Sweep from Superfluid to Mott phase in the Bose-Hubbard model

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Motivation

- Equilibrium properties well understood
  for many systems, e.g., near a phase transition
- Dynamical (time-dependent) phase transitions
  $\longrightarrow$ response time typically diverges
  $\longrightarrow$ non-equilibrium properties
- Analogy to expanding universe
  $\longrightarrow$ effective horizon (universal behaviour)
  $\longrightarrow$ loss of causal contact
- Amplification of quantum fluctuations
  $\longrightarrow$ seeds for pattern formation etc.
- Physical example: Bose-Hubbard model
- Relation to real cosmic inflation?

Bose-Hubbard Model

$$\hat{H} = J(t) \sum_{\alpha\beta} M_{\alpha\beta} \hat{a}_\alpha^\dagger \hat{a}_\beta + \frac{U}{2} \sum_\alpha (\hat{a}_\alpha^\dagger)^2 \hat{a}_\alpha^2$$

Interaction U, tunnelling rate $J(t)$, lattice matrix $M_{\alpha\beta}$

Large integer filling $n = \langle \hat{n}_\alpha \rangle = \langle \hat{a}_\alpha^\dagger \hat{a}_\alpha \rangle \gg 1$

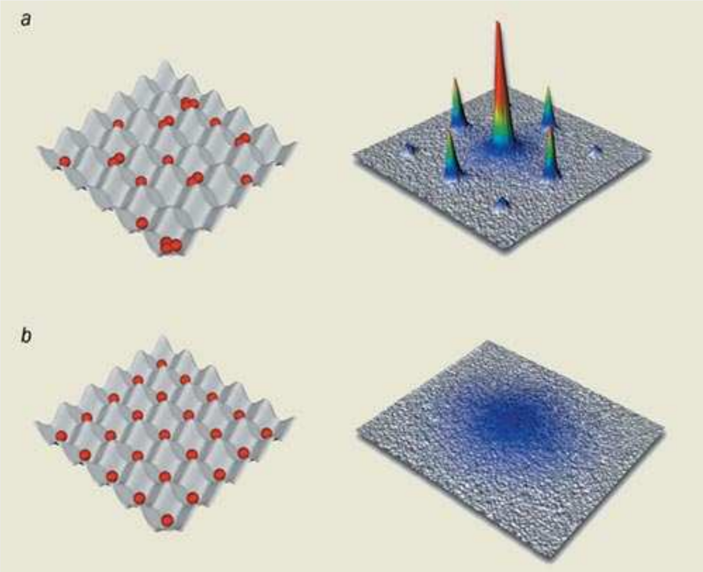
Superfluid→Mott phase transition at $J_c = \mathcal{O}(U/n)$

$J \gg U/n \longrightarrow$ superfluid phase

$$|\Psi\rangle_{\rm sf} \propto \left(\sum_\alpha \hat{a}_\alpha^\dagger\right)^N |0\rangle$$

$J \ll U/n \longrightarrow$ Mott insulator

$$|\Psi\rangle_{\rm Mott} \propto \bigotimes_\alpha (\hat{a}_\alpha^\dagger)^n |0\rangle$$



