Percolation transition and dissipation in Ising magnets

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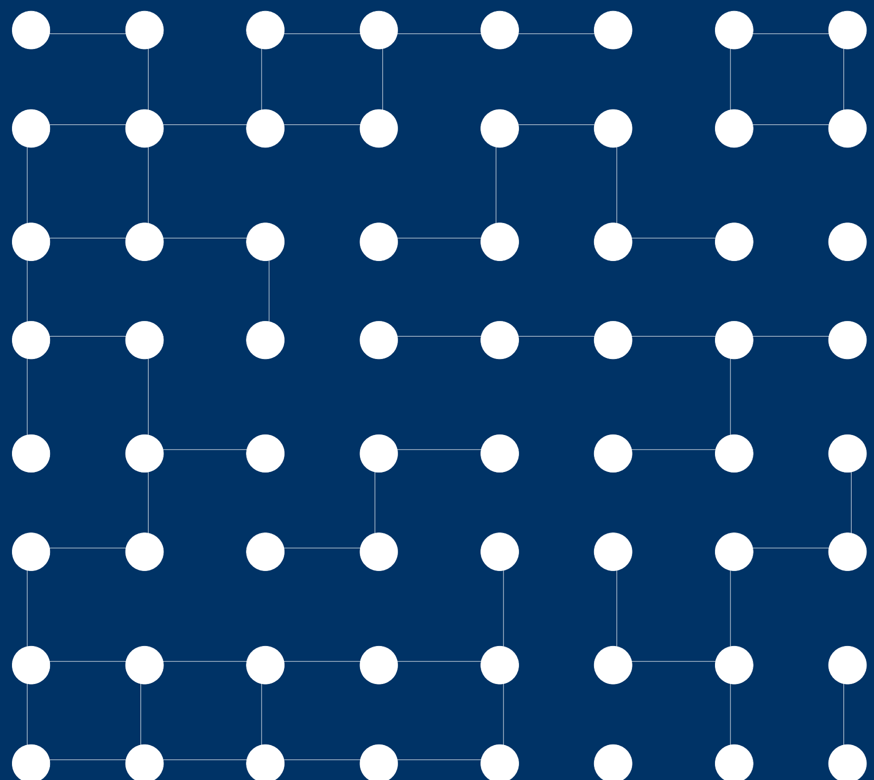
The system



Ising quantum magnet ($d > 1$)

$$H = \sum -J S_i^z S_{i+1}^z - h S_i^x + H_l(\alpha)$$

p = Prob. of a missing bond.

α = Dissipation strength

$$H_l(\alpha) = \sum \nu_n a^\dagger_n a_n + \lambda_{n,i} S_i^z (a^\dagger_n + a_n)$$

Each spin coupled to an independent ohmic bath.

Spectral function: $\mathcal{E}(\omega) = \pi \sum \lambda_n^2 \delta(\omega - \nu_n)/\nu_n = 2\pi\alpha\omega \exp\{-\omega/\omega_c\}$

About percolation

$p > p_c$ Disconnected clusters

$$\xi \sim |p - p_c|^{-\nu}$$

$p = p_c$ Clusters are fractals with dimension $D < d$.

$p < p_c$ Infinite cluster: $P_\infty \sim |p - p_c|^{\beta}$

Cluster size distribution
(# of clusters with size s):

