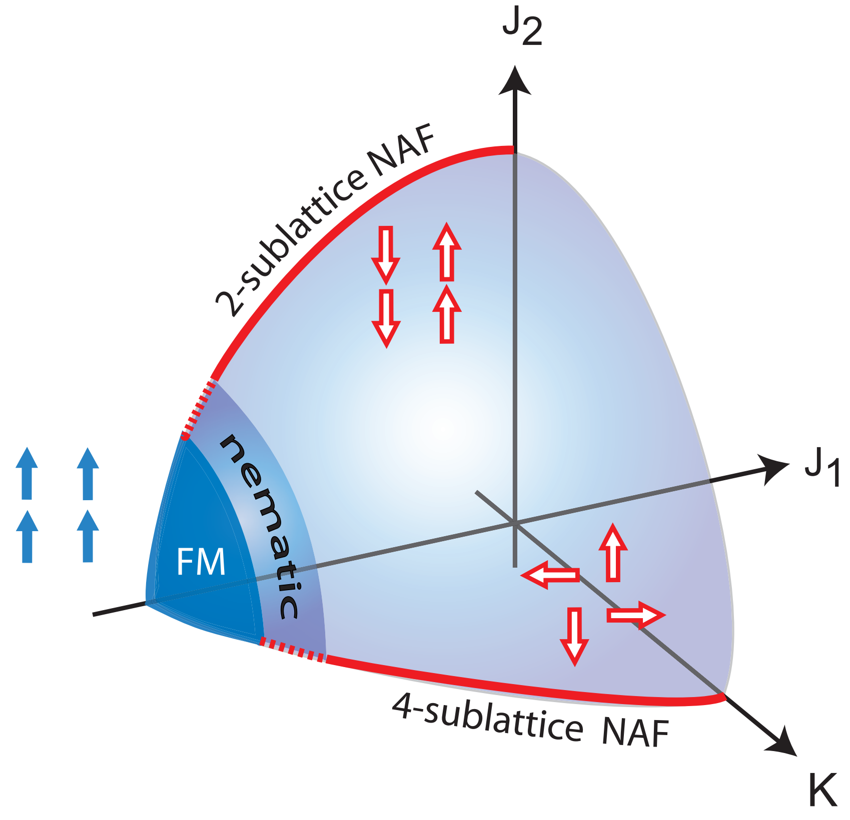
(How to have fun with) **two-dimensional frustrated ferromagnets**



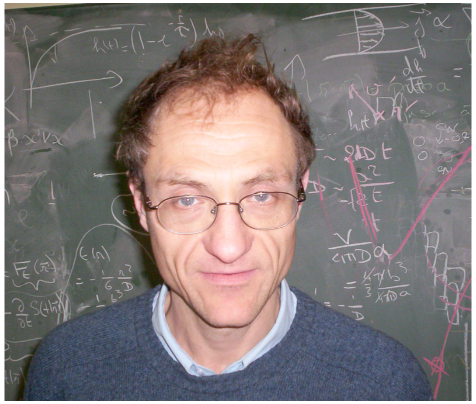
nic shannon
(Bristol, UK)

Les Houches 20/6/6

thanks to...

Tsutomu Momoi

RIKEN



Philippe Sindzingre

Paris VI, Jussieu

MAX-PLANCK-INSTITUT FÜR CHEMISCHE PHYSIK FESTER STOFFE

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RIKEN

JST Japan Science and Technology Agency

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Institut du Développement et des Ressources en Informatique Scientifique

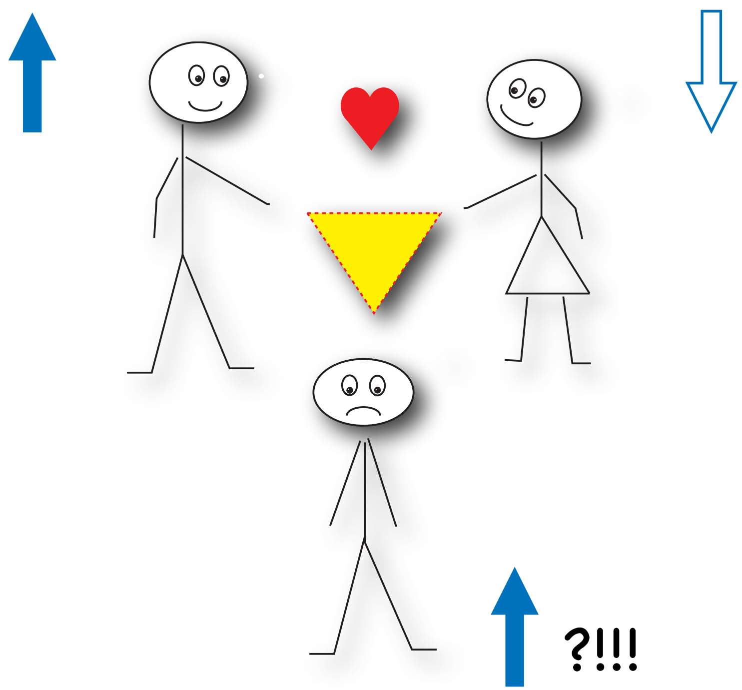
spin-1/2 on a triangular lattice

- spin liquids and the RVB idea -

spin-1/2 on a triangular lattice

- spin liquids and the RVB idea -

the eternal triangle



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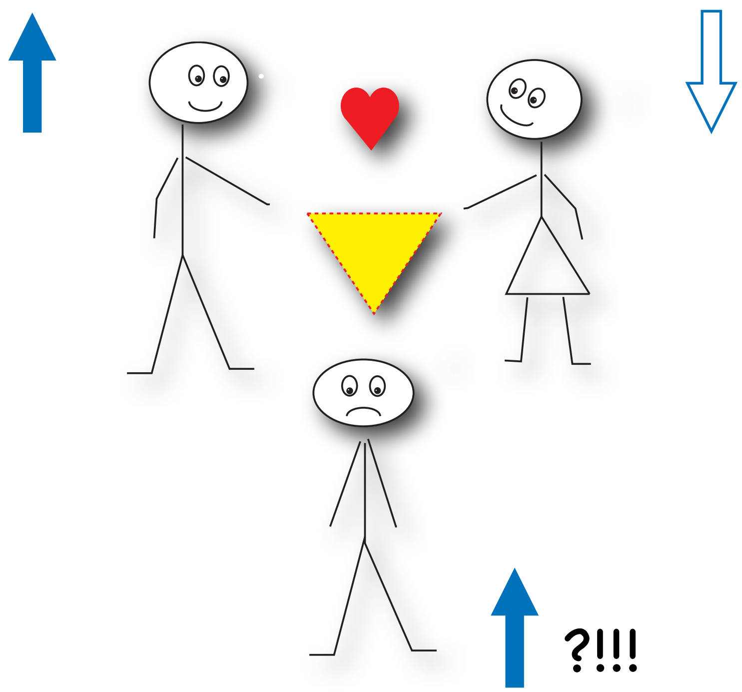

frustration, i.e. you can't please all of the spins, all of the time

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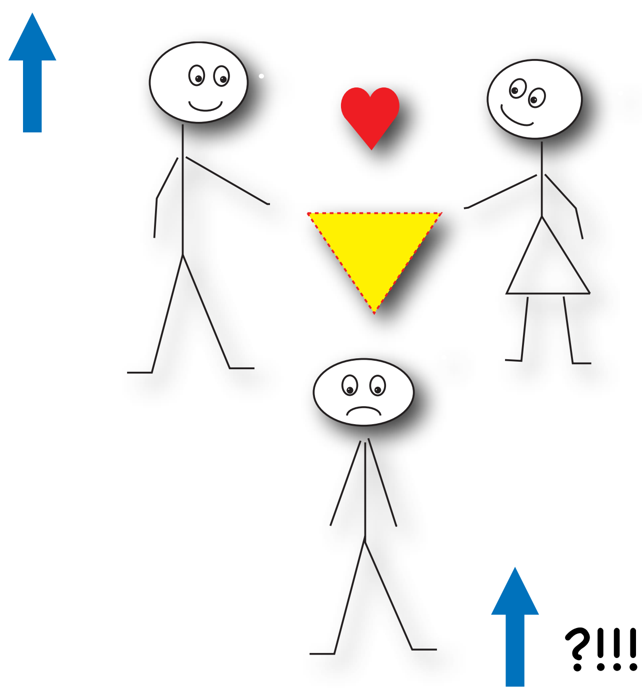

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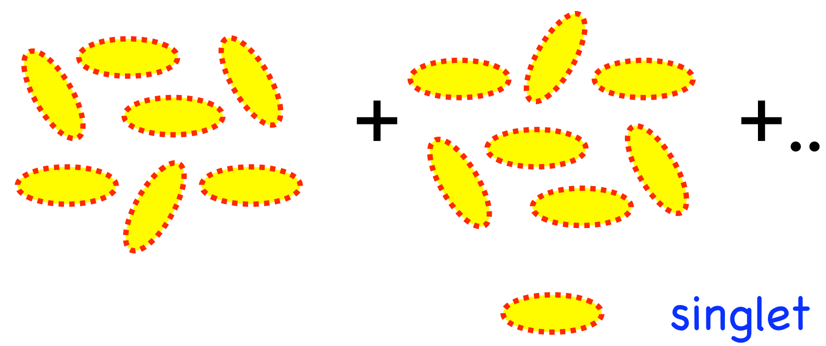
Anderson's resonating valance bond (**RVB**) state



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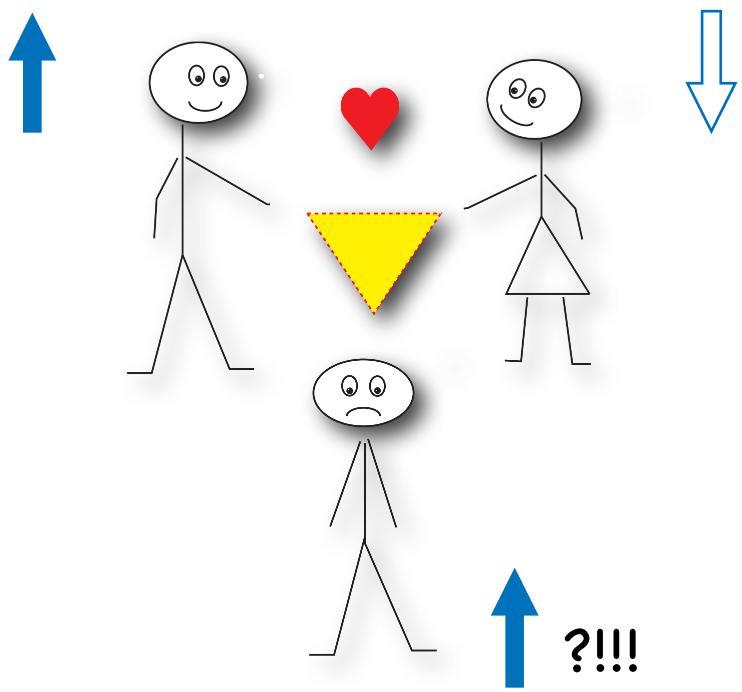

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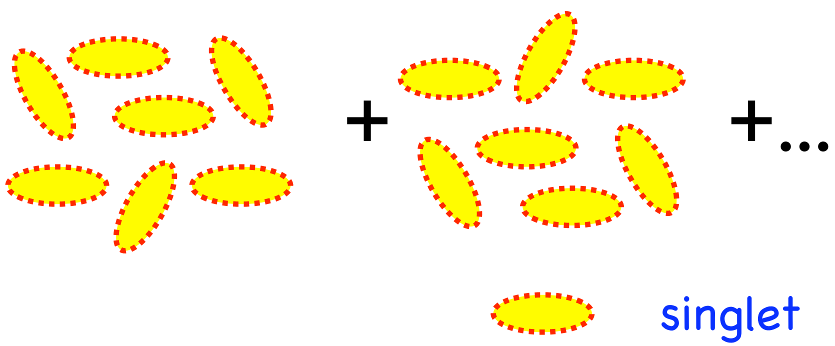
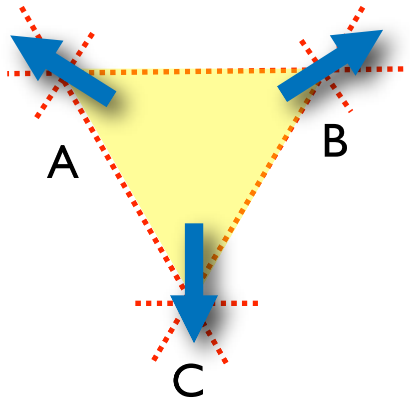


actual ground state of **Heisenberg model** on triangular lattice



3-sublattice Néel state

spin-1/2 on a triangular lattice

- spin liquids and the RVB idea -

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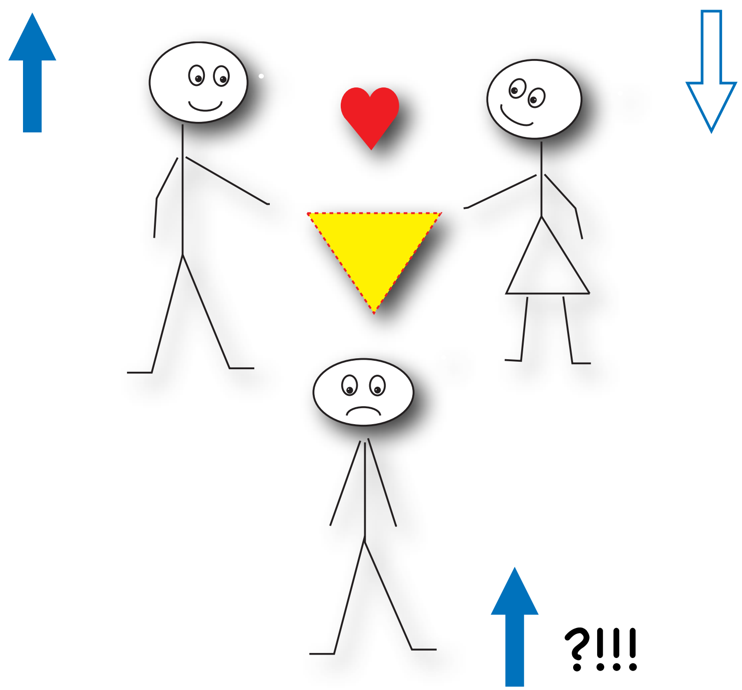

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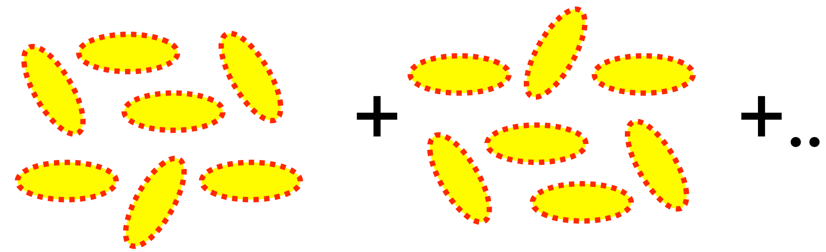

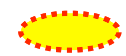 singlet

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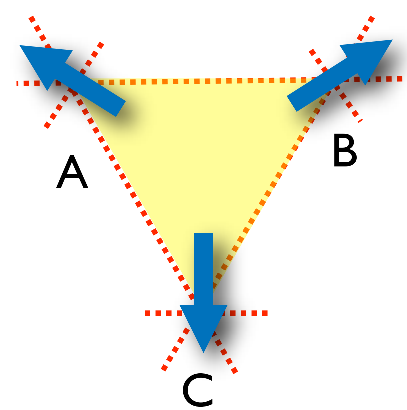

3-sublattice Néel state

for a review, see e.g. **Misguich and Lhullier** in "Quantum Spin Systems" (2004 Diep)

in the beginning, God created He III...

the most perfect correlated Fermi system known to man

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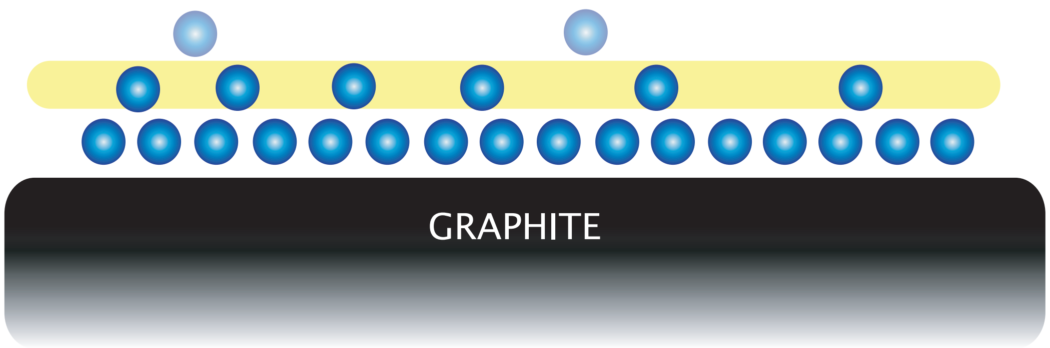
2D incarnation - He III on graphite



3rd layer - ignore

2nd layer - 2D FL/magnet

1st layer - paramagnetic solid

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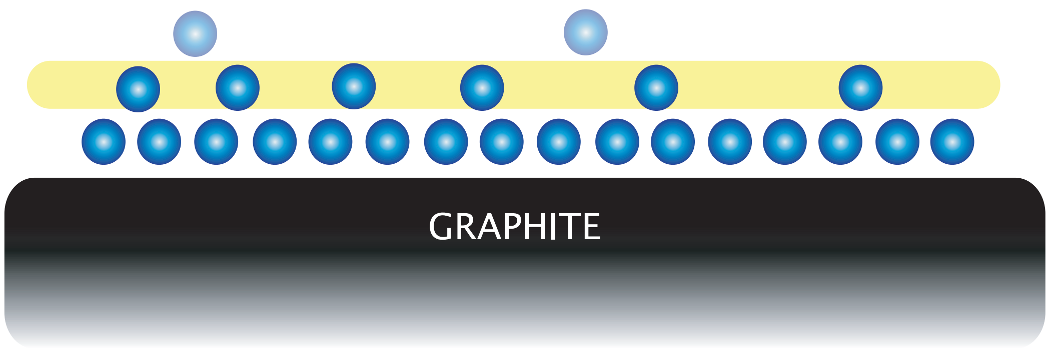

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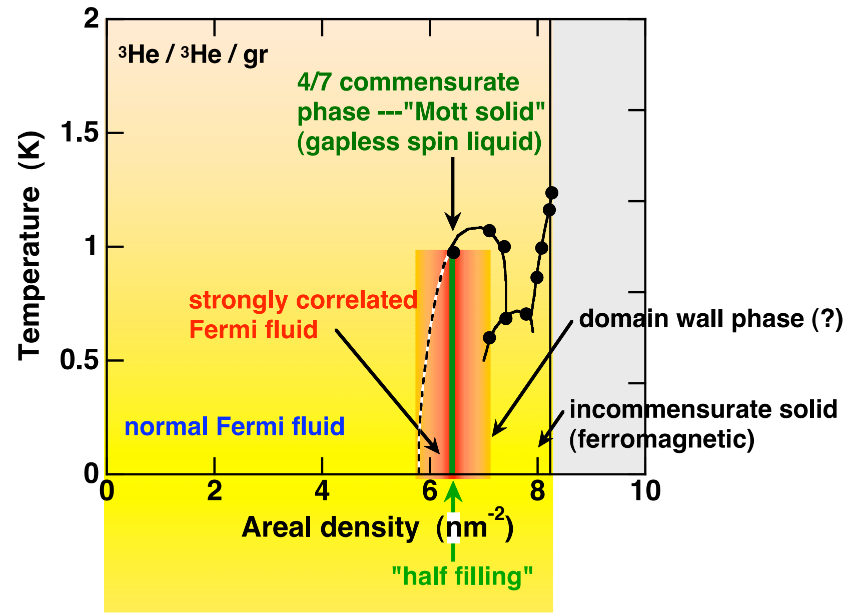
Fermi liquid in second layer becomes magnetic solid with increasing density :

high density solid is FM

something very special happens at low densities...



the first true spin liquid...

...in a 2D triangular lattice frustrated FM

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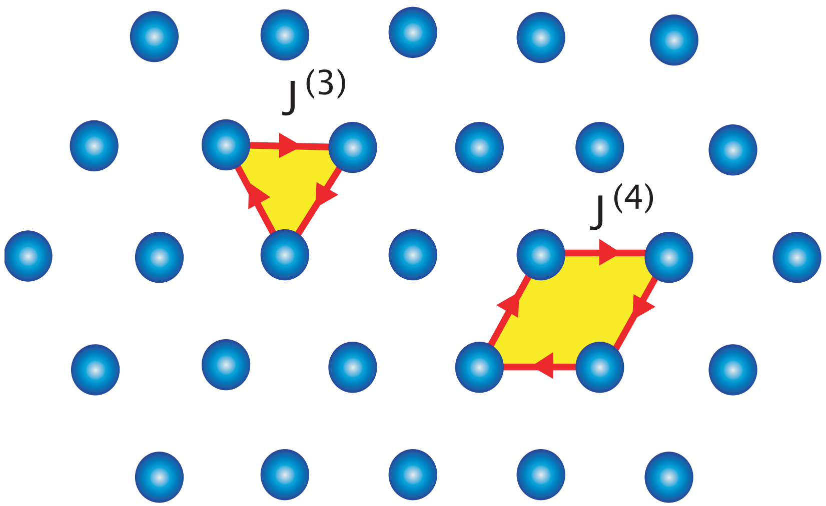

2nd layer magnetism controlled
by competition between
FM 3-spin exchange
and AF 4-spin exchange

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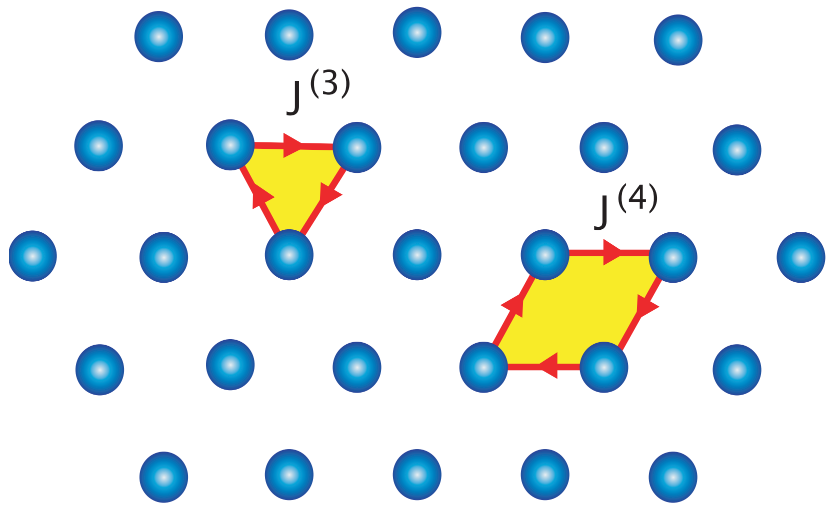

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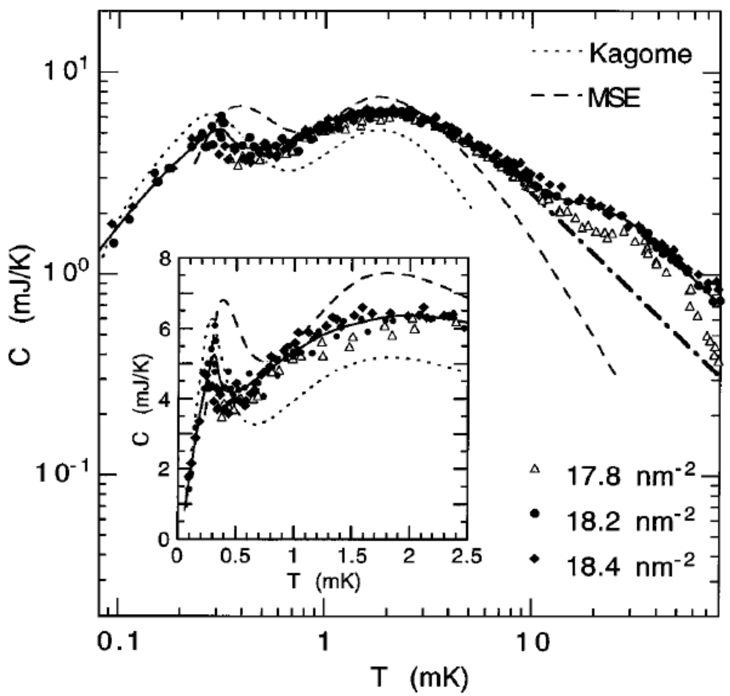

no magnetic order down to 0.1 mK !!!

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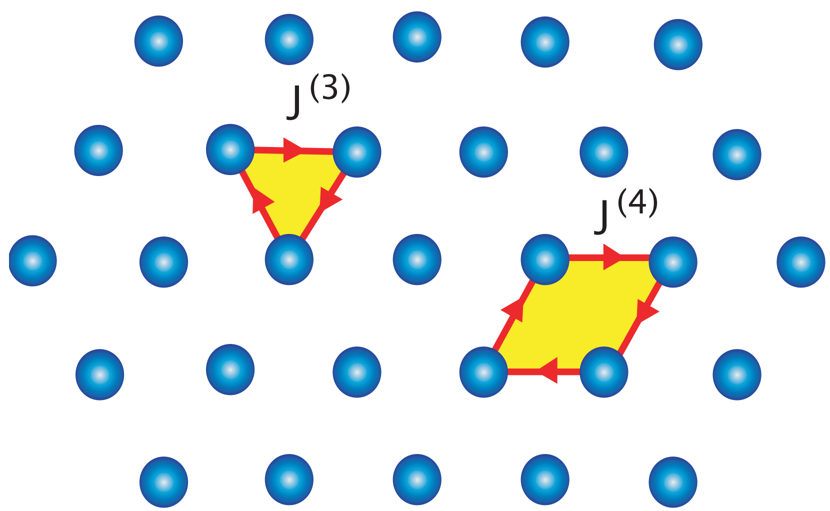

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linear specific heat
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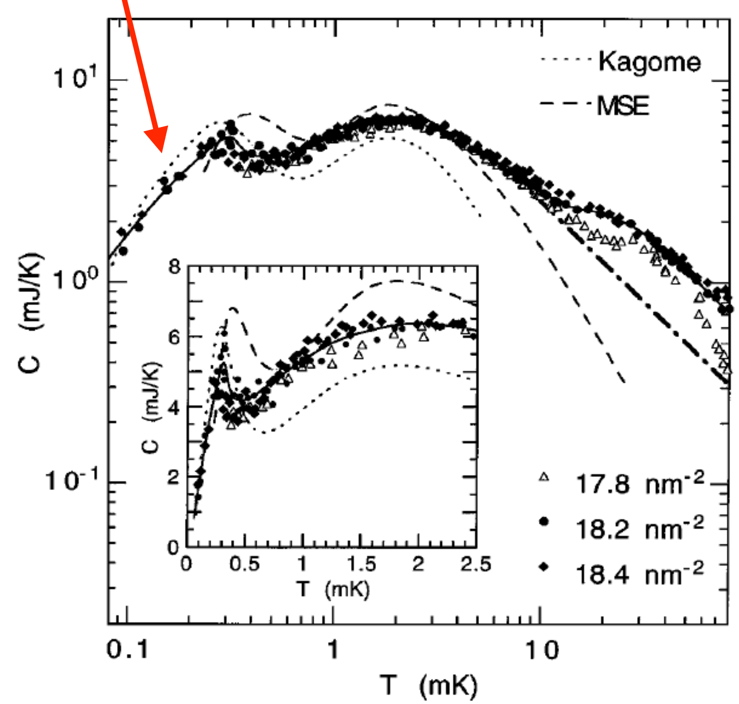

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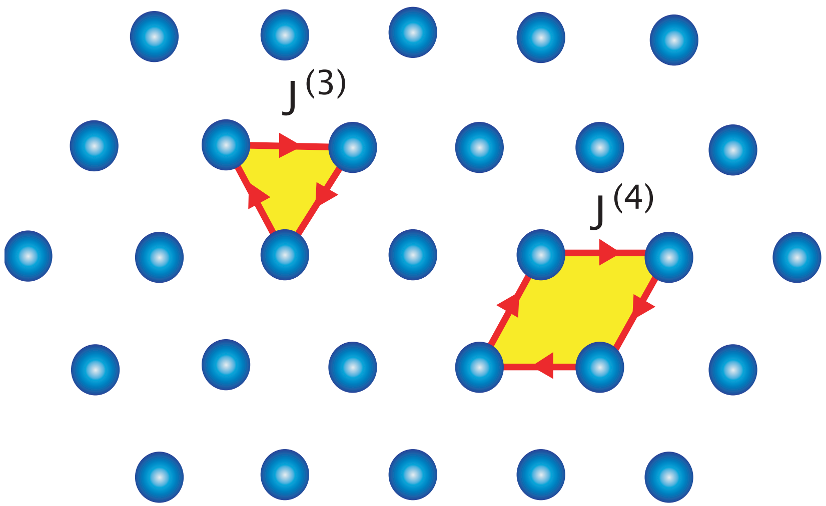

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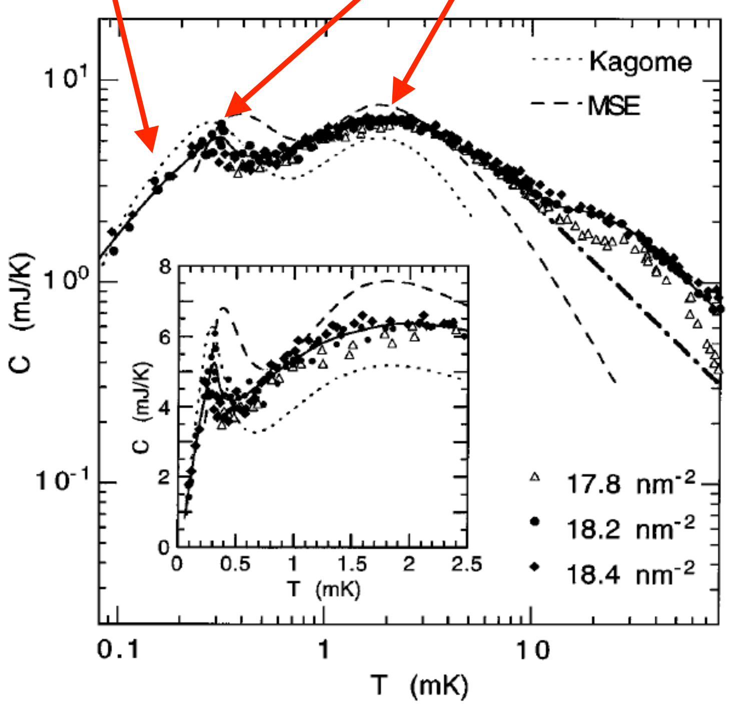
double peak structure



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$Pb_2VO(PO_4)_2$

first example of a square lattice frustrated FM

E. Kaul *et al.*, JMMM **272-276 (II)**, 922 (2004)

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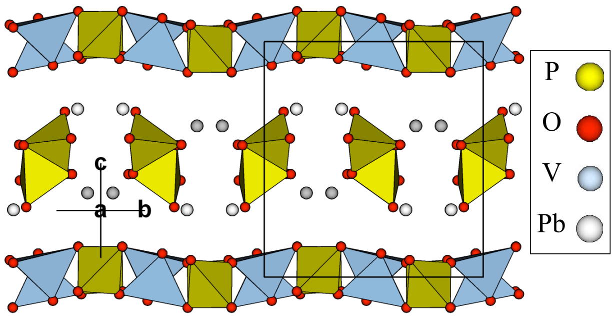
$Pb_2VO(PO_4)_2$: Structure



spin-1/2 V^{4+} in layered pyramids

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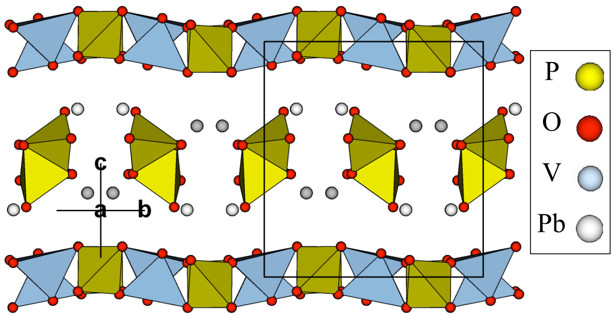

