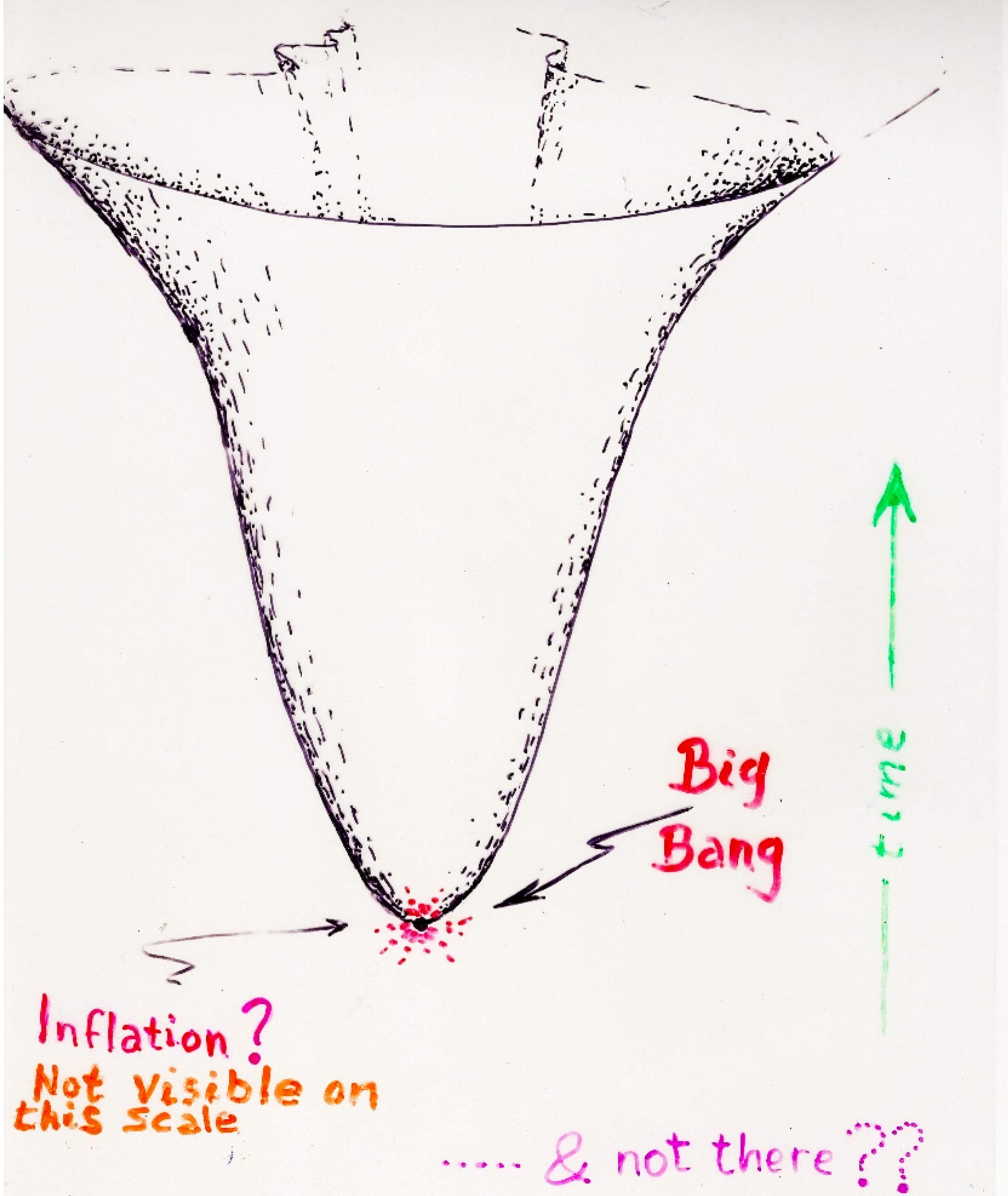
Why Gravitize Quantum Mechanics?

The Galilean Unruh Effect

R. Penrose
Galiano Island 2013

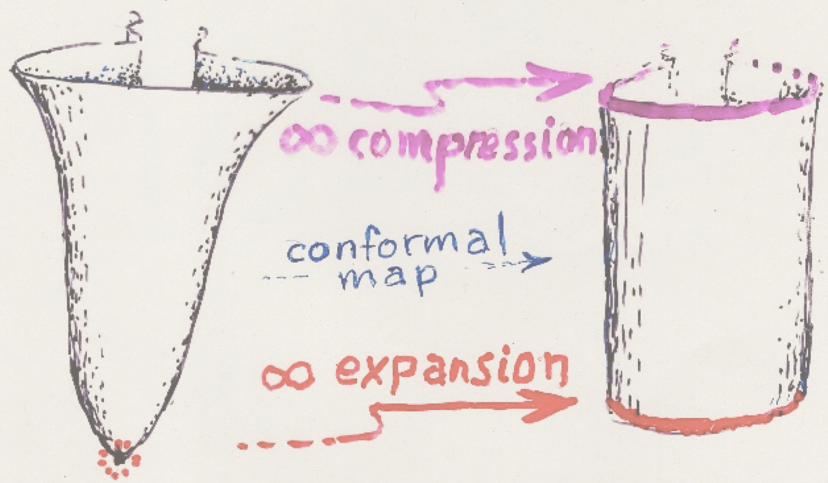
- Cosmology & the 2nd Law
- Space-time singularities
- CCC, black holes & information loss

- Quantum measurement, a clash between the principles of QM & GR,
  - General covariance
  - Principle of equiv. (Galilean Unruh effect)


Big Bang
time
Inflation?
Not visible on this scale
..... & not there??