$$n_s \sim s^{-\tau} f\left((p - p_c)^{1/\sigma} s\right)$$

$f(x) \sim \exp\{-B_1 x\}$, $p > p_c$

$f(x) = const$, $p = p_c$

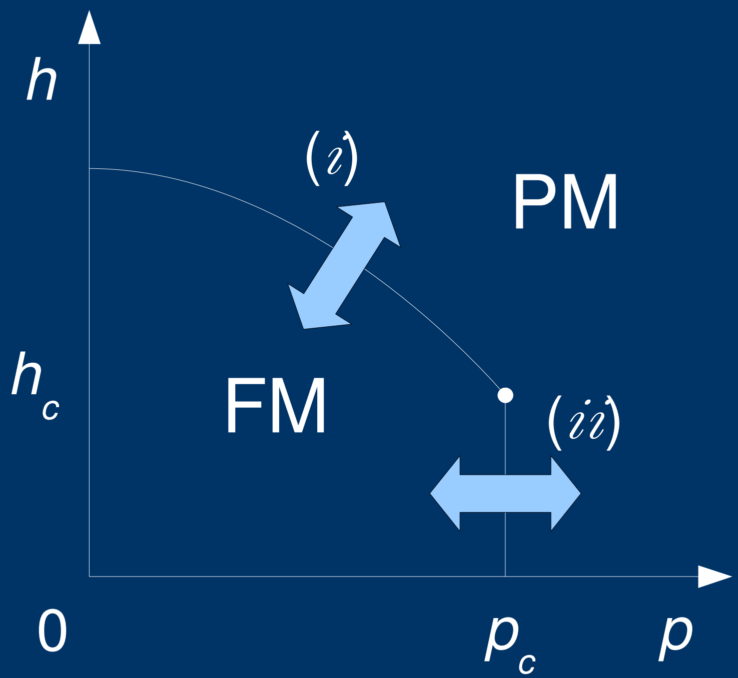
$f(x) \sim \exp\{-(B_2 x)^{1-1/d}\}$, $p < p_c$

$p > p_c$

$p = p_c$

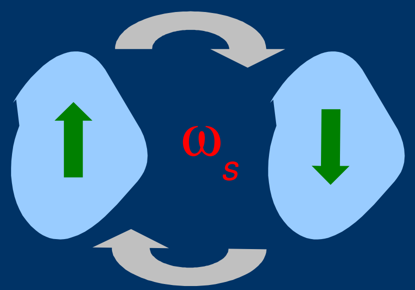
$p < p_c$

Without dissipation (α=0)



(i) Expected to be in the same universality class of random bond quantum Ising transition.

(ii) Dominated by the geometrical fluctuations.



At the disordered side of the transition (ii)

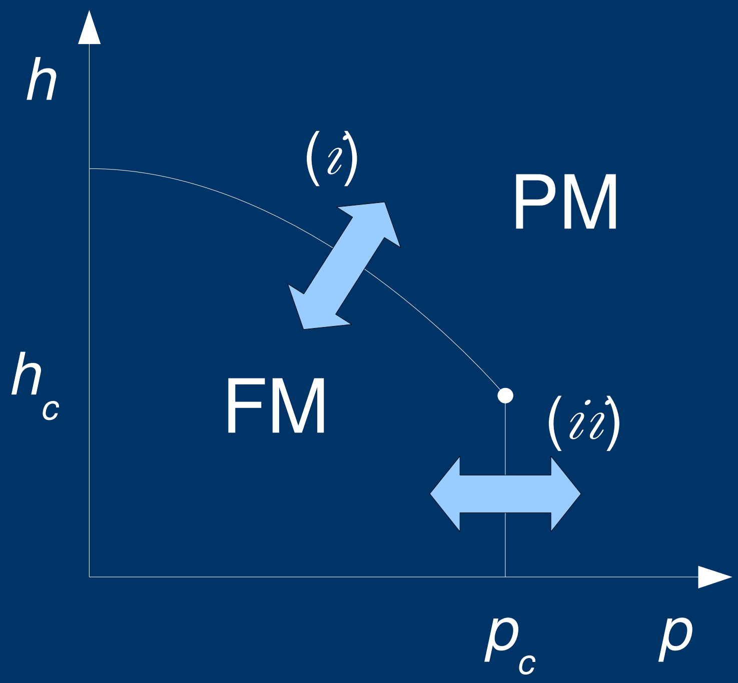
- Non interacting slowly fluctuating clusters.

A cluster of size s fluctuates with a rate $\omega_s \approx h \exp\{-cs\}$.

$c = \ln(J/h)$

Low-energy density of states: $\rho(\omega) \sim \int n_s \delta(\omega - \omega_s) ds = n_{s(\omega)}/\omega$.

Without dissipation (α=0)



Low-energy density of states:
$\rho(\omega) \sim f(c^{-1}\ln(h/\omega)/\xi^{D})/(\omega \ln^{\tau}(h/\omega))$.

Note the scaling variable: $\ln(h/\omega)/\xi^{D}$.
Activated dynamical scaling!

