $\rho(T)$
Unitarity may be lost since one ends up with a density matrix that is a superposition of density matrices associated with different values of *t*

The underlying unitary evolution of the evolving constants in the ideal time *t* is crucial, yet unobservable. All we observe are the correlations in physical time, then it is not surprising that they present a fundamental level of loss of coherence due to the Intrinsically limitations of real clocks.

If we assume the "real clock'' is behaving semi-classically.

$$\mathcal{P}_t(T) = \delta(T-t) + a(T)\delta'(T-t) + b(T)\delta''(T-t) + \ldots$$

The Schrödinger evolution is modified: RG, R. Porto, JP, NJP 6, 45 (2004)

$$-i\hbar\frac{\partial\rho}{\partial T} = [\hat{H},\rho] + \sigma(T)[\hat{H},[\hat{H},\rho]] + \ldots \qquad \sigma(T) = \partial b(T)/\partial T.$$

If we assume σ is constant, the equation can be solved exactly and one gets that the density matrix in an energy eigen-basis evolves as

$$\rho_{nm}(t) = \rho_{nm}(0)e^{-i\omega_{nm}t}e^{(-\sigma(\omega_{nm})^2)t} \qquad \omega_{mn} = E_m - E_n$$

Has this been observed?

The effect can be made arbitrarily large simply choosing “lousy clocks” to do physics. This is not usually done, but an interpretation of experiments with Rabi oscillations indicates the effect is there,

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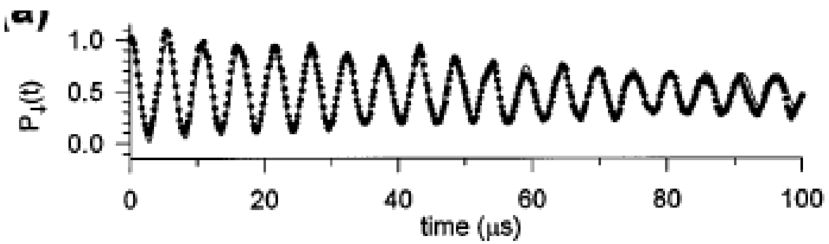

Therefore, the off-diagonal elements of the density matrix decay to zero exponentially, and pure states generically evolve into mixed states. Quantum mechanics with real clocks therefore does not have a unitary evolution.

The effects are more pronounced the worse the clock is. Which raises the question: is there a fundamental limitation to how good a clock can be?

There are many phenomenological arguments based on quantum and gravitational considerations that lead to estimates of such a limitation,
(Salecker-Wigner and Ng, Karolyhazy, Lloyd, Hogan, Amelino Camelia) $\delta T = T^{1/3} t_P^{2/3}$
They seem to survive the introduction of contracting states and other techniques to beat the standard quantum limit (Ozawa, Kasugi).

We will not enter into the analysis of these phenomenological estimations, (which have been questioned in the literature). But it is important to remark that the evolution with real clocks will not be unitary if the spread in the error of the clock grows with time with some power of *T*.

That is, if $\delta T = T_{planck}$ he evolution is unitary, but if $\delta T = T^a T_{Planck}^{1-a}$ with a>0 there will Exist a fundamental loss of unitarity.

Putting this together with the formula we had for the evolution:

$$\rho = \rho(0)\, e^{-i\omega_{nm}t} e^{-t_{Planck}{}^{4/3} t^{2/3} \omega_{nm}{}^{2}}$$

Where the omega's are the Bohr frequencies associated with the eigenvalues of H.

$$\omega_{mn} = E_m - E_n$$

It would require "Schrödinger cat" type of states to observe this. BECs?

Alternatives?

Is the proposal for the probability we postulated the only one?

$$\mathcal{P}\left(O \in [O_0 - \Delta O, O_0 + \Delta O] | T \in [T_0 - \Delta T, T_0 + \Delta T]\right) = \lim_{\tau \to \infty} \frac{\int_{-\tau}^{\tau} dt \, \mathrm{Tr}\left(P_{O_0}(t) P_{T_0}(t) \rho P_{T_0}(t)\right)}{\int_{-\tau}^{\tau} dt \, \mathrm{Tr}\left(P_{T_0}(t) \rho\right)}$$

Anastopoulos and Hu (CQG 25, 154003 (2008)) have proposed an alternative for the joint probability

To construct the correct expression for the probability that "$O \in \Delta_O, T \in \Delta_T$ at some time t_i" one notes that this proposition is the negation of the proposition that "$O \notin \Delta_O, T \notin \Delta_T$ at *all* times t_i". Hence, the probabilities for these two alternatives add up to unity.

If one works out this proposal in detail one gets an expression that coincides with ours at leading order but also involves many crossed products of projectors at different times. It may be that the extra terms restore unitarity, but at the cost of a very complicated expression for the probability that has to be analyzed case by case. So our approach can be seen as postulating that the simpler expressions are the ones that describe nature. This seems to lead to a coherent axiomatic for quantum mechanics (Stud. Hist. Phil. Mod. Phys. 42, 256 (2011)). In the end, which is the correct description for systems without time will have to be settled by experiment.

Conclusions:

- Using evolving constants of the motion in the conditional probability interpretation of Page and Wootters allows to correctly compute the propagator and assign probabilities to histories.
- The resulting description is entirely in terms of Dirac observables.
- There are corrections to the propagator due to the use of “real clocks and rods” to measure space and time associated with loss of quantum coherence.

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