Two Mathematical Tricks

1. Squash down future infinity to get smooth future boundary

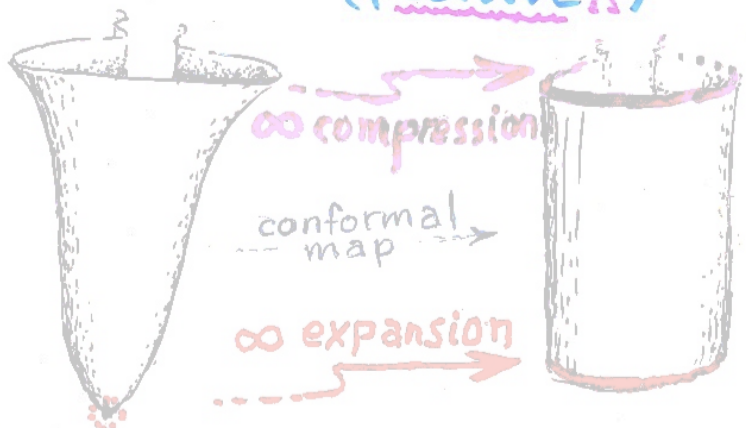


2. Stretch out Big Bang singularity to get smooth initial boundary

Two Mathematical Tricks

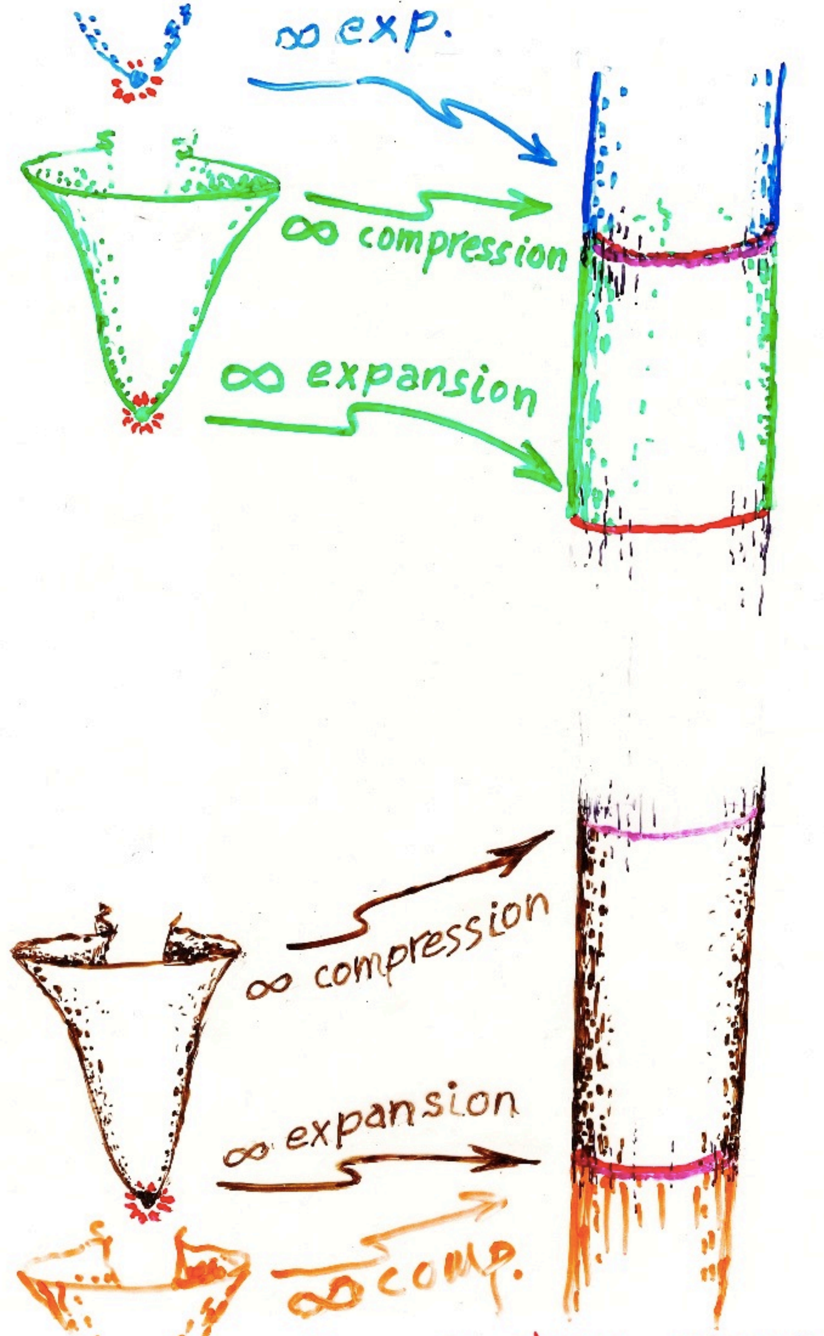
1. Squash down future infinity to get smooth future boundary

- Works under very general circumstances (H. Friedrich)
  (positive Λ)



2. Stretch out Big Bang singularity to get smooth initial boundary

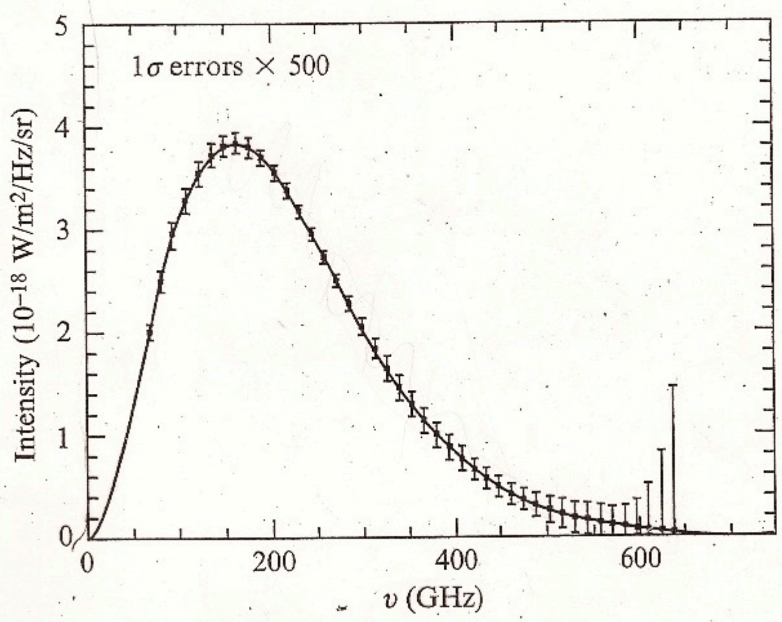
- Extremely strong restriction suppressing gravitational degrees of freedom (K.P. Tod)



Conformal cyclic cosmology (ccc)

Spectrum of the Cosmic Microwave Background

CMB



Note: error bars are exaggerated by a factor of 500.

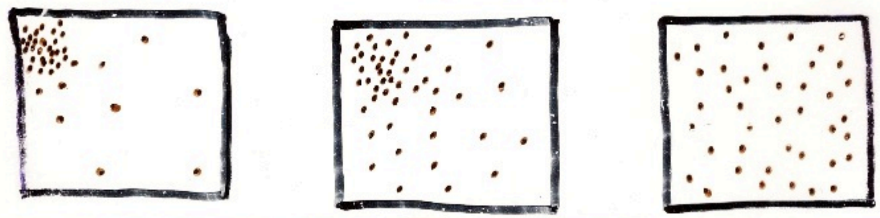
The solid curve displays the Planck black body spectrum of *thermal equilibrium*.

2nd Law of Thermodynamics

Entropy increases with time

= "disorder" (roughly speaking)