$\rho(\omega) \sim 1/[\omega^{1-\varphi}\ln^{\tau}(h/\omega)]$, $\varphi \sim 1/\xi^{D}$, for $p>p_c$

$\rho(\omega) \sim \delta(\omega) + \exp\{-\kappa\ln(h/\omega)\}^{1-1/d}/[\omega\ln^{\tau}(h/\omega)]$, $\kappa \sim \xi^{D(1-1/d)}$, for $p>p_c$

In both sides of the phase transition the system is GAPLESS!

Turning on the dissipation ($\alpha>0$)

Effective dissipation strength:

$$\alpha_s = \alpha s$$

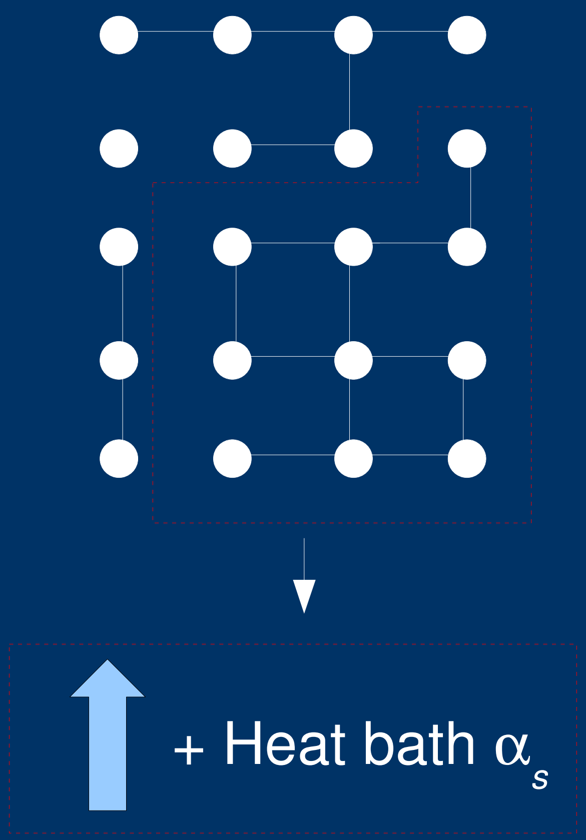
If $\alpha_s > 1$, FROZEN (static) clusters

$$\tilde{\omega}_s = 0$$

If $\alpha_s < 1$, FLUCTUATING (dynamical) clusters

$$\tilde{\omega}_s = \omega_s \, (\omega_s/\omega_c)^{\alpha s/(1-\alpha s)} = h \exp\{-bs/(1-\alpha s)\}.$$

At $\alpha_s = 1$, Kosterlitz-Thouless phase transition



3 types of clusters:
Infinity cluster, dynamical clusters, static clusters.

Turning on the dissipation: DOS

Low-energy density of states:

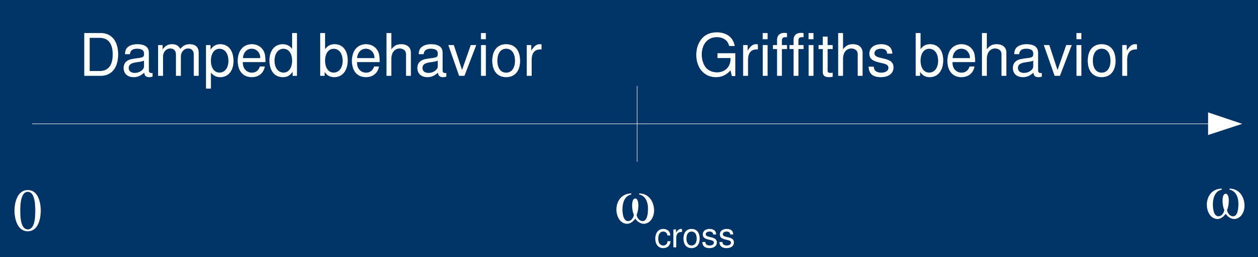
$$\rho(\omega) = \rho_\infty(\omega) + \rho_{dy}(\omega) + \rho_{st}(\omega).$$

$$\rho_{dy}(\omega) \sim \int n_s \delta(\omega - \tilde{\omega}_s) ds = n_{s(\omega)}/[\omega\,(c + \alpha \ln(h/\omega))^2].$$

$$\omega_{cross} = h \exp\{-c/\alpha\}$$

$\rho_{dy}(\omega) \sim 1/\omega^{1-\varphi}$, for $\omega >> \omega_{cross}$ and $p > p_c$