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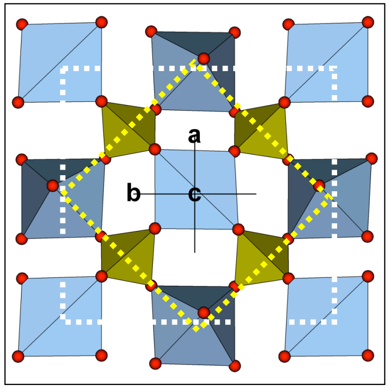

two different exchange paths
- both n.n. and n.n.n. bonds -

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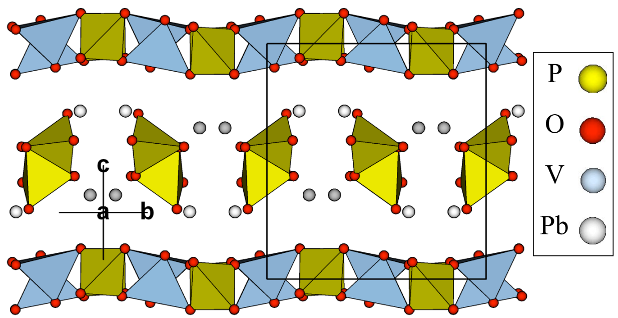

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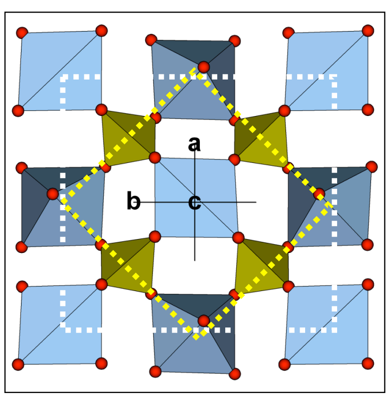

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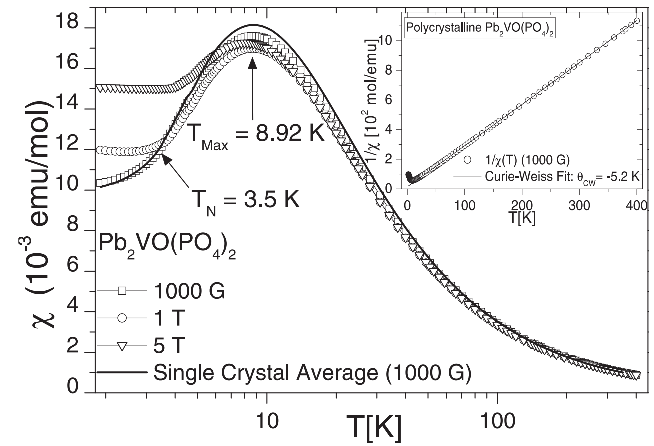

linear χ-inverse ⇒ frustrated magnet

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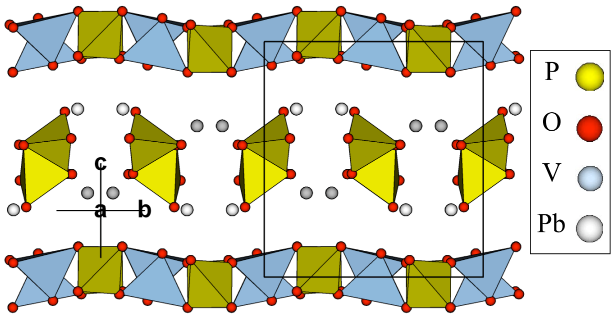

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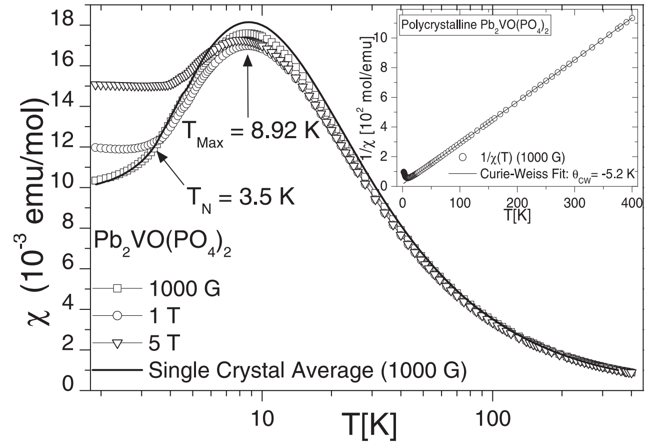
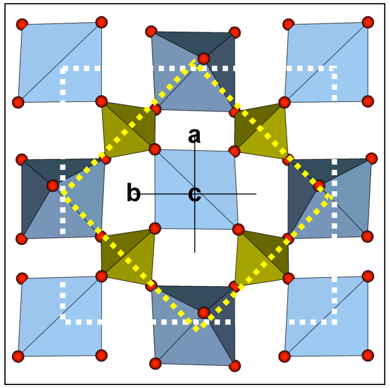


linear χ-inverse ⇒ frustrated magnet

$J2/J1 \approx -2$



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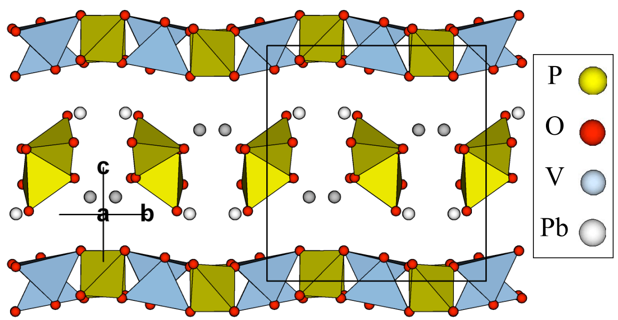

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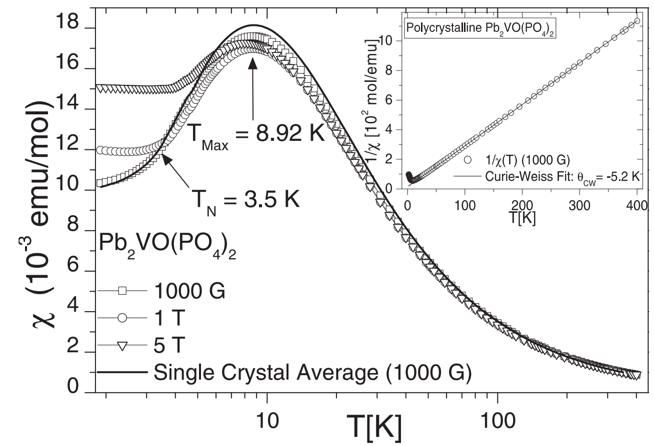

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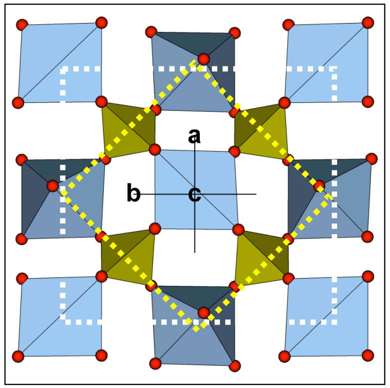


J2/J1 ≈ -2

two different exchange paths

- both n.n. and n.n.n. bonds -

ground state

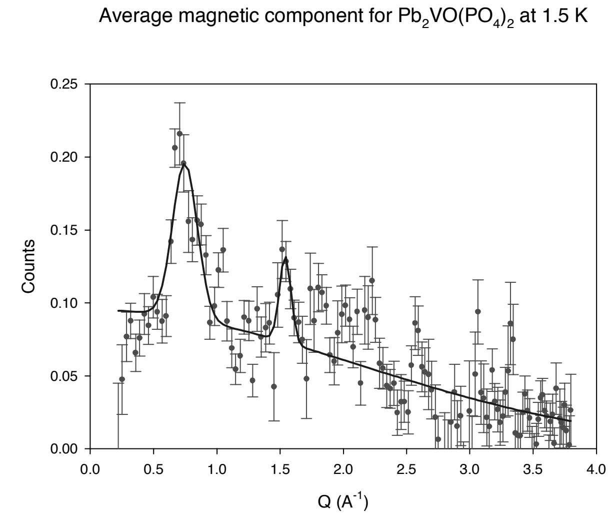
is (π,0)

collinear AF

with

reduced

moment



the “simplest” frustrated ferromagnet

extended FM Heisenberg model on square lattice

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extended FM Heisenberg model on square lattice

$$\mathcal{H} = 2J_1 \sum_{\langle ij \rangle_1} \mathbf{S}_i \mathbf{S}_j + 2J_2 \sum_{\langle ij \rangle_2} \mathbf{S}_i \mathbf{S}_j + K \sum_{\langle 1234 \rangle} P_{1234} + P_{1234}^{-1}$$

the "simplest" frustrated ferromagnet

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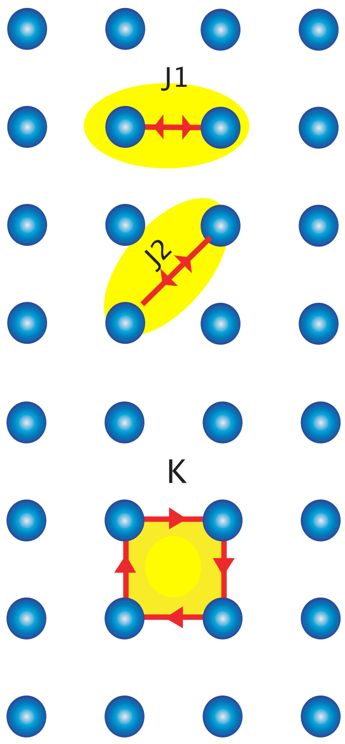

FM n.n. interaction
J1 < 0

AF n.n.n. interaction
J2 > 0

AF 4-spin cyclic exchange
K > 0

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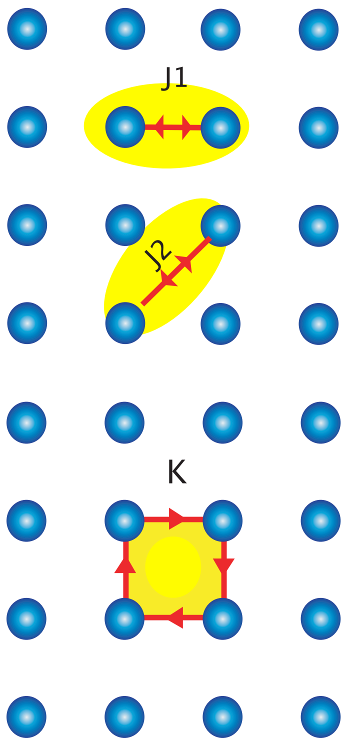

FM n.n. interaction
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K > 0

N.B. $P_{ijkl} + P_{ijkl}^{-1} = \vec{S}_i \cdot \vec{S}_j + \ldots + 4\left(\vec{S}_i \cdot \vec{S}_j\right)\left(\vec{S}_k \cdot \vec{S}_l\right) + \ldots$

the "simplest" frustrated ferromagnet

extended FM Heisenberg model on square lattice

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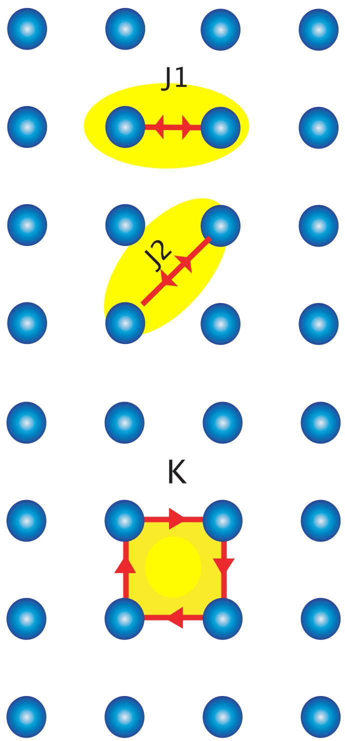

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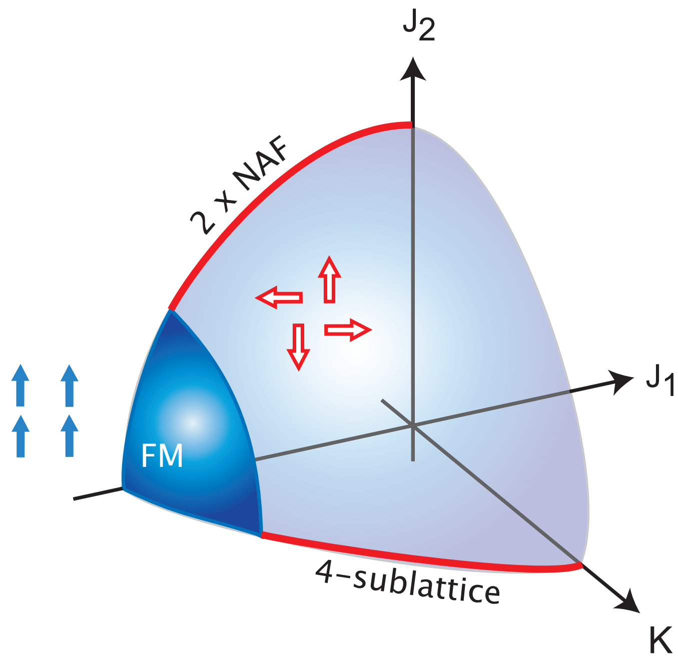

mean field phase diagram

N.B. $$P_{ijkl} + P_{ijkl}^{-1} \;=\; \vec{S}_i \cdot \vec{S}_j + \ldots + 4\left(\vec{S}_i \cdot \vec{S}_j\right)\left(\vec{S}_k \cdot \vec{S}_l\right) + \ldots$$

quantum critical points

- and ferromagnetism -

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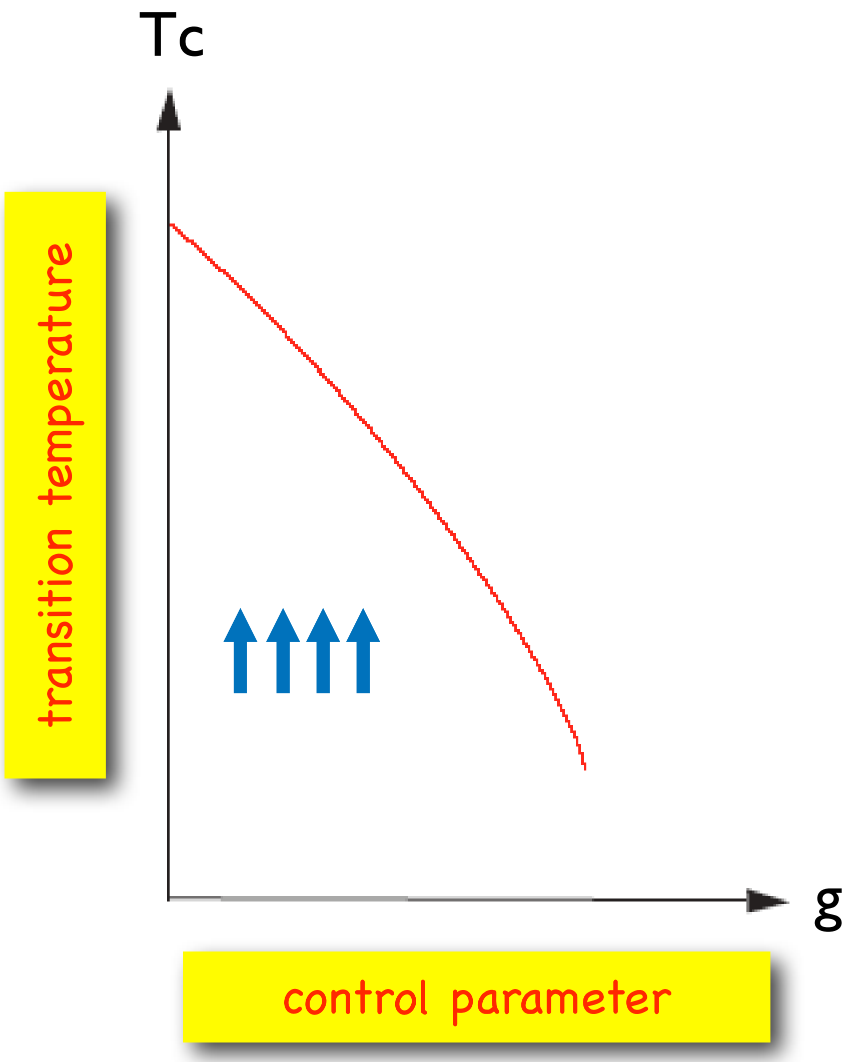

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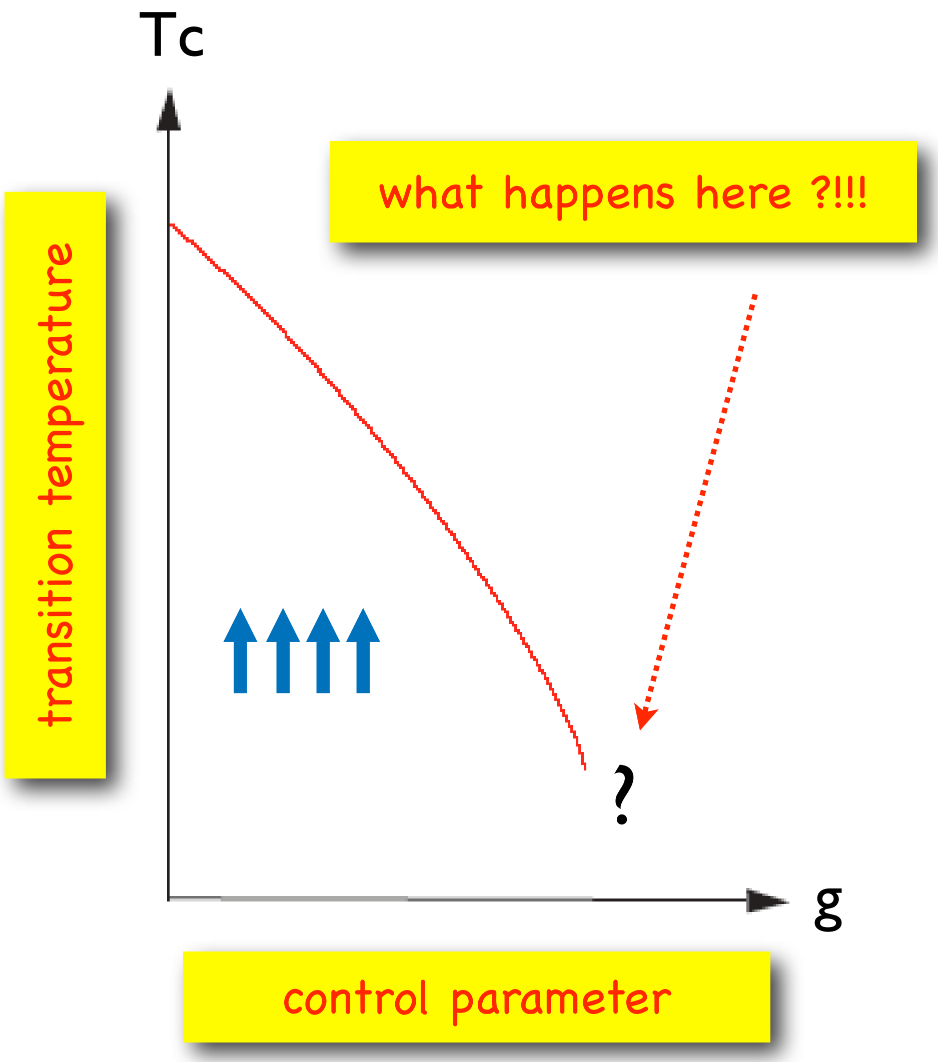

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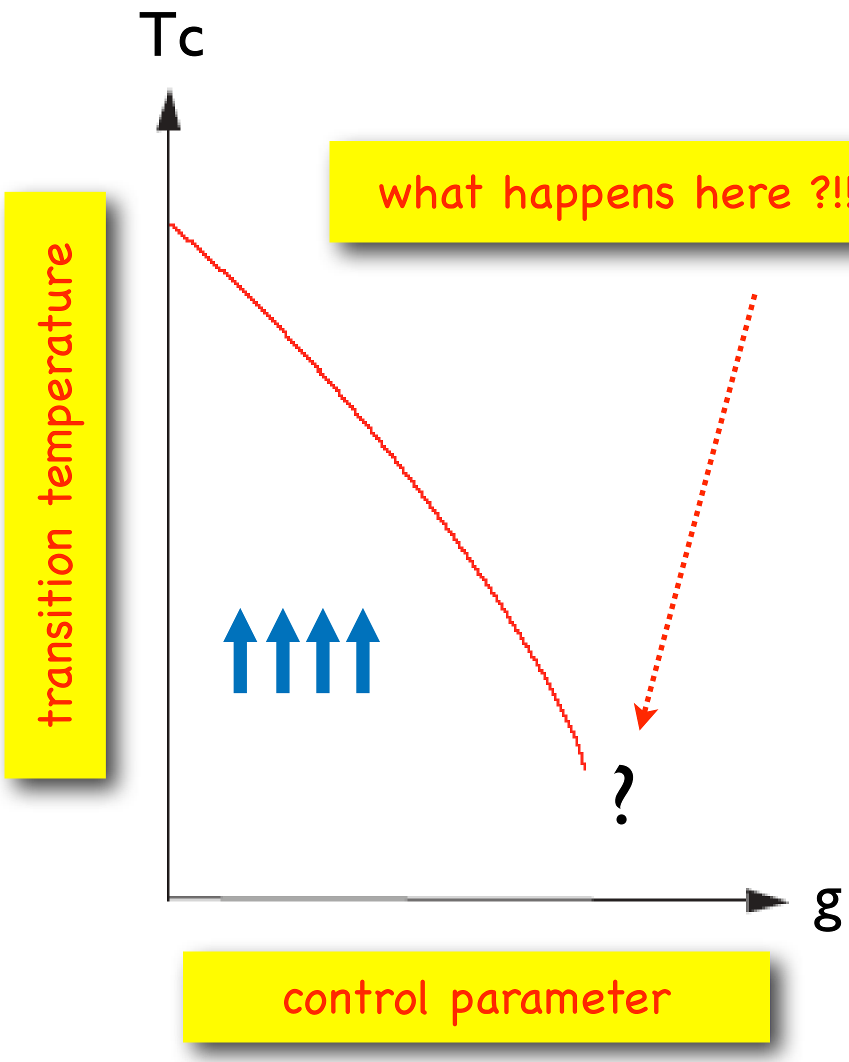

manganites

(g=doping)

- phase separation

quantum critical points

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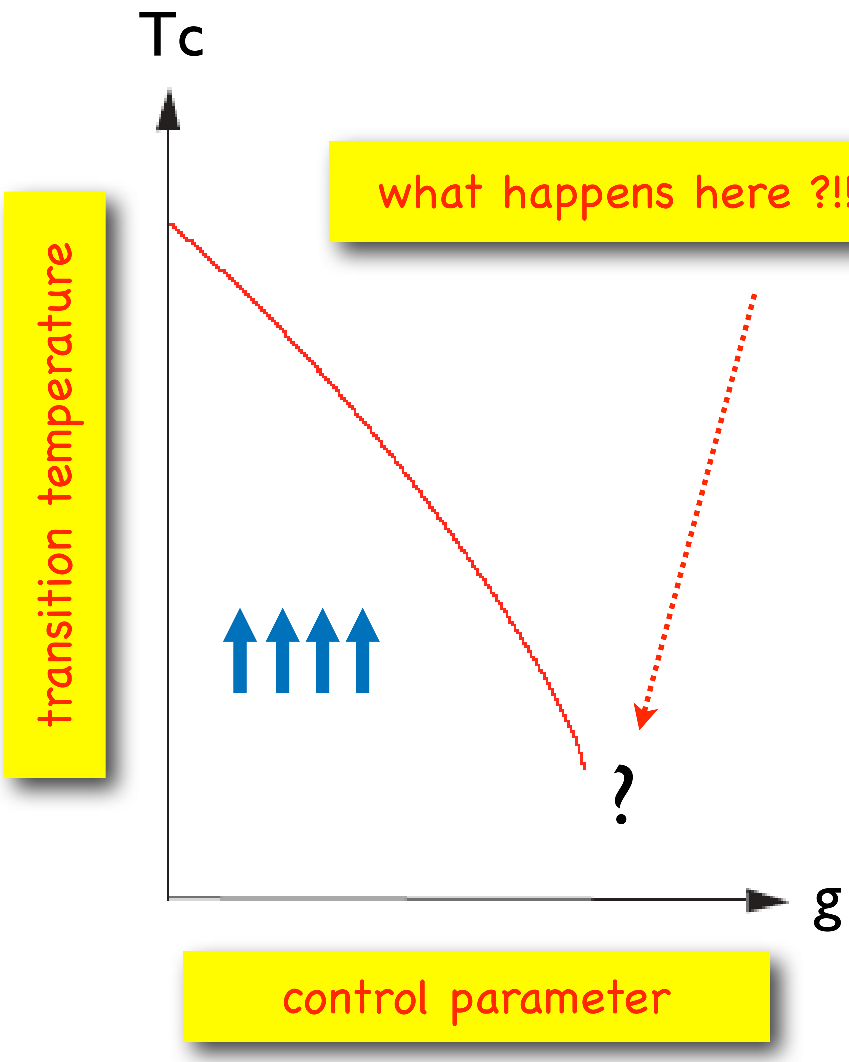

manganites
(g=doping)

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weak itinerant ferromagnets
(g=pressure)

- superconductivty

quantum critical points

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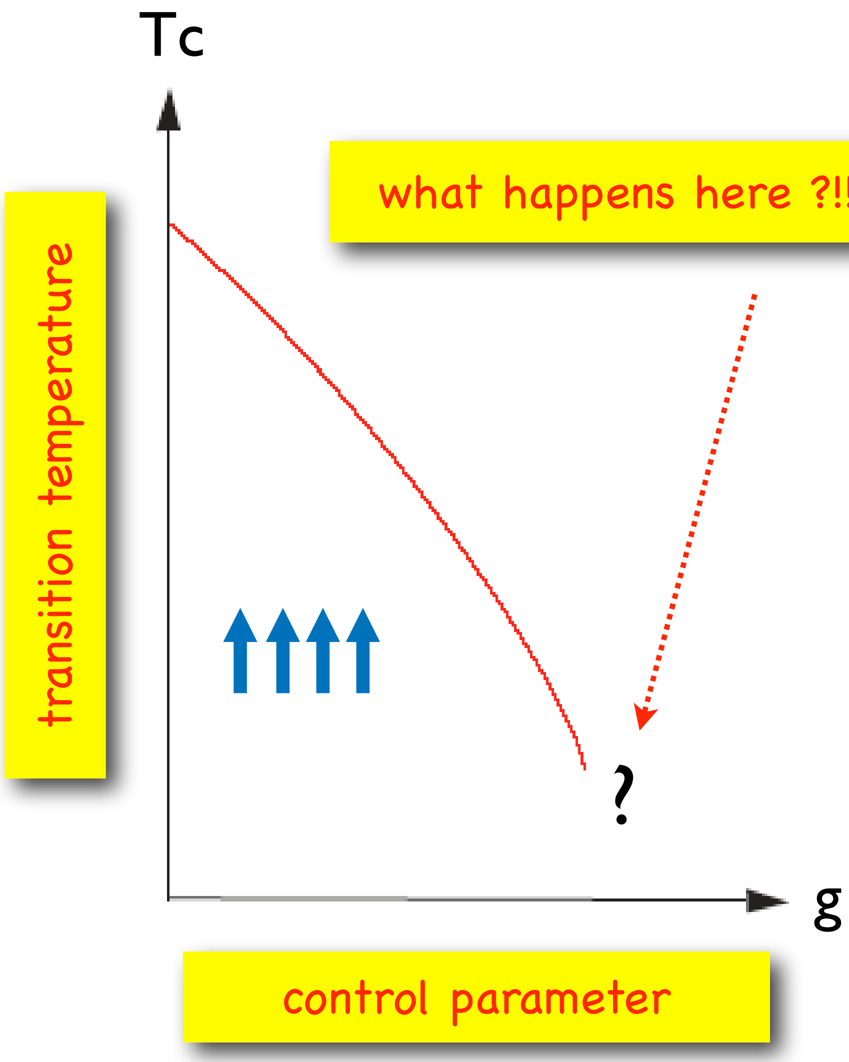

manganites
(g=doping)

- phase separation

weak itinerant ferromagnets
(g=pressure)

- superconductivty

frustrated quantum spin systems
(g=density, chemical pressure)

- spin liquid ?!!!

how does the FM die ?

- nature of spin excitations at boundary with AF -

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- nature of spin excitations at boundary with AF -

"one magnon" dispersion :

$$\omega(\mathbf{q}) \quad = \quad 8(|J_1| - 2K - J_2) - 4(|J_1| - 2K)[\cos q_x + \cos q_y] + 8J_2 \cos q_x \cos q_y$$

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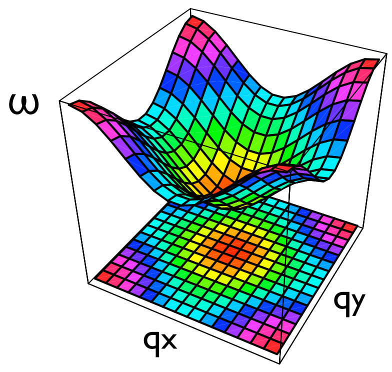

J=-1, J2=K=0

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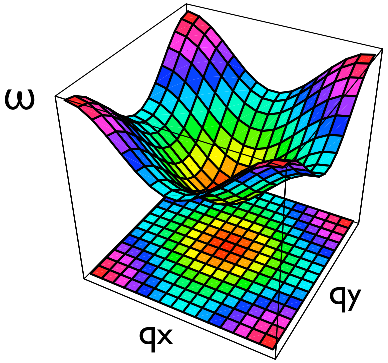

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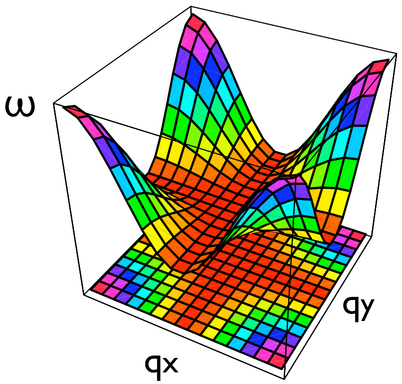

J=-1, J2=1/2, K=0

how does the FM die ?

