(How to have fun with)
two-dimensional frustrated ferromagnets

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Les Houches 20/6/6
thanks to...

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spin-1/2 on a triangular lattice
- spin liquids and the RVB idea -
spin-1/2 on a triangular lattice
- spin liquids and the RVB idea -

the eternal triangle

?!!!
spin-1/2 on a triangular lattice
- spin liquids and the RVB idea -

the eternal triangle

frustration, i.e. you can't please all of the spins, all of the time
spin-1/2 on a triangular lattice
- spin liquids and the RVB idea -

the eternal triangle

Anderson’s resonating valence bond (RVB) state

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Anderson's resonating valence bond (RVB) state

actual ground state of Heisenberg model on triangular lattice

3-sublattice Néel state
spin-1/2 on a triangular lattice
- spin liquids and the RVB idea -

the eternal triangle

Anderson’s resonating valence bond (RVB) state

frustration, i.e. you can’t please all of the spins, all of the time

actual ground state of Heisenberg model on triangular lattice

for a review, see e.g. Misguich and Lhullier in “Quantum Spin Systems” (2004 Diep)
in the beginning, God created He III...
the most perfect correlated Fermi system known to man
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2D incarnation - He III on graphite

Fermi liquid in second layer becomes magnetic solid with increasing density:

high density solid is FM

something very special happens at low densities...
the first true spin liquid...
...in a 2D triangular lattice frustrated FM

K. Ishida et al., PRL 79, 3451 (1997)
the first true spin liquid...
...in a 2D triangular lattice frustrated FM

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2nd layer magnetism controlled by competition between FM 3-spin exchange and AF 4-spin exchange
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no magnetic order down to 0.1 mK !!!
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no magnetic order down to 0.1 mK !!!

linear specific heat (c.f. 2D FM)
double peak structure
Pb$_2$VO(PO$_4$)$_2$

first example of a square lattice frustrated FM

E. Kaul et al., JMMM 272-276 (II), 922 (2004)
Pb\textsubscript{2}VO(PO\textsubscript{4})\textsubscript{2}

first example of a square lattice frustrated FM

E. Kaul et al., JMMM 272-276 (II), 922 (2004)

\textbf{Pb\textsubscript{2}VO(PO\textsubscript{4})\textsubscript{2} : Structure}

spin-1/2 V\textsuperscript{4+} in layered pyramids
Pb2VO(PO4)2
first example of a square lattice frustrated FM
E. Kaul et al., JMMM 272-276 (II), 922 (2004)

Pb2VO(PO4)2: Structure

spin-1/2 V4+ in layered pyramids

two different exchange paths
- both n.n. and n.n.n. bonds -
**Pb$_2$VO(PO$_4$)$_2$**

*first example of a square lattice frustrated FM*


**Pb$_2$VO(PO$_4$)$_2$ : Structure**

- spin-1/2 V$_4^+$ in layered pyramids
- two different exchange paths
  - both n.n. and n.n.n. bonds

**χ(T) Behavior**

- $T_{\text{Max}} = 8.92$ K
- $T_N = 3.5$ K

**χ-Inverse**

- linear $\chi$-inverse $\Rightarrow$ frustrated magnet
**Pb2VO(PO4)2**

first example of a square lattice frustrated FM

E. Kaul et al., JMMM 272-276 (II), 922 (2004)

**Pb2VO(PO4)2 : Structure**

spin-1/2 V4+ in layered pyramids

linear $\chi$-inverse $\Rightarrow$ frustrated magnet

$\frac{J_2}{J_1} \approx -2$

two different exchange paths
- both n.n. and n.n.n. bonds -
**Pb2VO(PO4)2**

first example of a square lattice frustrated FM

E. Kaul et al., JMMM 272-276 (II), 922 (2004)

**Pb2VO(PO4)2 : Structure**

spin-1/2 V⁴⁺ in layered pyramids

linear $\chi$-inverse $\Rightarrow$ frustrated magnet

$J_2/J_1 \approx -2$

ground state is $(\pi, 0)$
collinear AF
with reduced moment

two different exchange paths
- both n.n. and n.n.n. bonds -
the "simplest" frustrated ferromagnet
extended FM Heisenberg model on square lattice
the “simplest” frustrated ferromagnet
extended FM Heisenberg model on square lattice