$\rho_{dy}(\omega) \sim n_{s=1/\alpha}/[\omega \ln^{\phi}(h/\omega)]$, for $\omega << \omega_{cross}$, with ϕ=3 and for all *p.*



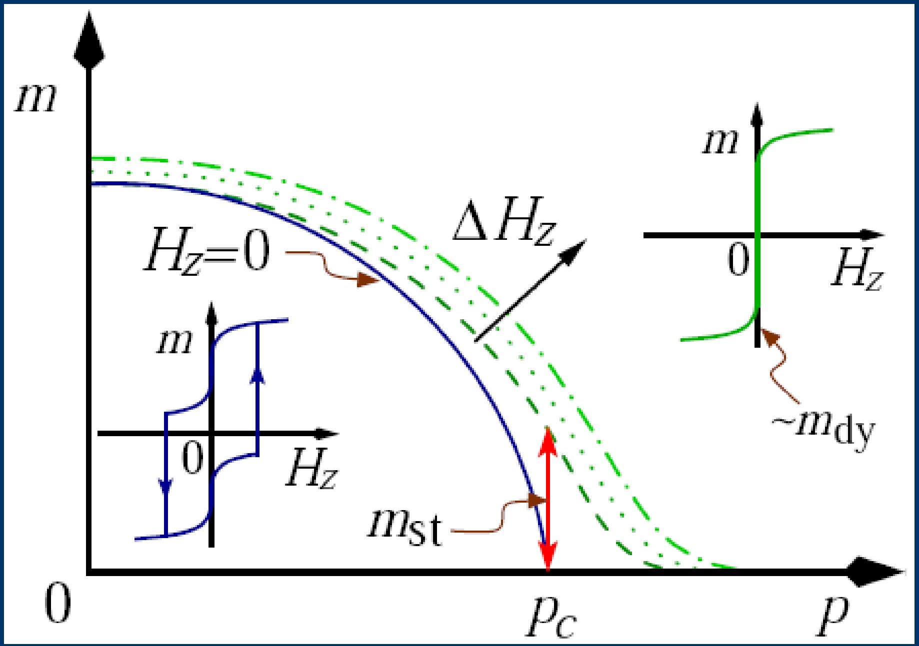
Turning on the dissipation: Magnetization

Magnetization:

At zero external field,

$$m \sim P_{\infty} \sim |p - p_c|^{\beta}$$

At an applied H_z external field,



$$m_{st} = \int_{s > 1/\alpha} n_s ds = (1 - p_c)\alpha^{\tau-2}, \text{ near } p_c$$

$$m_{dy} = \int \rho_{dy}(\omega) \left[H_z s^2(\omega)/(H_z^{\,2} s^2(\omega)+\omega^2)^{1/2} \right] d\omega \sim 1/\ln^{\phi-1}(h/H_z), \text{ for weak fields.}$$

$$\sim H_z^{\varphi}, \text{ for strong fields } (p>p_c).$$

Turning on the dissipation: Thermodynamics

Magnetic susceptibility:

$$\chi_{st}(T) = \int_{s > 1/\alpha} n_s\, s^2/T\, ds \sim |p - p_c|^{-\gamma}/T,$$

$$\chi_{dy}(T) = \int \rho_{dy}(\omega)\, [\, s^2(\omega) \tanh(\omega/T)\,]\, d\omega \sim 1/[T \ln^{\phi-1}(h/H_z)], \text{ for low } T,$$

$$\sim 1/T^{\varphi-1} \text{ for high } T\, (p>p_c).$$

Specif heat:

$$C_{st}(T) \sim C_{dy}(T) \sim 1/\ln^{\phi-1}(h/T), \text{ for low } T,$$

$$\sim T^{\varphi}, \text{ for high } T\, (p>p_c).$$

Conclusions

Phase diagram: