Correlation effects on transport through quantum dots and wires

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Outline

– Introduction: impurities in Luttinger liquids

– Method: functional renormalization group (fRG)

– Results: *local density of states of single impurity*

$\longrightarrow$ spinless fermions

$\longrightarrow$ spin-$\frac{1}{2}$ fermions

transport through quantum dot

$\longrightarrow$ interplay of correlation effects:

Luttinger-liquid behavior and Kondo physics

Introduction: impurities in Luttinger liquids

– *Luttinger liquid:*

→ effective low-energy model of correlated electrons in 1D

→ power laws with interaction-dependent exponents ($K_\rho < 1$)

– *impurity effects:*

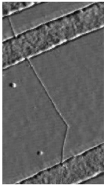
→ at low energy scales impurity effectively cuts the chain

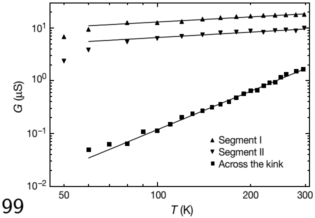
→ physical observables determined from *open-chain fixed point*

local DOS: $D_j \sim |\omega|^{\alpha_B}$ $\qquad\qquad$ $\alpha_B = (K_\rho^{-1} - 1)/2 > 0$

conductance: $G \sim T^{2\alpha_B}$

Kane, Fisher '92





Yao *et al.* '99

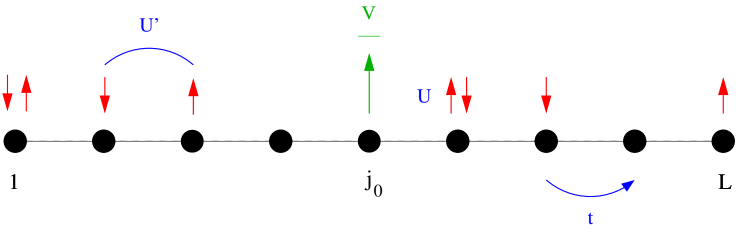
conductance through kink in carbon nanotube: power law

Aim:

development of quantitative theory for *microscopic* models of interacting Fermi systems:

- computation of observables on *all* energy scales, providing also **non-universal** properties
- determination of **scale** at which universal asymptotics sets in

Microscopic model



$$H = -t \sum_{j,\sigma} (c^\dagger_{j+1,\sigma} c_{j,\sigma} + c^\dagger_{j,\sigma} c_{j+1,\sigma})$$

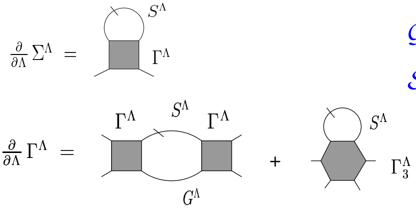
$$+ U \sum_j n_{j\uparrow} n_{j\downarrow} + U' \sum_j n_j n_{j+1} + H_{\mathrm{imp}}$$

site impurity: $H_{\mathrm{imp}} = V n_{j_0}$ (j_0 impurity site)

hopping impurity: $H_{\mathrm{imp}} = (t - t')(c^\dagger_{j_0+1,\sigma} c_{j_0,\sigma} + \mathrm{h.c.})$

Method: functional renormalization group (fRG)

- general formulation of Wilson's RG idea
- generating functional of m-particle interaction
- introduction of IR-cutoff Λ in $\mathcal{G}_0^\Lambda(i\omega) = \Theta(|\omega| - \Lambda)\mathcal{G}_0(i\omega)$
- exact infinite *hierarchy* of coupled flow equations:



$$\mathcal{G}^\Lambda = [(\mathcal{G}_0^\Lambda)^{-1} - \Sigma^\Lambda]^{-1}$$

$$\mathcal{S}^\Lambda = \mathcal{G}^\Lambda \, [\partial_\Lambda(\mathcal{G}_0^\Lambda)^{-1}] \, \mathcal{G}^\Lambda$$

- *initial conditions:* Σ^{Λ_0} = bare impurity potential
  Γ^{Λ_0} = bare interaction
- truncation of hierarchy: $\Gamma_3^\Lambda = \Gamma_3^{\Lambda_0} = 0$