\[ H = 2J_1 \sum_{\langle ij \rangle_1} S_i S_j + 2J_2 \sum_{\langle ij \rangle_2} S_i S_j + K \sum_{\langle 1234 \rangle} P_{1234} + P_{1234}^{-1} \]
the "simplest" frustrated ferromagnet
extended FM Heisenberg model on square lattice

\[ \mathcal{H} = 2J_1 \sum_{\langle ij \rangle_1} S_i S_j + 2J_2 \sum_{\langle ij \rangle_2} S_i S_j + K \sum_{\langle 1234 \rangle} P_{1234} + P_{1234}^{-1} \]

- **FM n.n. interaction**
  - \( J_1 < 0 \)
- **AF n.n.n. interaction**
  - \( J_2 > 0 \)
- **AF 4-spin cyclic exchange**
  - \( K > 0 \)
the "simplest" frustrated ferromagnet
extended FM Heisenberg model on square lattice

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\[ P_{ijkl} + P_{ijkl}^{-1} = \vec{S}_i \cdot \vec{S}_j + \ldots + 4 \left( \vec{S}_i \cdot \vec{S}_j \right) \left( \vec{S}_k \cdot \vec{S}_l \right) + \ldots \]

N.B.

FM n.n. interaction
\[ J_1 < 0 \]

AF n.n.n. interaction
\[ J_2 > 0 \]

AF 4-spin cyclic exchange
\[ K > 0 \]
the "simplest" frustrated ferromagnet
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- FM n.n. interaction
  - \( J_1 < 0 \)
- AF n.n.n. interaction
  - \( J_2 > 0 \)
- AF 4-spin cyclic exchange
  - \( K > 0 \)

mean field phase diagram

\[ P_{ijkl} + P_{ijkl}^{-1} = \vec{S}_i \cdot \vec{S}_j + \ldots + 4 \left( \vec{S}_i \cdot \vec{S}_j \right) \left( \vec{S}_k \cdot \vec{S}_l \right) + \ldots \]
quantum critical points

- and ferromagnetism -
quantum critical points
- and ferromagnetism -
quantum critical points
- and ferromagnetism -

what happens here ?!!!
quantum critical points
- and ferromagnetism -

what happens here ?!!!

manganites (g=doping)
- phase separation
quantum critical points
- and ferromagnetism -

- phase separation
- superconductivity

manganites
(g=doping)

weak itinerant ferromagnets
(g=pressure)

what happens here ???

control parameter

transition temperature
quantum critical points
- and ferromagnetism -

- phase separation
- superconductivity
- spin liquid ?!!!
manganites
(g=doping)
weak itinerant
ferromagnets
(g=pressure)
frustrated
quantum spin
systems
(g=density, chemical pressure)
control parameter
transition temperature
what happens here ?!!!
how does the FM die?
- nature of spin excitations at boundary with AF -
how does the FM die?
- nature of spin excitations at boundary with AF -

“one magnon” dispersion:

\[
\omega(q) = 8(|J_1| - 2K - J_2) - 4(|J_1| - 2K)[\cos q_x + \cos q_y] + 8J_2 \cos q_x \cos q_y
\]
how does the FM die?
- nature of spin excitations at boundary with AF -

"one magnon" dispersion:

\[ \omega(q) = 8(|J_1| - 2K - J_2) - 4(|J_1| - 2K)[\cos q_x + \cos q_y] + 8J_2 \cos q_x \cos q_y \]

J=-1, J2=K=0
how does the FM die?
- nature of spin excitations at boundary with AF -

“one magnon” dispersion:

\[ \omega(q) = 8(|J_1| - 2K - J_2) - 4(|J_1| - 2K)[\cos q_x + \cos q_y] + 8J_2 \cos q_x \cos q_y \]

\( J = -1, J_2 = K = 0 \)