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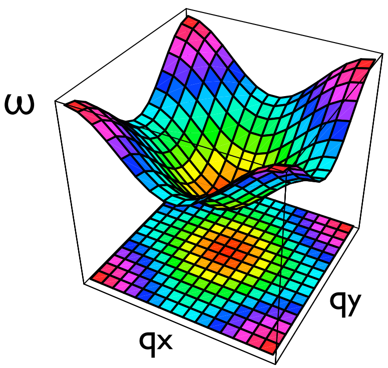
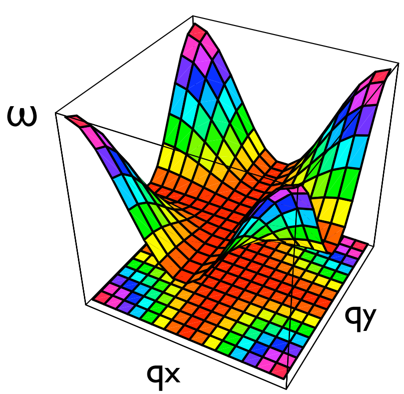


J=-1, J2=K=0



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limiting case #1 :

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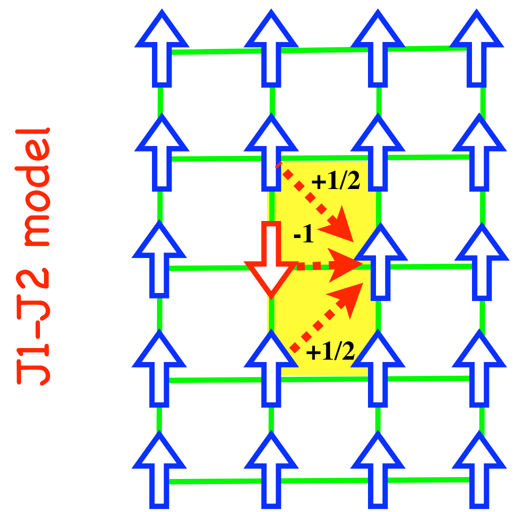

line zeros for qx = 0, qy = 0

how does the FM die ?

- nature of spin excitations at boundary with AF -

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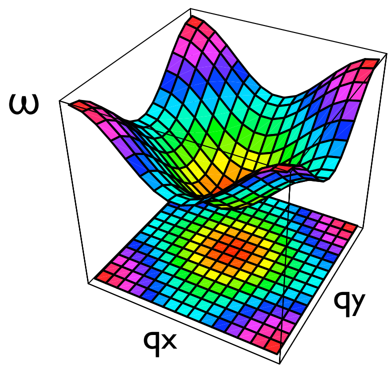

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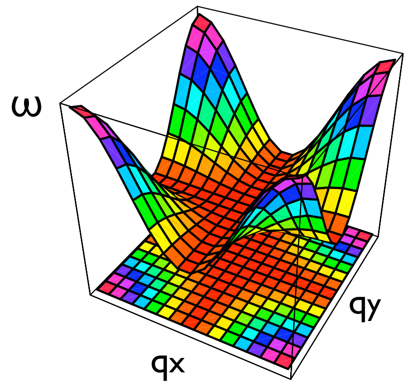

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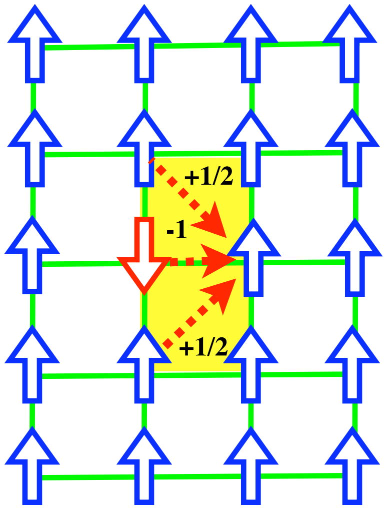
J1-J2 model



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limiting case #2 :

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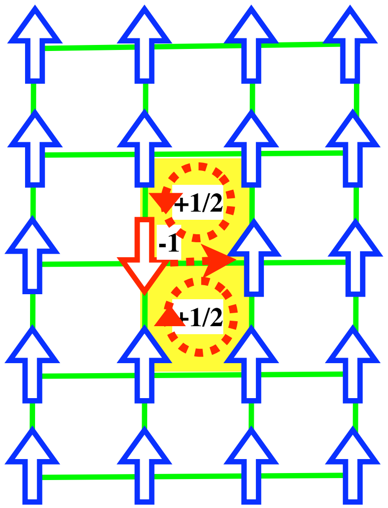
square lattice MSE model



entire dispersion vanishes !!!

what kind of excitation works ?

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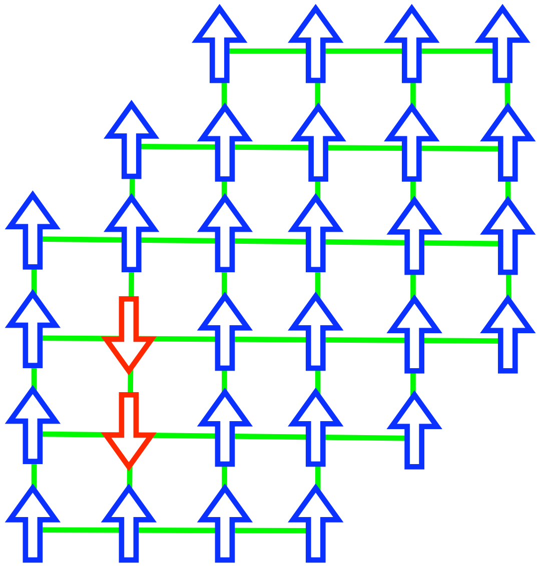

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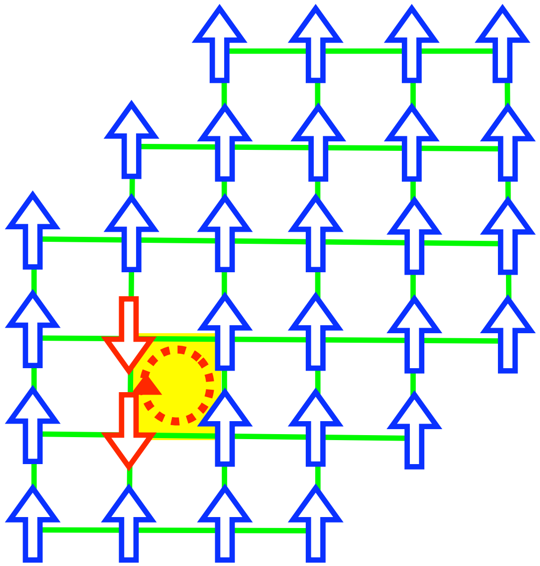

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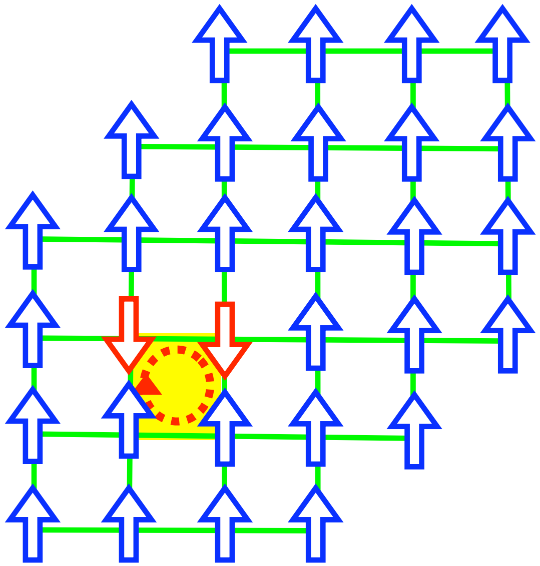

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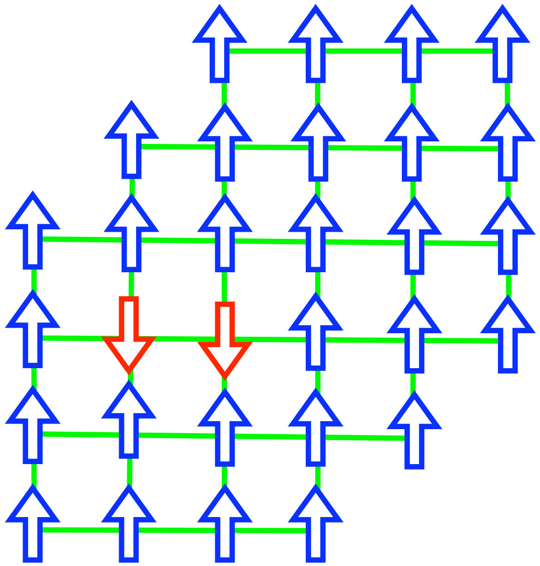

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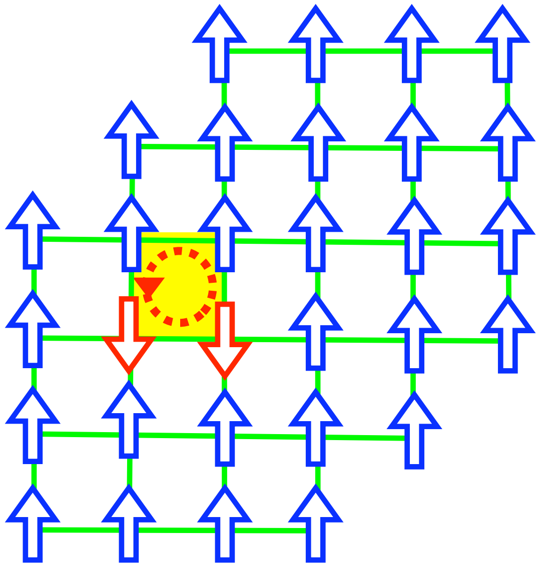

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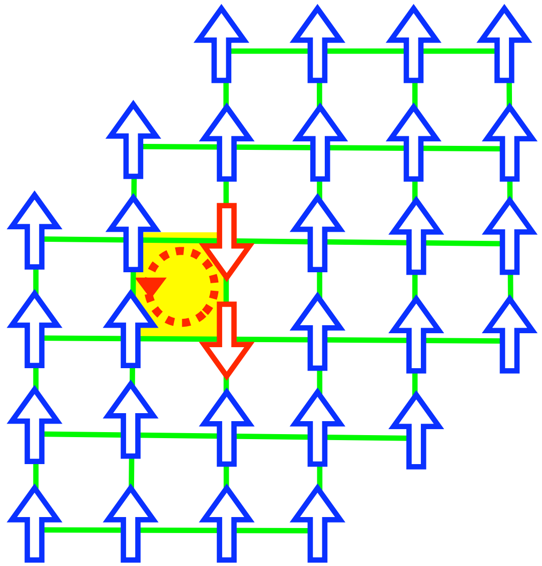

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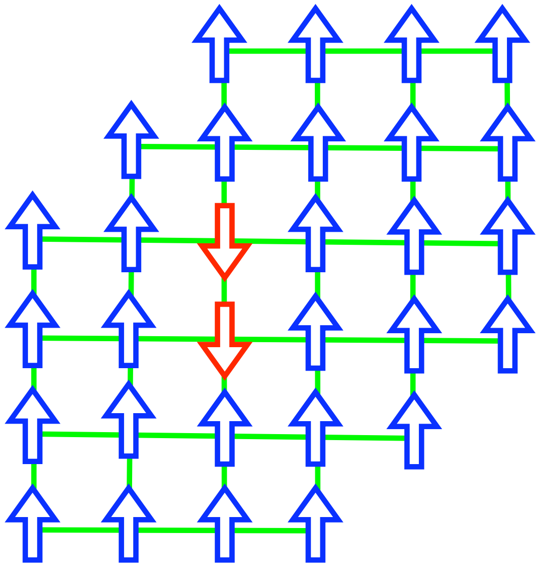

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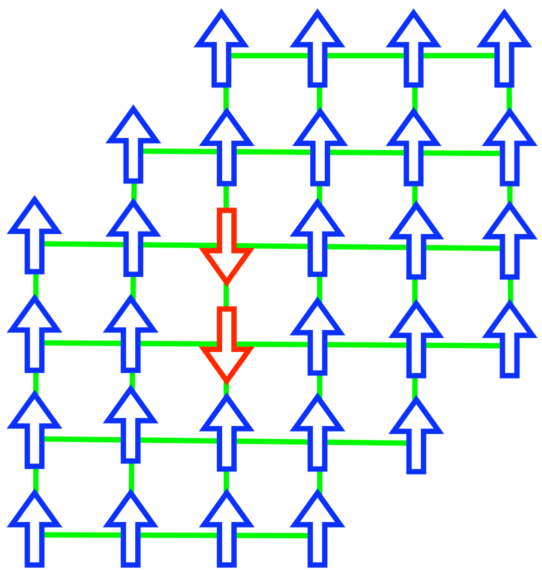

individual magnons are localized

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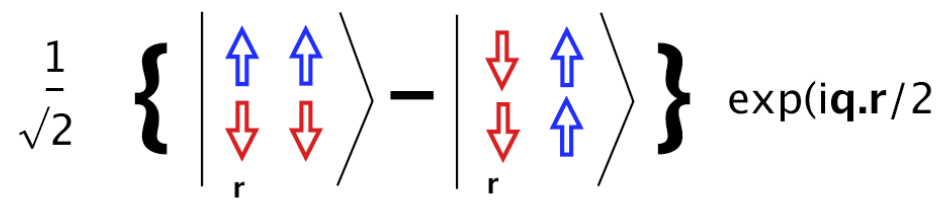
simple trial wave function for two-magnon bound state :



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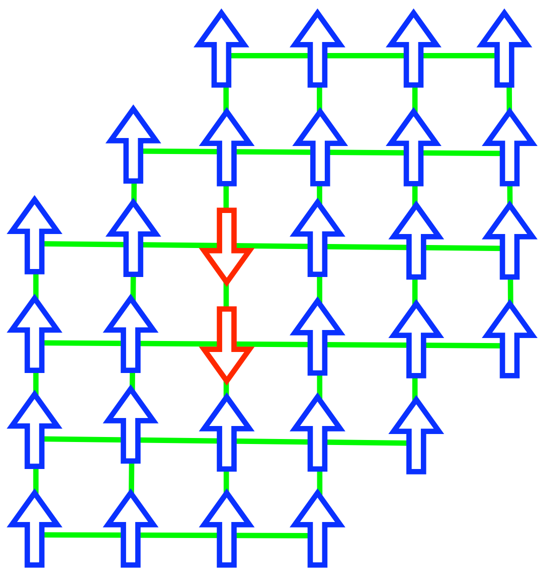

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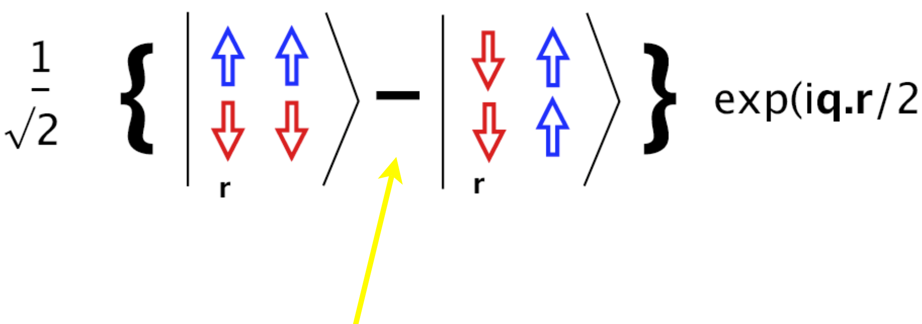

d-wave symmetry

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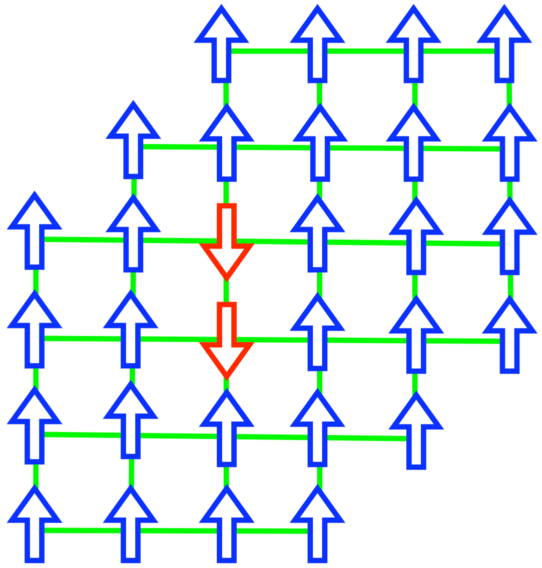

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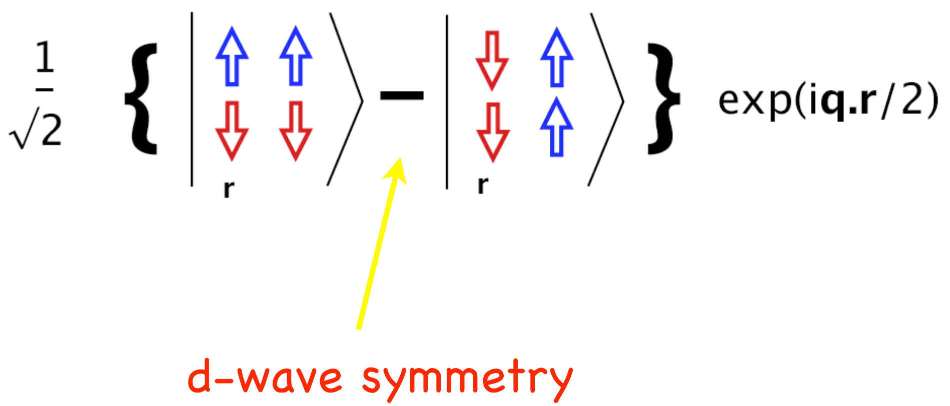

for special point

J1=-1, J2 = 0, K=1/2

this wave function is an

exact eigenstate

so what is the first instability of the FM ?

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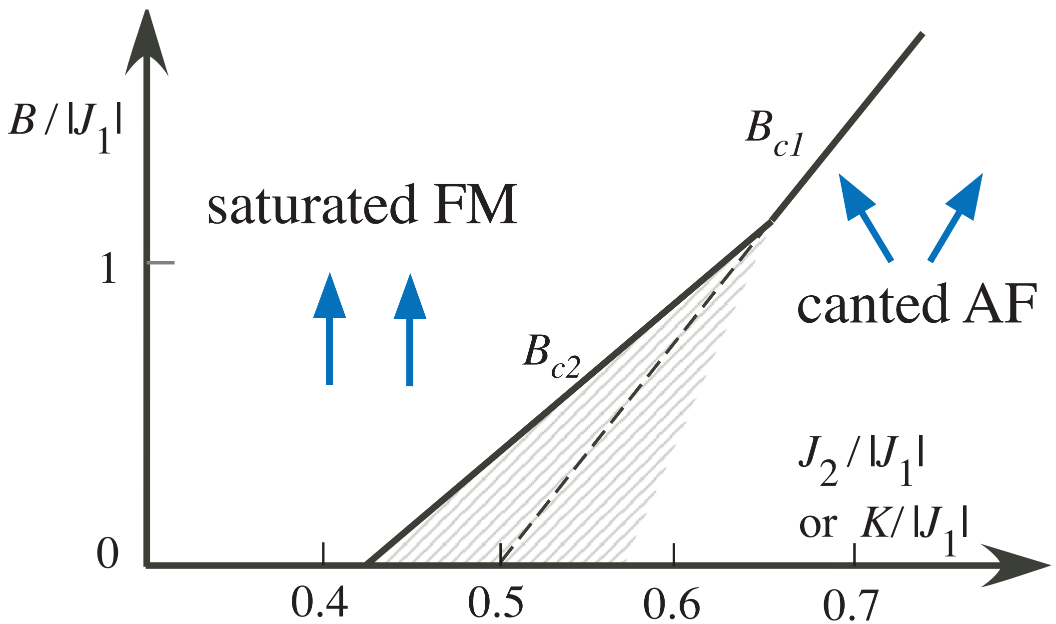

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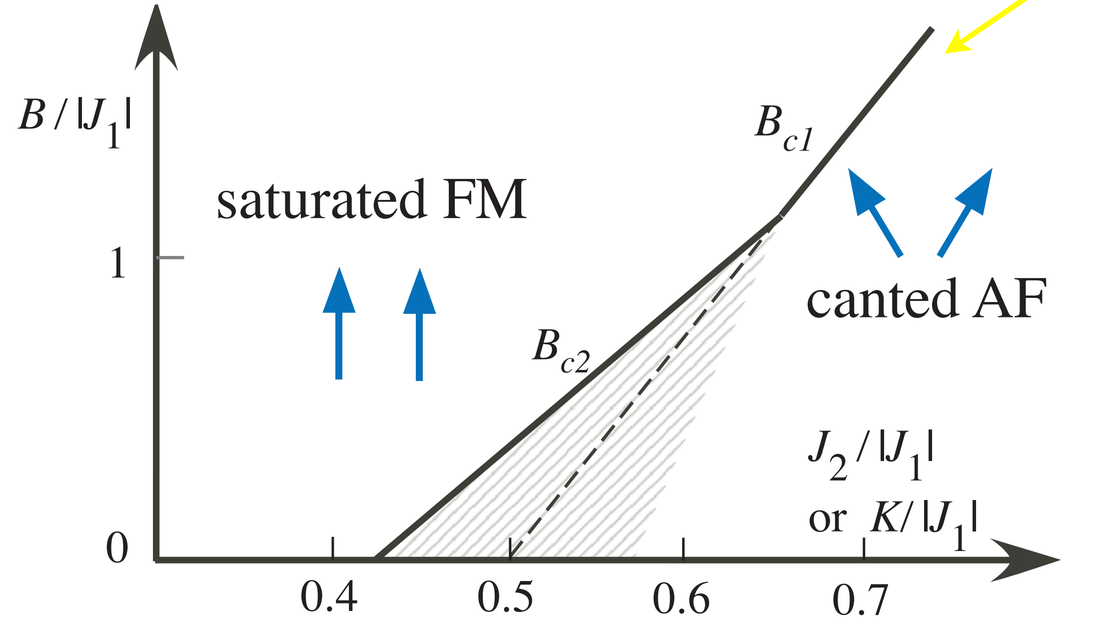

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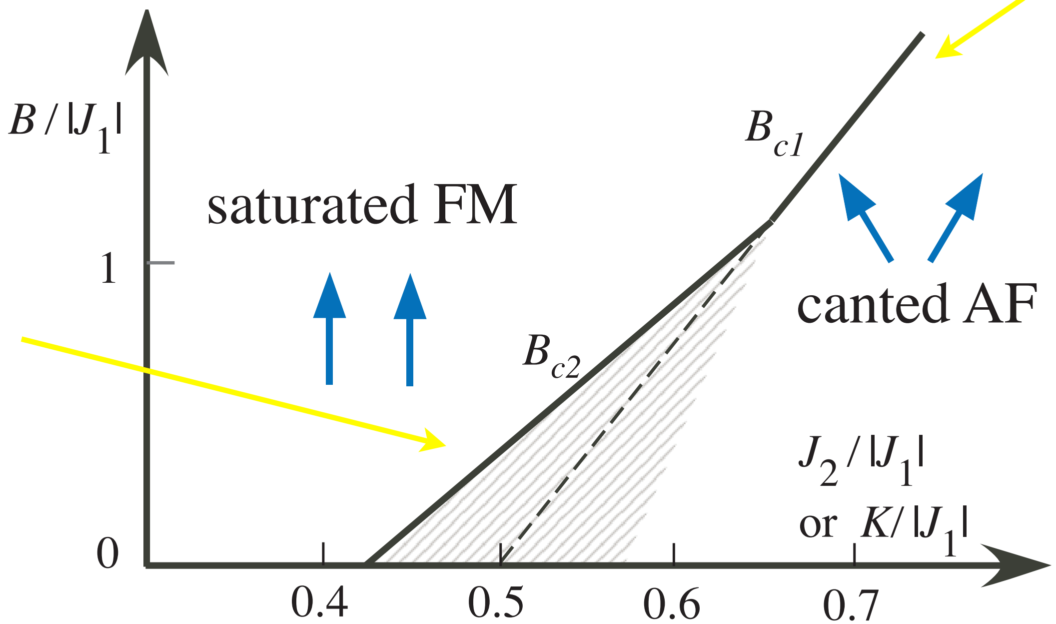

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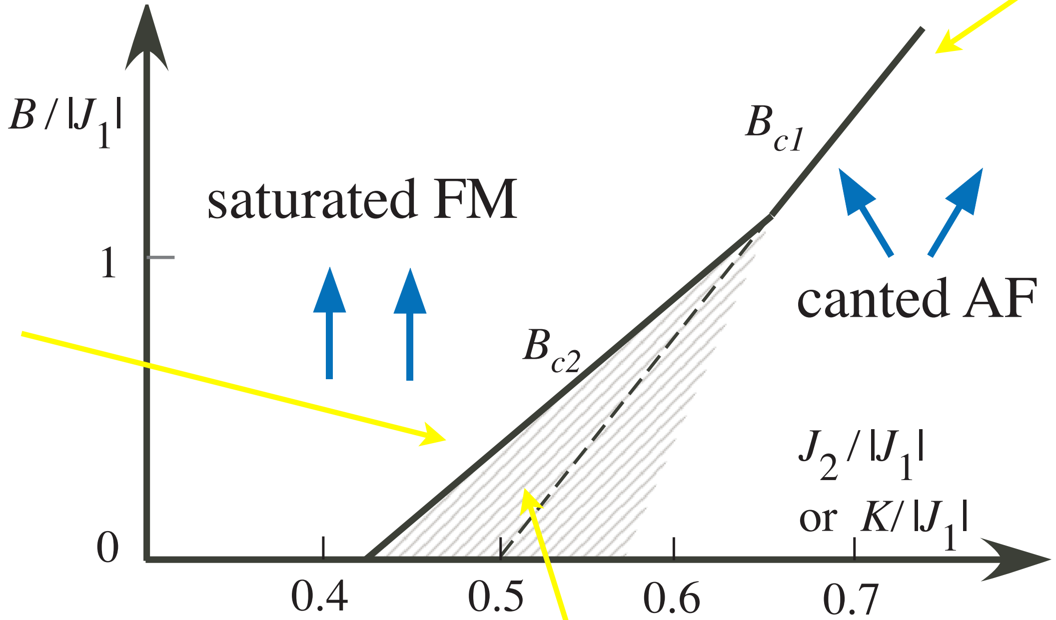

what is the nature of this phase ?!!!

a new idea - nematic order

- systems that don't know up from down -

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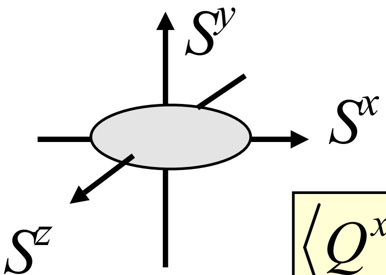
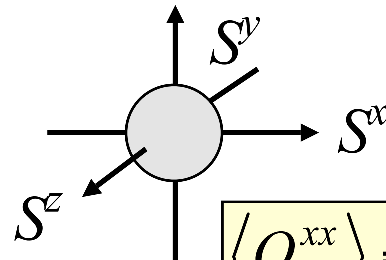
nematic (quadropolar) order :



$$\langle Q^{xx} \rangle > \langle Q^{yy} \rangle = \langle Q^{zz} \rangle$$



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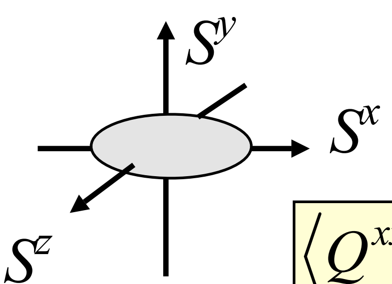

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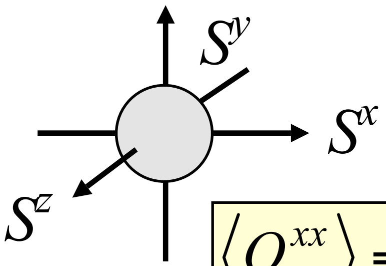

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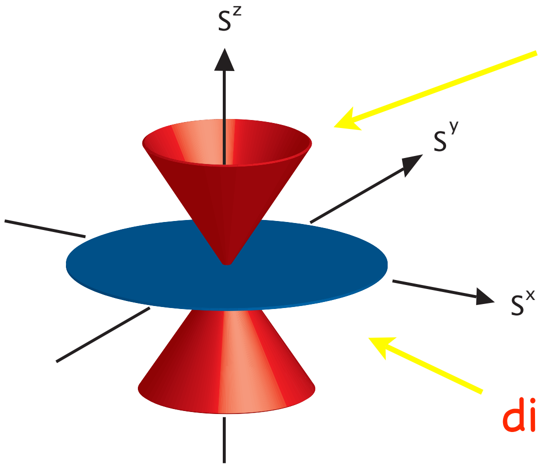
site-wise nematic works for spin-1 :



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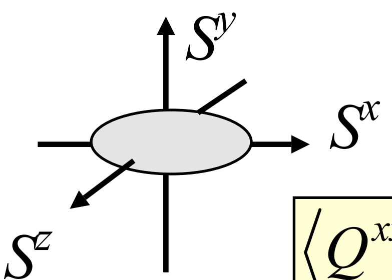

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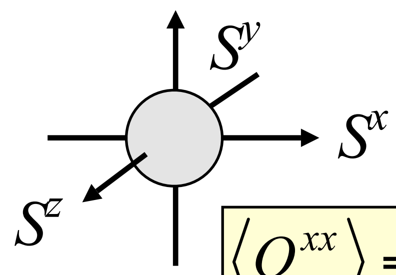

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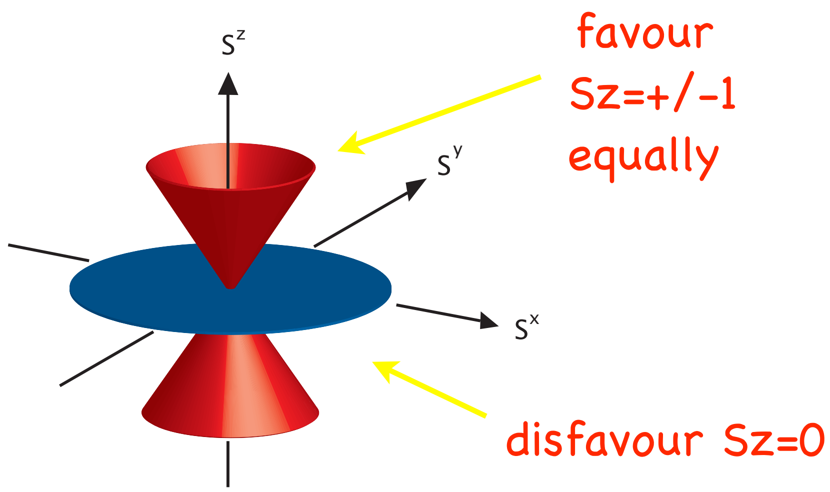

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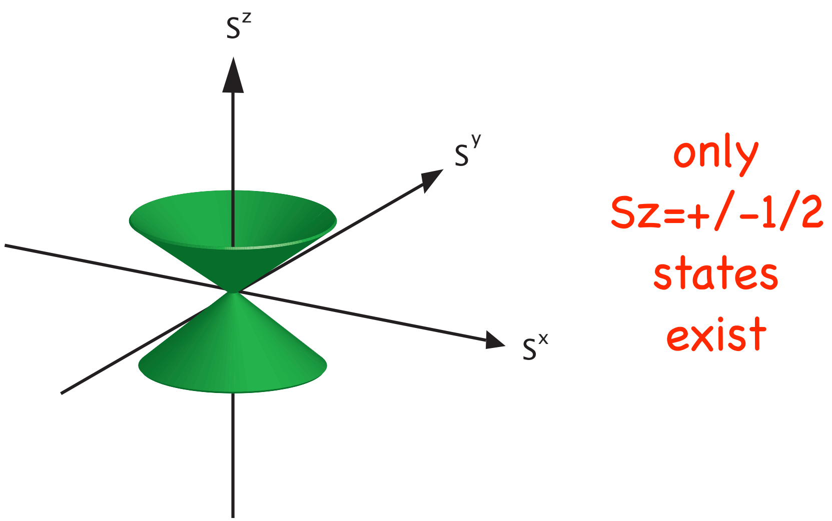

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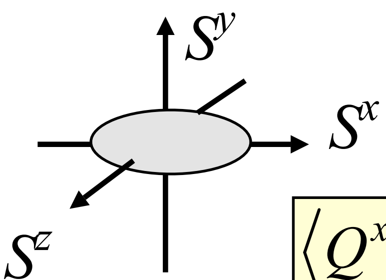

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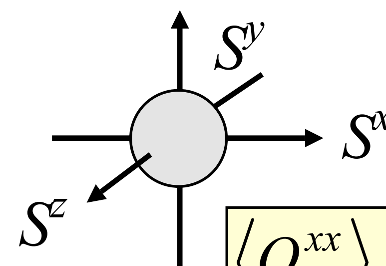

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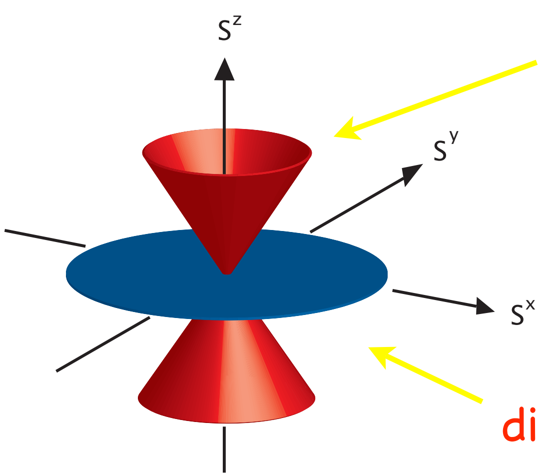

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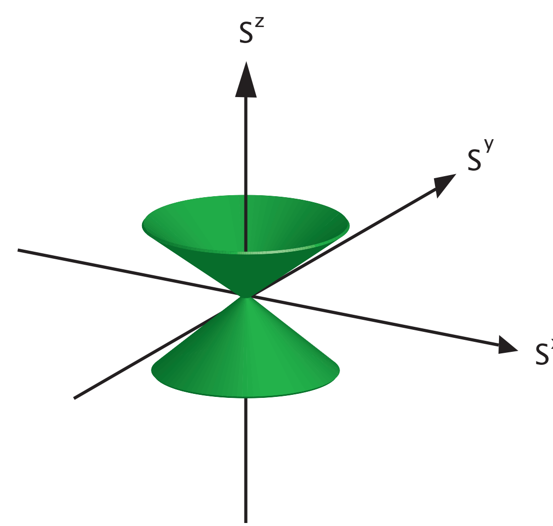

only Sz=+/-1/2 states exist

for a spin-1 example see, e.g. : K. Harada and N. Kawashima, PRB **65**, 052403(2002)

so what...?