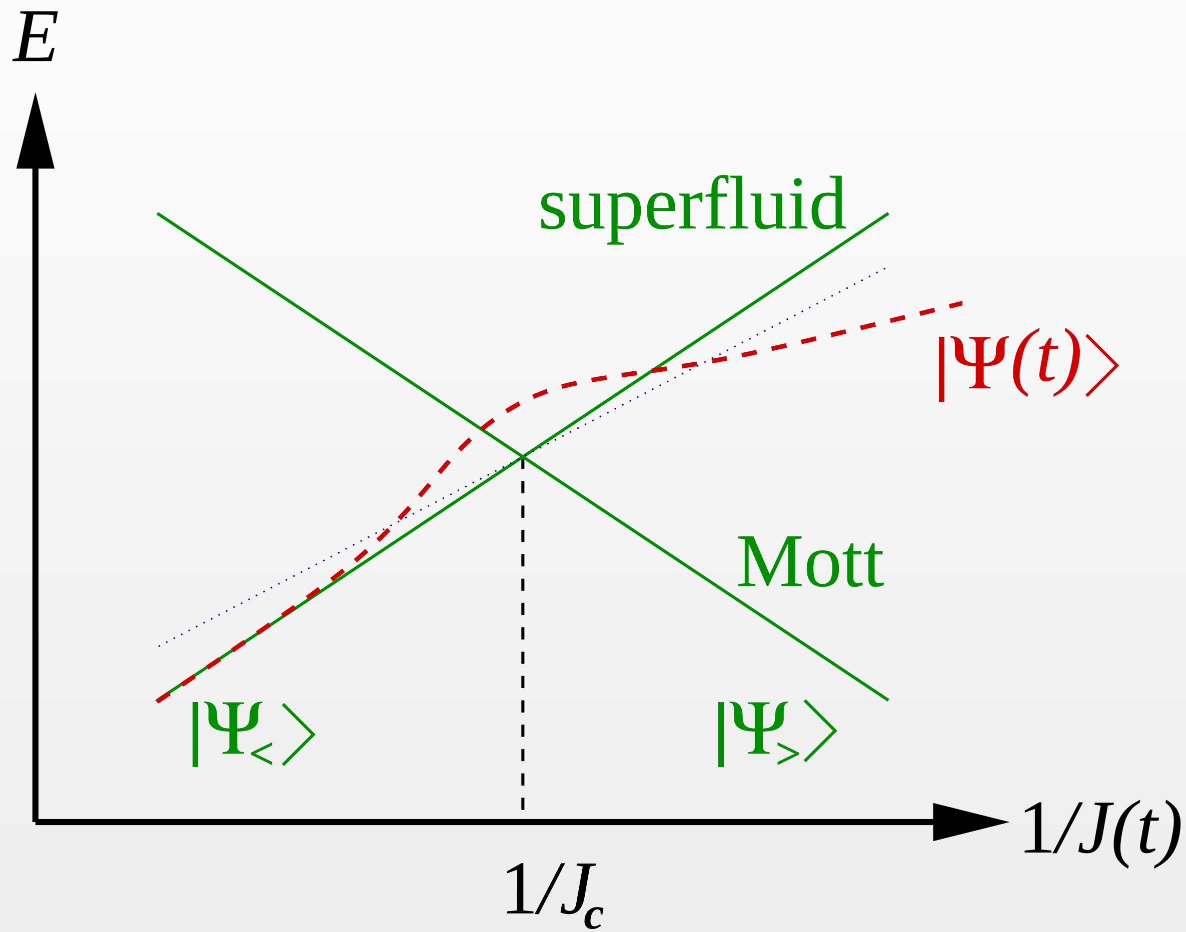
Dynamical Phase Transition

Zero temperature
$T = 0$

External parameter
$J = J(t)$

Two competing ground states
$|\Psi_<\rangle$ and $|\Psi_>\rangle$

Actual quantum state $|\Psi(t)\rangle$



Sweeping through phase transition (critical point J_c)

$\rightarrow$ non-equilibrium dynamics $J(t)$

Number Fluctuations

Normal-mode expansion with label κ, eigenvalues λ_κ

$$\left(\frac{\partial}{\partial t}\frac{1}{J(t)}\frac{\partial}{\partial t} + 8\lambda_\kappa\left[Un + 2J(t)\lambda_\kappa\right]\right)\delta\hat{n}_\kappa = 0$$

Exponential sweep of tunnelling rate (experiments)

$$J(t) = J_0 \exp\{-\gamma t\}$$

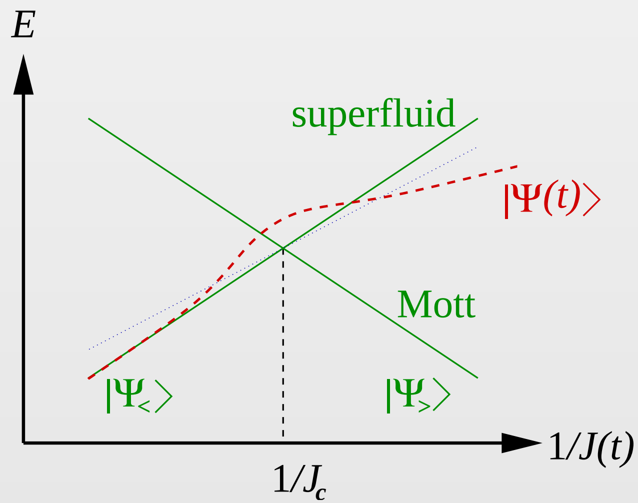
Scaling solution with $\tau_\kappa = -4\lambda_\kappa J(t)/\gamma$

$$\left(\frac{\partial^2}{\partial\tau_\kappa^2} + \left[1 - \frac{2}{\tau_\kappa}\frac{Un}{\gamma}\right]\right)\delta\hat{n}_\kappa = 0$$

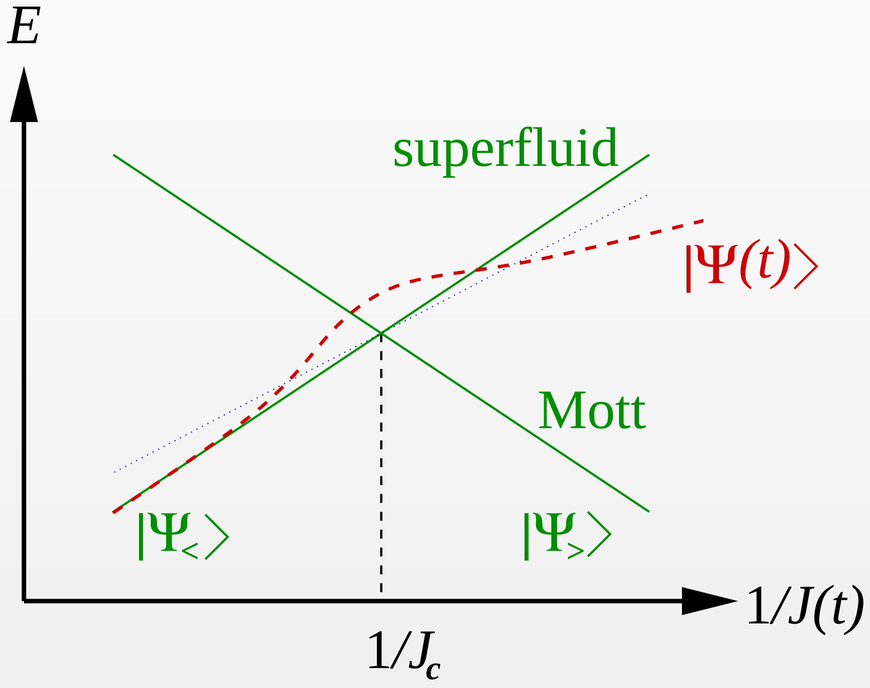
Adiabaticity parameter

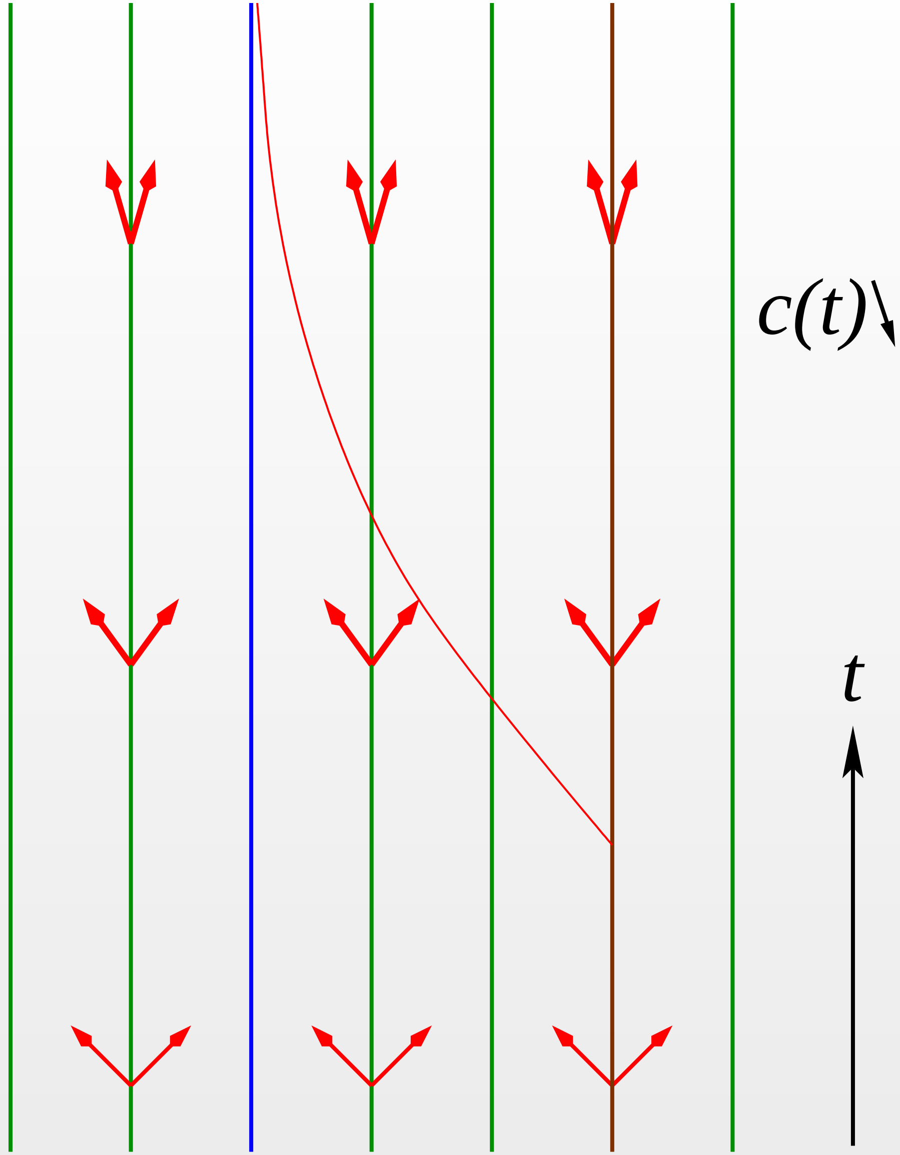
$$\nu = \frac{Un}{\gamma} = \frac{\mu}{\gamma}$$

Fast $\nu \ll 1$ vs slow $\nu \gg 1$ sweep



"Cosmic" Horizon

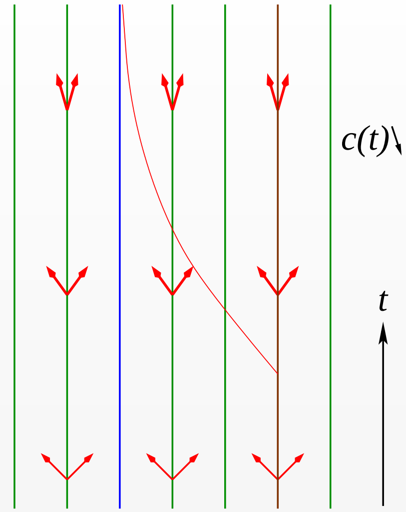




Analogue of cosmic horizon

$$\Delta_{\mathrm{horizon}}(t) = \int_{t}^{\infty} dt' \sqrt{J(t')Un}$$

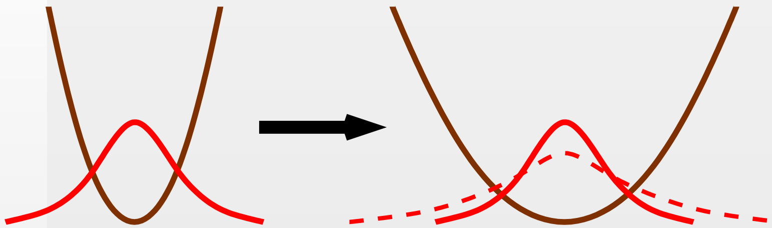
Quantum Fluctuations

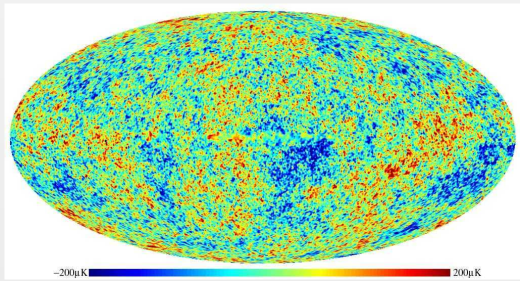


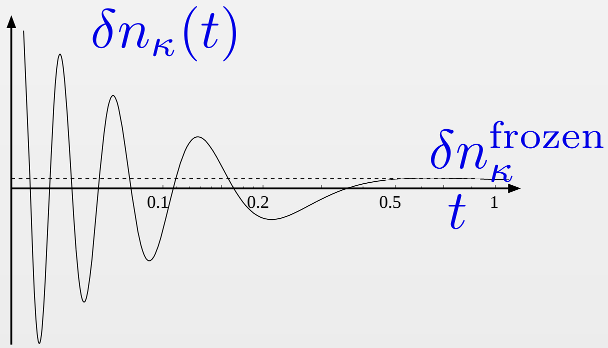
Size of “cosmic” horizon always decreases

$$\frac{d}{dt}\Delta_{\text{horizon}}(t) < 0$$

Oscillation $\lambda \ll \Delta r(t)$ $\rightarrow$ horizon crossing $\rightarrow$
$\rightarrow$ freezing $\lambda \gg \Delta r(t)$ and squeezing







Amplification of quantum fluctuations $\hbar\omega/2$
Analogous to early universe (WMAP)

Frozen Number Fluctuations

$$\langle \delta \hat{n}_\kappa^2 \rangle = n \frac{1 - e^{-2\pi\nu}}{2\pi\nu} + \mathcal{O}(t \lambda_\kappa e^{-\gamma t})$$

Off-site number correlations decay exponentially

$$\langle \hat{n}_\alpha \hat{n}_\beta \rangle - \langle \hat{n}_\alpha \rangle \langle \hat{n}_\beta \rangle = \langle \delta \hat{n}_\alpha \delta \hat{n}_\beta \rangle = \mathcal{O}(\gamma t \, e^{-\gamma t})$$

On-site number variations

$$\Delta^2(n_\alpha) = \langle \hat{n}_\alpha^2 \rangle - \langle \hat{n}_\alpha \rangle^2 = \langle \delta \hat{n}_\alpha^2 \rangle = n \frac{1 - e^{-2\pi\nu}}{2\pi\nu}$$

Fast $\nu \ll 1$ sweep:

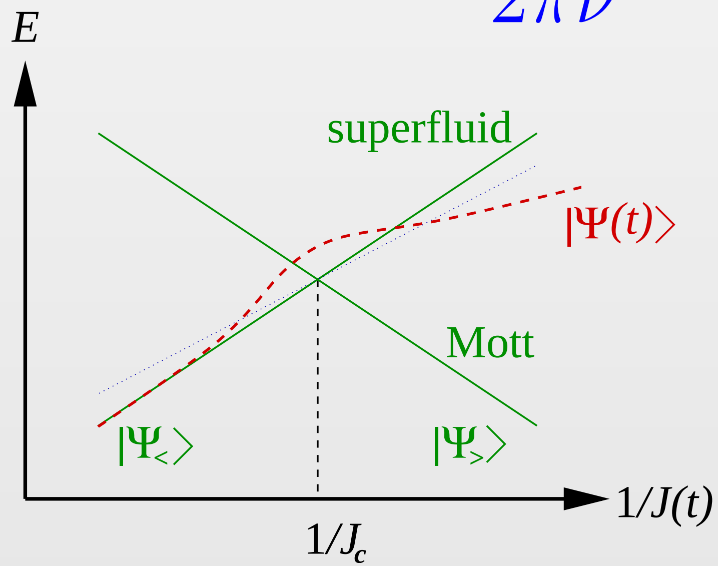
$\Delta^2(n_\alpha) \to n$

(Poissonian $\to$ superfluid)

Slow $\nu \gg 1$ sweep:

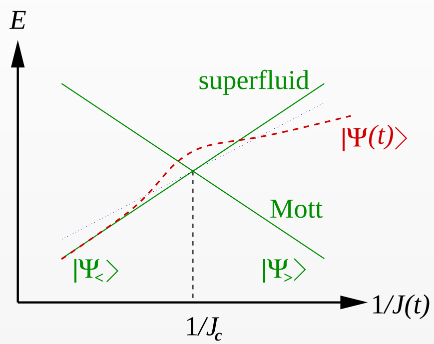
$\Delta^2(n_\alpha) \to 0$

($\to$ Mott)



Off-diagonal long-range order

Phase fluctuations grow (instability)



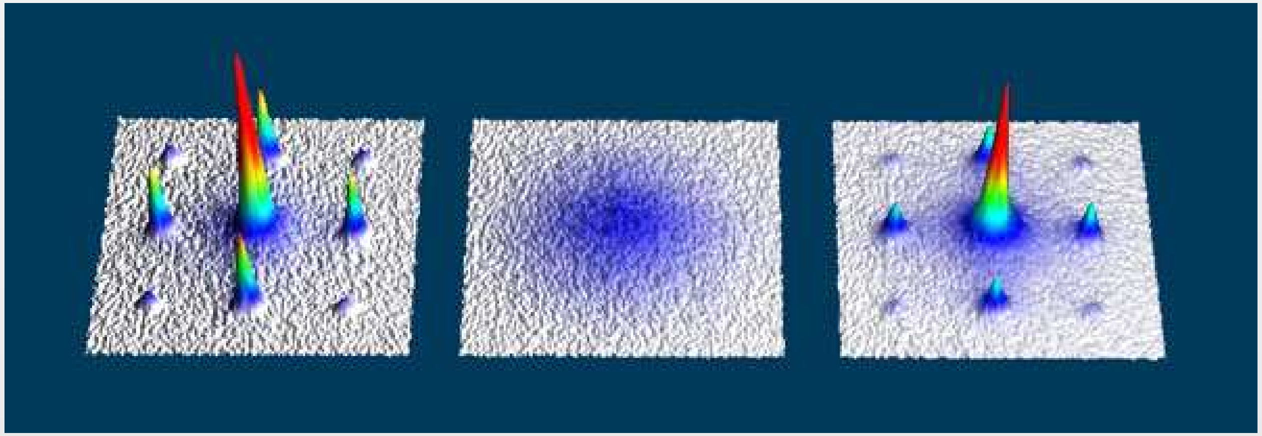
$$\langle\delta\hat{\phi}_\kappa^2\rangle = \nu\frac{1-e^{-2\pi\nu}}{2\pi n}\gamma^2 t^2 + \mathcal{O}(\gamma t \ln \lambda_\kappa)$$

Decay of off-diagonal long-range order

$$\langle\hat{a}_\alpha^\dagger(t)\hat{a}_\beta(t)\rangle \approx n\exp\{-U^2t^2\Delta^2(n_\alpha)\}$$

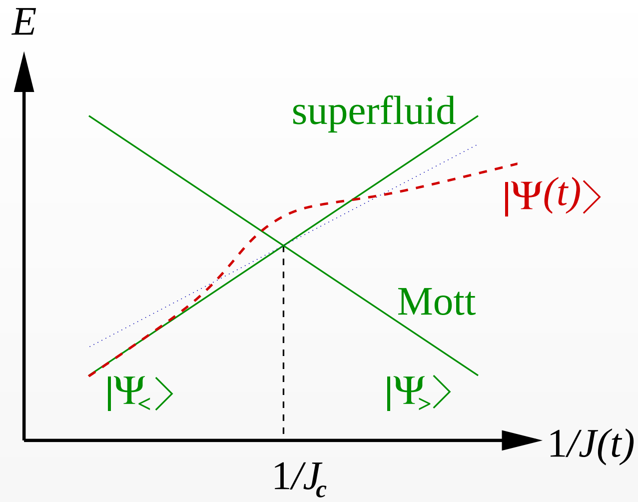
Loss of phase coherence due to horizon

$\longrightarrow$ peak at $k=0$ decreases (Greiner *et al*)



Note: revival for $Ut\in 2\pi\mathbb{N}$ (similar to spin echo)

Decay of Superfluid



Superfluid fraction defined via $\boldsymbol{j} = \varrho_{\rm sf} \boldsymbol{\nabla} \Phi$ is determined by $\langle \hat{a}_\ell^\dagger \hat{a}_{\ell+1} \rangle$ and thus also decreases

$$\frac{n_{\rm sf}}{n} \approx \exp\left\{ -U^2 t^2 n \frac{1-e^{-2\pi\nu}}{2\pi\nu} \right\}$$

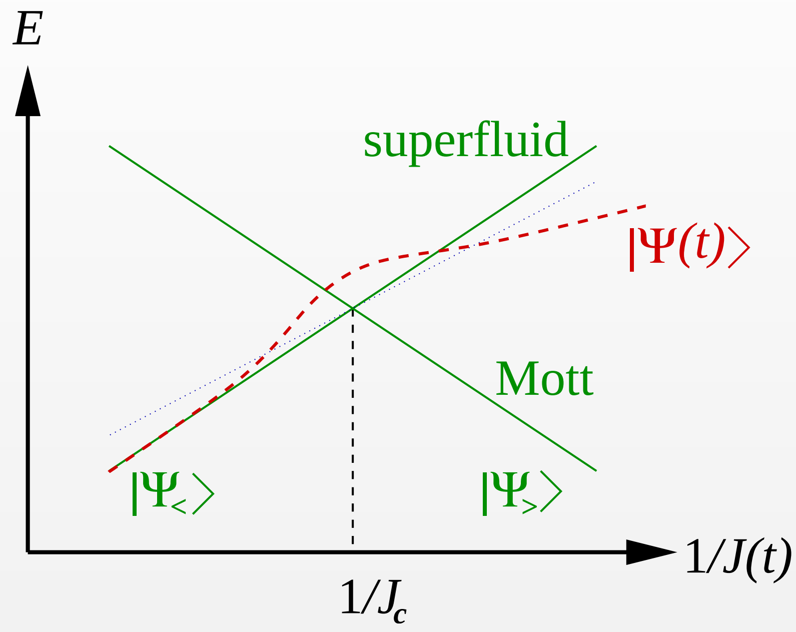
Rapid sweep $\nu \ll 1 \to$ decay with $\exp\{-nU^2t^2\}$ (independent of γ)

Adiabatic sweep $\nu \gg 1 \to$ decay of superfluid fraction much slower $\exp\{-nU^2t^2/(2\pi\nu)\}$

R. S., M. Uhlmann, Y. Xu and U. R. Fischer,
Sweeping from the superfluid to Mott phase in the Bose-Hubbard model,
`cond-mat/0605121`

Similarities to Cosmic Inflation

- Release of energy
  $\longrightarrow$ (p)re-heating
- Robust against initial (small-scale) perturbations
- Universality (no fine-tuning)

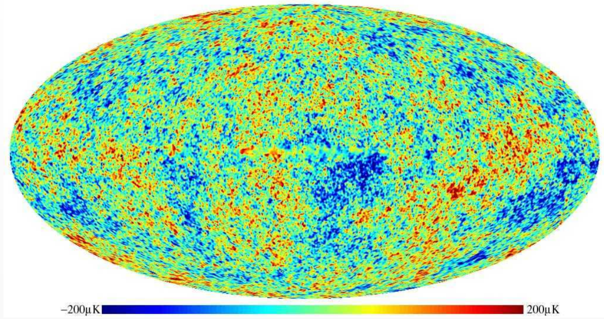


- Amplification of quantum fluctuations

But: different spectrum in general

- Preferred frame (rest frame of medium)
- No unique/constant propagation speed
- Neglect of (quantum) back-reaction

Speculations...



Postulate:

- No (locally) preferred frame
- Unique/constant propagation speed

$$\mathcal{A} = \frac{1}{2} \int dt\, d^3r\, \frac{\dot{\Phi}^2 - (\boldsymbol{\nabla}\Phi)^2}{t^2}$$

- $\leftrightarrow$ scale-invariance $\mathcal{A}[\lambda t, \lambda \boldsymbol{r}] = \mathcal{A}[t, \boldsymbol{r}]$
- Dominated by (quantum) back-reaction?

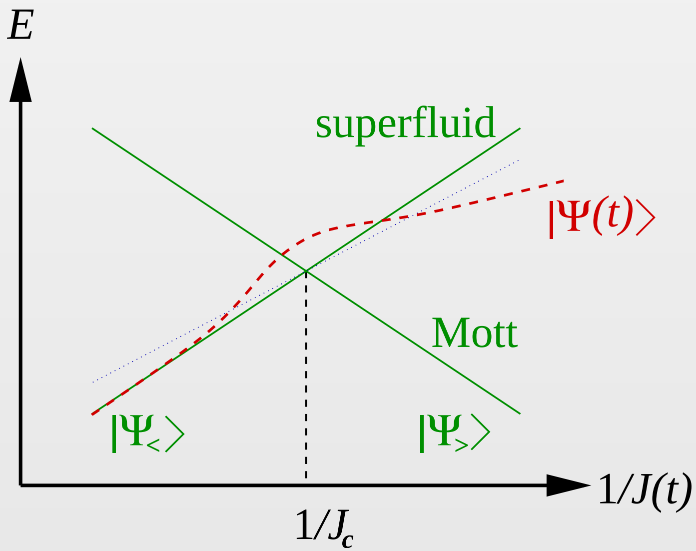
$\rightarrow$ correct $1/k^3$-spectrum (conformal de Sitter metric)

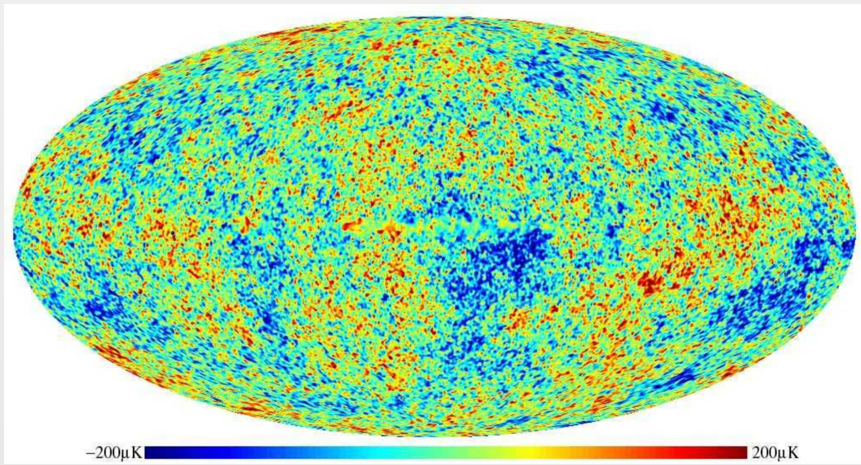
Was cosmic inflation just a phase transition?

R. S., Phys. Rev. Lett. **95**, 135703 (2005)

Summary

- Analogy between cosmic inflation and dynamical quantum phase transitions
- Effective "cosmic" horizons
  $\rightarrow$ loss of causal contact
  $\rightarrow$ non-adiabatic behaviour
  $\rightarrow$ amplification of quantum fluctuations
- Relation to real cosmic inflation?





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