\( J = -1, J_2 = 1/2, K = 0 \)
how does the FM die?
- nature of spin excitations at boundary with AF -

“one magnon” dispersion:

$$\omega(q) = 8(|J_1| - 2K - J_2) - 4(|J_1| - 2K)[\cos q_x + \cos q_y] + 8J_2 \cos q_x \cos q_y$$

limiting case #1:

J1=-1, J2 = 1/2, K=0

line zeros for qx = 0, qy = 0
how does the FM die?
- nature of spin excitations at boundary with AF -

“one magnon” dispersion:

$$\omega(q) = 8(|J_1| - 2K - J_2) - 4(|J_1| - 2K)[\cos q_x + \cos q_y] + 8J_2 \cos q_x \cos q_y$$

limiting case #1:
J1=-1, J2 = 1/2, K=0

J=-1, J2=K=0

J1-J2 model

line zeros for qx = 0, qy = 0

limiting case #2:
J1=-1, J2 = 0, K=1/2

square lattice MSE model

entire dispersion vanishes !!!
what kind of excitation works?
- two magnons are better than one -
what kind of excitation works?  
- two magnons are better than one -

square lattice MSE model  
$J_1 = -1, J_2 = 0, K = 1/2$
what kind of excitation works?

- two magnons are better than one -

square lattice MSE model

$J_1 = -1, J_2 = 0, K = 1/2$
what kind of excitation works?
- two magnons are better than one -

square lattice MSE model
\[ J_1 = -1, \ J_2 = 0, \ K = 1/2 \]
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square lattice MSE model
$J_1 = -1, J_2 = 0, K = 1/2$
what kind of excitation works?
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square lattice MSE model

$J_1=-1$, $J_2=0$, $K=1/2$
what kind of excitation works?
- two magnons are better than one -

square lattice MSE model
$J_1 = -1, J_2 = 0, K = 1/2$
What kind of excitation works?  
- Two magnons are better than one -

Square lattice MSE model  
\( J_1 = -1, J_2 = 0, K = 1/2 \)

Simple trial wave function for two-magnon bound state:

\[
\frac{1}{\sqrt{2}} \left\{ \left\langle \uparrow \uparrow \downarrow \downarrow \right| - \left| \downarrow \downarrow \uparrow \uparrow \right\rangle \right\} \exp(iq.r/2)
\]

Individual magnons are localized, but **pairs** of magnons can propagate coherently.
what kind of excitation works?
- two magnons are better than one -

square lattice MSE model
\[ J_1 = -1, \quad J_2 = 0, \quad K = 1/2 \]

simple trial wave function for two-magnon bound state:
\[ \frac{1}{\sqrt{2}} \left\{ \begin{array}{c|c}
\uparrow & \uparrow \\
\downarrow & \downarrow \\
\end{array} \right\} - \left\{ \begin{array}{c|c}
\downarrow & \uparrow \\
\uparrow & \downarrow \\
\end{array} \right\} \exp(iq.r/2) \]

d-wave symmetry

individual magnons are localized
but **pairs** of magnons can propagate coherently
what kind of excitation works? 
- two magnons are better than one -

square lattice MSE model
\( J_1 = -1, J_2 = 0, K = 1/2 \)

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simple trial wave function
for two-magnon bound state:

\[
\frac{1}{\sqrt{2}} \left\{ \langle \uparrow \uparrow \rangle - \langle \downarrow \downarrow \rangle \right\} \exp(iq.r/2)
\]

d-wave symmetry

for special point
\( J_1 = -1, J_2 = 0, K = 1/2 \)
this wave function is an **exact eigenstate**
so what is the first instability of the FM?