- what do nematics and spin-1/2 FM's have in common ? -

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*c.f. 1D example : A. V. Chubukov, Phys. Rev. B. **44**, 4693 (1991)

can we see the nematic in numerics ?

- two-magnon instability in applied field -

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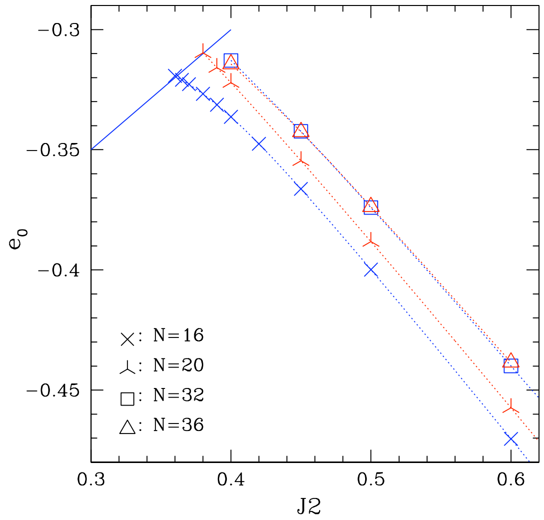
first establish extent of FM



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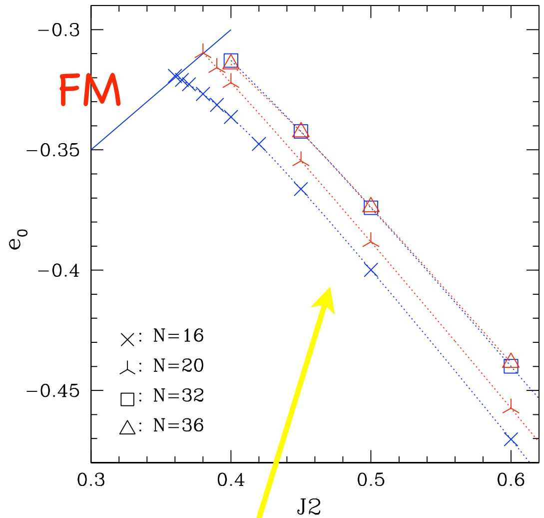

ground state energy
(S=0) of finite size cluster

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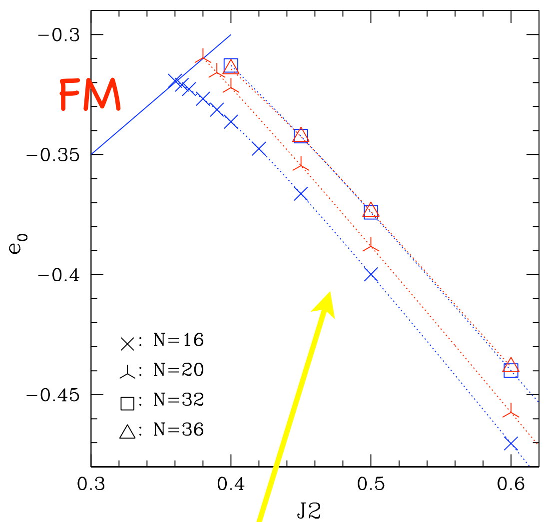

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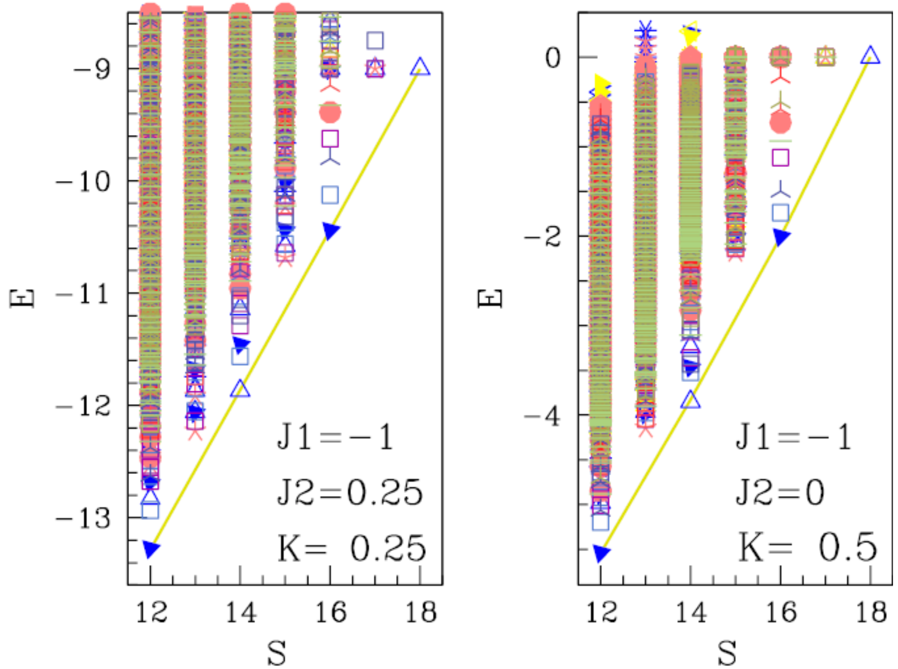

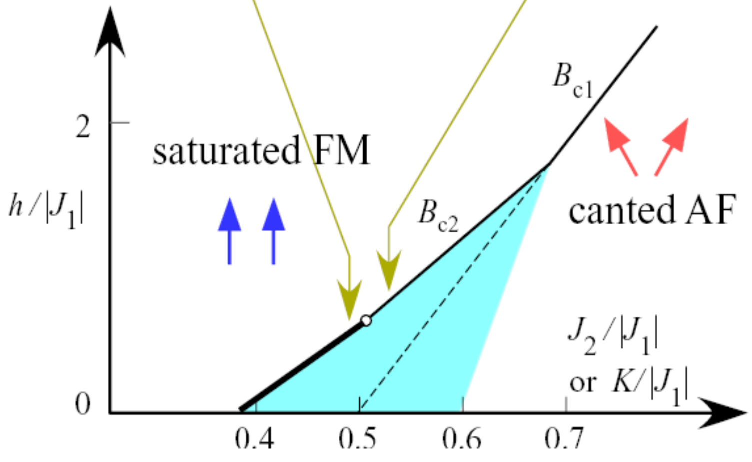

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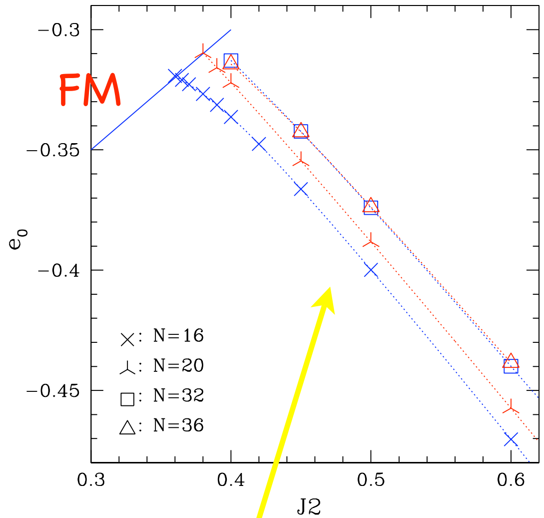

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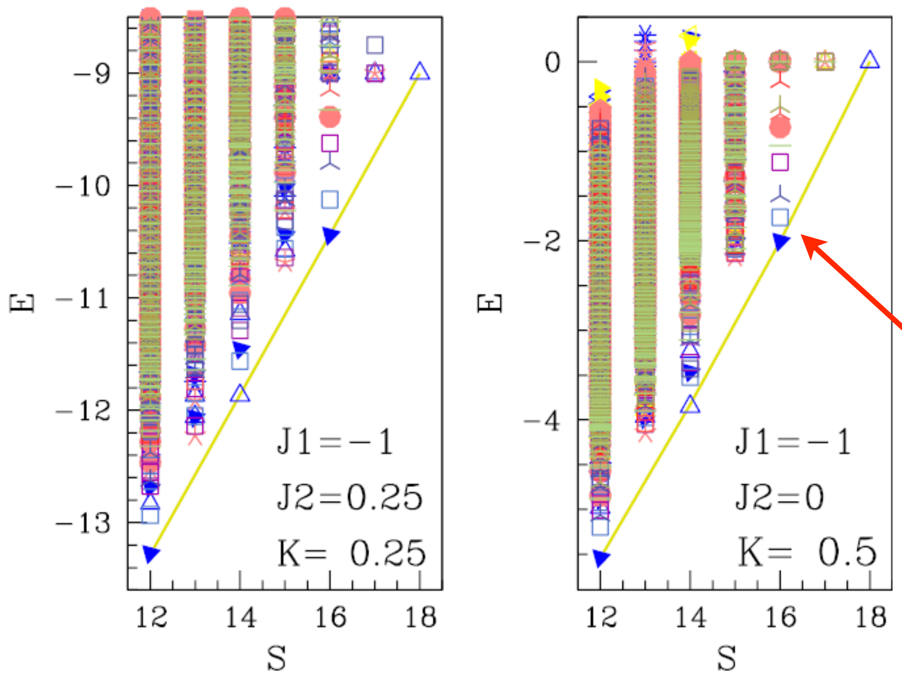

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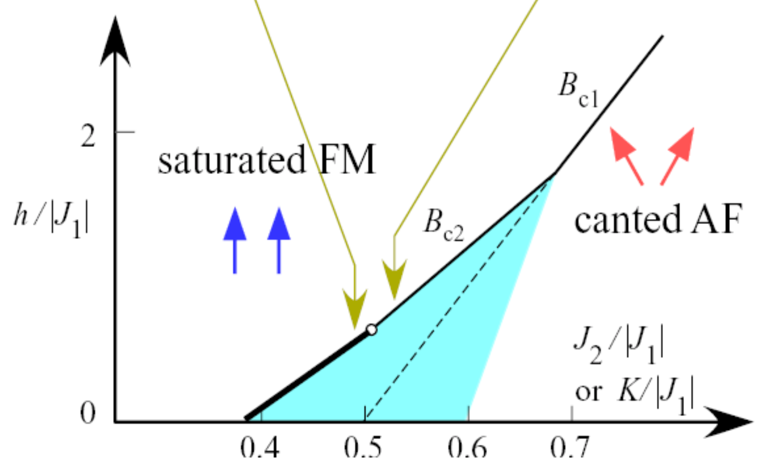

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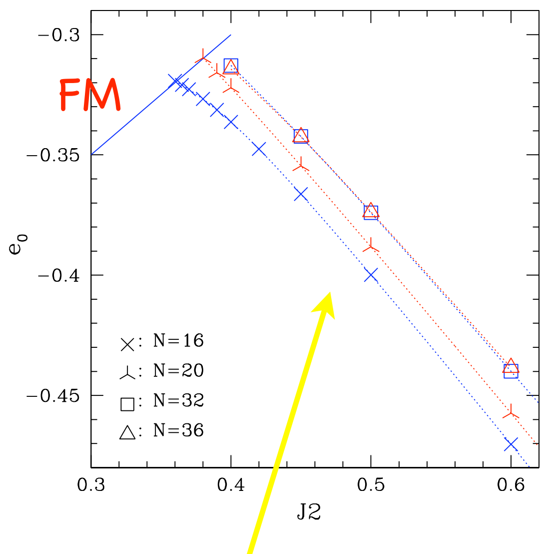

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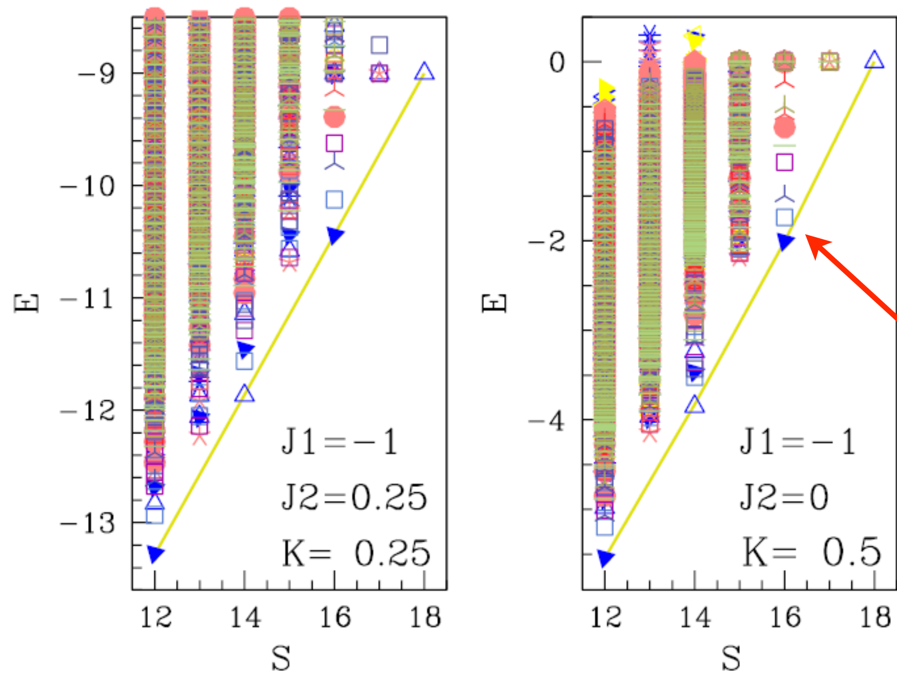

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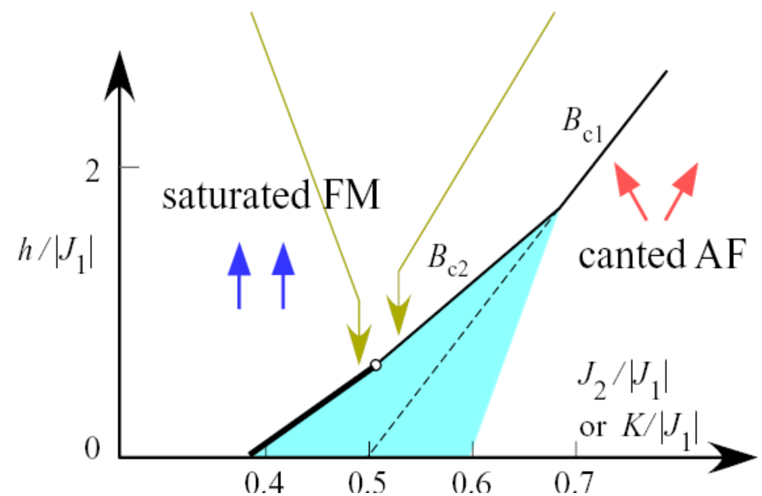

Bose-Einstein condensation of bi-magnons at high field can be seen in numerics ✓

what about the ground state ?

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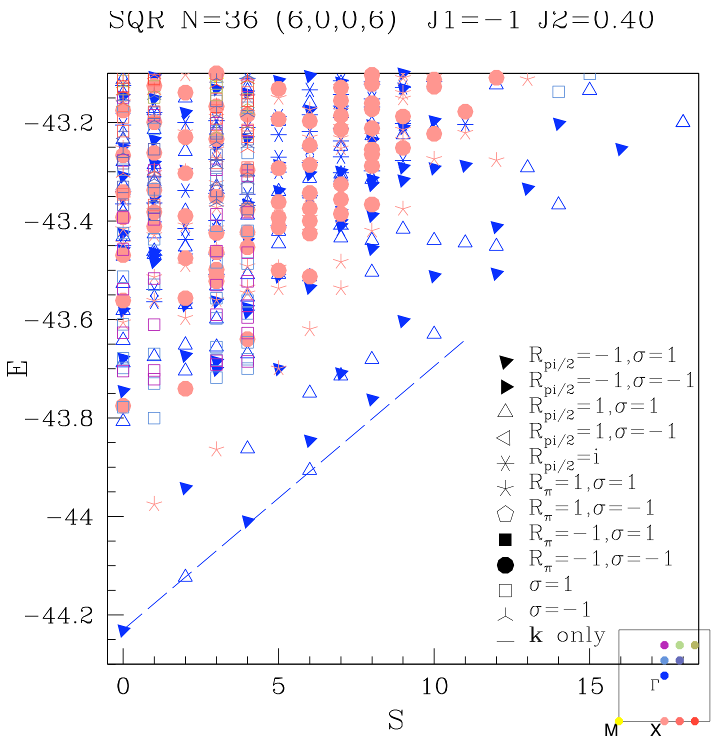

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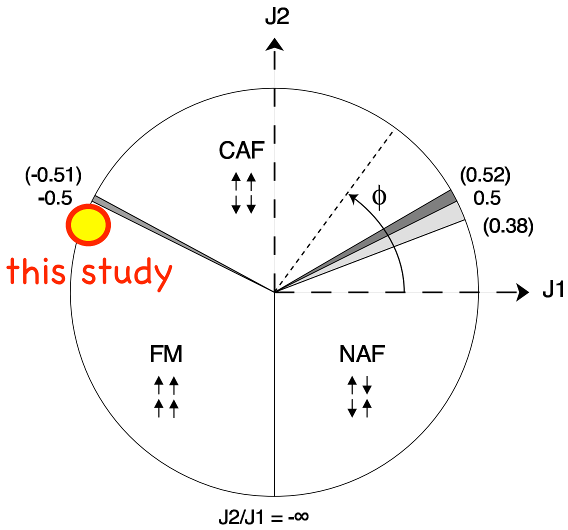

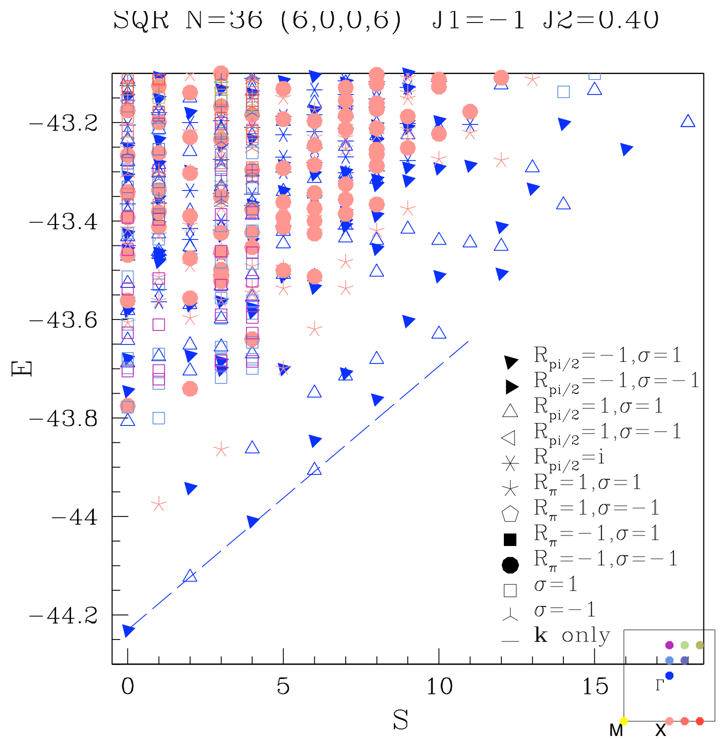

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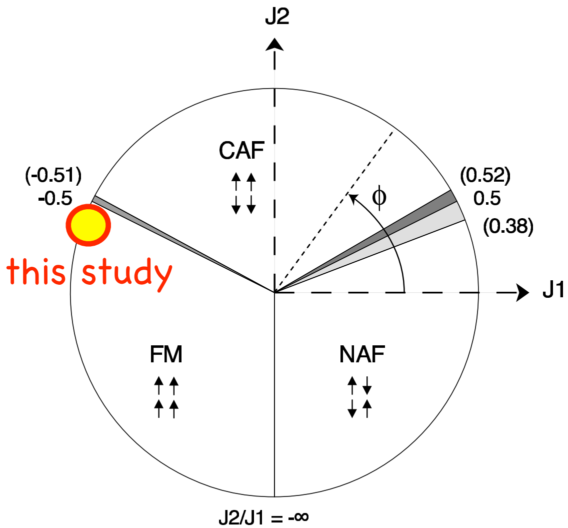

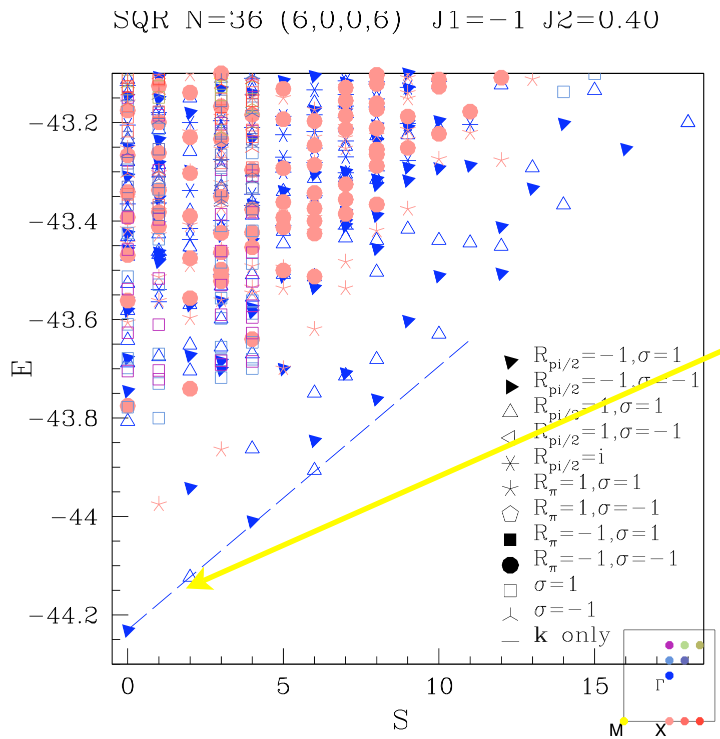

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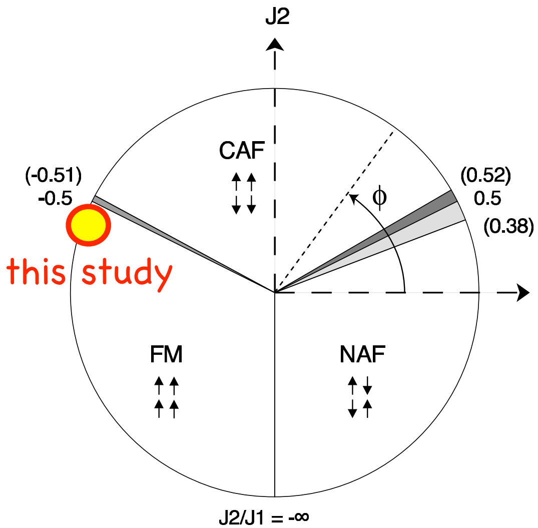

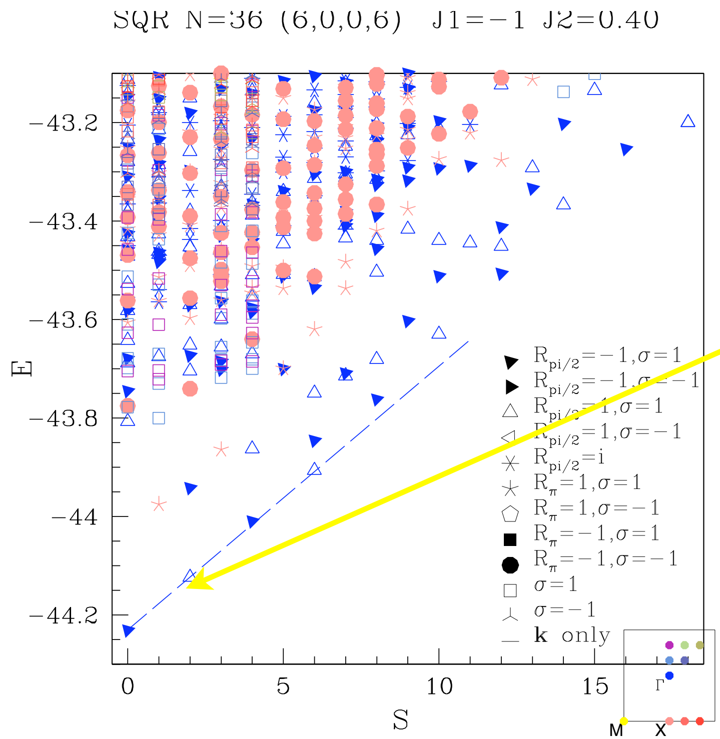

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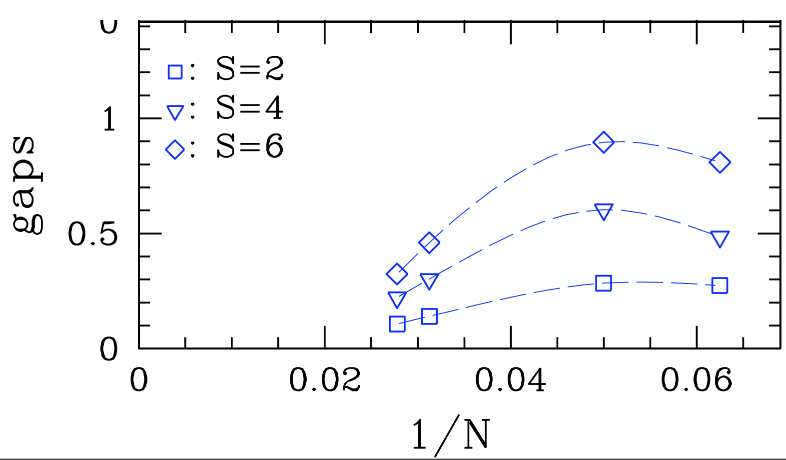

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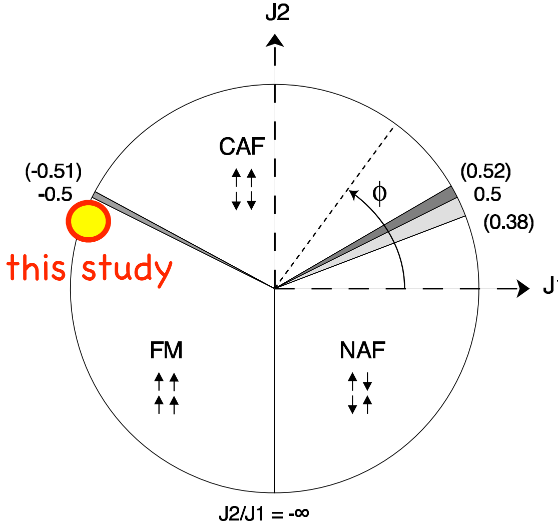

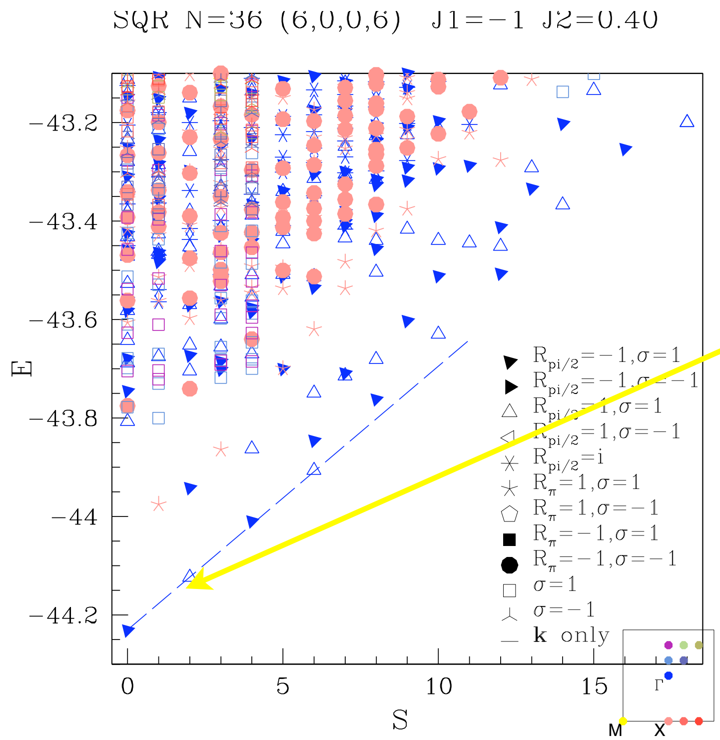

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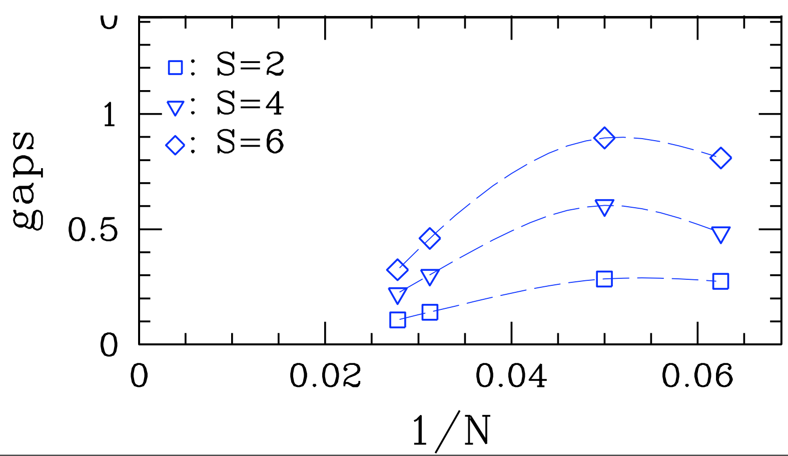

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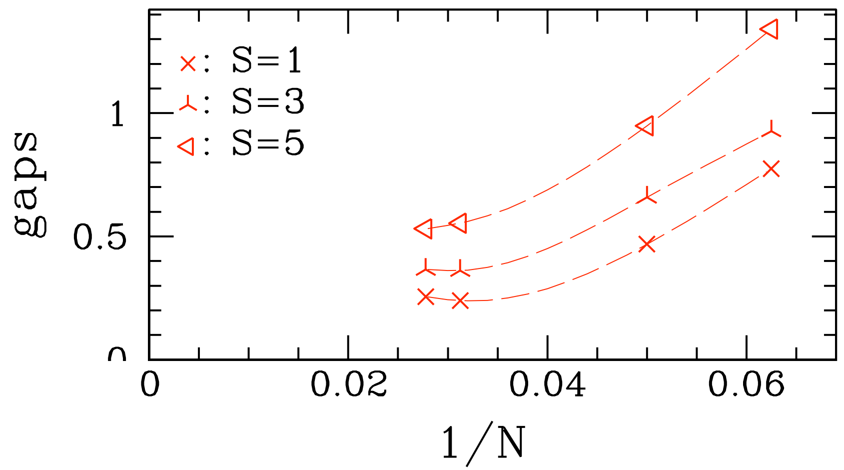

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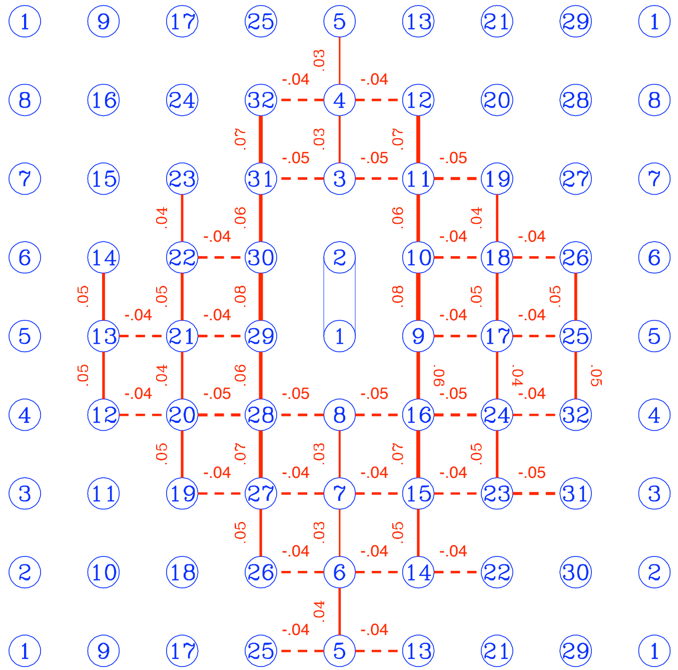

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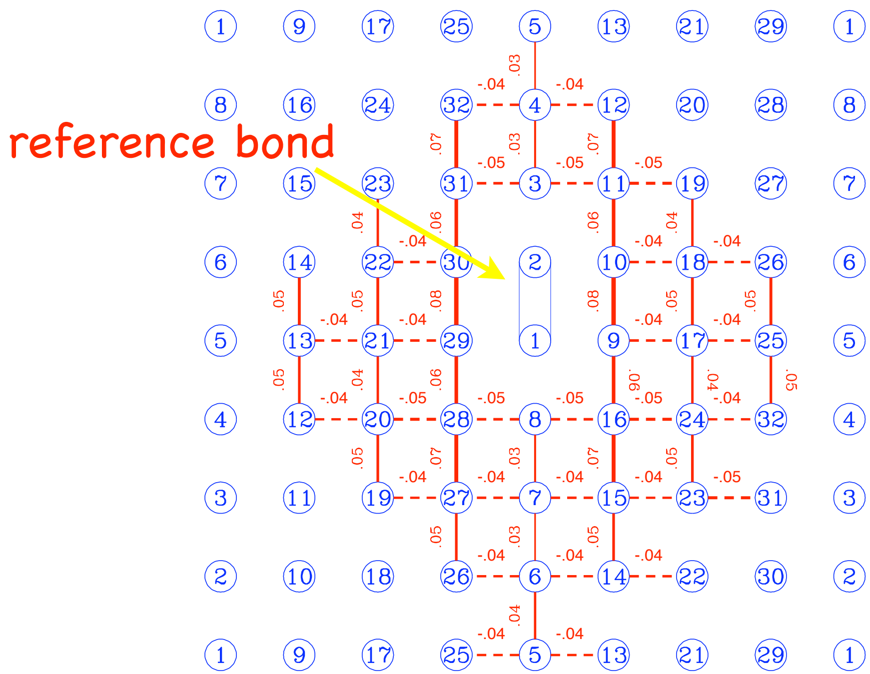

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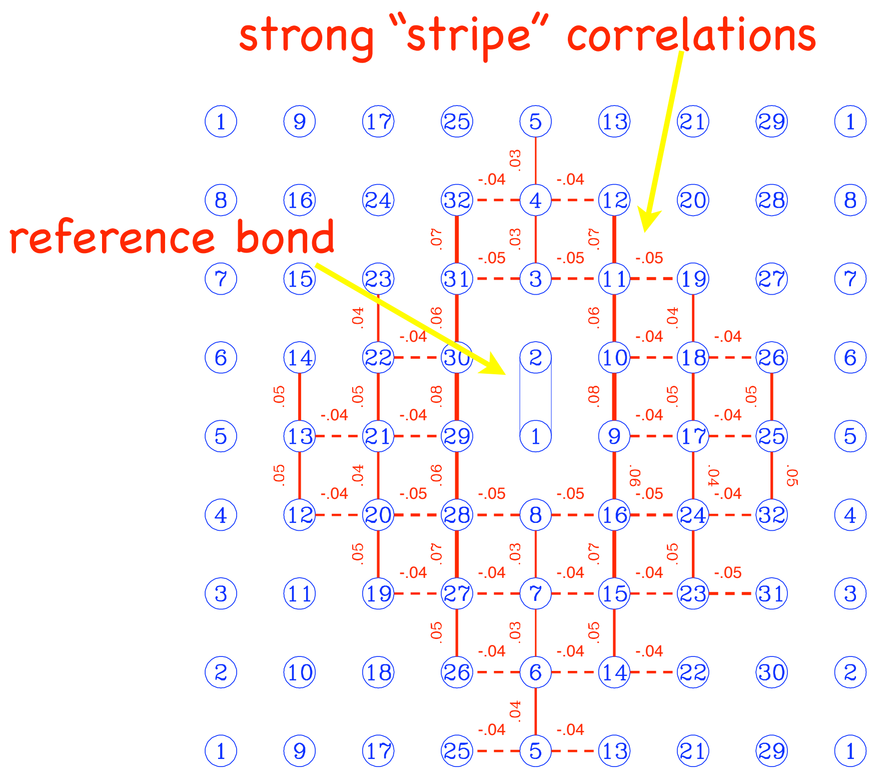

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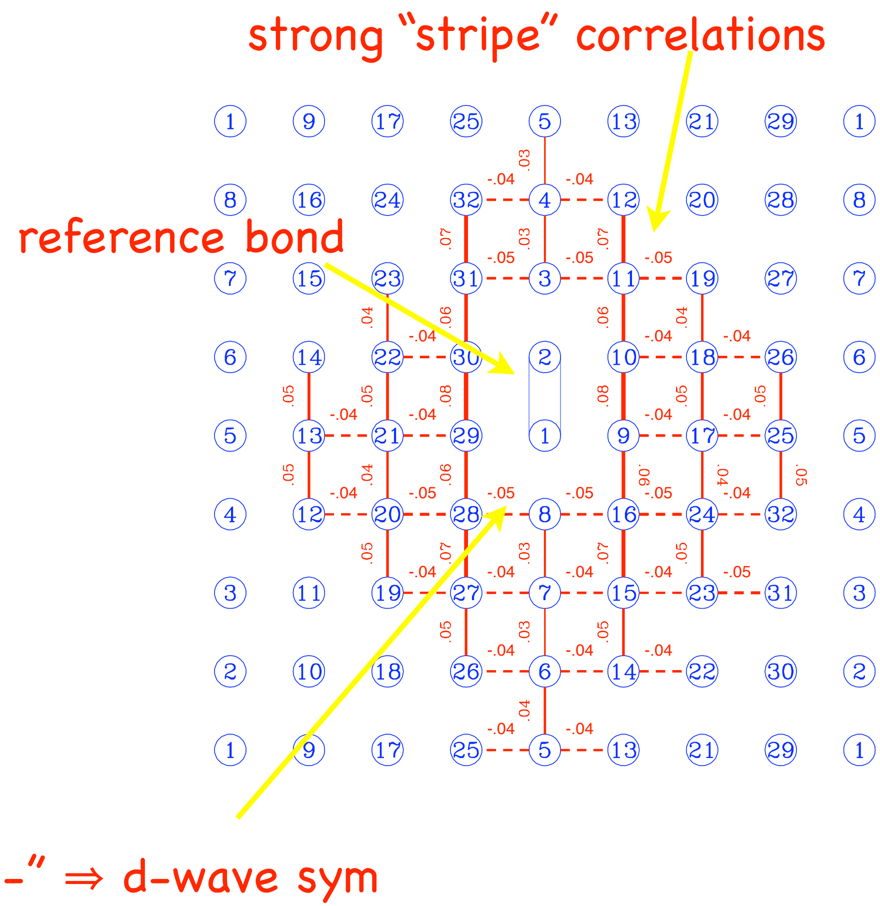

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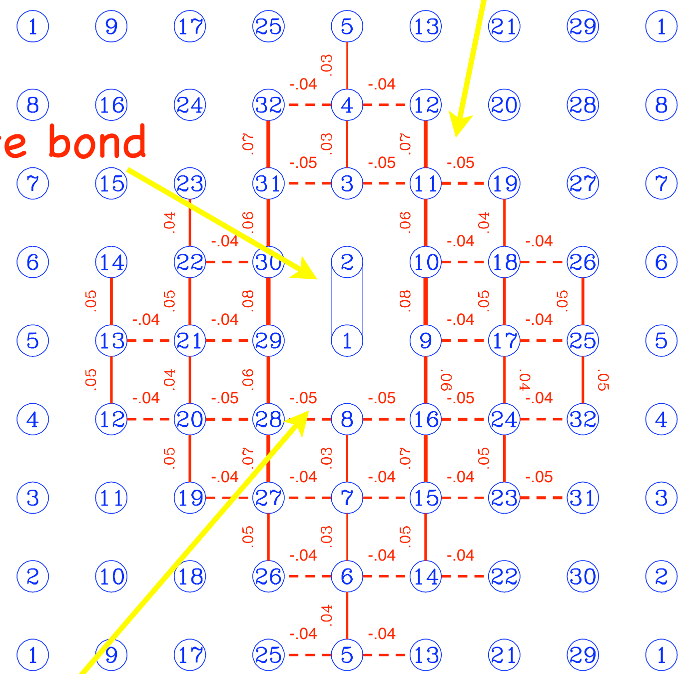

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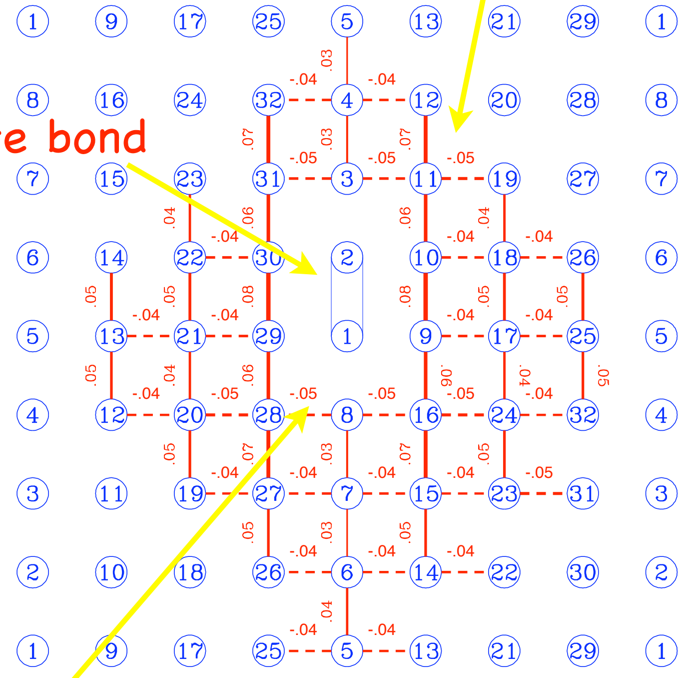

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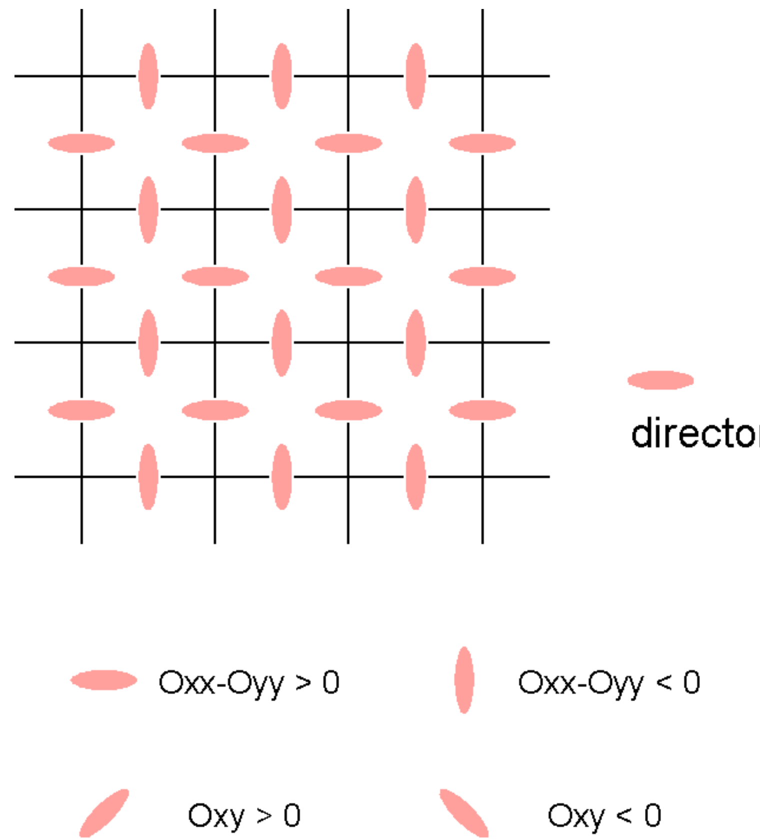

thermodynamics...

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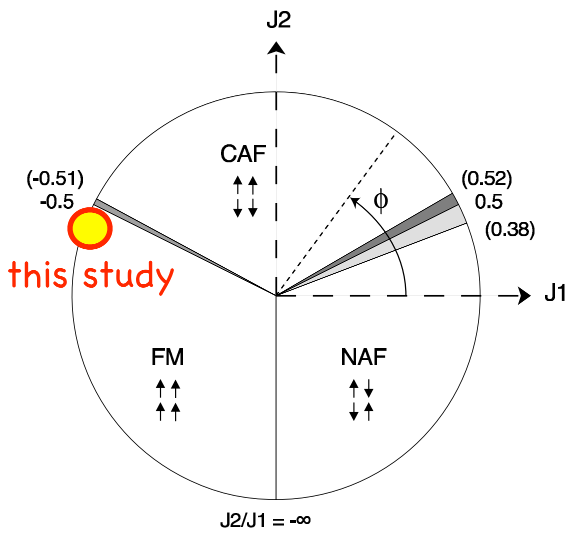

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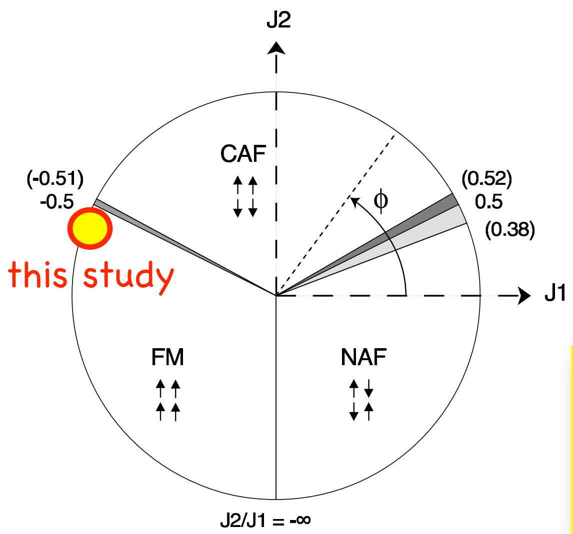
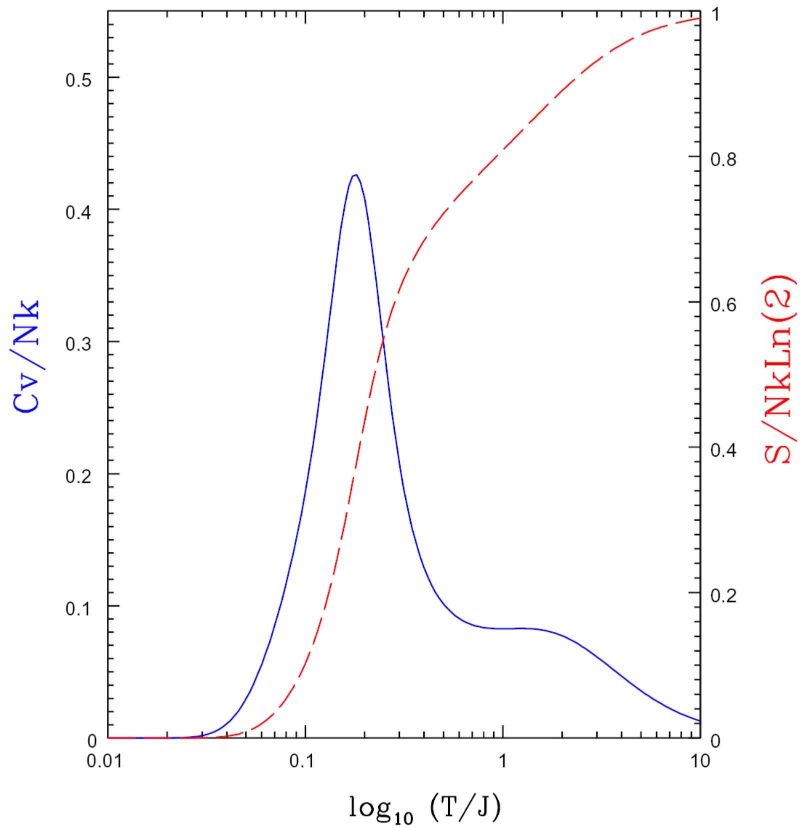


heat capacity



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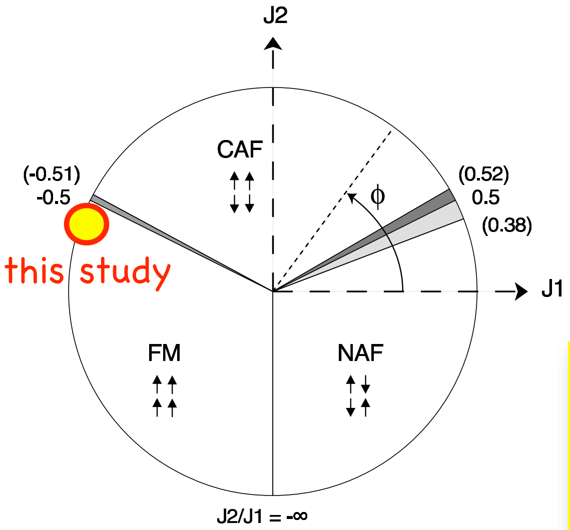
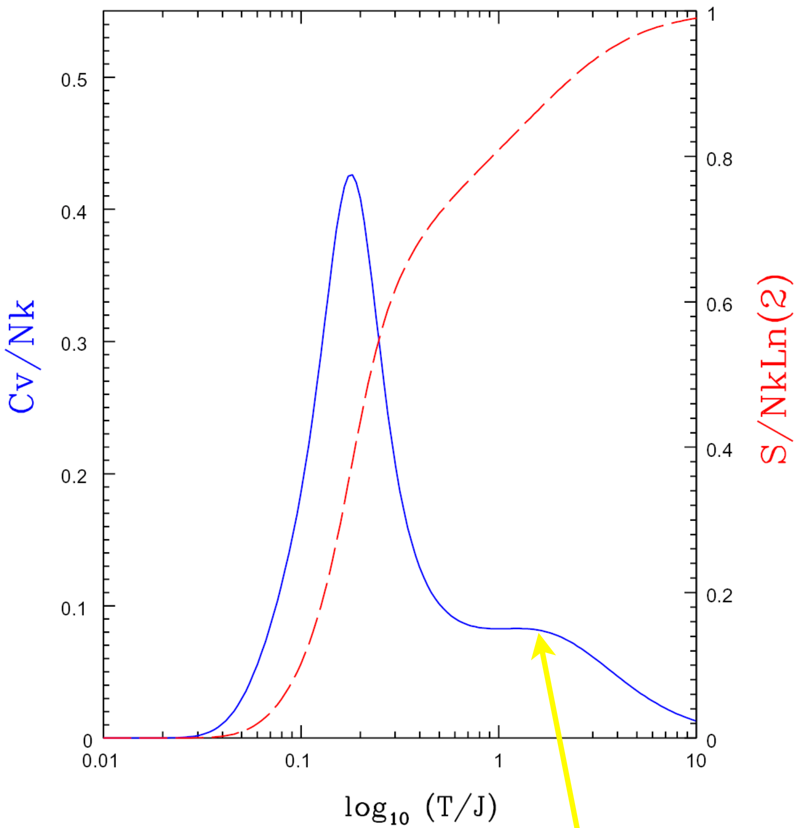


this study

heat capacity



natural energy scale ~J

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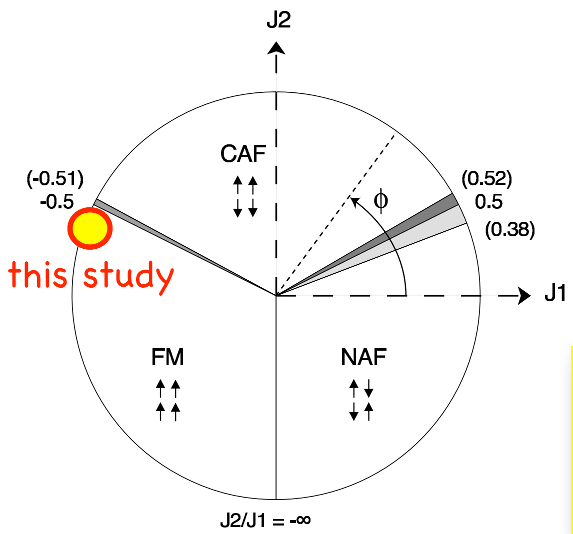

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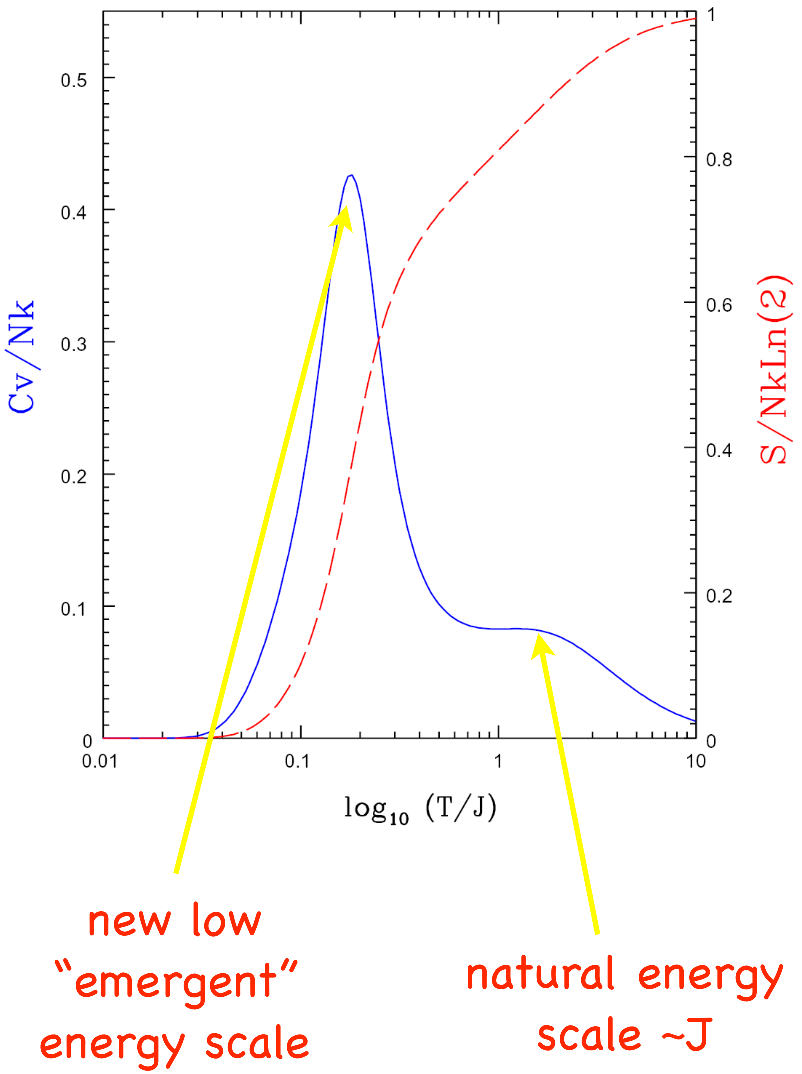

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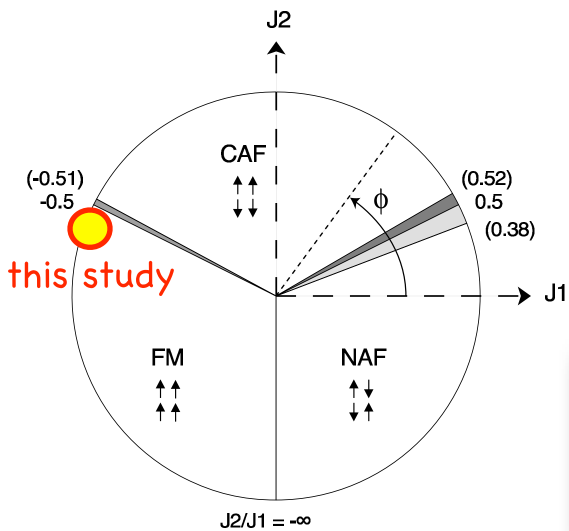

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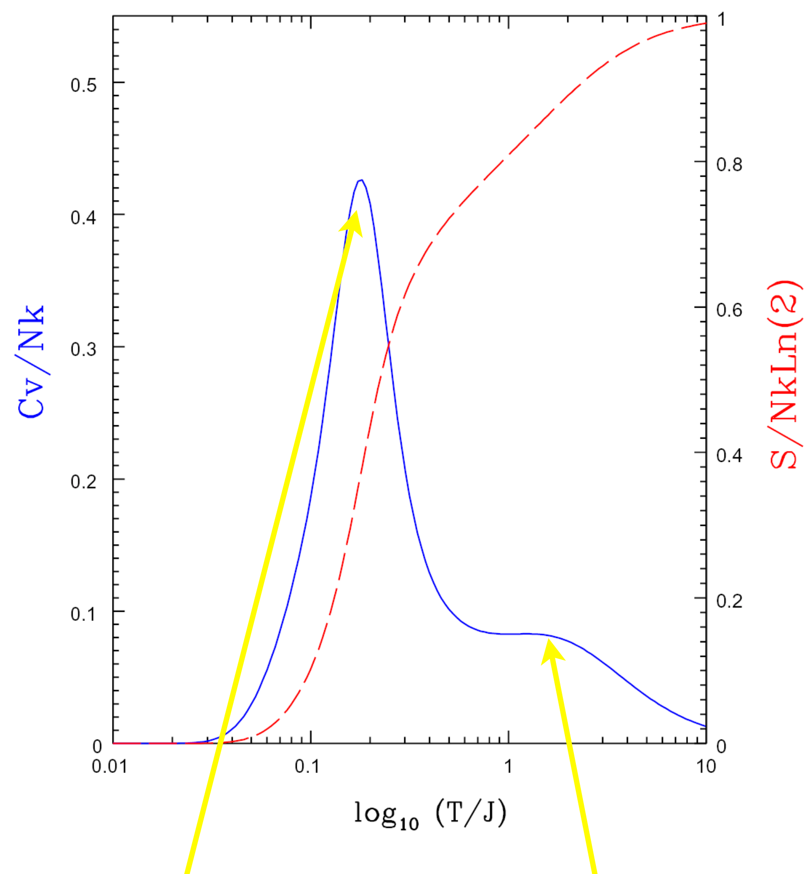

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new low "emergent" energy scale

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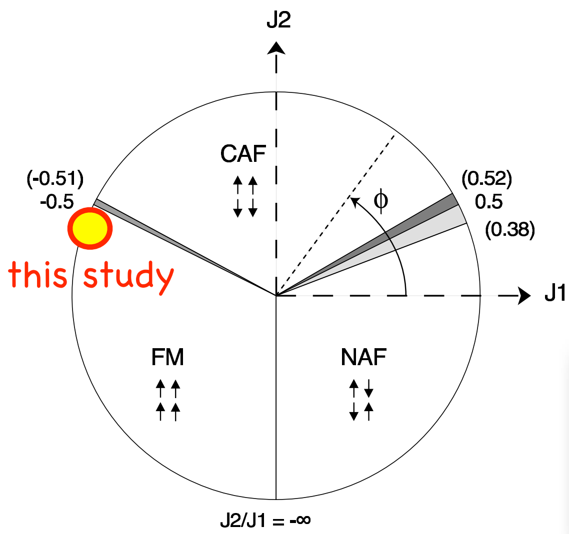

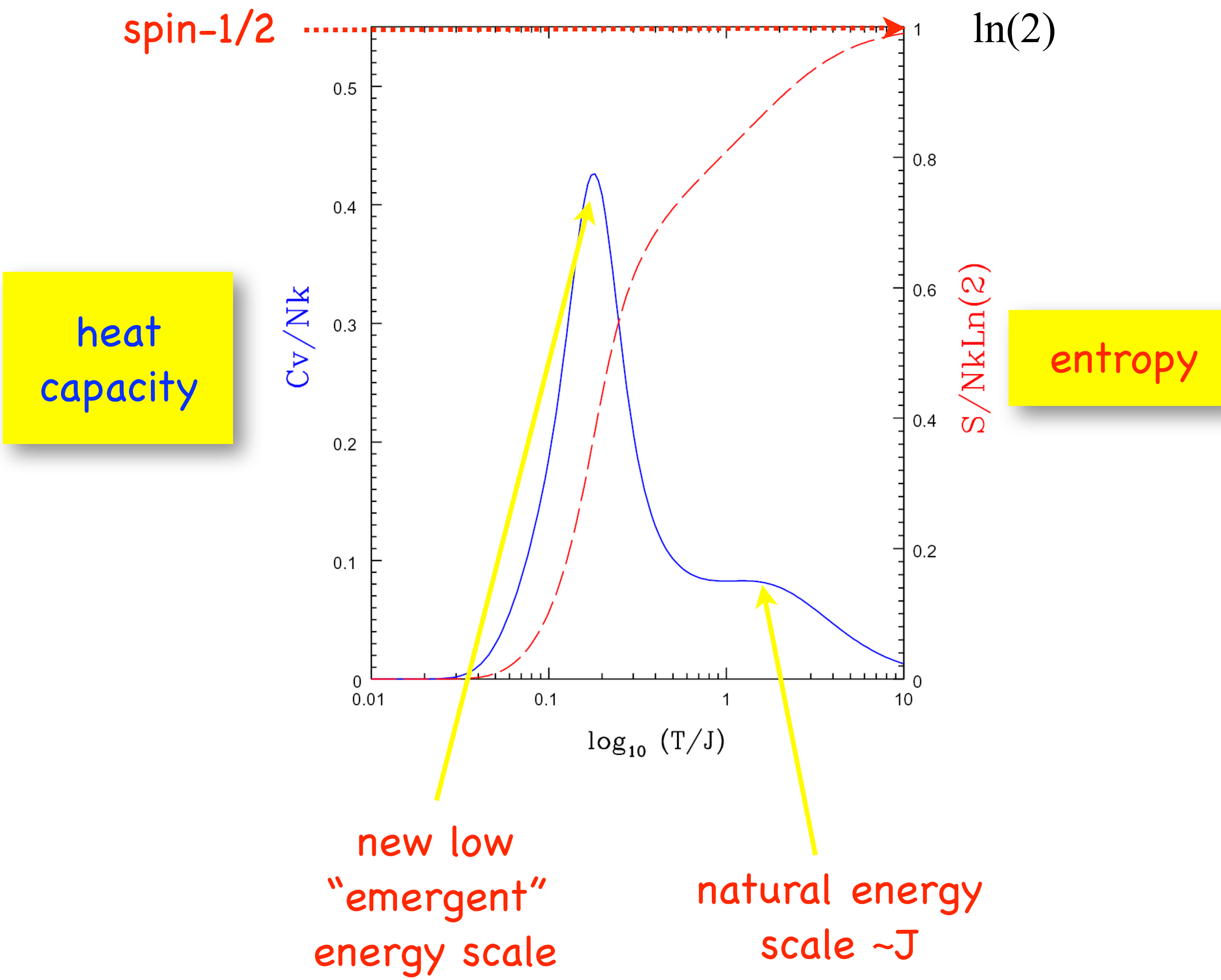

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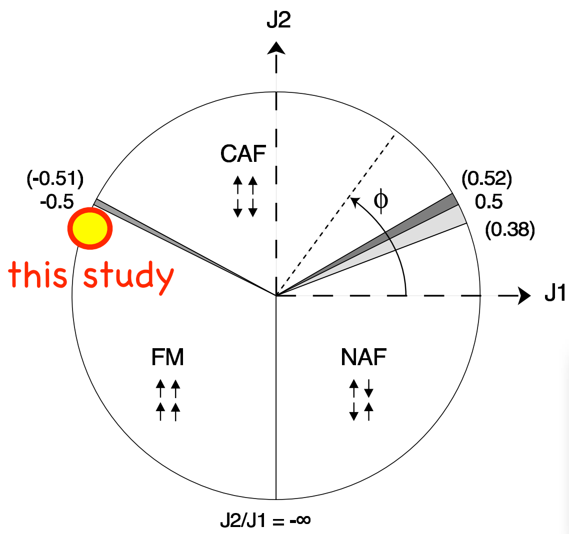

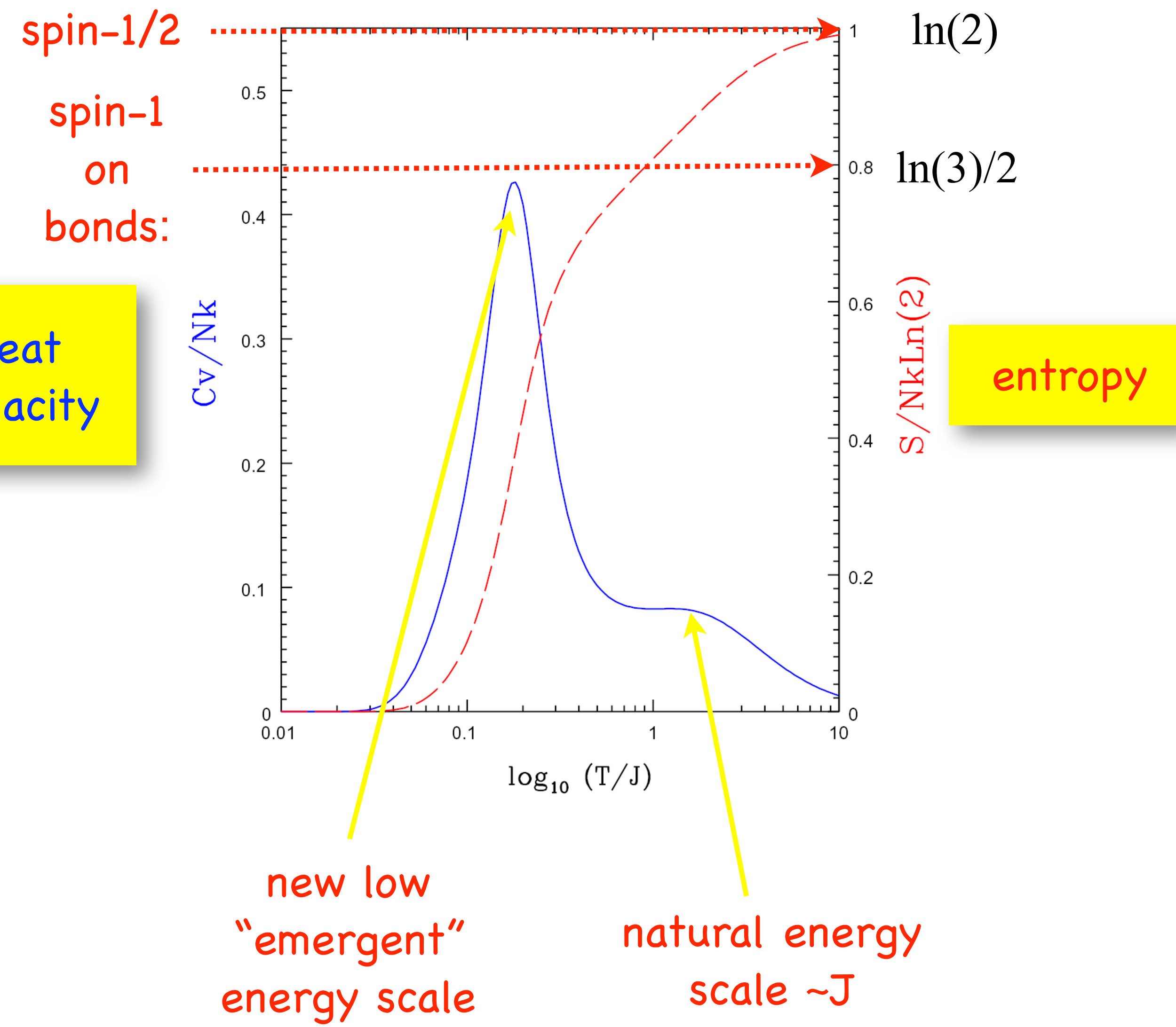

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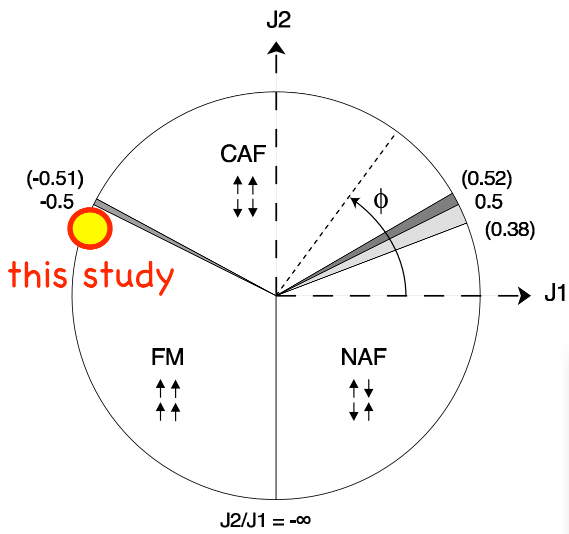

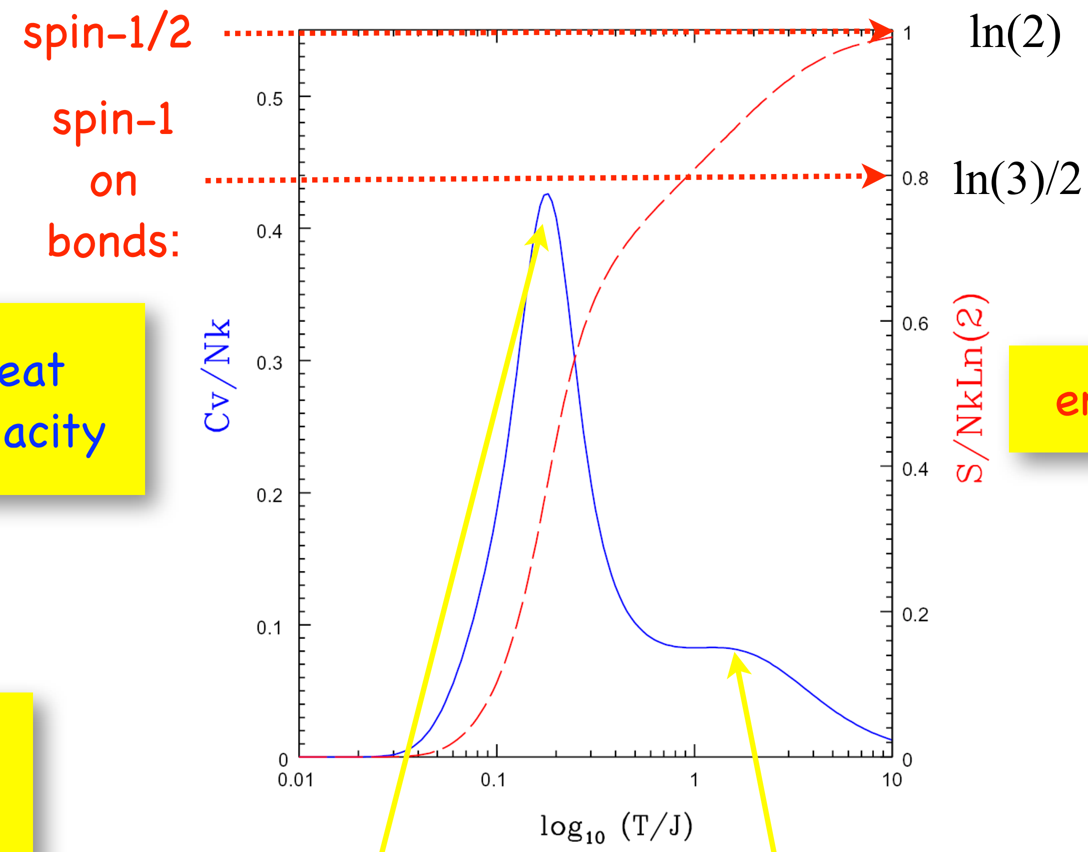

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entropy in peak is associated with nematic ordering of **spin-1** bond degrees of freedom

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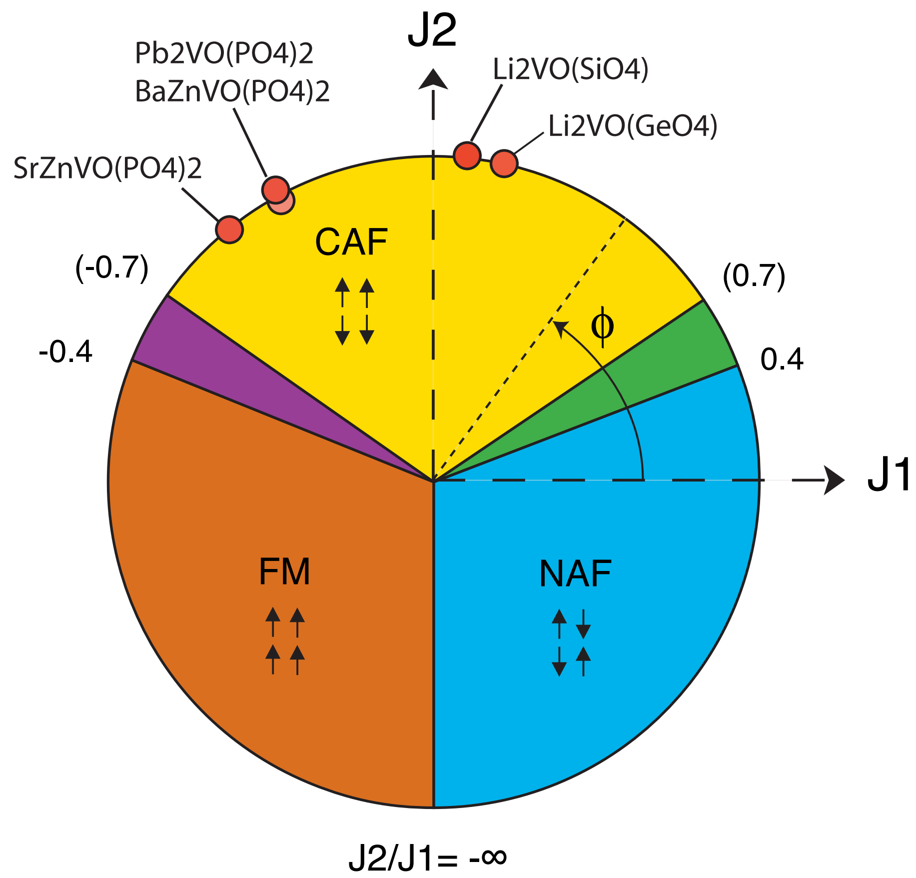

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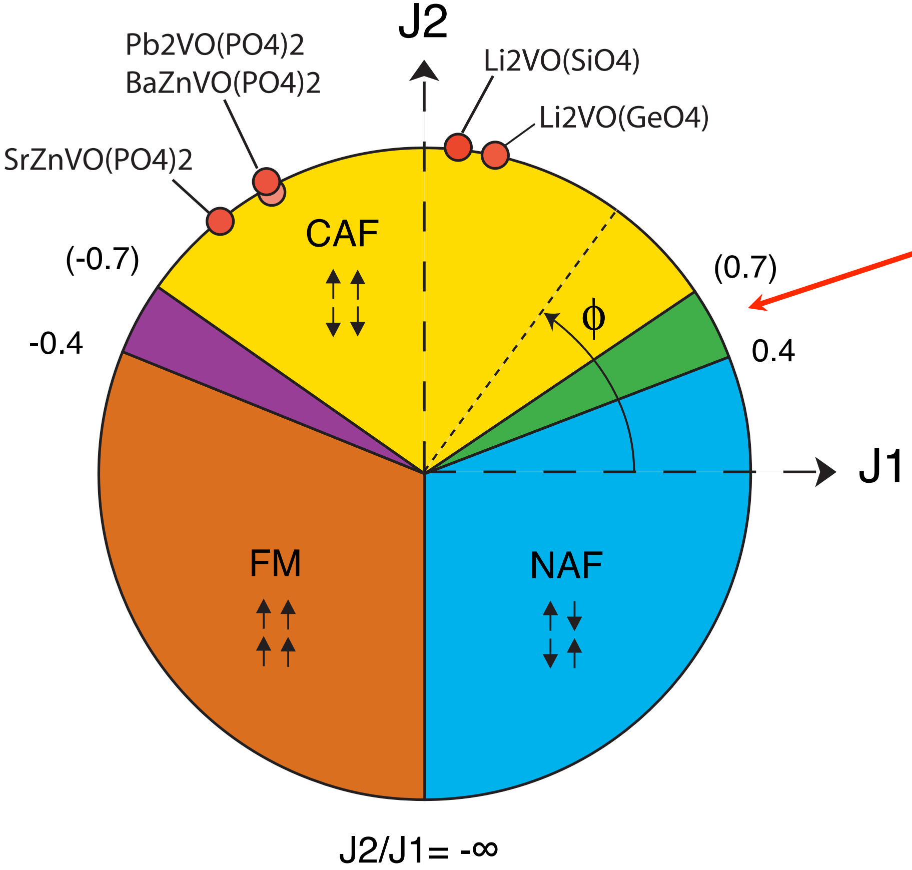
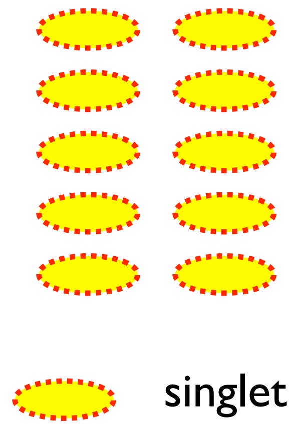


gapped, staggered dimer phase
[P. Sindzingre, Phys. Rev. B **69**, 094418 (2004), and references therein]



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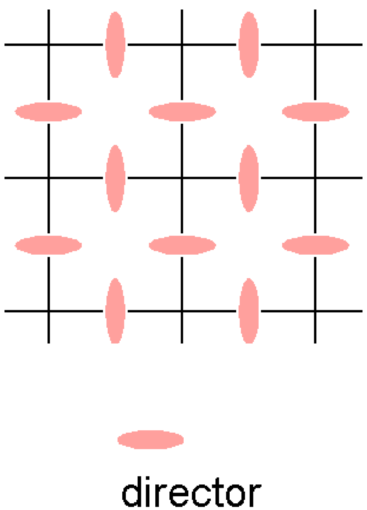

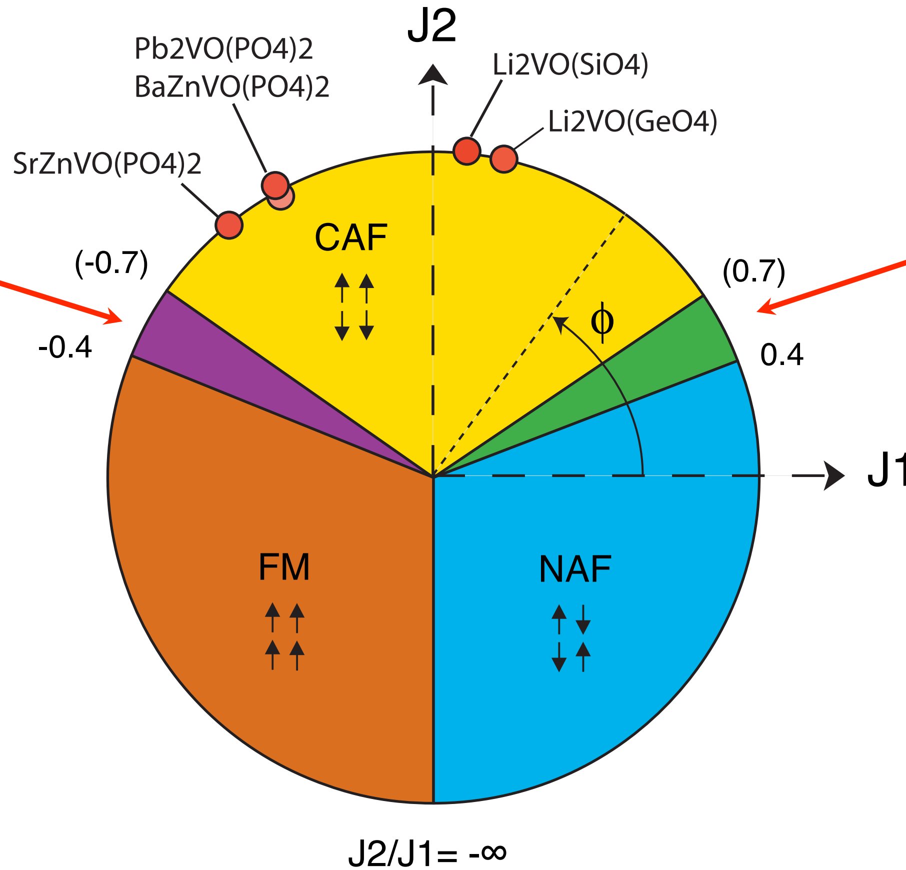

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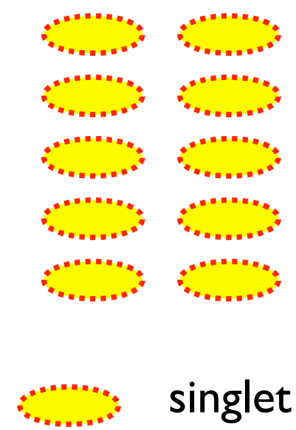

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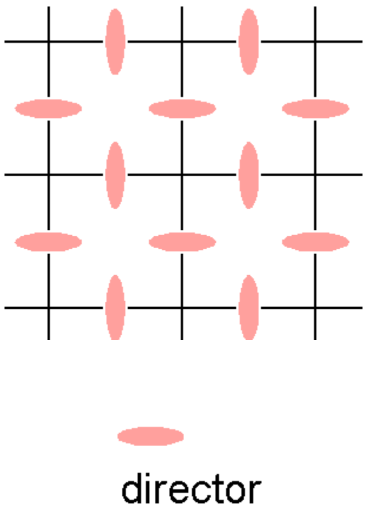

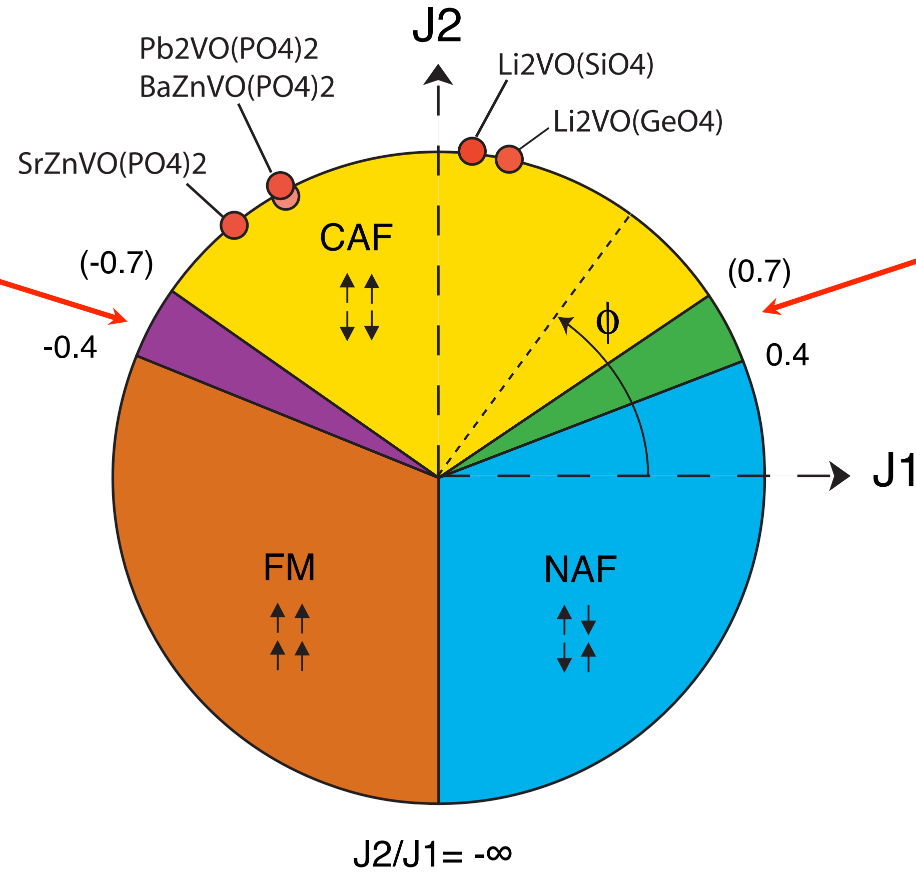

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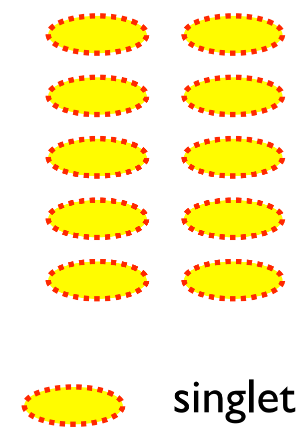

new compound CaZnVO(PO4)2 looks promising...

so what happens on a triangular lattice ?

- modeling solid 2D films of He III -

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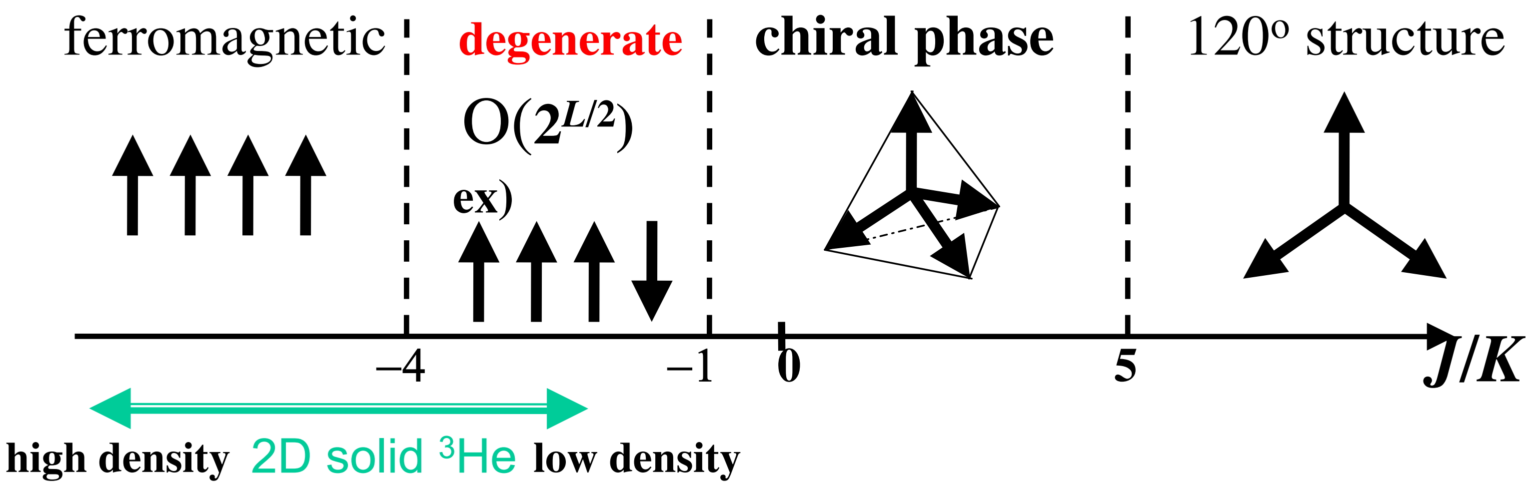

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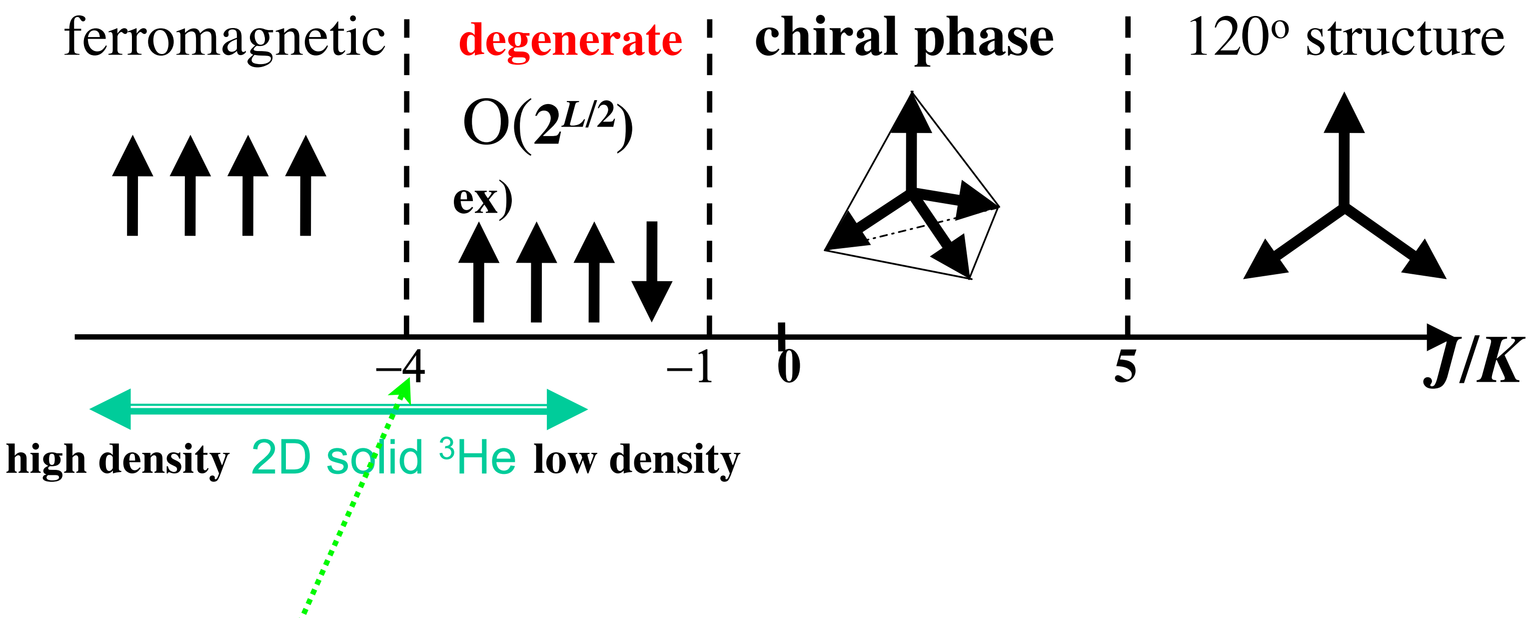

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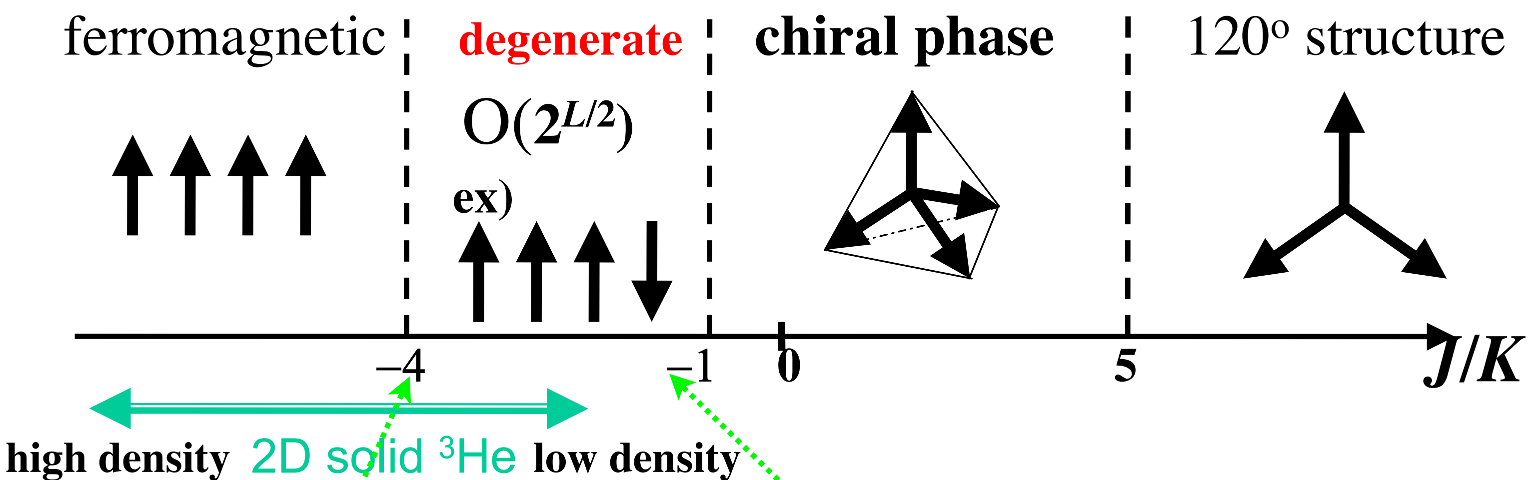

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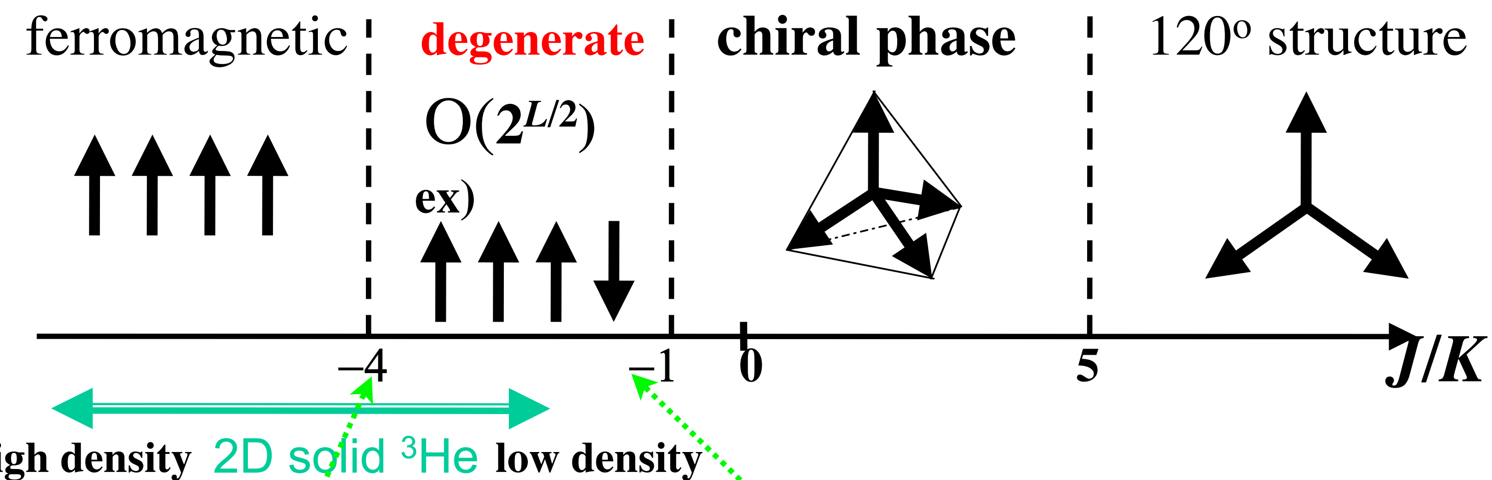

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experiment sees a **gapless** spin liquid bordering on the FM
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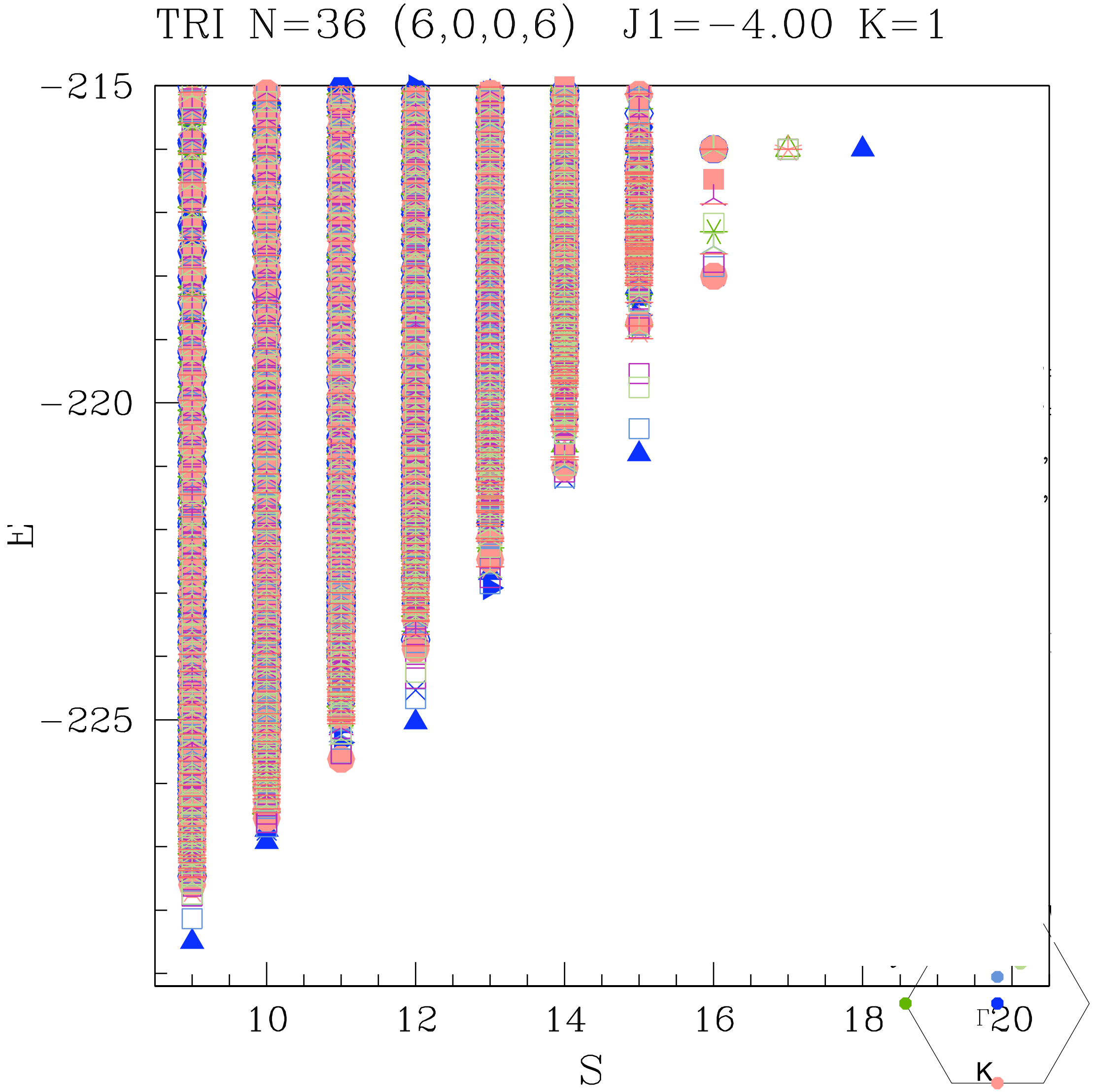
- going beyond nematic structure -



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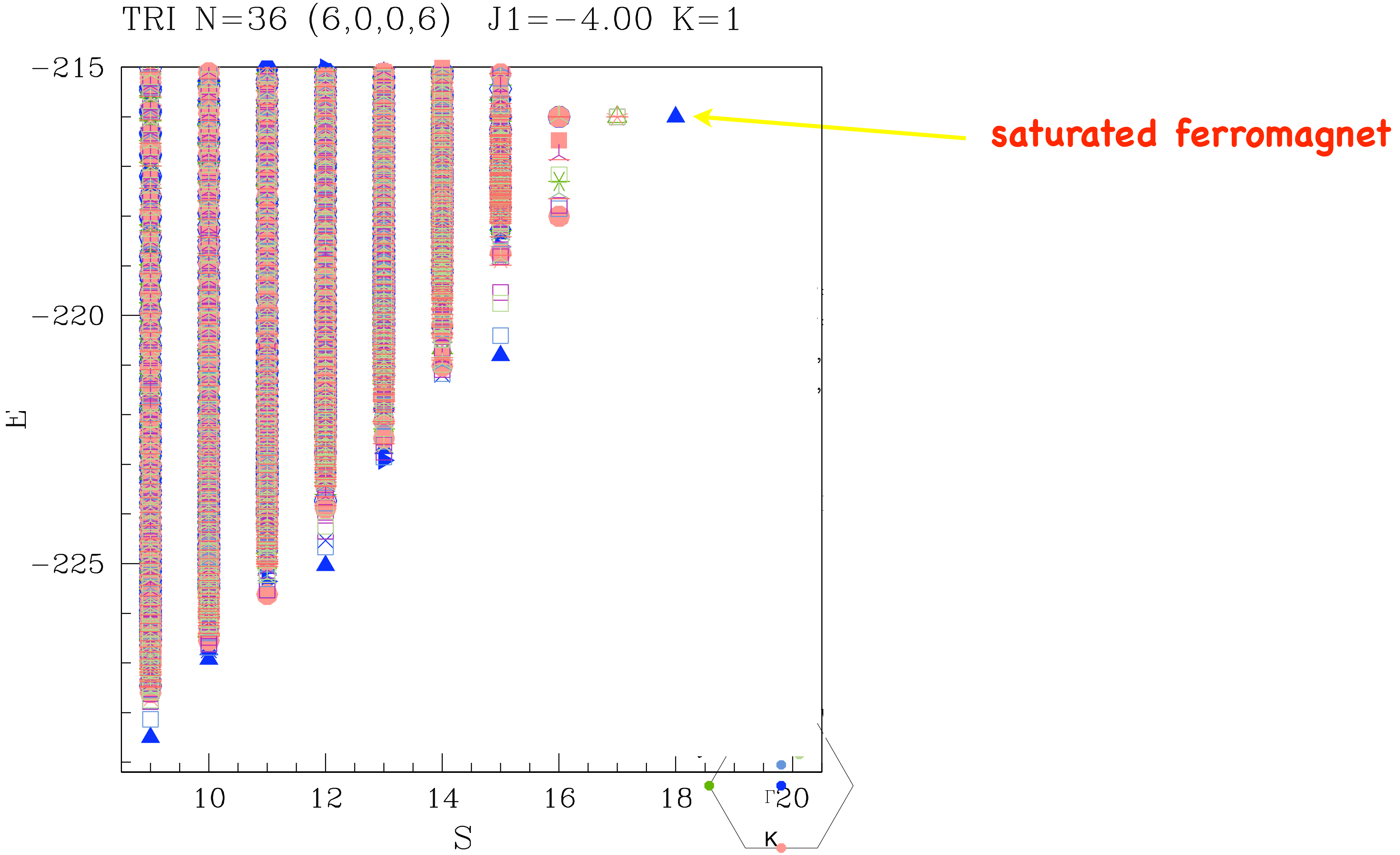

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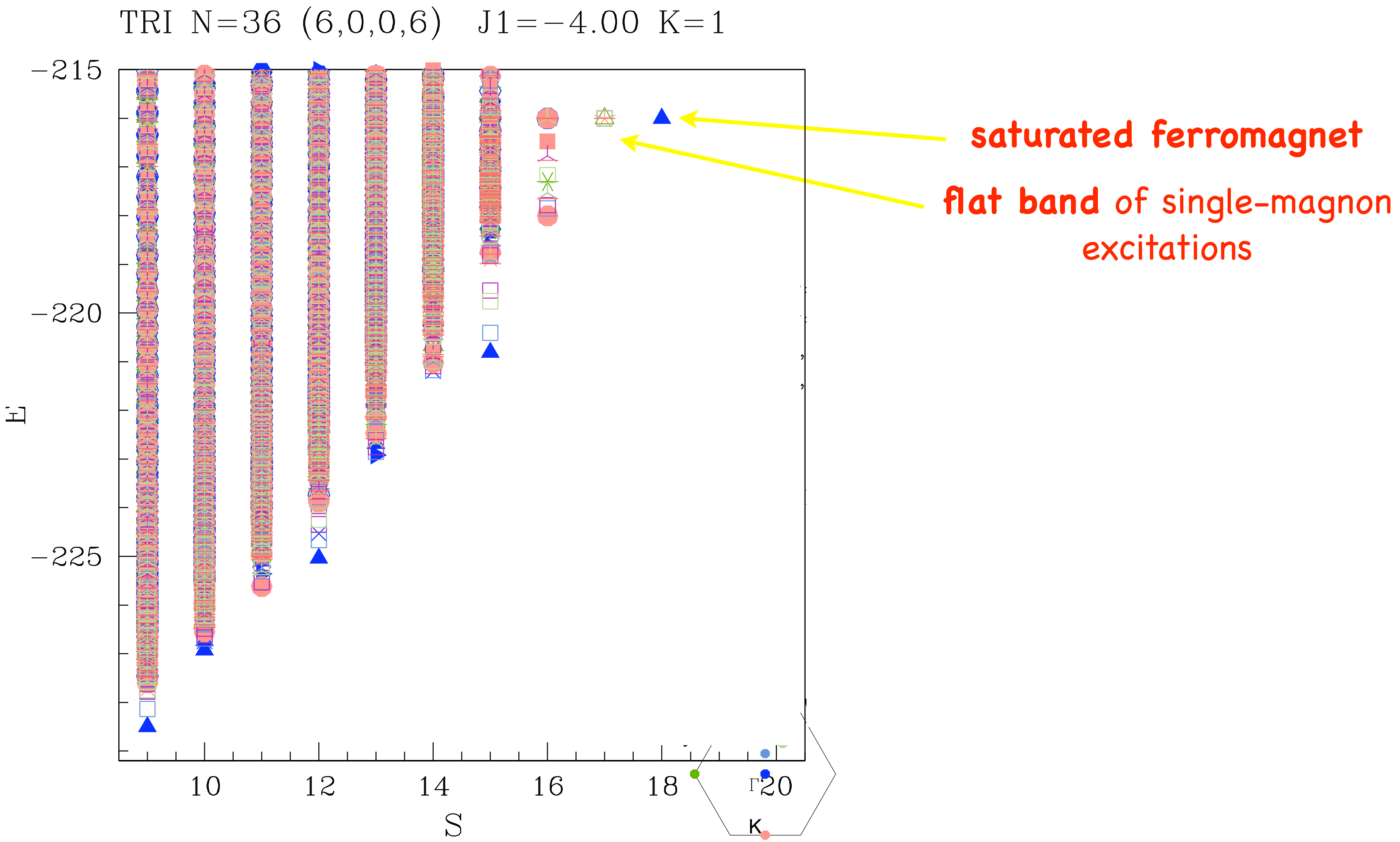

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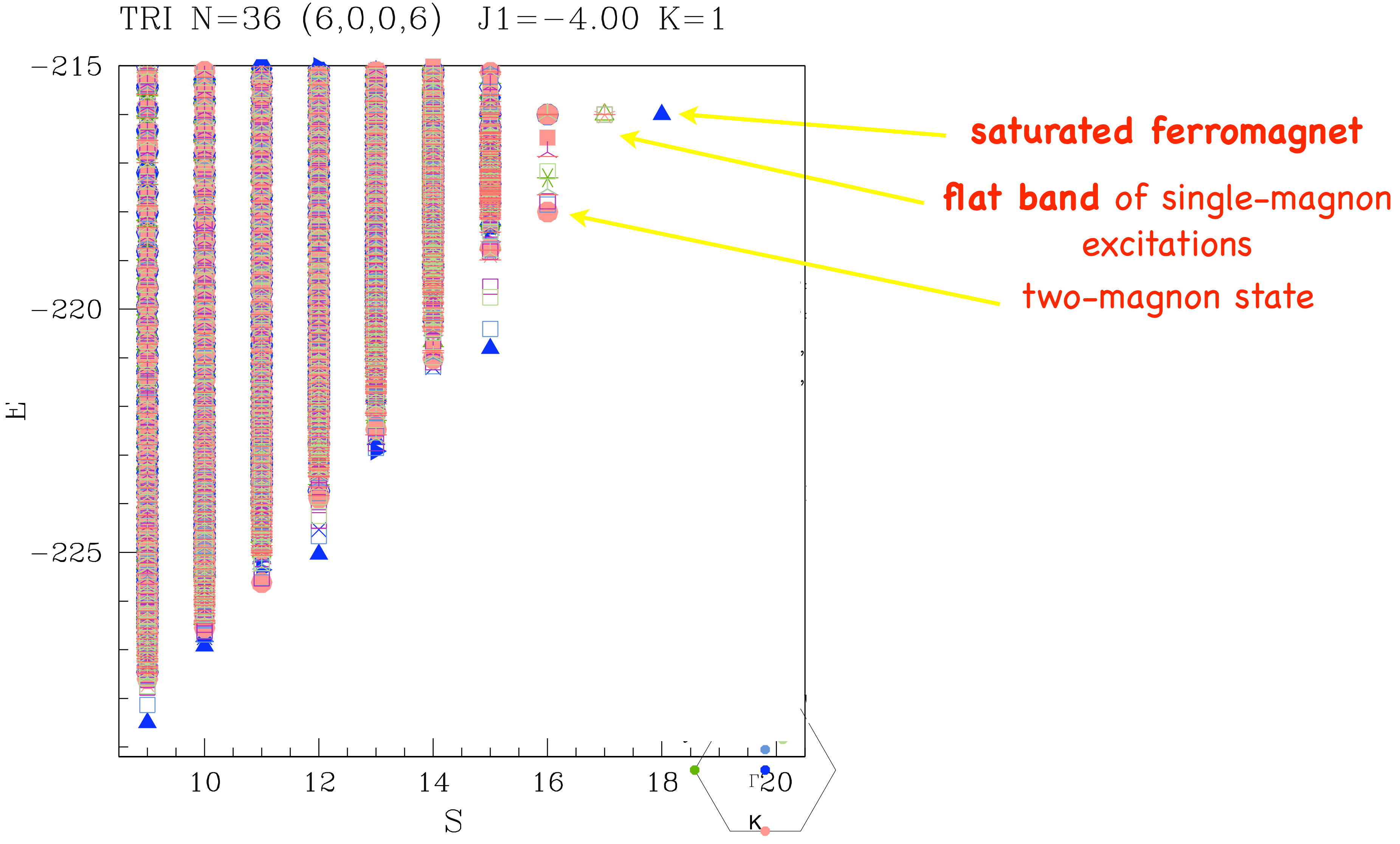

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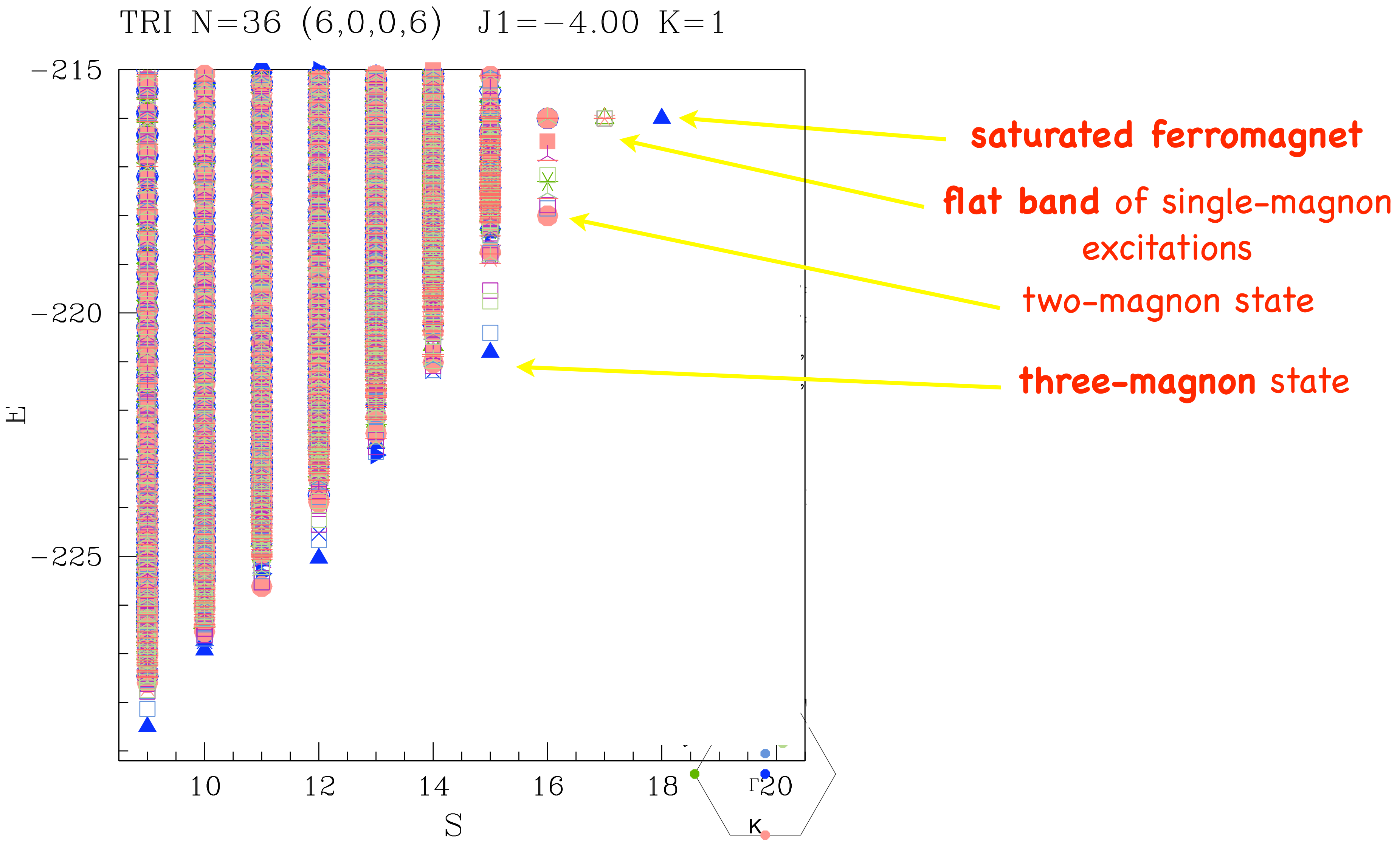

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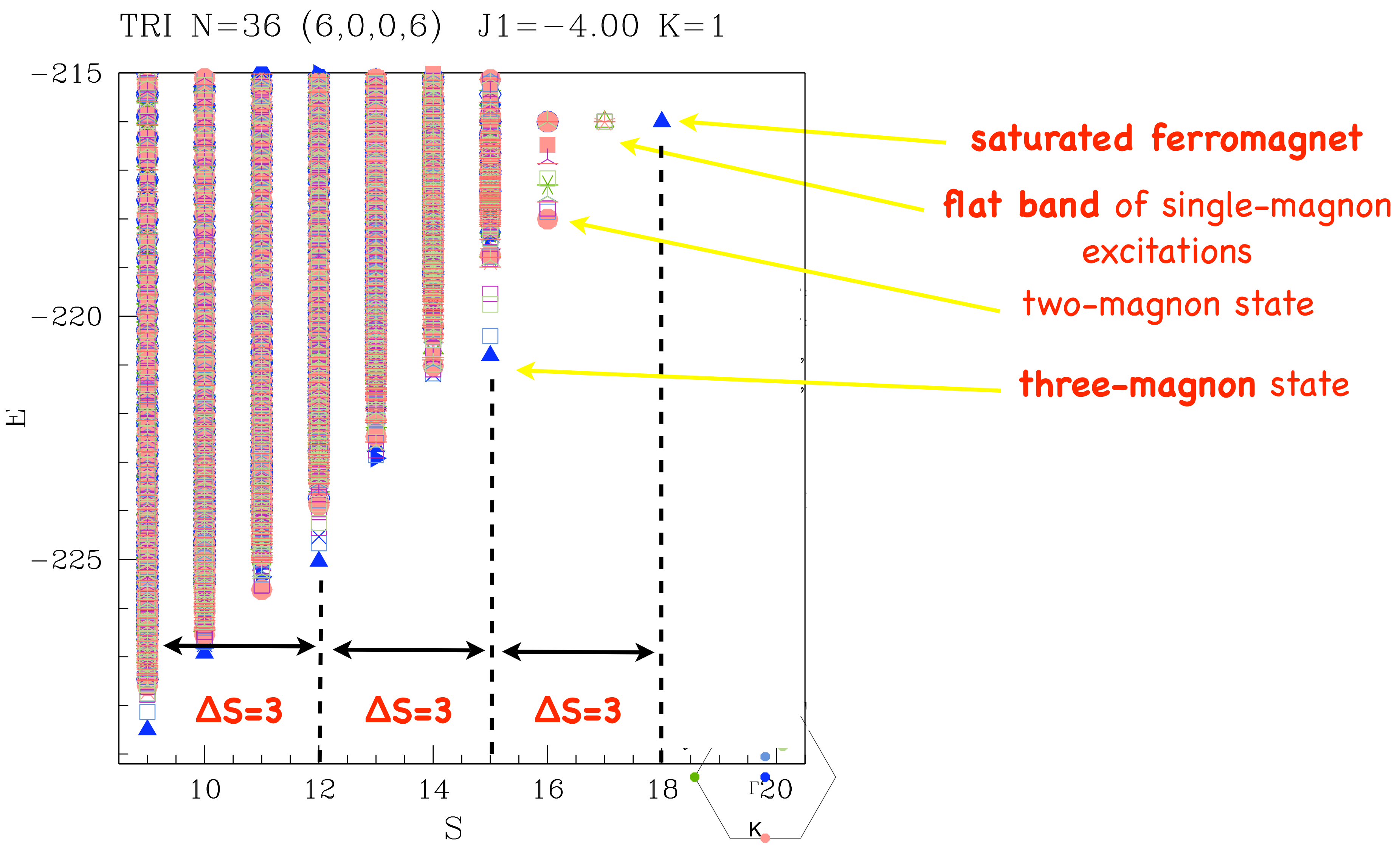

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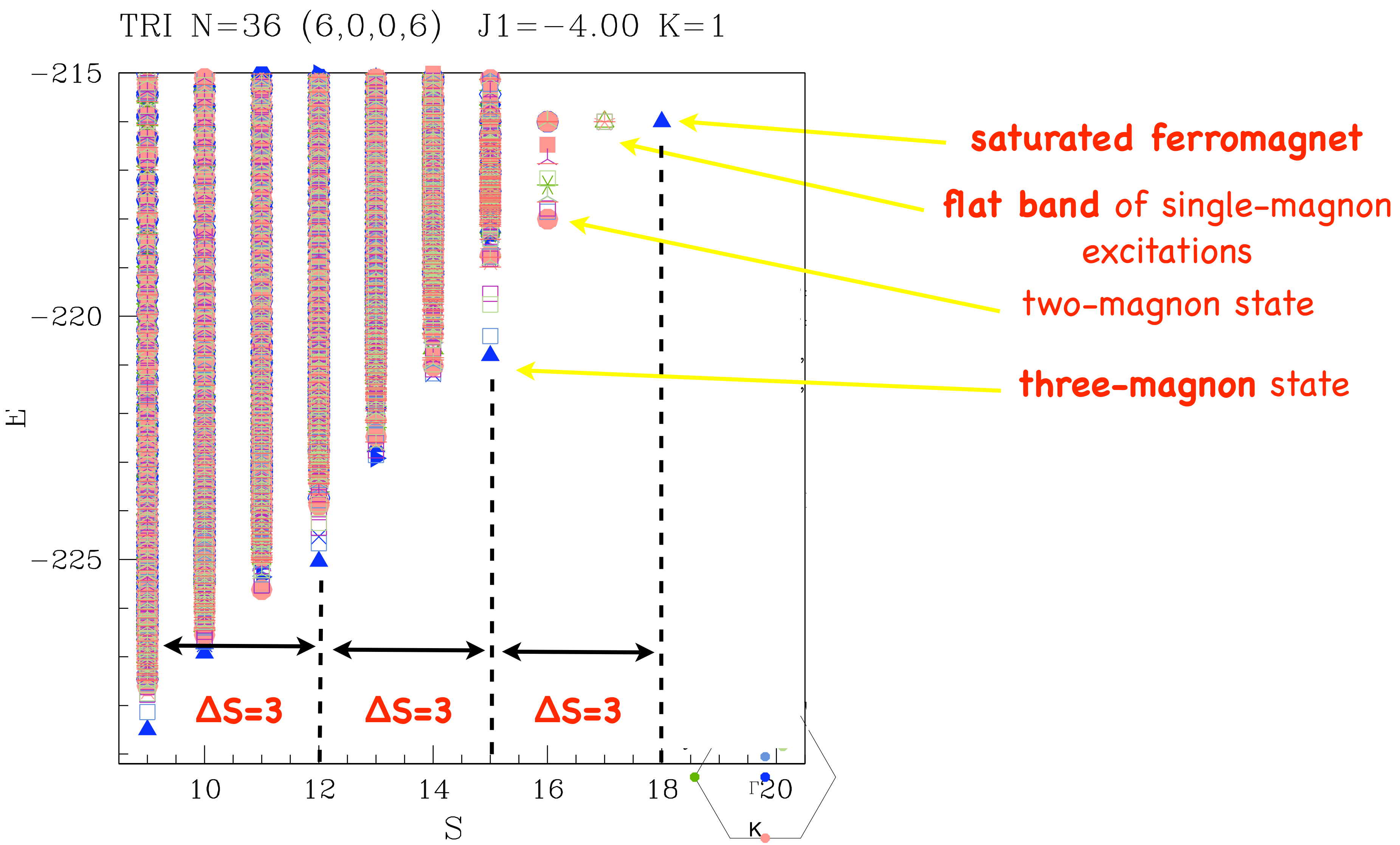

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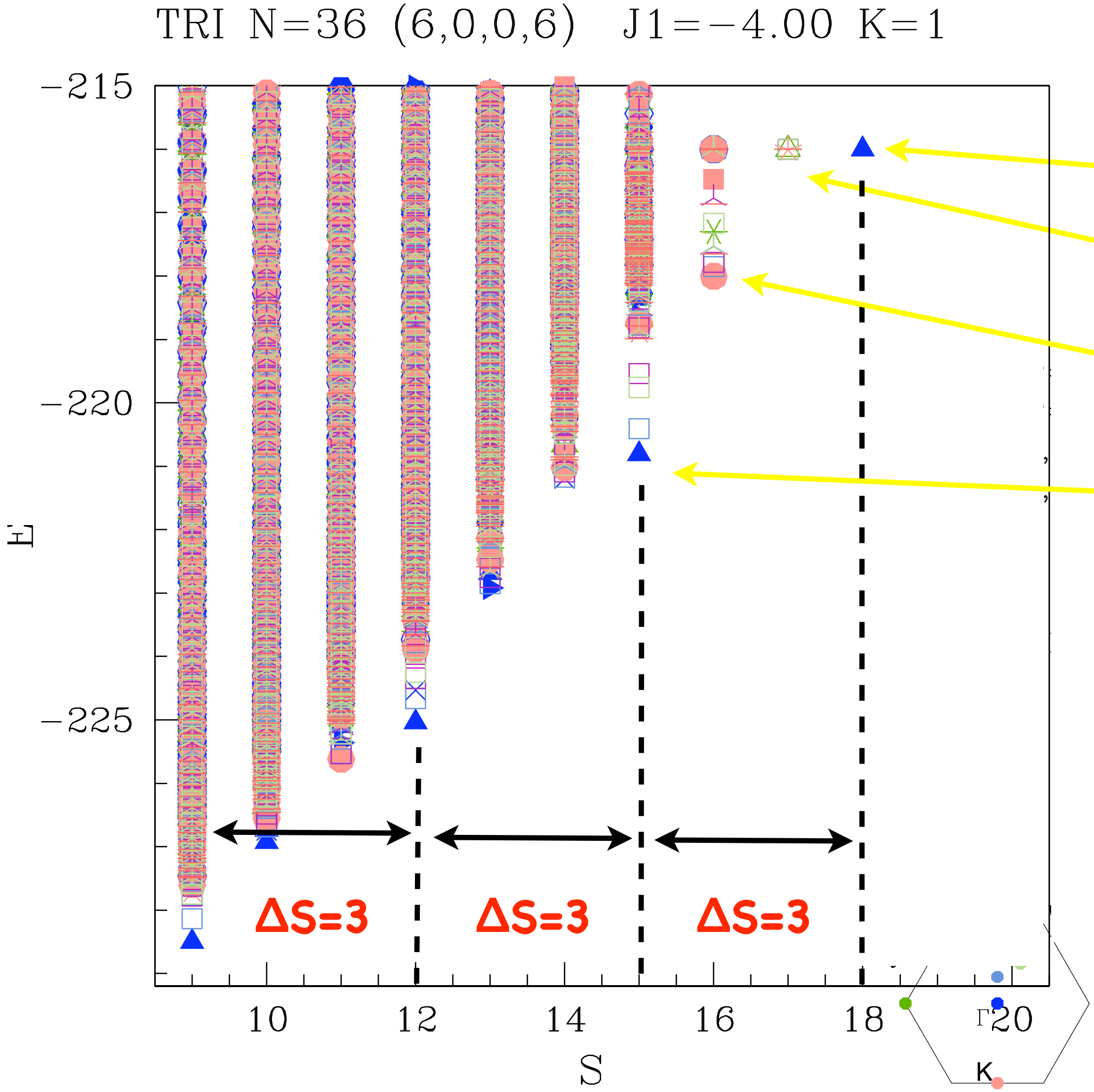

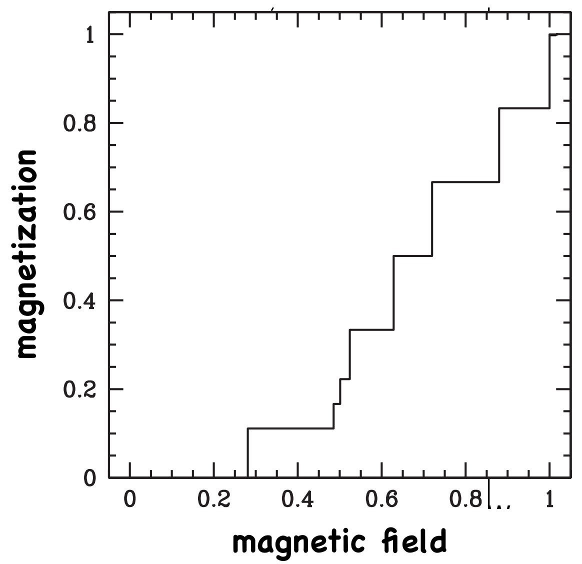

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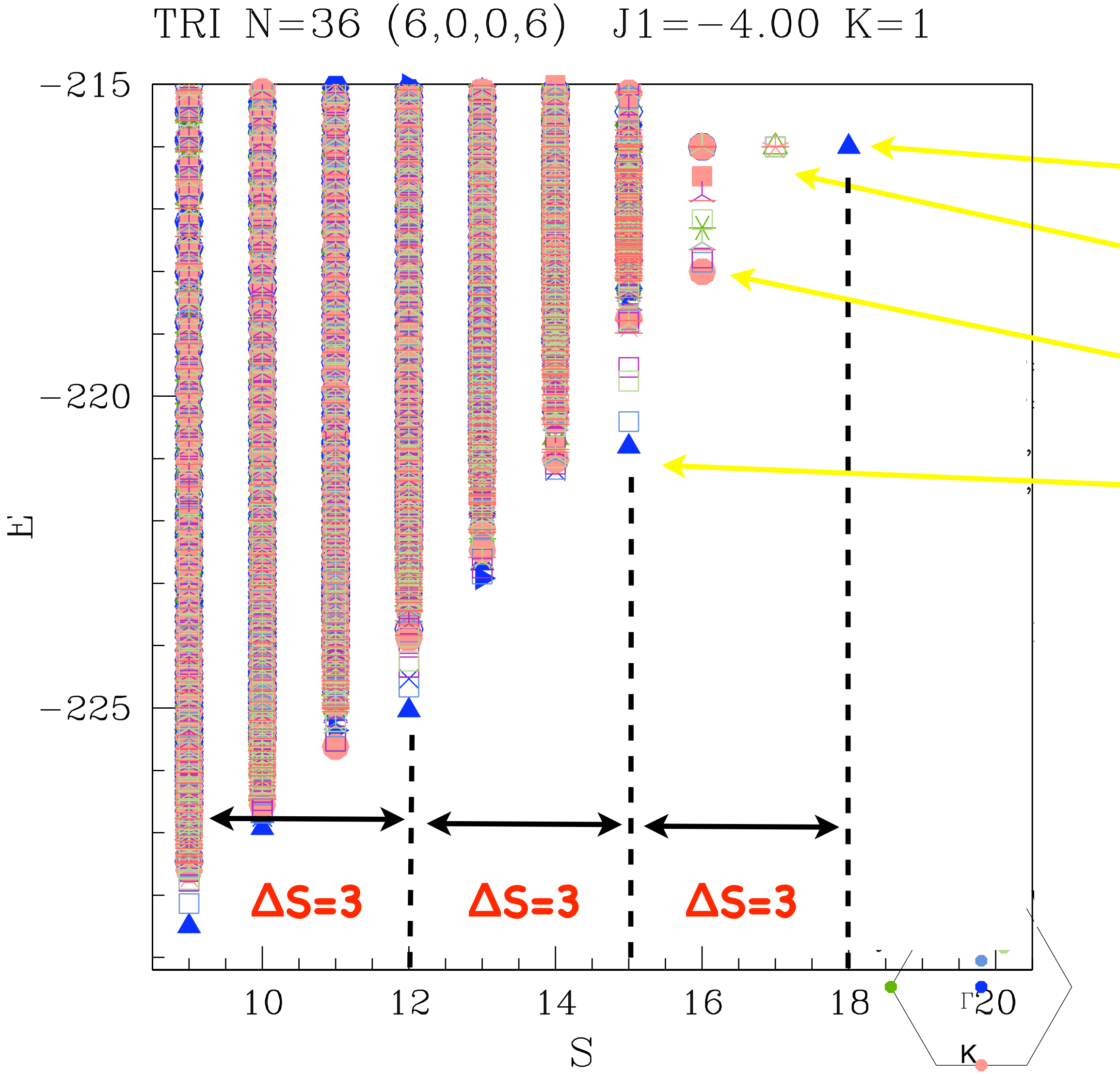

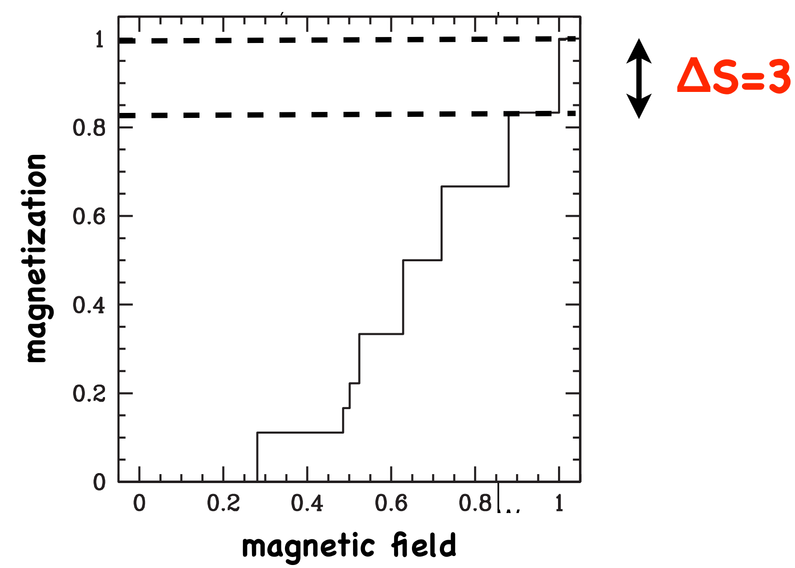

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another new quantum phase !

- three-spin bound states at high magnetic field -

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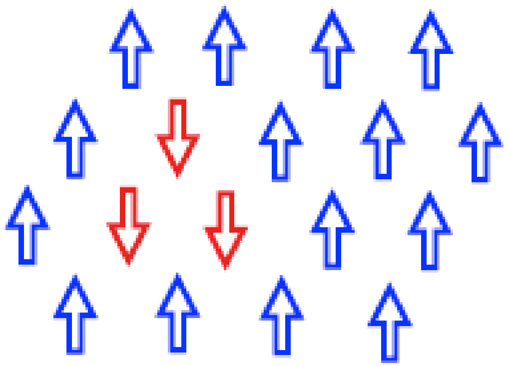

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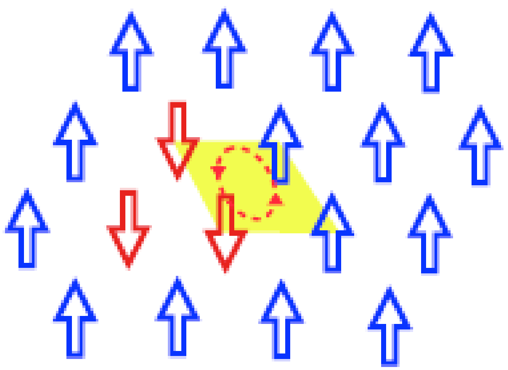

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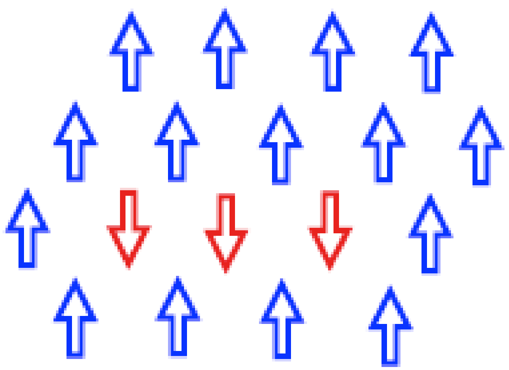

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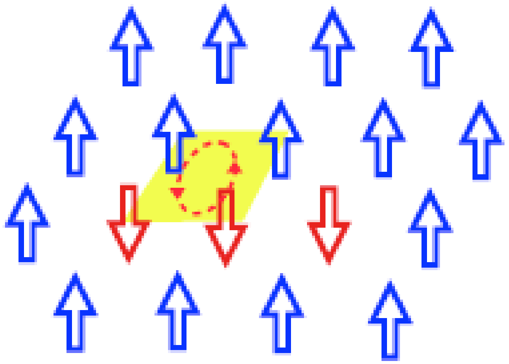

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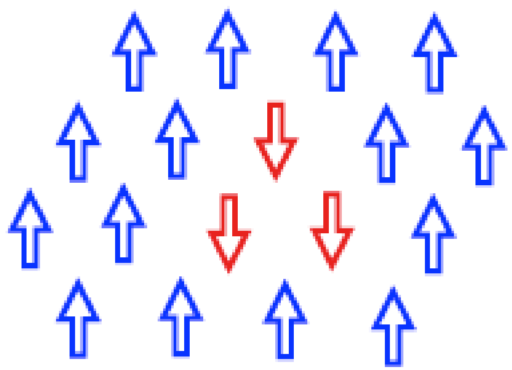

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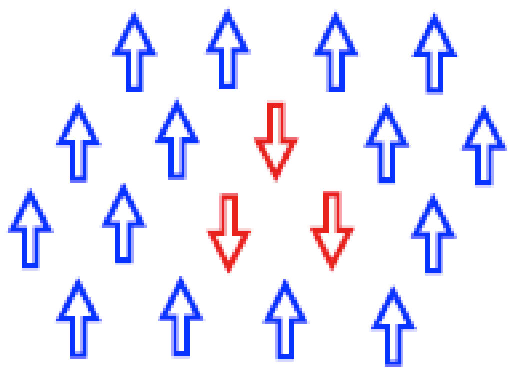

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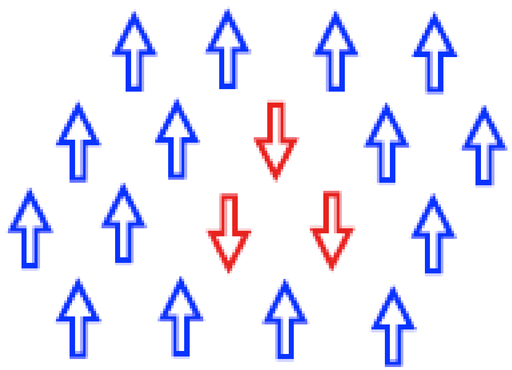

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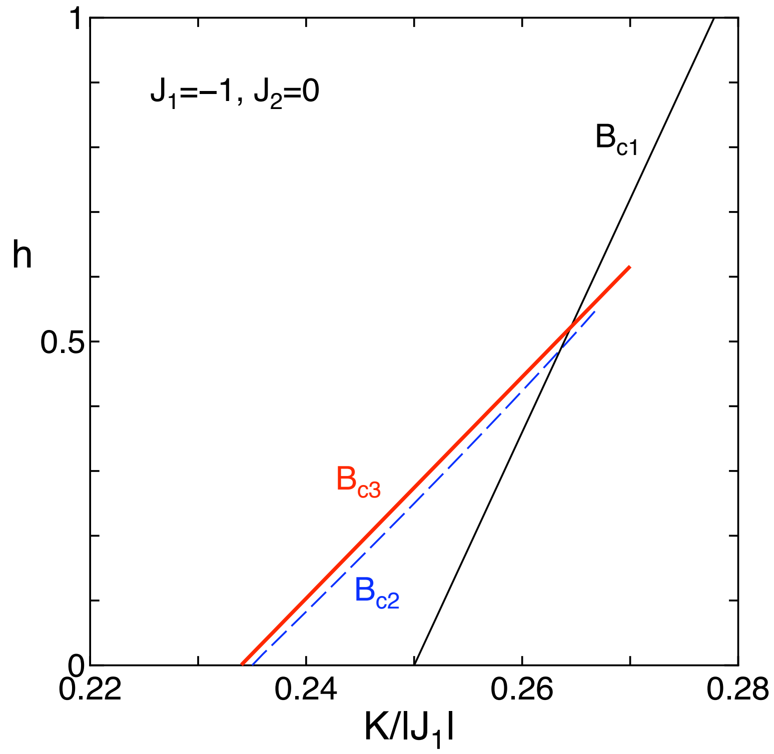

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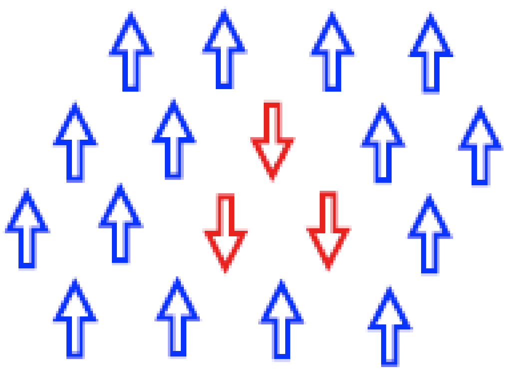

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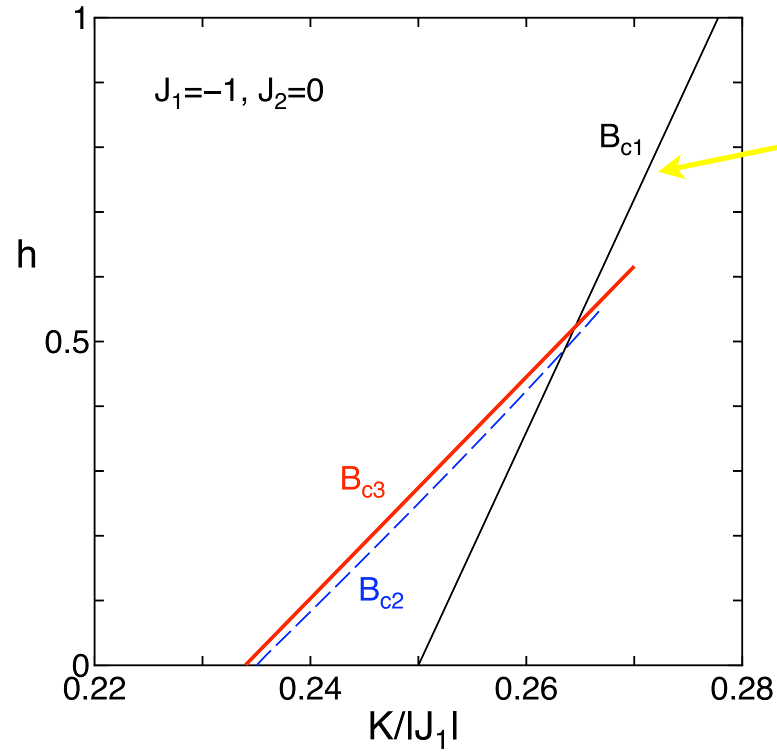

1-magnon instability

i.e. **canted Néel** state (exact)

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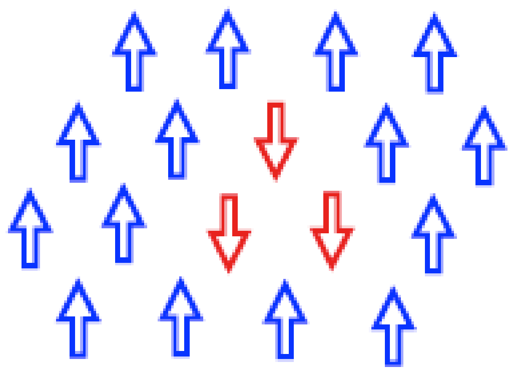

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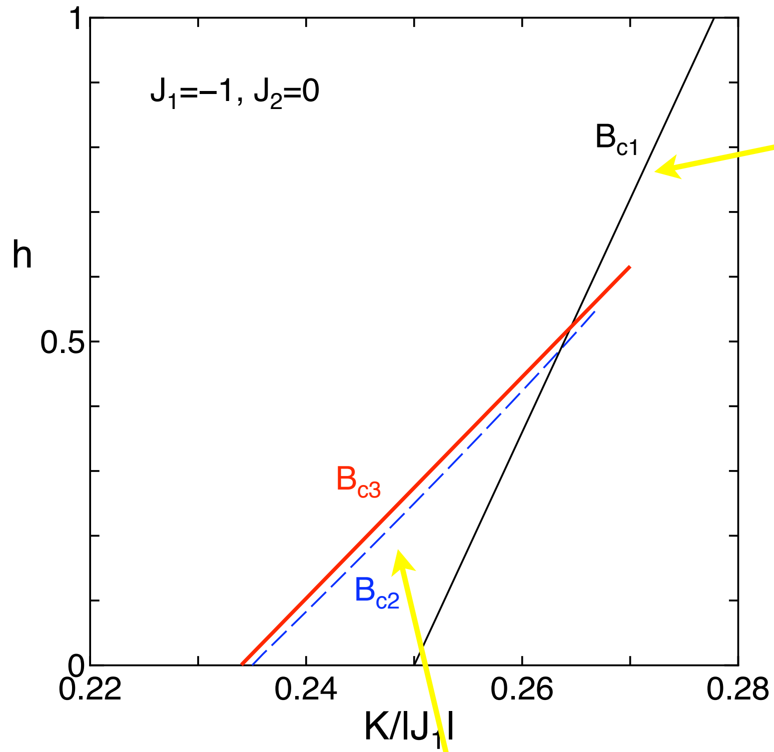

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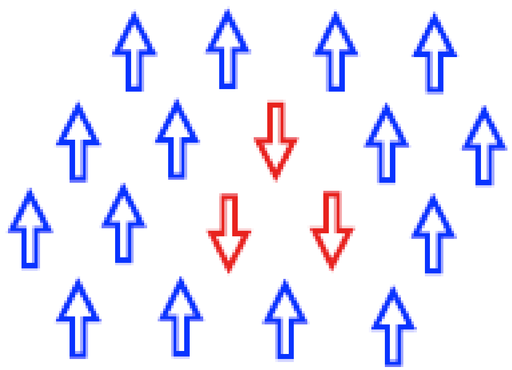

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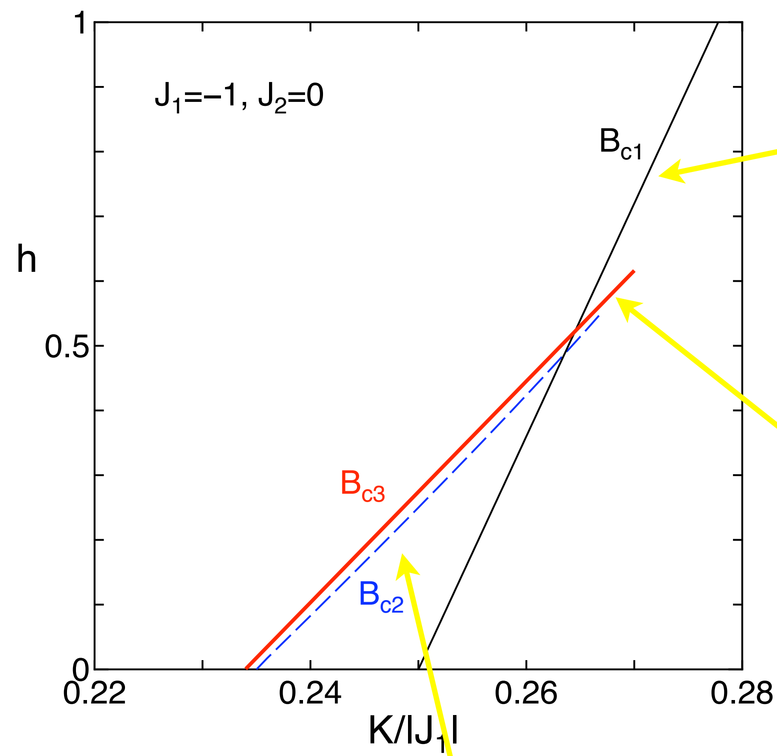

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3-magnon instability

i.e. "**triatic**" state (variational lower bound)

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symmetries of the triatic phase

- three-spin bound states at high magnetic field -

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order parameter is **rank 3 tensor :**

$$\mathcal{R}e\left\{S_i^- S_j^- S_k^-\right\} = 2S_i^x S_j^x S_k^x - 2S_i^x S_j^y S_k^y - 2S_i^y S_j^x S_k^y - 2S_i^y S_j^y S_k^x$$

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matrix elements of order parameter are linear combinations of fully **symmetrized** spin operators on triangular plaquette

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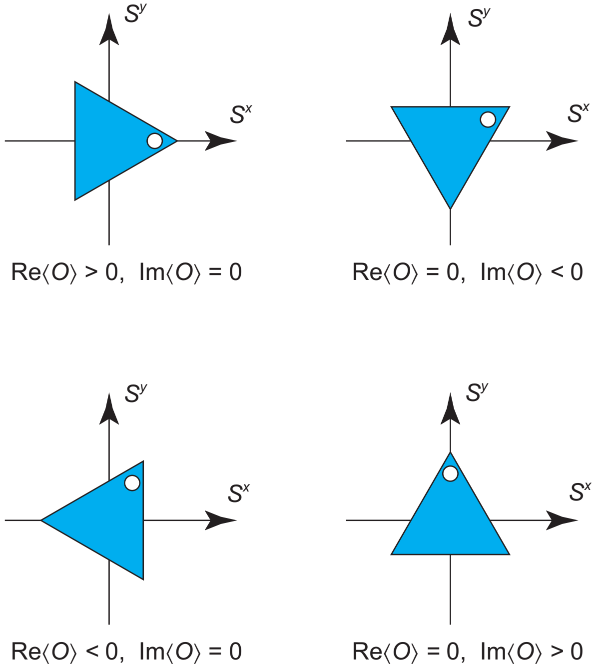

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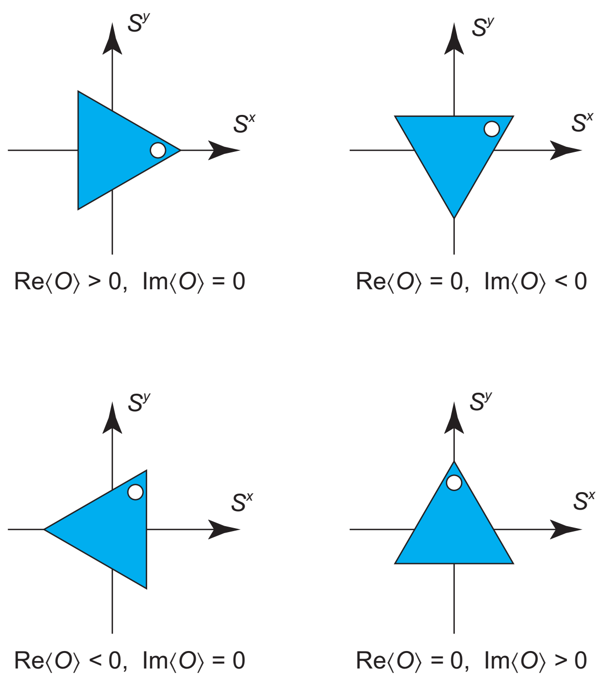

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FM octopolar order naively has k^2 dispersion

⇒ **cV ∝ T** in 2D

c.f HeIII on graphite

conclusions

- new quantum phases for all the family -

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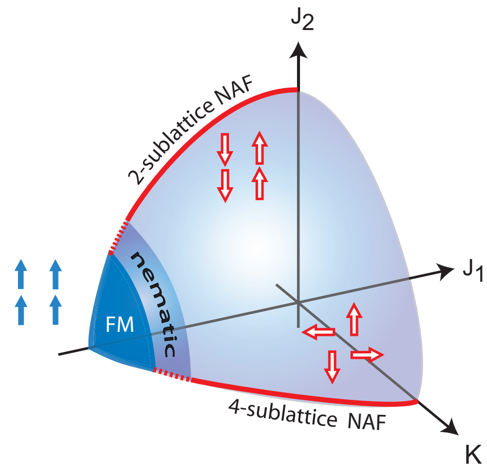

new **nematic** phase in
square lattice
frustrated ferromagnets
(c.f. quasi-2D vanadates)

Shannon, Momoi + Sindzingre,
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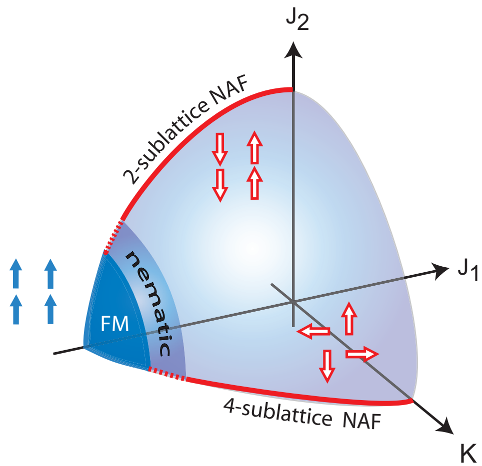

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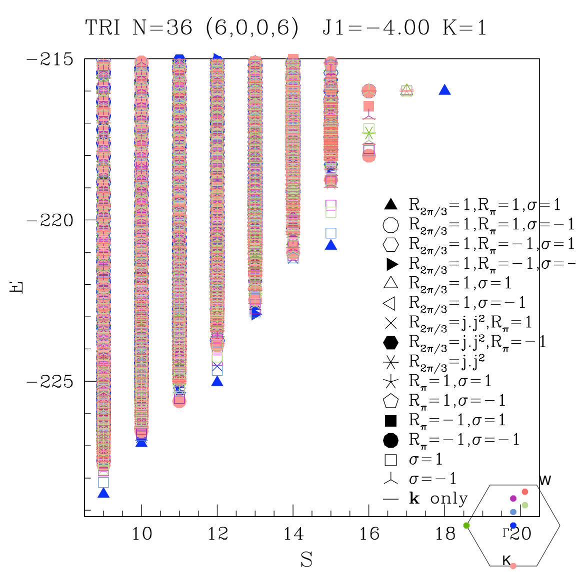

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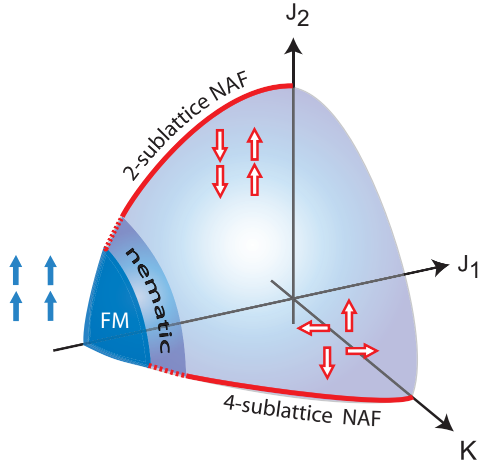

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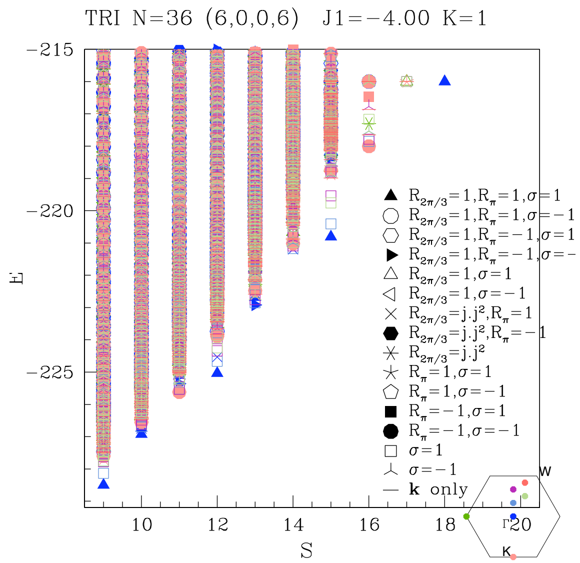
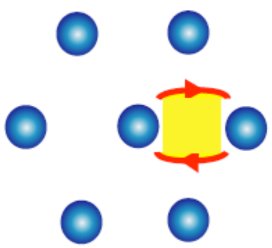


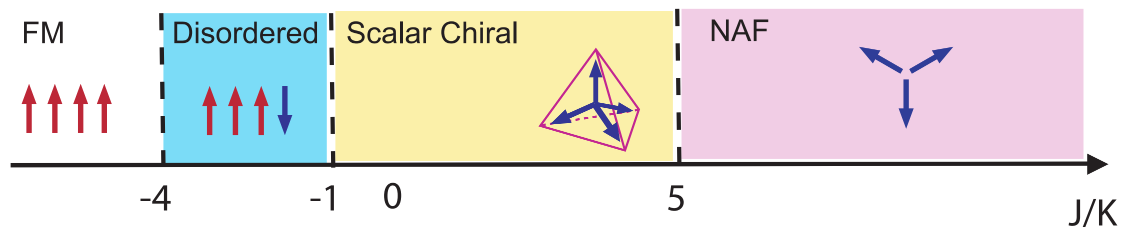
frustrated ferromagnets are fun and there's lots still to learn that's all folks...

multiple spin exchange on the triangular lattice

$$\mathcal{H} = J \sum_{\langle ij \rangle} P_{ij} + K \sum_{\langle ijkl \rangle} \left[P_{ijkl} + P_{ijkl} \right]$$



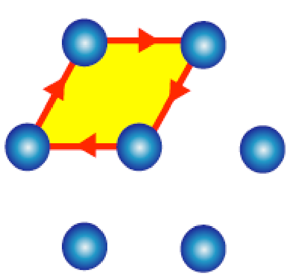
Classical limit S = ∞ K > 0

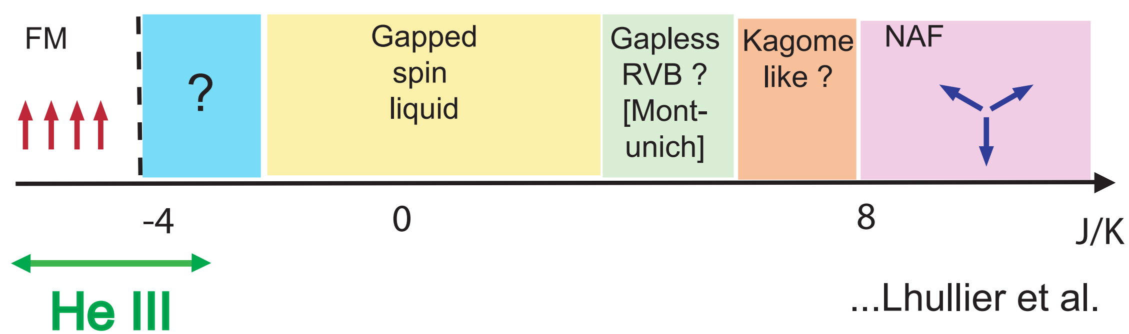


...Momoi et al.

$$P_{ij} = \left[\frac{1 + \vec{\sigma}_i \cdot \vec{\sigma}_j}{2} \right] = 2\vec{S}_i \cdot \vec{S}_j + \ldots$$

Quantum limit S = 1/2 K > 0





...Lhullier et al.

$$P_{ijkl} + P_{ijkl}^{-1} = \vec{S}_i \cdot \vec{S}_j + \ldots + 4 \left(\vec{S}_i \cdot \vec{S}_j \right) \left(\vec{S}_k \cdot \vec{S}_l \right) + \ldots$$