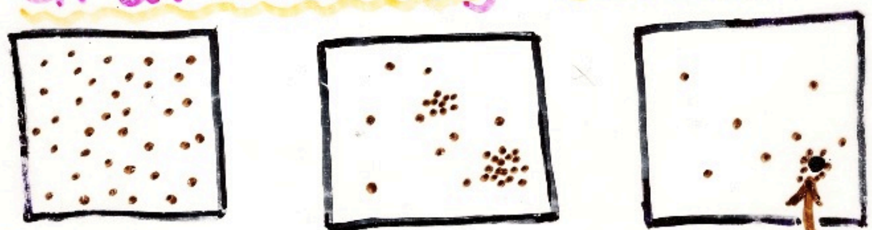
Gas in a box



time increases →

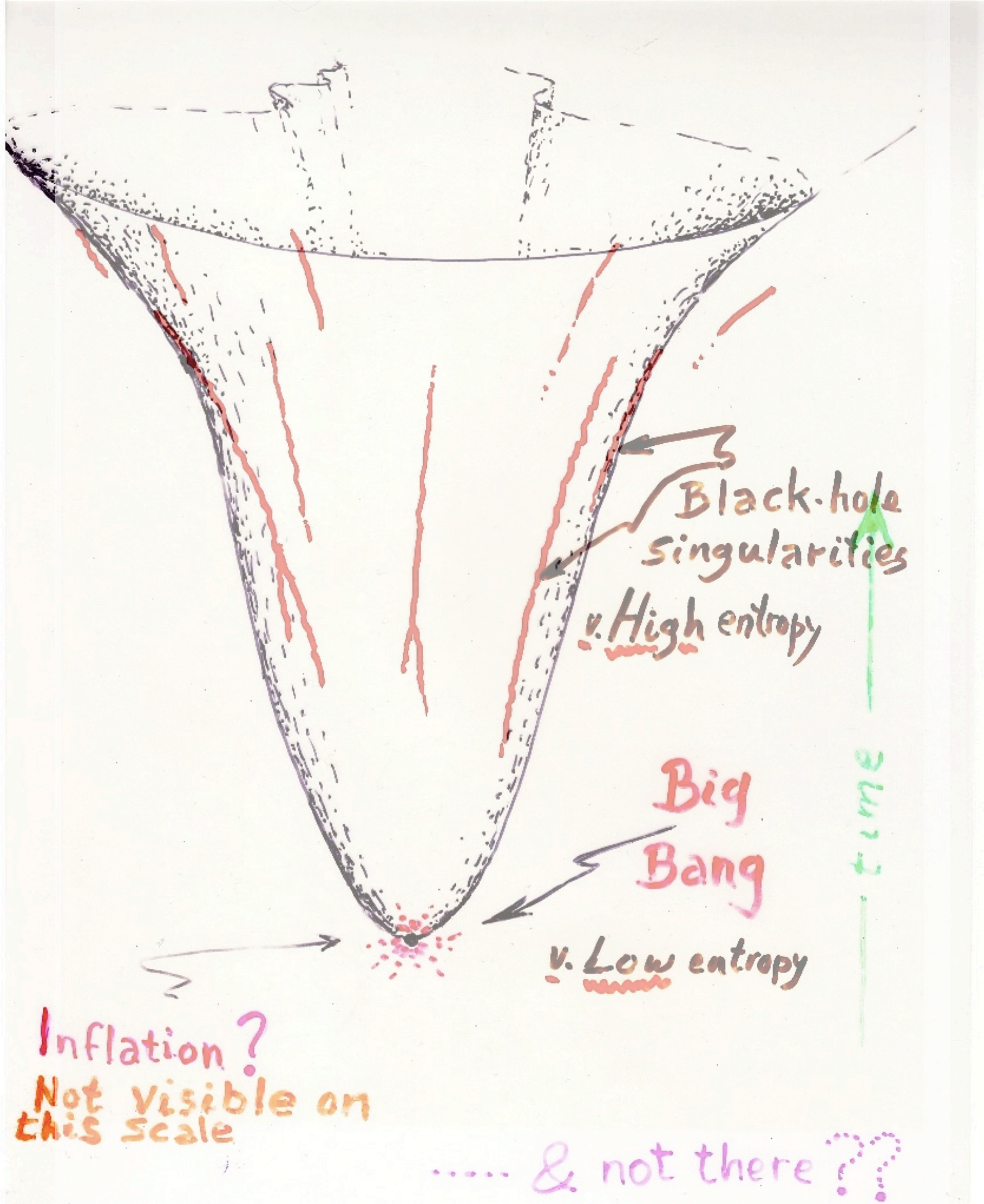
entropy increases →

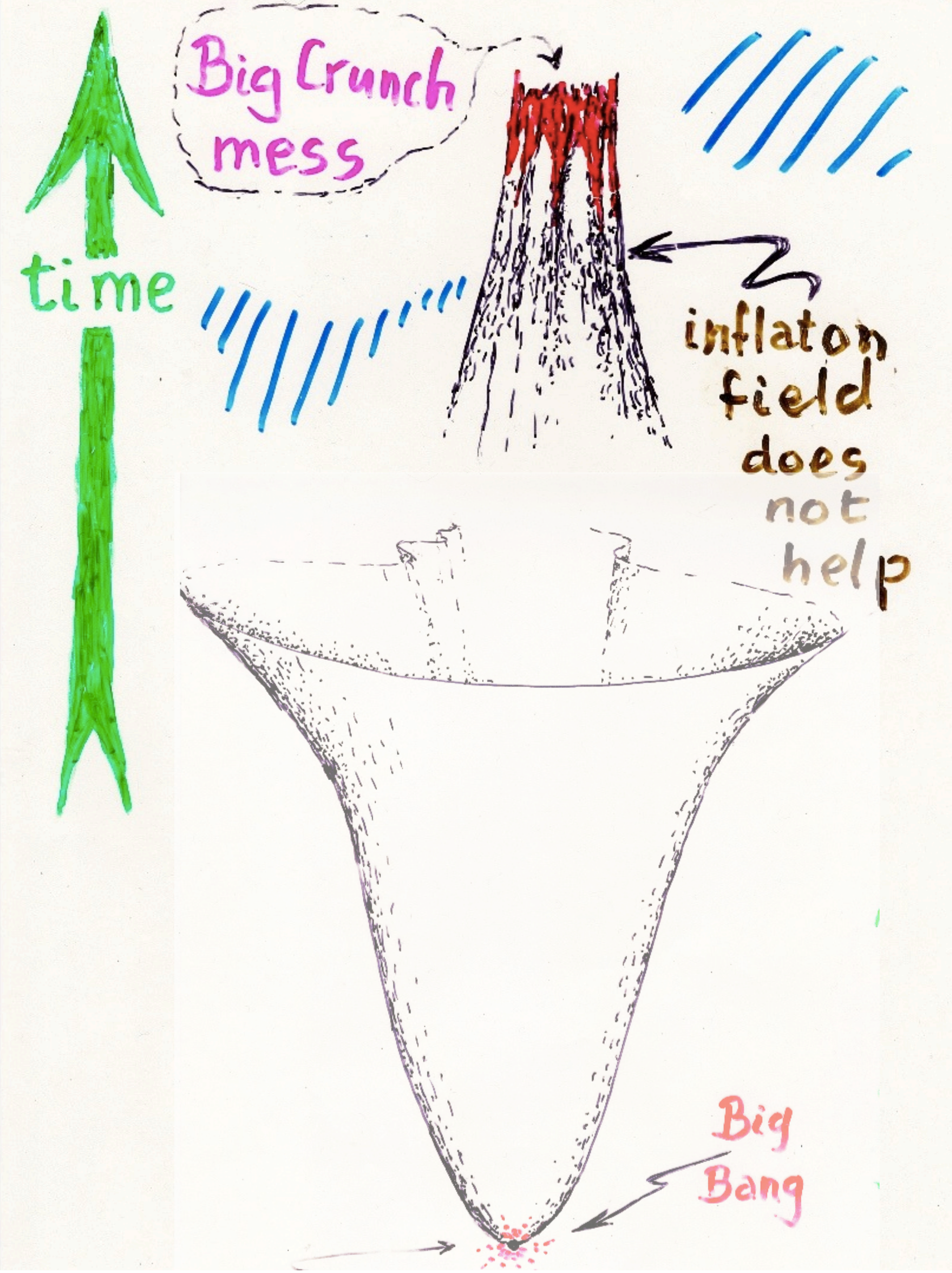
Gravitating bodies



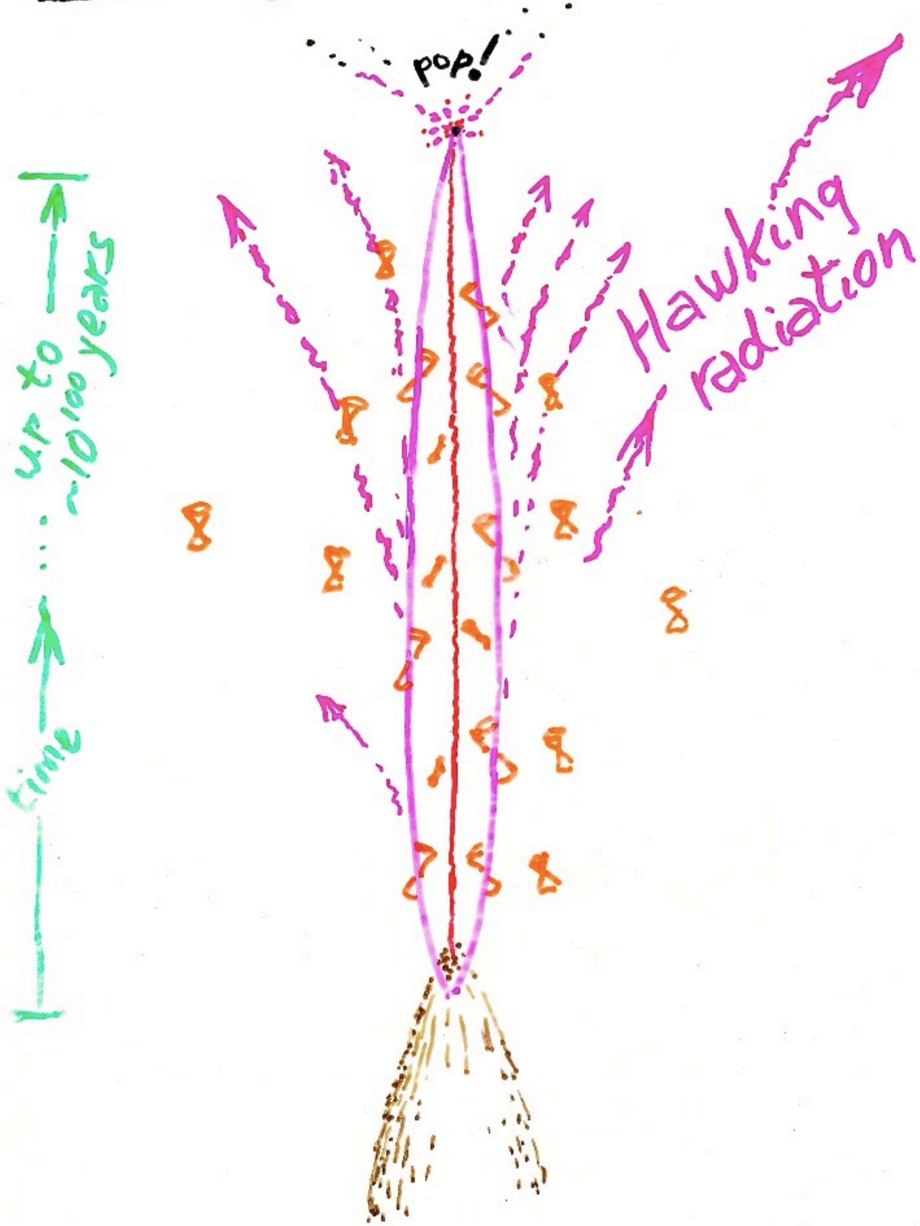
Maximum entropy:

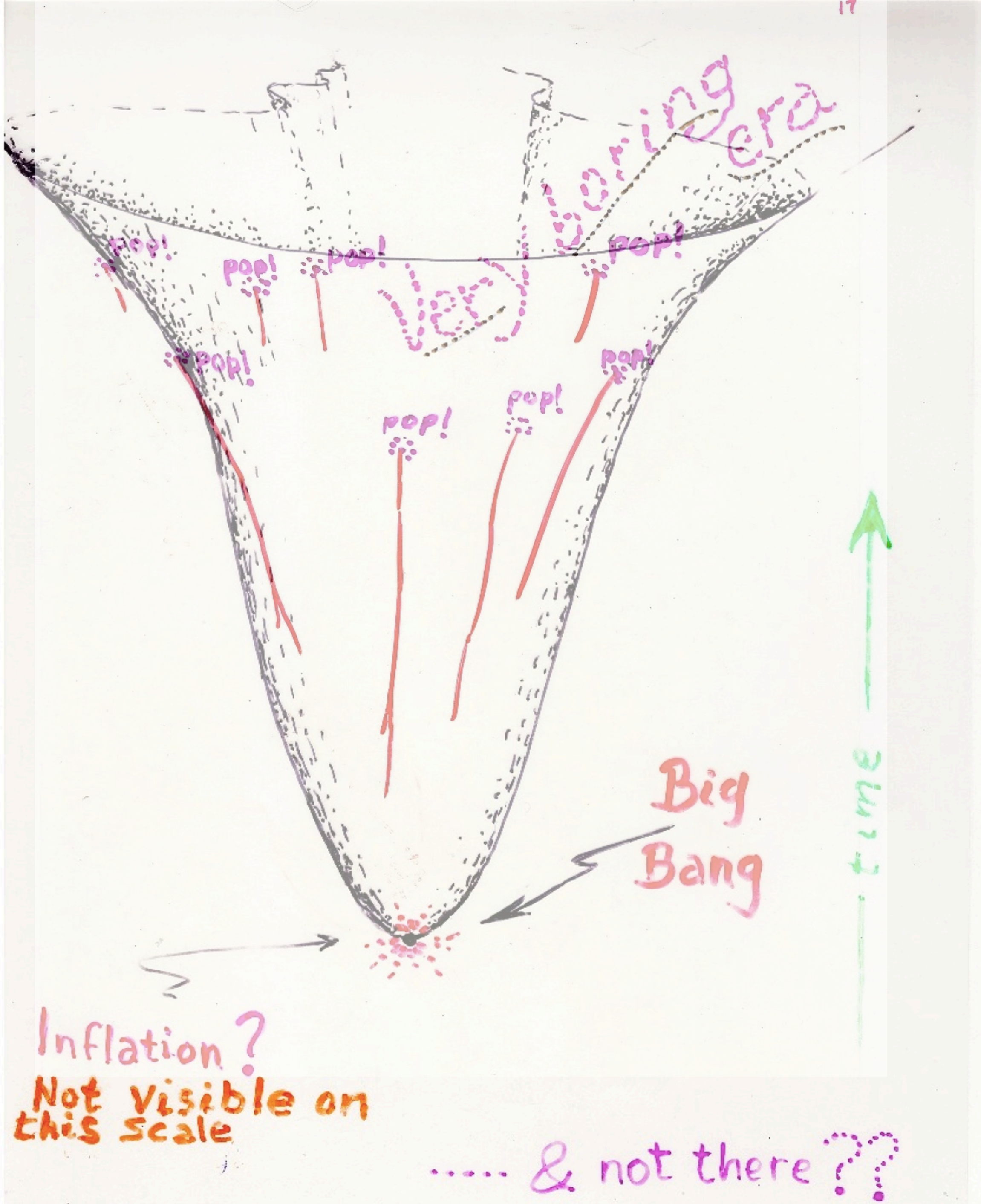
BLACK HOLE


Black-hole
singularities
v. High entropy
Big
Bang
v. Low entropy
time
Inflation?
Not visible on
this scale
..... & not there??


Big Crunch mess
time
inflaton field does not help
Big Bang

Hawking evaporating black hole




Very boring era
pop!
pop!
pop!
pop!
pop!
pop!
pop!
pop!
Big Bang
time
Inflation?
Not visible on this scale
..... & not there ??

The second law of thermodynamics: how can this make sense in a cyclic universe?

Degrees of freedom (i.e. "information") get LOST at black holes' singularities

Agrees with the young (1976) Hawking; disagrees with the old (2004) Hawking!

By FAR the greatest entropy around today is in huge super-massive black holes

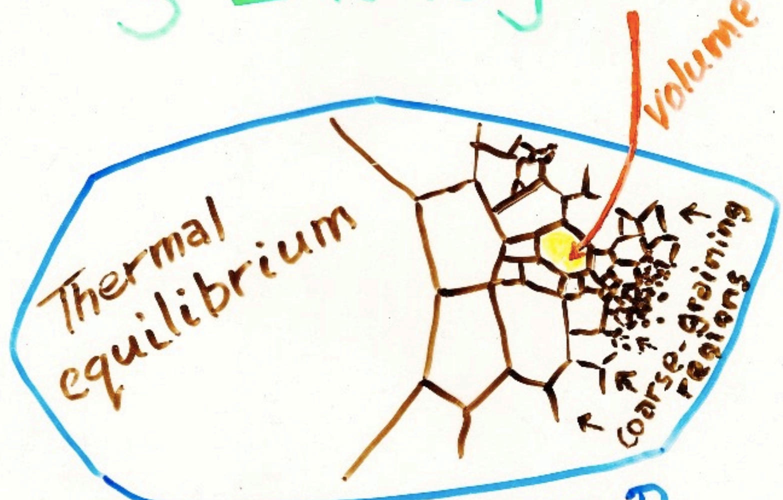
As these black holes eventually evaporate away, their lost degrees of freedom no longer contribute to the total entropy value

The ZERO of entropy is then re-set

The 2nd Law is "transcended", not violated

Boltzmann's definition of ENTROPY

$$S = k \log V$$

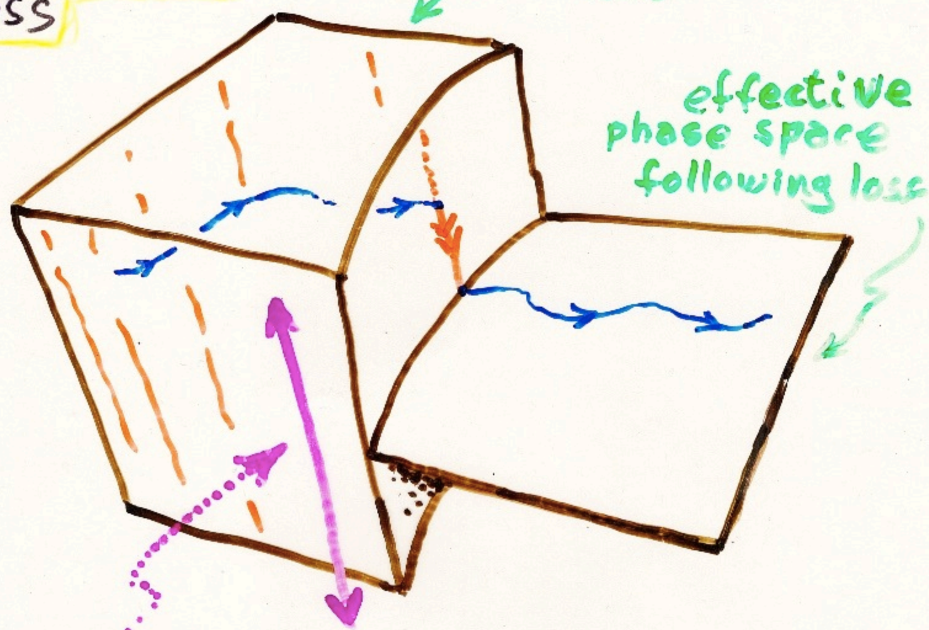


phase space $\mathcal{P}$

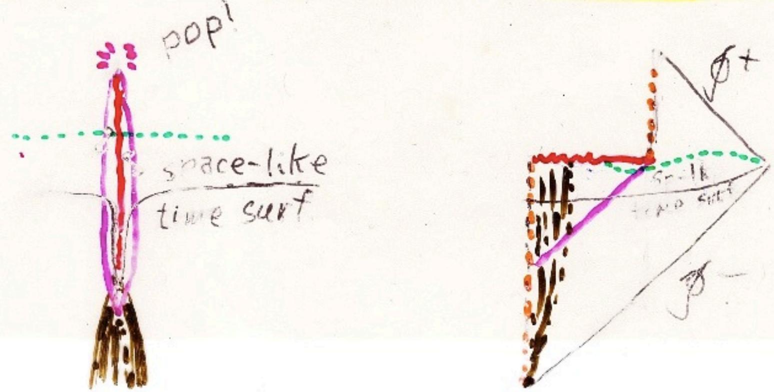
Note: $\log(VW) = \log V + \log W$

so entropy is additive for independent systems

phase space with info. loss

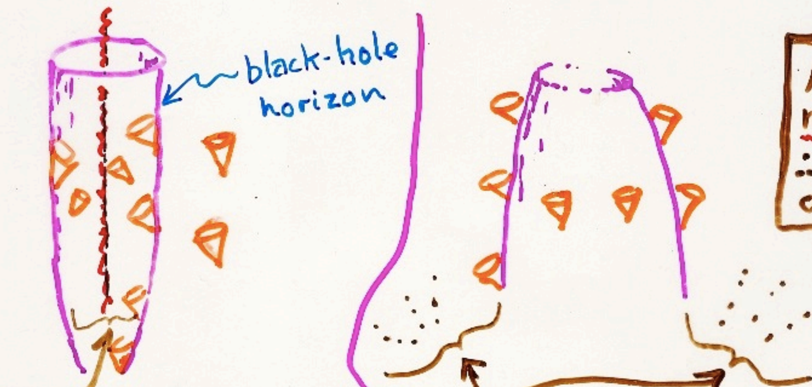


degrees of freedom lost in the black hole



Why I don't believe in

(a) The Λ-entropy



Also:
no singul.
∴ no loss
of info.

Bekenstein-Hawking
entropy refers to
this region

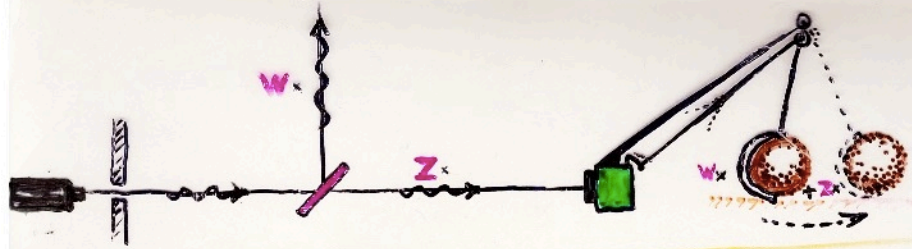
∴ cosmological "entropy"
refers to this region
~INFINITE, for open univ.
Violates 2nd Law!

BLACK HOLE

Λ-COSMOLOGY

(b) The Λ-temperature

Free-fall observer feels no local temperature
Accelerating observer feels Unruh temperature.



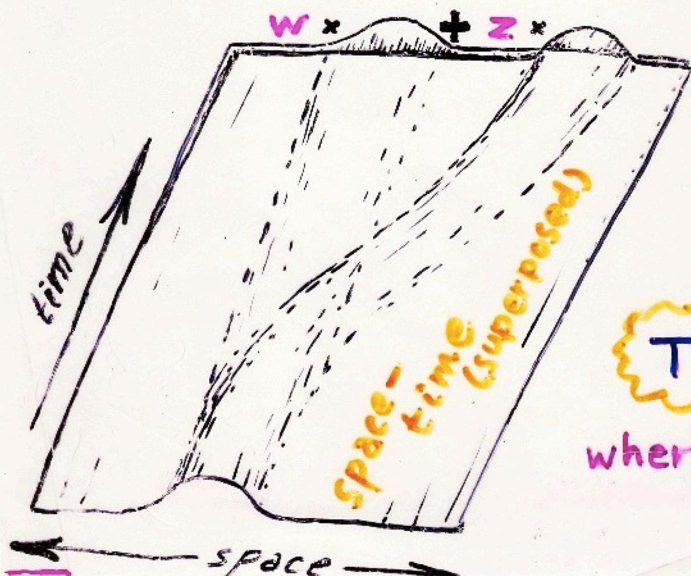
Is

a stationary state?

A problem arises when we consider the gravitational fields of the lumps

We need to superpose differing space-times

There is an inherent ill-definedness in the very notion of stationarity for the superposed state, leading to an instability of the state, with timescale



$$\boxed{T = \hbar / E_G}$$

where E_G is the

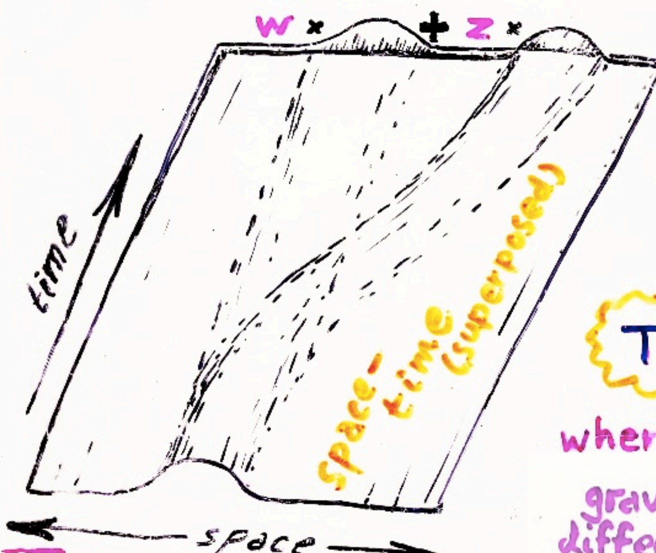
Also: Diósi

Is

a stationary state?