- two magnons are better than one -
so what is the first instability of the FM?  
- two magnons are better than one -

calculate energies of one-magnon band and two-magnon trial wave function in applied magnetic field and see which becomes negative first:
so what is the first instability of the FM?
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calculate energies of one-magnon band and two-magnon trial wave function in applied magnetic field and see which becomes negative first:

![Graph showing saturated FM and canted AF regions with magnetic field and $J_2/|J_1|$ or $K/|J_1|$ axes.](image)
so what is the first instability of the FM?
- two magnons are better than one -

calculate energies of one-magnon band and two-magnon trial wave function in applied magnetic field and see which becomes negative first:

c conventional one-magnon instability
so what is the first instability of the FM?
- two magnons are better than one -

calculate energies of one-magnon band and two-magnon trial wave function in applied magnetic field and see which becomes negative first:

![Diagram showing magnetic field dependence and phase transitions between saturated FM, two-magnon instability, and canted AF states.](image-url)
so what is the first instability of the FM?

- two magnons are better than one -

calculate energies of one-magnon band and two-magnon trial wave function in applied magnetic field and see which becomes negative first:

conventional one-magnon instability

two-magnon instability

what is the nature of this phase?!!!
a new idea - nematic order
- systems that don’t know up from down -
a new idea - nematic order
- systems that don’t know up from down -

nematic (quadropolar) order:

\[
\langle Q^{xx} \rangle > \langle Q^{yy} \rangle = \langle Q^{zz} \rangle
\]

\[
\langle Q^{xx} \rangle = \langle Q^{yy} \rangle = \langle Q^{zz} \rangle
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a new idea - nematic order
- systems that don’t know up from down -

nematic (quadropolar) order:

\[
\langle Q^{xx} \rangle > \langle Q^{yy} \rangle = \langle Q^{zz} \rangle
\]

site-wise nematic works for spin-1:

favour \( Sz = +/-1 \)
equally

disfavour \( Sz = 0 \)
a new idea - nematic order
- systems that don’t know up from down -

nematic (quadropolar) order:

\[ \langle Q^{xx} \rangle > \langle Q^{yy} \rangle = \langle Q^{zz} \rangle \]

site-wise nematic works for spin-1:

favour \( S_z = \pm 1 \) equally
disfavour \( S_z = 0 \)

doesn’t work for spin-1/2:

only \( S_z = \pm 1/2 \) states exist
a new idea - nematic order
- systems that don’t know up from down -

nematic (quadropolar) order:

\[
\langle Q^{xx} \rangle > \langle Q^{yy} \rangle = \langle Q^{zz} \rangle
\]

site-wise nematic works for spin-1:
- favour Sz=+/−1 equally
- disfavour Sz=0

for a spin-1 example see, e.g.: K. Harada and N. Kawashima, PRB 65, 052403(2002)

doesn’t work for spin-1/2:
- only Sz=+/−1/2 states exist
so what...?

- what do nematics and spin-1/2 FM’s have in common? -
so what...?
- what do nematics and spin-1/2 FM's have in common? -

what if we project spin-1/2's into a spin-1 space?
so what...?
- what do nematics and spin-1/2 FM’s have in common? -

what if we project spin-1/2’s into a spin-1 space?

consider the traceless second rank tensor:

\[ \mathcal{O}^{\alpha\beta}(r_i, r_j) = \frac{1}{2}(S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) - \frac{1}{3}\delta^{\alpha\beta}\langle S_i \cdot S_j \rangle \]
so what...?
- what do nematics and spin-1/2 FM's have in common? -

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\]

symmetrized product of spin-1/2's
i.e. spin-1 object
so what...?
- what do nematics and spin-1/2 FM’s have in common? -

what if we project spin-1/2’s into a spin-1 space?

consider the traceless second rank tensor:

\[ O^{\alpha\beta}(r_i, r_j) = \frac{1}{2}(S_\alpha^i S_\beta^j + S_\beta^i S_\alpha^j) - \frac{1}{3}\delta^{\alpha\beta}\langle S_i \cdot S_j \rangle \]

symmerizerized product of spin-1/2’s
i.e. spin-1 object

removes trivial self-correlation
so what...?
- what do nematics and spin-1/2 FM’s have in common? -

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- symmeterized product of spin-1/2’s
  i.e. spin-1 object

relationship with wave function for two-magnon bound state through:

\[ S_i^- S_j^- = O^{xx} - O^{yy} - 2iO^{xy} \]

...i.e. bond nematic can form through bi-magnon condensation *
so what...?
- what do nematics and spin-1/2 FM’s have in common? -

what if we project spin-1/2’s into a spin-1 space?

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symmetrized product
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relationship with wave function for two-magnon bound state through:

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...i.e. bond nematic can form through bi-magnon condensation *

can we see the nematic in numerics?

- two-magnon instability in applied field -
can we see the nematic in numerics?
- two-magnon instability in applied field -

first establish extent of FM
can we see the nematic in numerics?
- two-magnon instability in applied field -

first establish extent of FM

ground state energy (S=0) of finite size cluster
can we see the nematic in numerics?
- two-magnon instability in applied field -

first establish extent of FM

ground state energy (S=0) of finite size cluster
can we see the nematic in numerics?

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ground state energy (S=0) of finite size cluster

first instability of saturated paramagnet is against two-magnon bound state
can we see the nematic in numerics?  
- two-magnon instability in applied field -

first establish extent of FM

ground state energy (S=0) of finite size cluster

Bose-Einstein condensation of bi-magnons at high field can be seen in numerics ✓

first instability of saturated paramagnet is against two-magnon bound state
what about the ground state?
- absence of Néel order in the FM J1-J2 model -
what about the ground state?
- absence of Néel order in the FM J1-J2 model -
what about the ground state? 
- absence of Néel order in the FM J1-J2 model -

This study

SQR N=36 (6,0,0,6) J1=-1 J2=0.40
what about the ground state?
- absence of Néel order in the FM J1-J2 model -

spectrum contains wrong set of low-lying states for a Néel order parameter
what about the ground state?
- absence of Néel order in the FM J1-J2 model -

Gaps in even spin sectors scale to zero:

spectrum contains wrong set of low-lying states for a Néel order parameter
what about the ground state?
- absence of Néel order in the FM J1-J2 model -

Gaps in even spin sectors scale to zero:

Gaps in odd spin sectors do not:

Spectrum contains wrong set of low-lying states for a Néel order parameter
but is it really the same state?
- nematic correlation in ground state -
but is it really the same state?
- nematic correlation in ground state -

nematic correlation function:

\[
C(i, j, k, l) = \sum_{\alpha \beta} \langle \mathcal{O}_{\alpha \beta}(r_i, r_j) \mathcal{O}_{\alpha \beta}(r_k, r_l) \rangle
\]
but is it really the same state?
- nematic correlation in ground state -

nematic correlation function:

\[ C(i, j, k, l) = \sum_{\alpha\beta} \langle O^{\alpha\beta}(r_i, r_j) O^{\alpha\beta}(r_k, r_l) \rangle \]
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but is it really the same state?

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nematic correlation function:

$$C(i, j, k, l) = \sum_{\alpha\beta} \langle O^{\alpha\beta}(r_i, r_j) O^{\alpha\beta}(r_k, r_l) \rangle$$

strong “stripe” correlations

reference bond
but is it really the same state?
- nematic correlation in ground state -

nematic correlation function:

\[ C(i, j, k, l) = \sum_{\alpha \beta} \langle O^{\alpha \beta}(r_i, r_j) O^{\alpha \beta}(r_k, r_l) \rangle \]

strong “stripe” correlations

reference bond

"-" \Rightarrow d-wave sym
but is it really the same state?
- nematic correlation in ground state -

nematic correlation function:

\[ C(i, j, k, l) = \sum_{\alpha \beta} \langle \mathcal{O}^{\alpha \beta}(r_i, r_j) \mathcal{O}^{\alpha \beta}(r_k, r_l) \rangle \]

strong “stripe” correlations
reference bond

“-” \( \Rightarrow \) d-wave sym
slow decay of correlations
but is it really the same state?
- nematic correlation in ground state -

nematic correlation function:

\[ C(i, j, k, l) = \sum_{\alpha \beta} \langle O^{\alpha \beta}(r_i, r_j) O^{\alpha \beta}(r_k, r_l) \rangle \]

strong "stripe" correlations

reference bond

"-" \(\Rightarrow\) d-wave sym

slow decay of correlations
thermodynamics...
- formation of triplets and “emergent” nematic order -
thermodynamics...
- formation of triplets and “emergent” nematic order -

\[
\begin{align*}
&\text{FM} \quad \uparrow \uparrow \\
&\text{NAF} \quad \uparrow \downarrow \\
&\text{CAF} \quad \downarrow \downarrow \\
&\text{this study} \quad \phi
\end{align*}
\]
thermodynamics...
- formation of triplets and “emergent” nematic order -
thermodynamics...
- formation of triplets and “emergent” nematic order -