Wetterich '93, Morris '94, Metzner '99, Salmhofer and Honerkamp '01

Results: Local DOS at impurity

impurity induces long-range

$2k_F$ oscillations

$\Downarrow$

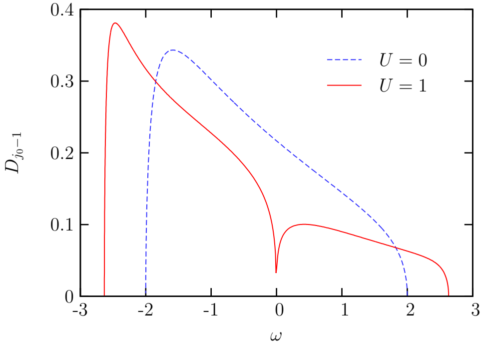
strong suppression

of DOS at Fermi energy:

$$D_{j_0-1} \sim |\omega|^{\alpha_B}$$

boundary exponent α_B

independent of impurity potential



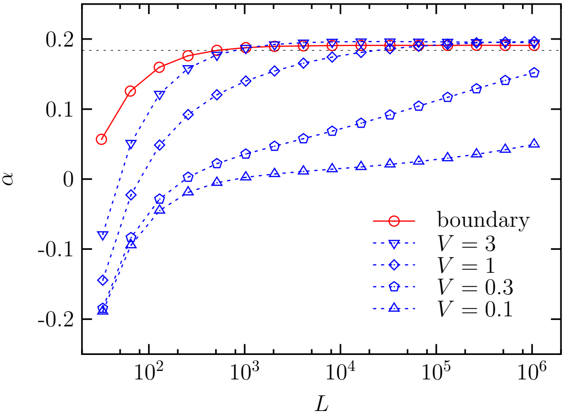
$n = 1/2, \ V = 1.5, \ L = 1025$

spinless fermions

Results:

Spinless fermions: effective exponents

Dependence on impurity potential



⟶ convergence to universal boundary exponent in general

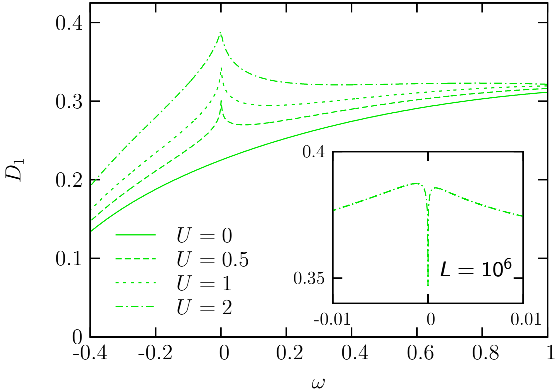
very slow

$$\frac{1}{L} \sim V^{\frac{1}{1-K_\rho}}$$

⟶ *non-universal* behavior relevant!

Results:

Spin-$\frac{1}{2}$ fermions: local DOS at boundary



parameters:

$U' = 0$
$n = 1/4$
$L = 4096$

– clear *increase* instead of expected suppression in contrast to low-energy description!

– effect of 2-particle backscattering

Discussion: Effect of 2-particle backscattering $\tilde{V}(2k_F)$

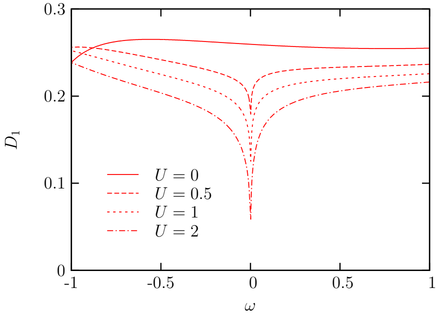
Hartree-Fock:

$$D_j(\omega) = D_j^0(\omega) \left[1 + \frac{\tilde{V}(0) - 2\tilde{V}(2k_F)}{2\pi v_F} \log |\omega/\epsilon_F| + \mathcal{O}(\tilde{V}^2) \right]$$

– bare Hubbard model: $\tilde{V}(0) - 2\tilde{V}(2k_F) = -U < 0$ → increase
suppression through $\mathcal{O}(\tilde{V}^2)$

– extended Hubbard model: $\tilde{V}(0) - 2\tilde{V}(2k_F) = 2U'[1 - 2\cos(2k_F)] - U$

for $\tilde{V}(2k_F) = 0$

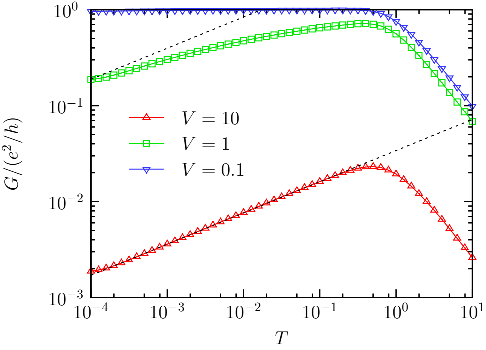


→ similar behavior as
for spinless fermions

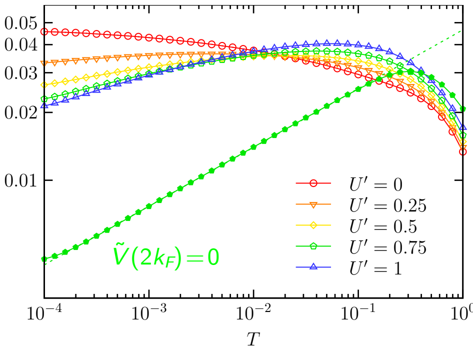
Results: linear conductance

$$G(T) = -2\,\frac{e^2}{h}\int d\varepsilon\; f'(\varepsilon)\,|t(\varepsilon)|^2 \quad \text{with} \quad |t(\epsilon)|^2 \sim |G_{1,N}(\epsilon)|^2$$

spinless fermions



spin-$\frac{1}{2}$ fermions

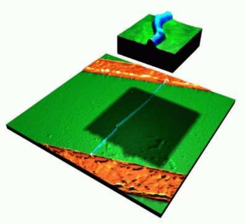


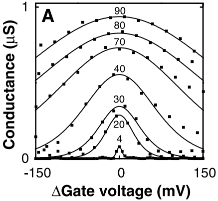
development of clear power laws parameter dependent,

in general *non-universal* behavior relevant!

PRB **73**, 045125 (2006), Enss *et al.* '04, Meden *et al.* '04

Resonant tunneling through a quantum dot



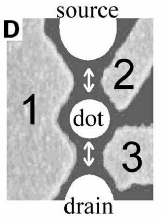


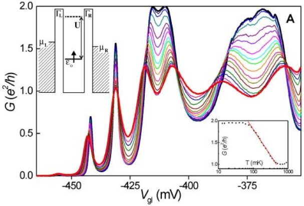
Luttinger - liquid behavior

in quantum wire

width $w \sim N^{(K_\rho - 1)/2}$

Postma *et al.* '01





Kondo physics

in quantum dot

resonance plateau

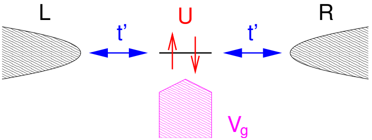
$w \sim U$

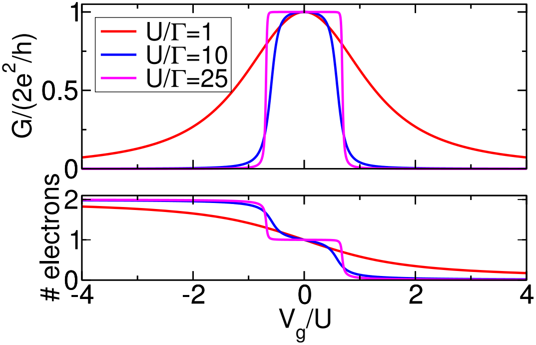
(independent of N)

Goldahber-Gordon *et al.* '98 Cronenwett *et al.* '98

$\rightarrow$ interplay of correlation effects

Conductance through a single dot: Kondo physics





Lorentzian *w* at $U = 0$

$$\Downarrow$$

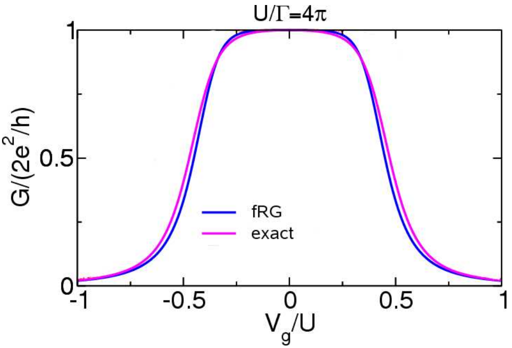
exponential pinning at μ:

$$V = V_g \exp[-U/(\pi\Gamma)]$$

Kondo '64; Glazman, Raikh '88; Ng, Lee '88; PRB **73**, 153308 (2006)

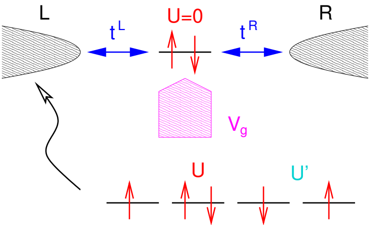
Conductance through a single dot: Kondo physics

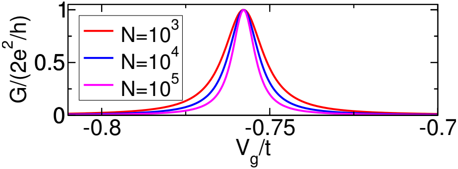
Comparison with exact results

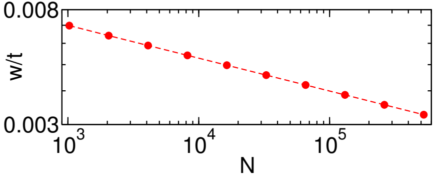


Gerland *et al.* '00, Karrasch *et al.* '06

Tunneling with Luttinger - liquid leads







Luttinger liquid

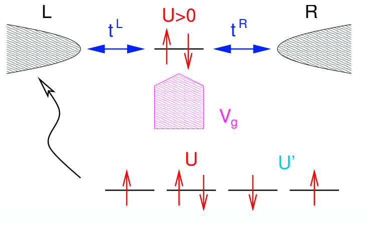
with characteristic power laws

$$w \sim N^{(K_\rho - 1)/2}$$

$$K_\rho^{fRG} = 0.760$$
$$K_\rho^{DMRG} = 0.749$$

Kane, Fisher '92; Ejima *et al.* '05

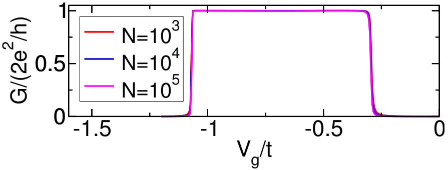
Kondo effect *and* Luttinger - liquid leads



competing effects:

– Luttinger liquid: $w \to 0$

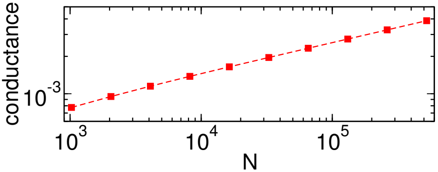
– Kondo effect: $w \sim U$



V_g on plateau:

$V_g = V_g^r$: $G(V_g^r)/(2e^2/h) = 1$

$V_g \neq V_g^r$: $1 - G(V_g)/(2e^2/h) \sim N^{1-K_\rho}$



$\rightarrow$ low-energy limit:

Luttinger liquid!

PRB **73**, 153308 (2006)

Conclusions

- Analysis of spectral and transport properties with fRG technique:
  - flexible *microscopic modeling* of geometries, leads and contacts
  - determination of *relevant energy scales* and *non-universal behavior*
- Results:

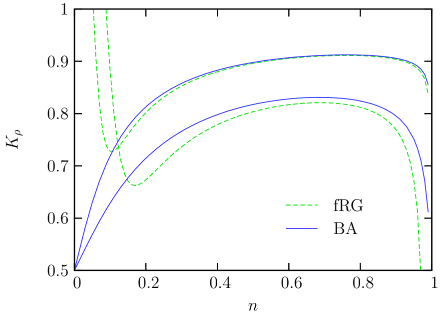
  $\longrightarrow$ for moderate interaction and impurity parameters
  large systems required to reach low-energy asymptotics

  $\longrightarrow$ spin-$\frac{1}{2}$: effects of 2-particle backscattering
  – deviation from low-energy description
  – logaritmic corrections

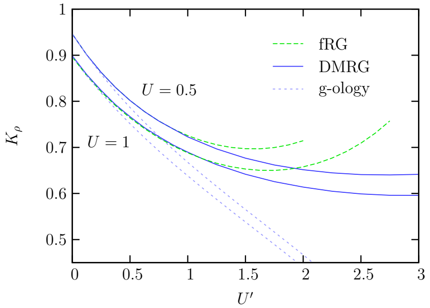
  $\longrightarrow$ double barrier: Kondo effect relevant on experimentally
  accessible length scales

Luttinger-liquid parameter K_ρ

Comparison with exact results



bare Hubbard model
$U = 1, U = 2$



extended Hubbard model
$U = 1, U = 2, n = 1/4$