A problem arises when we consider the gravitational fields of the lumps

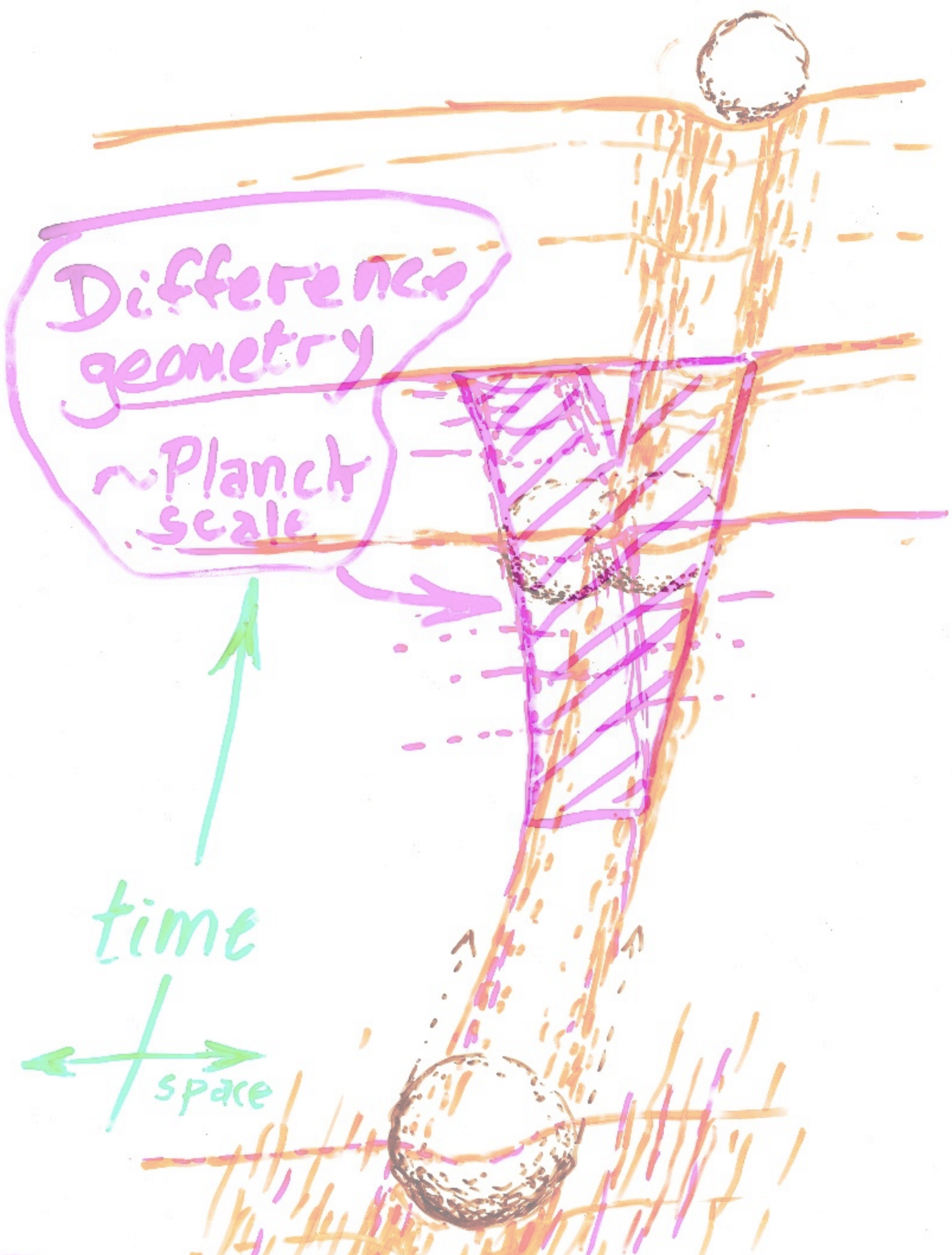
We need to superpose differing space-times
There is an inherent ill-definedness in the very notion of stationarity for the superposed state, leading to an instability of the state, with timescale