This study

Heat capacity

Natural energy scale ~J
thermodynamics...
- formation of triplets and “emergent” nematic order -

![Diagram of phase transitions and heat capacity curves](image)

- This study
- heat capacity
- new low “emergent” energy scale
- natural energy scale ~J
thermodynamics...

- formation of triplets and “emergent” nematic order -

[Diagram showing critical points and phase transitions]

- Heat Capacity
- Entropy
- New low “emergent” energy scale
- Natural energy scale ~J
thermodynamics...
- formation of triplets and "emergent" nematic order -

spin-1/2

heat capacity

entropy

new low "emergent" energy scale

natural energy scale ~J
thermodynamics...
- formation of triplets and “emergent” nematic order -

spin-1/2
spin-1
on bonds:

heat capacity

new low “emergent” energy scale

natural energy scale ~J

ln(2)
ln(3)/2

ln(2)

entropy

energy scale

log_{10} (T/J)
thermodynamics...

- formation of triplets and "emergent" nematic order -

spin-1/2
spin-1
on
bonds:

heat
capacity

entropy

entropy in peak is associated with nematic ordering of spin-1 bond degrees of freedom

new low "emergent" energy scale

natural energy scale ~J
could this ever be observed in nature?

- more quasi-2D vanadates! -
could this ever be observed in nature?

- more quasi-2D vanadates! -

\[
\begin{align*}
\phi & = J_2/J_1 = -\infty \\
\text{FM} & \quad \text{CAF} \\
\text{Pb}_2\text{VO}(\text{PO}_4)_2 \\
\text{BaZnVO}(\text{PO}_4)_2 \\
\text{SrZnVO}(\text{PO}_4)_2 \\
\text{Li}_2\text{VO}(\text{SiO}_4) \\
\text{Li}_2\text{VO}(\text{GeO}_4) \\
\text{NAF} \\
\end{align*}
\]
could this ever be observed in nature?

- more quasi-2D vanadates! -

\[ J_2/J_1 = -\infty \]

FM
CAF
Pb\(_2\)VO(PO\(_4\))\(_2\)
BaZnVO(PO\(_4\))\(_2\)
SrZnVO(PO\(_4\))\(_2\)
Li\(_2\)VO(SiO\(_4\))
Li\(_2\)VO(GeO\(_4\))
NAF
0.4
(0.7)
(-0.7)
-0.4

**gapped, staggered dimer phase**

[P. Sindzingre, Phys. Rev. B 69, 094418 (2004), and references therein]

singlet
could this ever be observed in nature?

- more quasi-2D vanadates! -

gapless nematic phase

gapped, staggered dimer phase
[P. Sindzingre, Phys. Rev. B 69, 094418 (2004), and references therein]

director
could this ever be observed in nature?
- more quasi-2D vanadates!

gapless nematic phase

gapped, staggered dimer phase
[P. Sindzingre, Phys. Rev. B 69, 094418 (2004), and references therein]

new compound CaZnVO(PO4)2 looks promising...
so what happens on a triangular lattice?

- modeling solid 2D films of He III -
so what happens on a triangular lattice?
- modeling solid 2D films of He III -

minimal model: \[ \mathcal{H} = 2J_1 \sum_{\langle ij \rangle} S_i S_j + K \sum_{\langle 1234 \rangle} P_{1234} + P_{1234}^{-1} \]

FM \( J_1 < 0 \)
AF \( K > 0 \)
so what happens on a triangular lattice?
- modeling solid 2D films of He III -

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ferromagnetic  degenerate  chiral phase  120° structure
\[ H = 2J_1 \sum_{\langle ij \rangle} S_i S_j + K \sum_{\langle 1234 \rangle} P_{1234} + P_{1234}^{-1} \]

FM  \( J_1 < 0 \)
AF  \( K > 0 \)

classical phase diagram:

[Phase diagram: Ferromagnetic, Degenerate, Chiral Phase, 120° Structure]

[T. Momoi and K. Kubo, PRB 65, 052403(2002)]
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ferromagnetic | degenerate | chiral phase | 120° structure

[O(2^{L/2})]

[high density 2D solid $^3$He low density]

classical critical point $|J_1| = K/4$

at which entire spin wave dispersion vanishes, i.e. $\omega(k) \equiv 0$

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ferromagnetic  \hspace{1cm} degenerate  \hspace{1cm} chiral phase  \hspace{1cm} 120^\circ \text{ structure}

<table>
<thead>
<tr>
<th>J/K</th>
<th>high density \hspace{1cm} 2D solid $^3$He \hspace{1cm} low density</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>up spins \hspace{1cm} uuud \hspace{1cm} uuud \hspace{1cm} uuud \hspace{1cm} uuud \hspace{1cm} uuud</td>
</tr>
<tr>
<td>-1</td>
<td>up spins \hspace{1cm} uuud \hspace{1cm} uuud \hspace{1cm} uuud \hspace{1cm} uuud \hspace{1cm} uuud</td>
</tr>
<tr>
<td>0</td>
<td>down spins \hspace{1cm} uudd \hspace{1cm} uudd \hspace{1cm} uudd \hspace{1cm} uudd \hspace{1cm} uudd</td>
</tr>
<tr>
<td>5</td>
<td>down spins \hspace{1cm} uudd \hspace{1cm} uudd \hspace{1cm} uudd \hspace{1cm} uudd \hspace{1cm} uudd</td>
</tr>
</tbody>
</table>

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valence bond crystal found in exact diagonalization for $S=1/2$, i.e. spin liquid state with large gap $\Delta \approx J$

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[Misguich et al., PRL 81, 1098 (1998)]
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ferromagnetic | degenerate \[ O(2L/2) \]

chiral phase | 120° structure

high density 2D solid \(^3\)He low density

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FM \(J_1 < 0\)
AF \(K > 0\)

experiment sees a gapless spin liquid bordering on the FM - could this be another nematic state?
who ordered that?

- going beyond nematic structure -

TRI N=36 (6,0,0,6) J1=-4.00 K=1
who ordered that?
- going beyond nematic structure -

TRI $N=36$ $(6,0,0,6)$ $J_1=-4.00$ $K=1$

saturated ferromagnet
**who ordered that?**

- going beyond nematic structure -

TRI $N=36$ (6,0,0,6) \( J_1=-4.00 \) \( K=1 \)

- saturated ferromagnet
- flat band of single-magnon excitations
who ordered that?
- going beyond nematic structure -

TRI $N=36$ (6,0,0,6)  $J_1=-4.00$  $K=1$

saturated ferromagnet
flat band of single-magnon excitations
two-magnon state
who ordered that?
- going beyond nematic structure -

TRI N=36 (6,0,0,6)  J1=-4.00  K=1

saturated ferromagnet
flat band of single-magnon excitations
two-magnon state
three-magnon state
who ordered that?

- going beyond nematic structure -

saturated ferromagnet

flat band of single-magnon excitations
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three-magnon state
who ordered that?  
- going beyond nematic structure -

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- structure is **period-3** not period-2 -
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- structure is period-3 not period-2 -

saturated ferromagnet
flat band of single-magnon excitations
two-magnon state
three-magnon state

ΔS=3

magnetization

magnetic field
another new quantum phase!
- three-spin bound states at high magnetic field -
another new quantum phase!
- three-spin bound states at high magnetic field -

“wiggle-on”

three spins propagate coherently under action of cyclic exchange
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\[ J_1 = -1, J_2 = 0 \]

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1-magnon instability
i.e. canted Néel state (exact)
another new quantum phase!
- three-spin bound states at high magnetic field -

three spins propagate coherently under action of cyclic exchange

"wiggle-on"

3-magnon instability
i.e. canted Néel state (exact)

2-magnon instability
i.e. nematic state (exact)
another new quantum phase!
- three-spin bound states at high magnetic field -

three spins propagate coherently under action of cyclic exchange

1-magnon instability
i.e. canted Néel state (exact)

3-magnon instability
i.e. "triatic" state (variational lower bound)

2-magnon instability
i.e. nematic state (exact)
symmetries of the triatic phase
- three-spin bound states at high magnetic field -
symmetries of the triatic phase
- three-spin bound states at high magnetic field -

order parameter is rank 3 tensor:

\[
\text{Re} \left\{ S_i^- S_j^- S_k^- \right\} = 2S_i^x S_j^x S_k^x - 2S_i^x S_j^y S_k^y - 2S_i^y S_j^x S_k^y - 2S_i^y S_j^y S_k^x
\]

\[
\text{Im} \left\{ S_i^- S_j^- S_k^- \right\} = 2S_i^x S_j^x S_k^y + 2S_i^x S_j^y S_k^x + 2S_i^y S_j^x S_k^x - 2S_i^y S_j^y S_k^y
\]
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\]

matrix elements of order parameter are linear combinations of fully symmetrized spin operators on triangular plaquette
symmetries of the triatic phase
- three-spin bound states at high magnetic field -

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\[
\begin{align*}
\text{Re} \{ S_i^- S_j^- S_k^- \} &= 2 S_i^x S_j^x S_k^x - 2 S_i^x S_j^y S_k^y - 2 S_i^y S_j^x S_k^y - 2 S_i^y S_j^y S_k^x \\
\text{Im} \{ S_i^- S_j^- S_k^- \} &= 2 S_i^x S_j^x S_k^y + 2 S_i^x S_j^y S_k^x + 2 S_i^y S_j^x S_k^x - 2 S_i^y S_j^y S_k^y
\end{align*}
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in applied magnetic field, order parameter is planar and maps onto itself under rotations through $2\pi/3$
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\]

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in applied magnetic field, order parameter is planar and maps onto itself under rotations through $2\pi/3$

FM octopolar order naively has $k^2$ dispersion

\[ \Rightarrow cV \propto T \text{ in } 2D \]

c.f HeIII on graphite
conclusions

- new quantum phases for all the family -
conclusions
- new quantum phases for all the family -

new \textit{nematic} phase in square lattice
frustrated ferromagnets
(c.f. quasi-2D vanadates)

Shannon, Momoi + Sindzingre,
PRL 2006

Shannon et al., EPJB 2004
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new **triatic** phase in
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frustrated ferromagnets
(c.f. He III)

Momoi, Sindzingre + Shannon,
in preparation

Momoi + Shannon, PTP 2005
conclusions
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(c.f. He III)
Momoi, Sindzingre + Shannon,
in preparation
Momoi + Shannon, PTP 2005

frustrated ferromagnets are fun and there's lots still to learn
that’s all folks...
multiple spin exchange on the triangular lattice

\[ \mathcal{H} = J \sum_{\langle ij \rangle} P_{ij} + K \sum_{\langle ijk \rangle} [P_{ijk} + P_{ikj}] \]

Classical limit S = \( \infty \)  
Quantum limit S = 1/2

Gapped spin liquid

FM
Disordered
Scalar Chiral
NAF

Kagome like ?

Gapless
RVB ?

[Montunich]

He III

...Momoi et al.

...Lhullier et al.

P_{ij} = \left[ \frac{1 + \vec{\sigma}_i \cdot \vec{\sigma}_j}{2} \right] 
= 2\vec{S}_i \cdot \vec{S}_j + \ldots

P_{ijkl} + P_{ijlk}^{-1} = \vec{S}_i \cdot \vec{S}_j + \ldots + 4 \left( \vec{S}_i \cdot \vec{S}_j \right) \left( \vec{S}_k \cdot \vec{S}_l \right) + \ldots