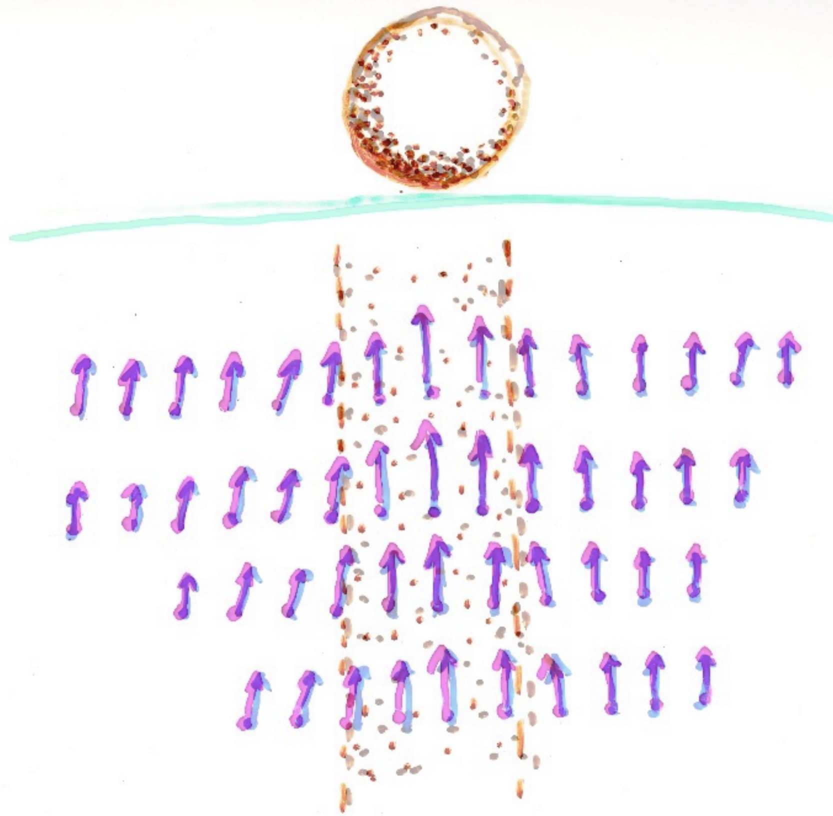


$$T = \hbar / E_G$$

Also: Diósi

where E_G is the grav. self-energy of the difference mass distribn

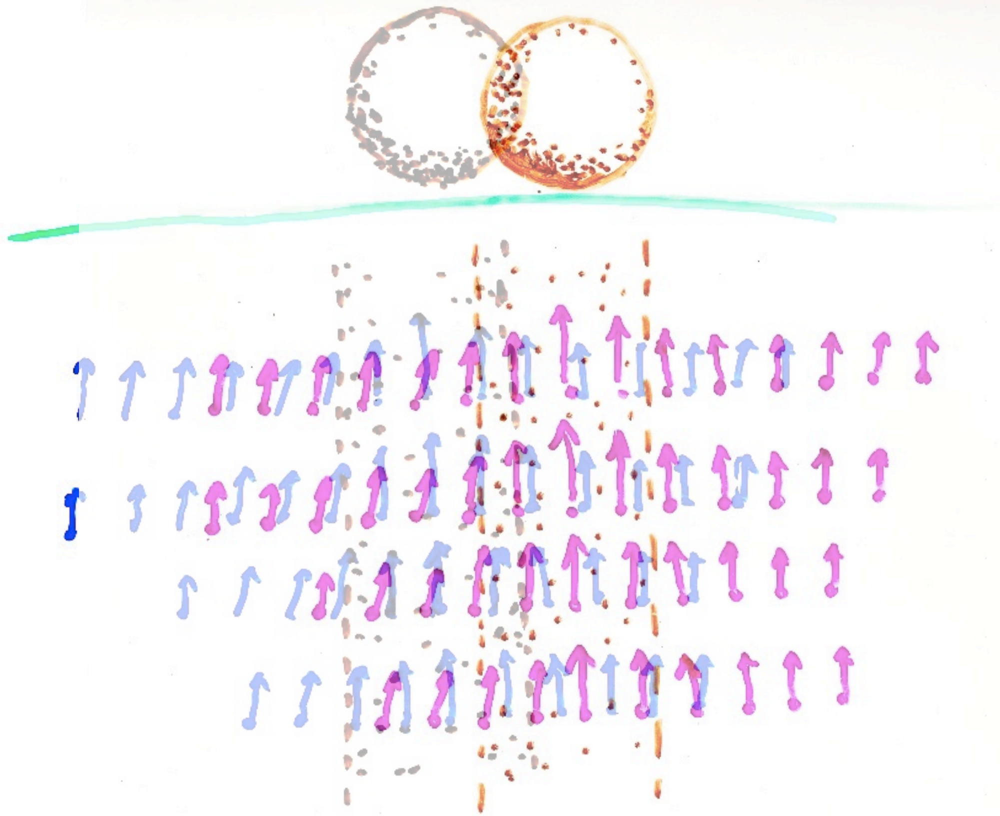

Difference geometry
~Planck scale
time
space



$$\frac{\partial}{\partial t'}$$

$t' = t$

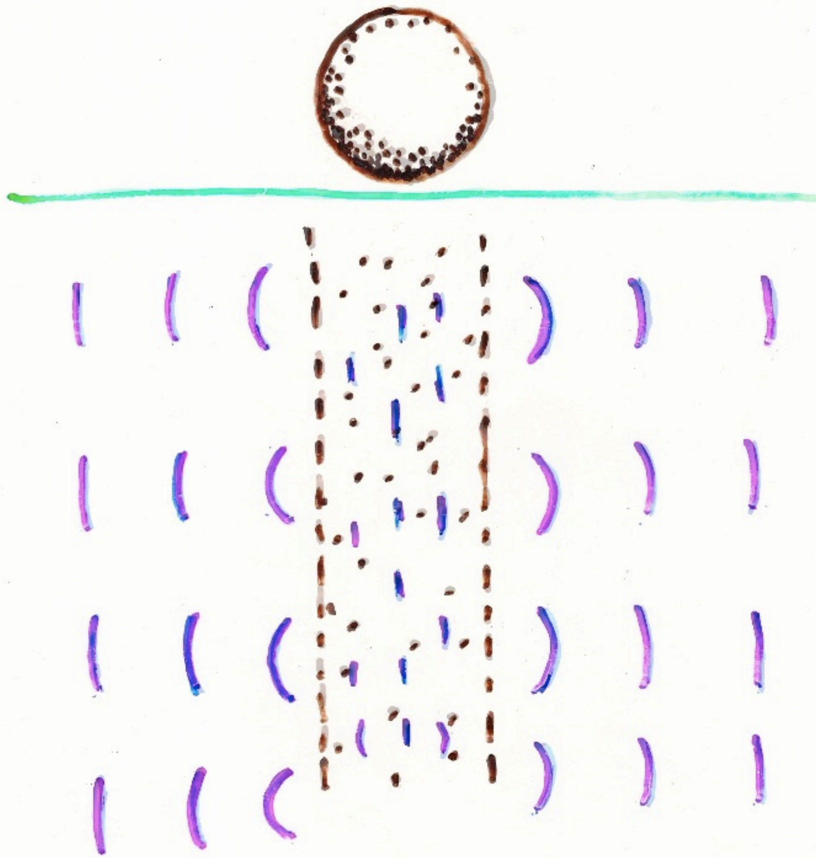
But: $\dfrac{\partial}{\partial t'} = \dfrac{\partial}{\partial t} + \dfrac{\partial x}{\partial t}\dfrac{\partial}{\partial x} + \dfrac{\partial y}{\partial t}\dfrac{\partial}{\partial y} + \dfrac{\partial z}{\partial t}\dfrac{\partial}{\partial z}$

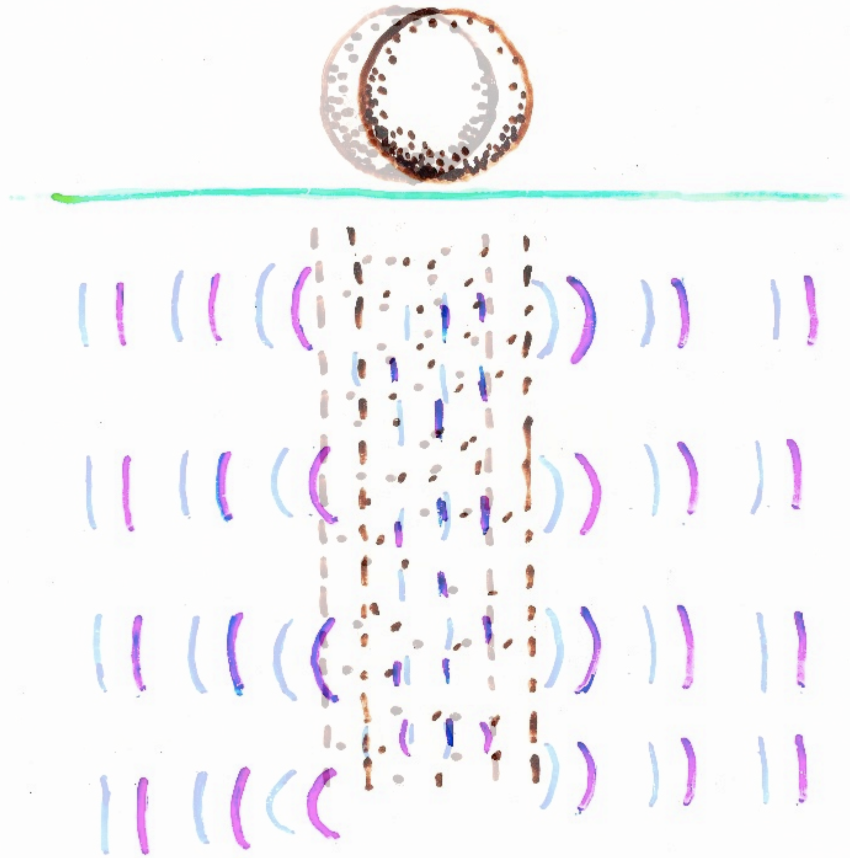


$$\frac{\partial}{\partial t} \quad \frac{\partial}{\partial t'}$$

$t' = t$

But: $$\frac{\partial}{\partial t'} = \frac{\partial}{\partial t} + \frac{\partial x}{\partial t}\frac{\partial}{\partial x} + \frac{\partial y}{\partial t}\frac{\partial}{\partial y} + \frac{\partial z}{\partial t}\frac{\partial}{\partial z}$$





Take "measure of ill-definedness"

(in Killing vector? in causality? in stationarity? in energy?)

as $\overset{1}{g}_{00}/\overset{2}{g}_{00}$ or $\phi_1 - \phi_2$

($\sqrt{g_{00}} \approx 1 + 2\phi$, with ϕ = Newtonian potential)

or, rather, as its spatial variation:

$\vec{\nabla}(\phi_1 - \phi_2)$

$\boxed{\text{Newtonian limit}}$

N.B. It is spatial variation that affects $\partial/\partial t$

$\boxed{G = 1}$

Total measure of ill-definedness =

$$\int \vec{\nabla}(\phi_1 - \phi_2) \cdot \vec{\nabla}(\phi_1 - \phi_2)\, d^3x$$

$$= -\int (\phi_1 - \phi_2)\, \nabla^2 (\phi_1 - \phi_2)\, d^3x$$

$$= 4\pi \int (\phi_1 - \phi_2)(\rho_1 - \rho_2)\, d^3x$$

$$= -4\pi \iint \frac{(\rho_1(x) - \rho_2(x))(\rho_1(y) - \rho_2(y))}{|x - y|}\, d^3x\, d^3y$$

= Grav. self-energy of difference mass distribution [R.P. (1996) GRG 28, 581]

Principle of Equivalence in Quantum Mechanics

Non-relativistic (take $c = \infty$). Consider a constant (in space and time) grav. field $\vec{g}$, in $(\vec{x}, t)$-system, whereas in the freely-falling $(\vec{X}, T)$-syst.

, the grav. field vanishes.

$$\boxed{\vec{x} = \vec{X} + \tfrac{1}{2} t^2 \vec{g} \qquad t = T}$$

ψ is the wavefunction in the $(\vec{x}, t)$-system

Ψ " " " " " $(\vec{X}, T)$-syst.

For a single free particle, mass m,

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi - m \vec{x} \cdot \vec{g}\, \psi$$

$$i\hbar \frac{\partial \Psi}{\partial T} = -\frac{\hbar^2}{2m} \nabla^2 \Psi \qquad \left(\text{N.B. } \vec{\nabla} = \vec{\nabla}\right).$$

Consistent if $$\boxed{\Psi = e^{\frac{i}{\hbar}\left(\frac{m t^3 g^2}{6} - m t \vec{x} \cdot \vec{g}\right)} \psi}$$

(Colella Overhauser Werner experiments) Ref. Berry Greenberger

For general system (many particles ...)

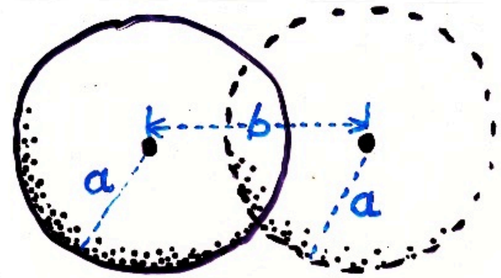
M = total mass; $\vec{x}, \vec{X}$ = posn. vect. of mass centre

$$\boxed{\Psi = e^{\frac{iM}{\hbar}\left(\frac{t^3 g^2}{6} - t \vec{x} \cdot \vec{g}\right)} \psi, \qquad \psi = e^{\frac{iM}{\hbar}\left(\frac{T^3 g^2}{3} + T \vec{X} \cdot \vec{g}\right)} \Psi}$$

Superposition of 2 gravitational fields

Assume that Nature "prefers" $(\vec{X},T)$-system, and that the ambient grav. field is a superposition.

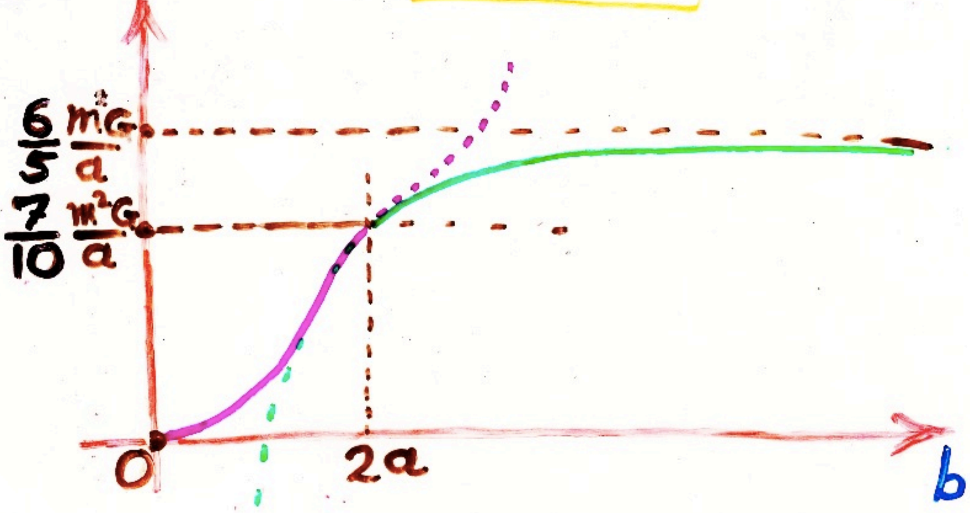
Then we have a wavefunction whose value at a point p is a sum of two parts, where passing from one free-fall frame to the other involves a "$e^{\frac{iM}{\hbar}\left(\frac{T^3}{3}g^2+T\vec{X}\cdot\vec{g}\right)}$"-type factor whereby there is a discrepancy of the form $e^{\frac{iM}{\hbar}\left(\frac{T^3}{3}(\vec{g}_2-\vec{g}_1)^2+T\vec{X}\cdot(\vec{g}_2-\vec{g}_1)\right)}$ between them. The notion of "positive frequency" differs between the two terms, so the sum is illegal — involving the superposition of different vacua. (Compare Unruh effect — but here not thermal (B.S. Kay)).

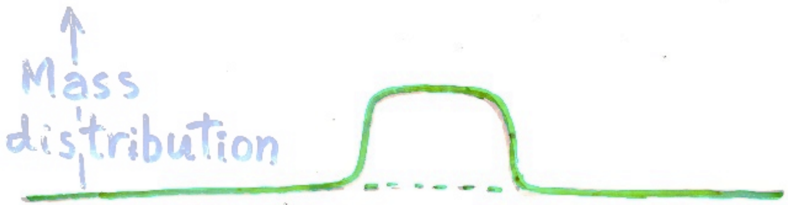


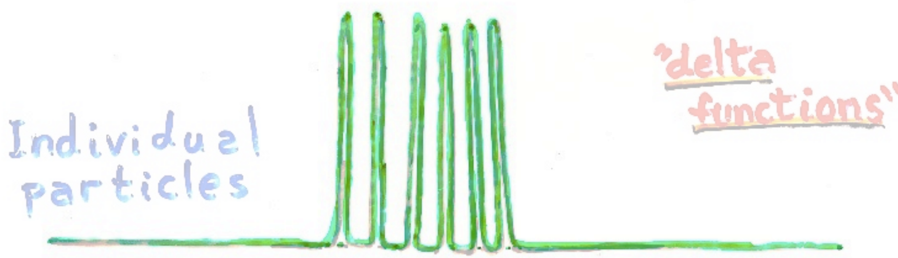
$$E_G =$$

$$\begin{cases} \dfrac{m^2G}{a}\left(2\lambda^2 - \dfrac{3}{2}\lambda^3 + \dfrac{1}{5}\lambda^5\right) & \boxed{0 \le \lambda \le 1} \\[2ex] \dfrac{m^2G}{a}\left(\dfrac{6}{5} - \dfrac{1}{2\lambda}\right) & \boxed{1 \le \lambda} \end{cases}$$

$$\boxed{\lambda = b/2a}$$

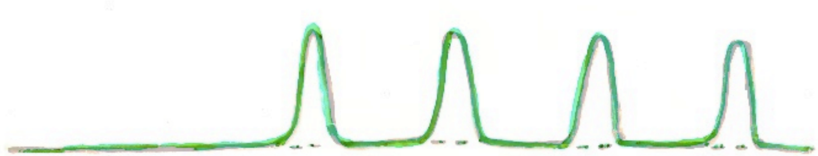


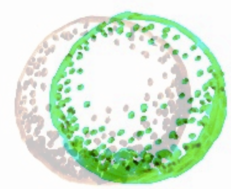


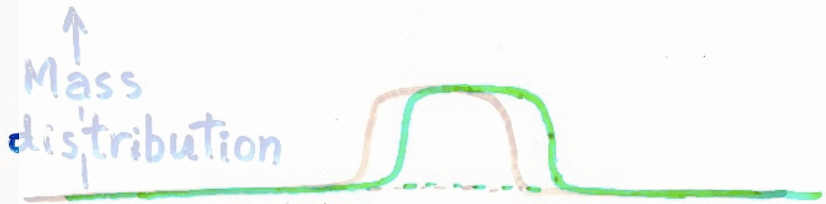


Would give *INFINITE* effect.....
but not stationary

Correct procedure: nuclei
spread, in stationary state

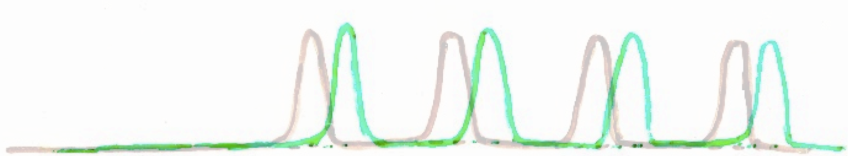






Would give *INFINITE* effect .....
but not stationary

Correct procedure: nuclei spread, in stationary state



Note: formula $T \sim \frac{\hbar}{E_G}$ does not involve the speed of light c — though Newton's constant G is implicitly involved in grav. energy E_G

$\therefore$ makes sense to use Schrödinger-Newton approximation ($c \to \infty$).

Stationary states are solutions of the Schrödinger eqn., where there is an additional term provided by a background Newtonian gravitational potential Φ, where Φ is determined by that mass distribution which is the mass expectation value of the wavefunction ψ. $\boxed{\text{Non-linear coupled eqns.}}$

Single particle, mass m.

$$i\hbar\frac{\partial\psi}{\partial t} = \left(\frac{p^2}{2m} + m\Phi\right)\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + m\Phi\psi$$

Stationary state: $i\hbar\frac{\partial\psi}{\partial t} = E\psi$

$\psi = e^{-iEt/\hbar}S$

$$\left(E + \frac{\hbar^2}{2m}\nabla^2 - m\Phi\right)S = 0$$

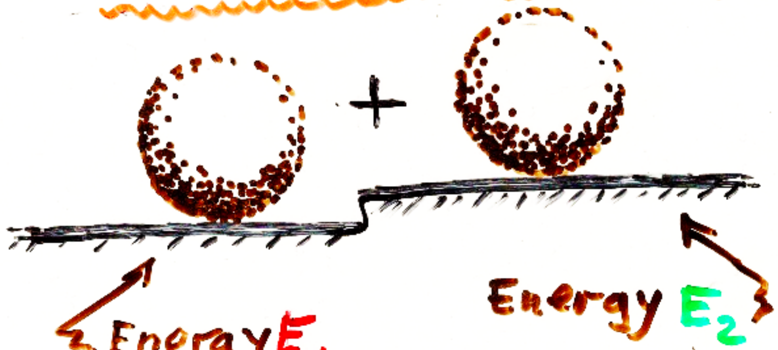
(Simplifying assumption: S real)

$$\nabla^2\Phi = 4\pi G m S^2$$

$V = -\Phi + \frac{E}{m}$

$$\boxed{\nabla^2\begin{pmatrix} S \\ V \end{pmatrix} = -\begin{pmatrix} \frac{2m^2}{\hbar^2} VS \\ 4\pi G m S^2 \end{pmatrix}}$$

Superposition of states with DIFFERING ENERGIES



Reduces to a classical oscillation

$a = tE_1/\hbar$ $\qquad$ $b = tE_2/\hbar$

$$\boxed{e^{ia} + e^{ib} = e^{i(a+b)} \times 2\cos\left(\frac{a-b}{2}\right)}$$